THE UNIVERSITY OF ZAMBIA
SCHOOL OF MINES,
GEOLOGY DEPARTMENT

GGY3049 Structural Geology

Introduction to strain

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Deformation

Deformation is the transformation from an initial to a final geometry, in response to stresses, by means of:

- Rigid body **translation**,
- Rigid body **rotation,**
- (**Distortion**) and/or
- Volume change **(Dilation).**

Strain

Strain or distortion is a **non-rigid body deformation** and is relatively simple to define:

- It is any change in shape, with or without change in volume, and it implies that particles in a rock have changed positions relative to each other.

*A change in **size** **(dilation)** and or **shape** **(distortion)** in response to stress.*

Non-rigid body deformation (Strain) cont'd

- **Strain**, expressed as **distortion** or **dilation,** results from a **change in configuration of points within a body.**
- A single body, during a single deformational event, may experience both **dilation** and **distortion.**
- Points within strained bodies do not retain their original spacing and configuration relative to one another.
- **The original spacing of points within the body is changed.**

Non-rigid body deformation (Strain) cont'd

A. Distortional Strain

- **Distortion** is an operation that involves a change in the spacing of points within a body of rock in such a way that the overall shape of the body is **altered,** with or without a change in volume.

- It describes changes of points in a body relative to each other;

  ❖Particle lines may rotate relative to an external coordinate system

  Example: squeezing a paste

Non-rigid body deformation (Strain) cont'd

Distortional Strain cont'd

- Where **pure dilation** takes place without change in shape:

- Internal points of reference spread apart or
- Pack closer together in such a way that
- Line-lengths between points become uniformly longer or shorter.

- The overall shape remains the same.

Non-rigid body deformation (Strain) cont'd

Distortional strain cont'd

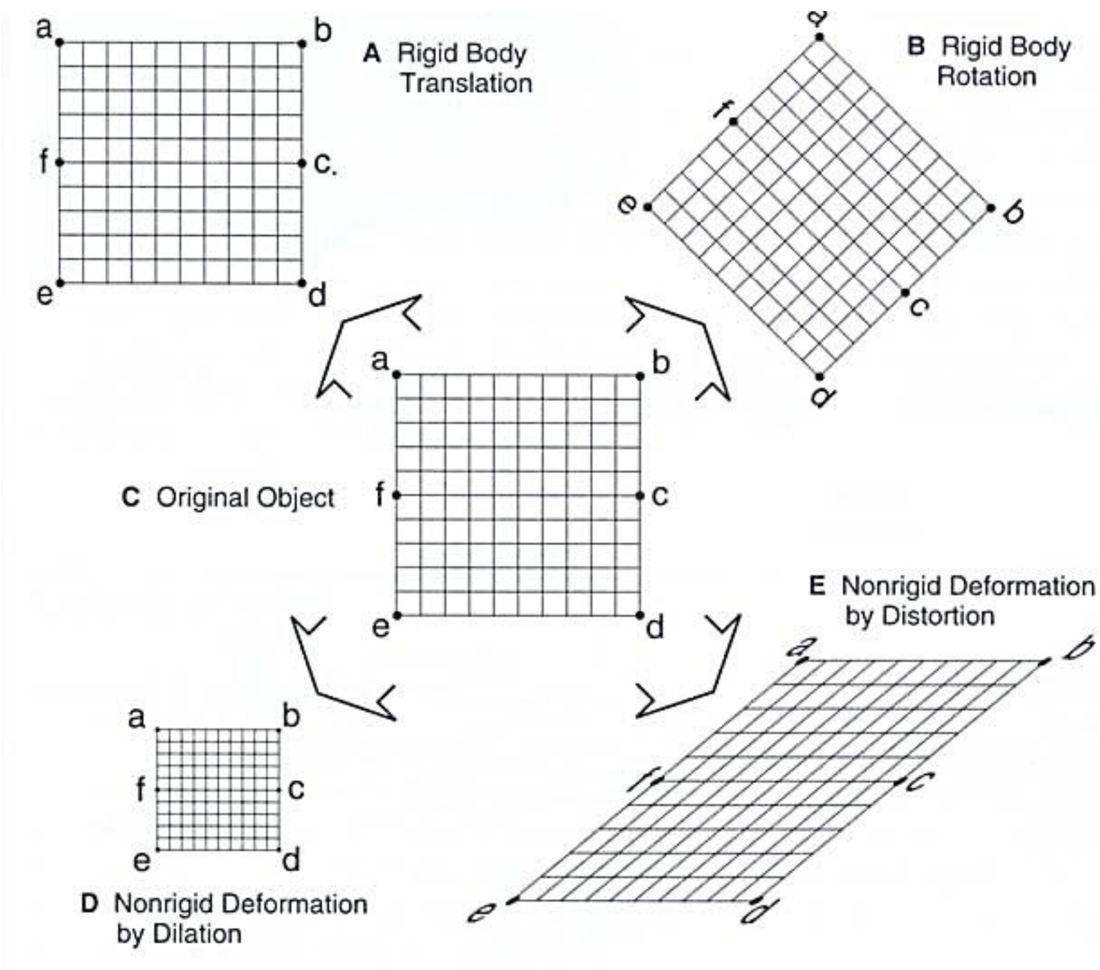
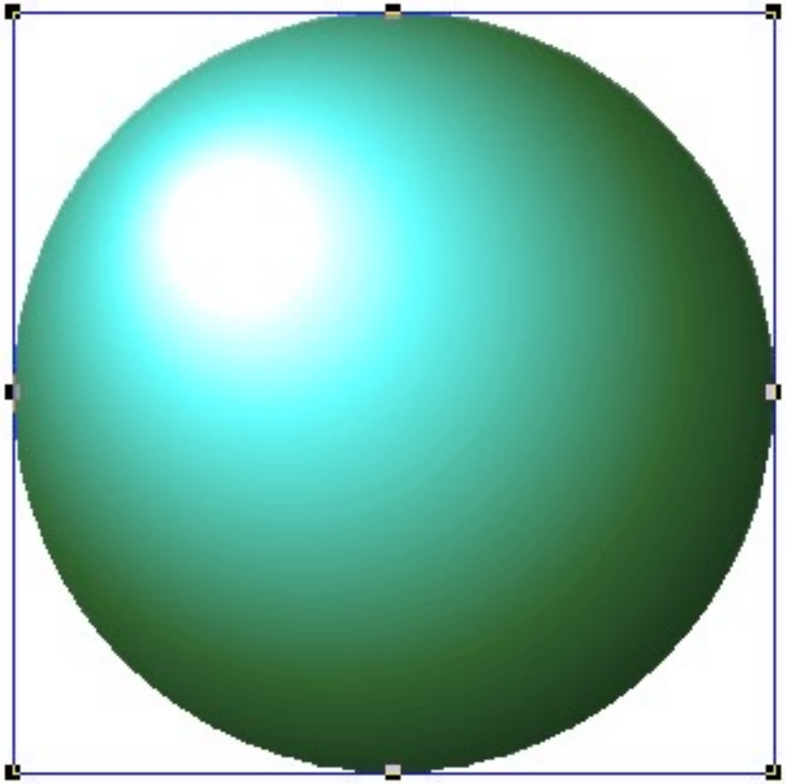


Fig. 2.2. Originally undeformed body (C) in center of diagram (i.e., square **abde**) is deformed by (A) rigid body translation, (B) rigid body rotation, (D) non-rigid body dilation , and (E) non-rigid body distortion.

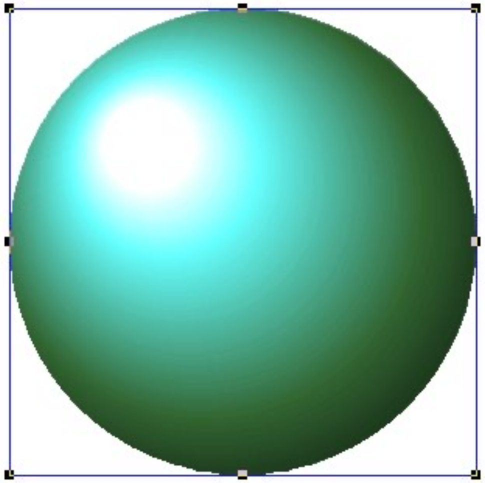
Non-rigid body deformation (Strain) cont'd

Distortion or Strain cont'd

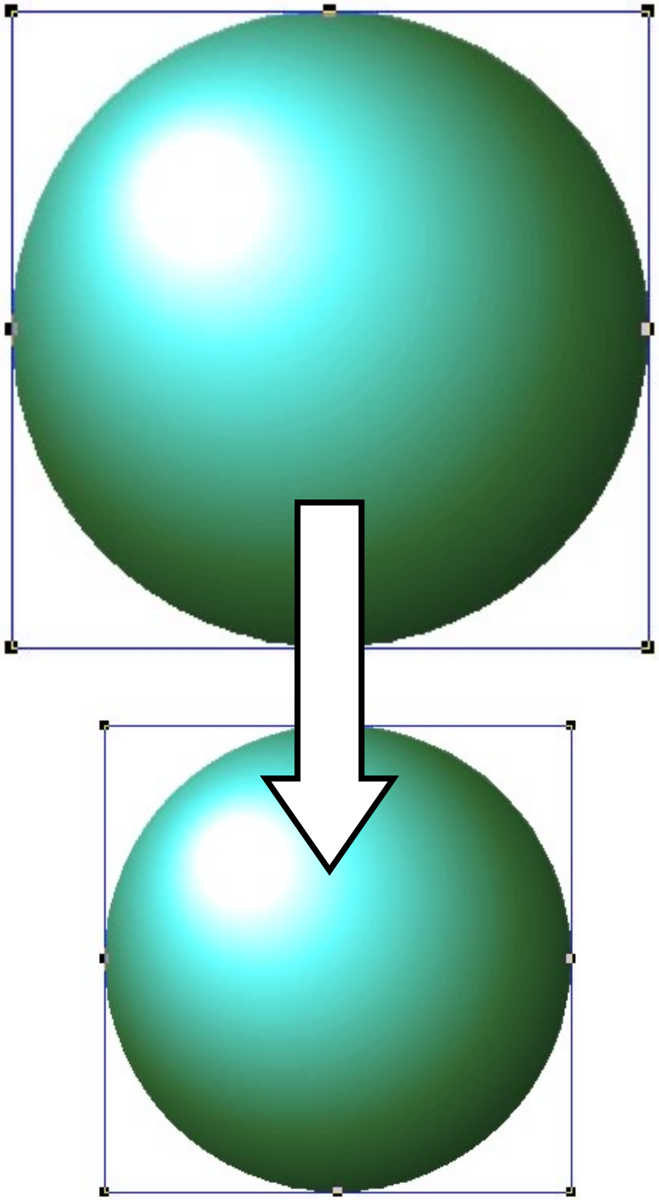


- Consider a sphere of some material immersed at the bottom of a swimming pool

- Now consider the same sphere after it has been immersed in the ocean at a depth of 5 km.



Non-rigid body deformation (Strain) cont'd



Distortional Strain cont'd

As hydrostatic pressures (confining stresses) increase, the <u>shape</u> of the sphere doesn't change — however, it may occupy a smaller, but still spherical volume.

By definition there is no change in shape (no distortion) — so no deformation …

Non-rigid body deformation (Strain) cont'd

Distortional Strain cont'd

- During **distortion**, the changes in spacing of points in a body are such that the **overall shape of the body is altered**, with or without a change in size/volume (Fig. 2.6):

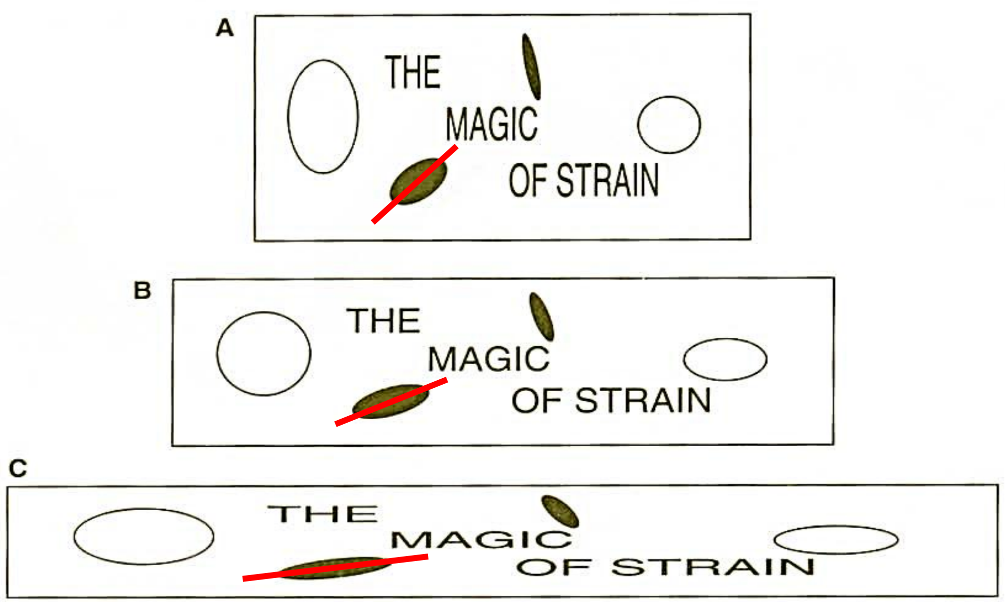


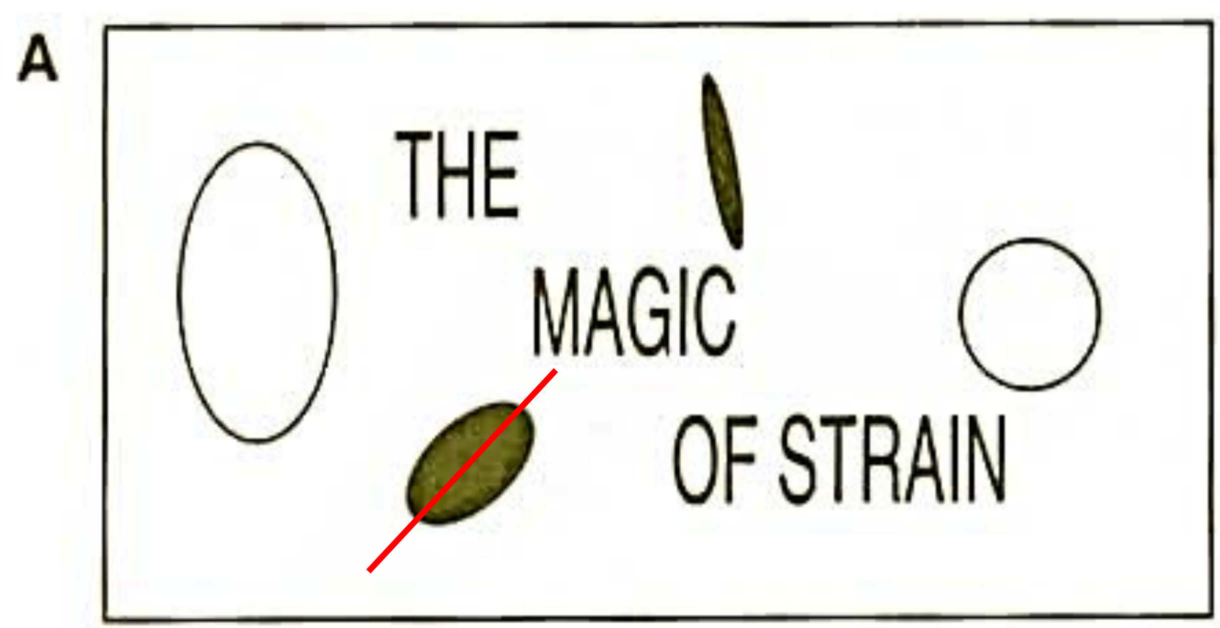
Fig. 2.22

Fig. 2.6: Here, a deformable block is imprinted with a circle, a vertically oriented ellipse, two black ellipses, and some words

Distortion or Strain cont'd

Fig. 2.26 (A):

- When flattened and extended, changes in the shapes and orientations of the reference objects on the front face of the block record the nature of the internal strain.



Distortion or Strain cont'd

Fig. 2.26 (B):

B

Non-rigid body deformation (Strain) cont'd

Distortional or Strain cont'd

Fig. 2.26 (C):

- With even more flattening, tighter and tighter;
- ❖ The two black ellipses continuously rotate toward the direction of stretching; and
- ❖ The letters of the words continuously change font.

Non-rigid body deformation (Strain) cont'd

Distortional or Strain cont'd

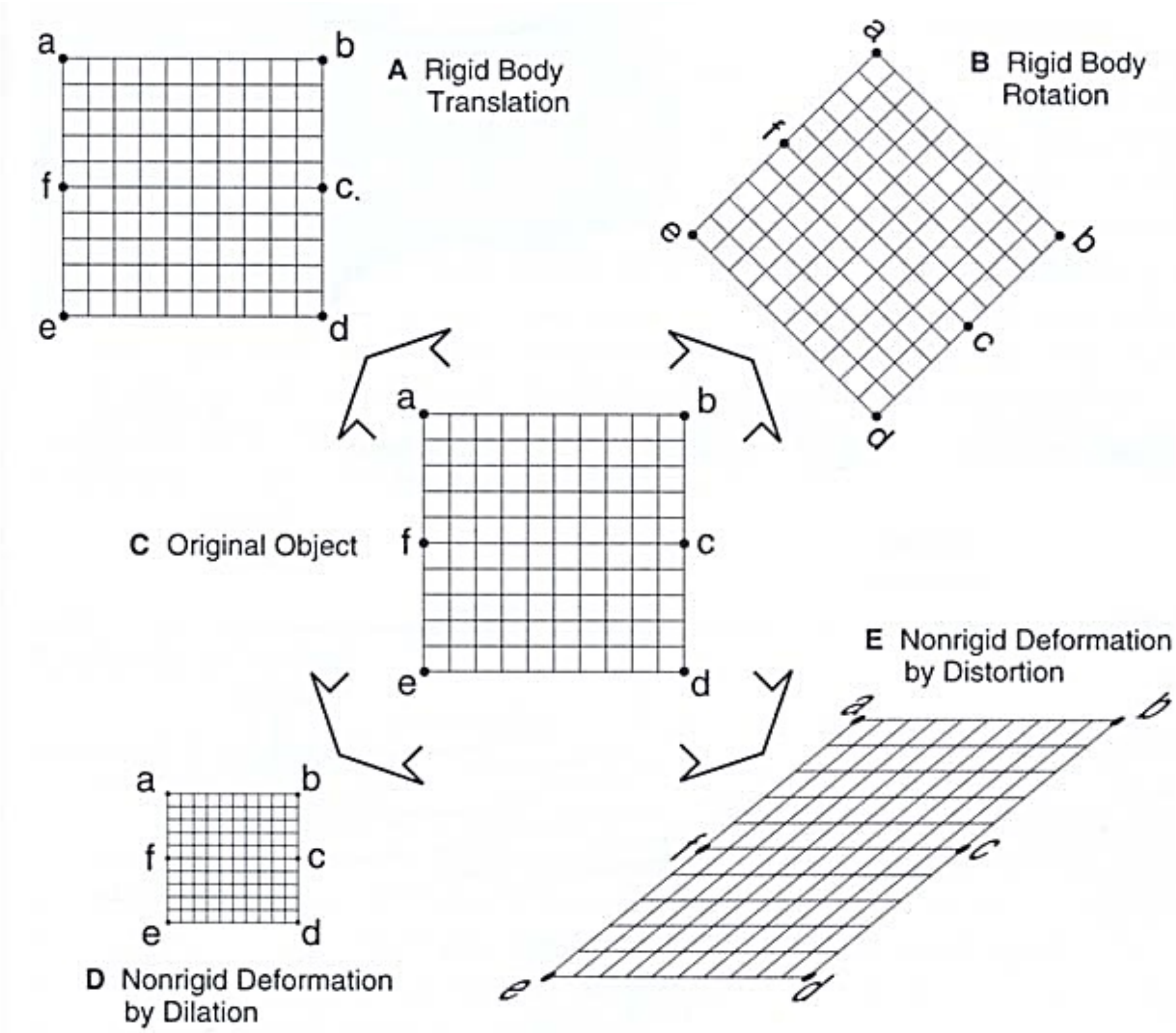


Fig. 2.2. Originally undeformed body **(C)** in centre of diagram (i.e. square ***abcd***) is deformed by (A) rigid body translation, (B) rigid body rotation, (D) non-rigid body dilation, and (E) non-rigid body distortion.

Distortional Strain cont'd

Fig. 2.2.:

Originally undeformed body **(C)** in Centre of diagram (i.e. square ***abcd*****)** is deformed by:

(A) Rigid body translation,

(B) Rigid body rotation,

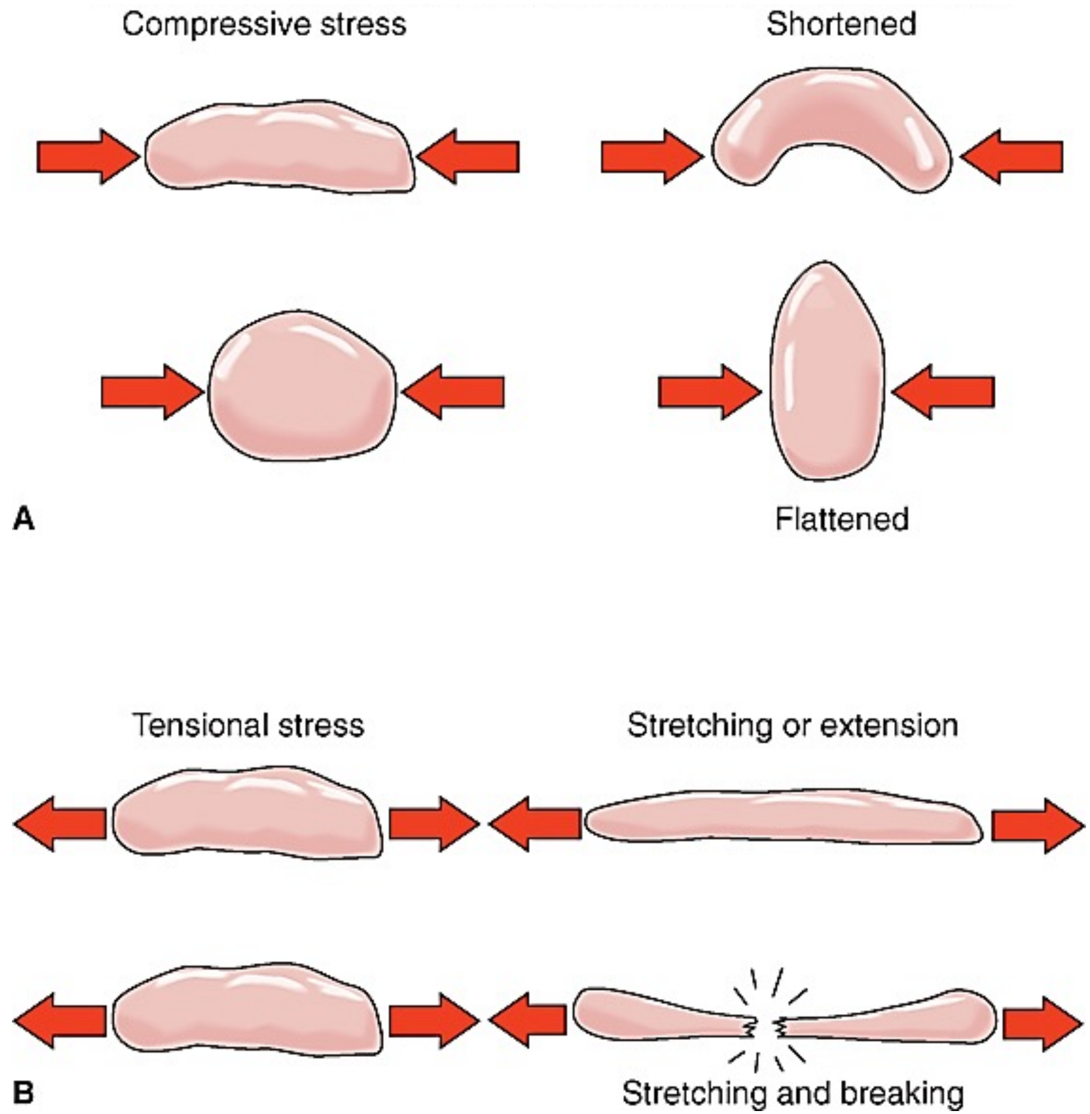
(D) Non-rigid body dilation, and

(E) Non-rigid body distortion.

Non-rigid body deformation (Strain) cont'd

Distortional Strain cont'd

Material Strain



Non-rigid body deformation (Strain) cont'd

Distortional or Strain cont'd

- **Pure distortion** is a change in a shape without a change in size.
- As shown in **Fig 2.2E**, the change in shape of a body from a **square** to a **rhomb** is made possible by systematic changes in spacing between points in the body.
- The distance between points ***a*** and ***e*** increases from **10** units to **16** units.
- Angular relations between alignment of points change as well.
- For example, angle ***aed*** is reduced from **90°** to **40°.**

Distortional Strain cont'd

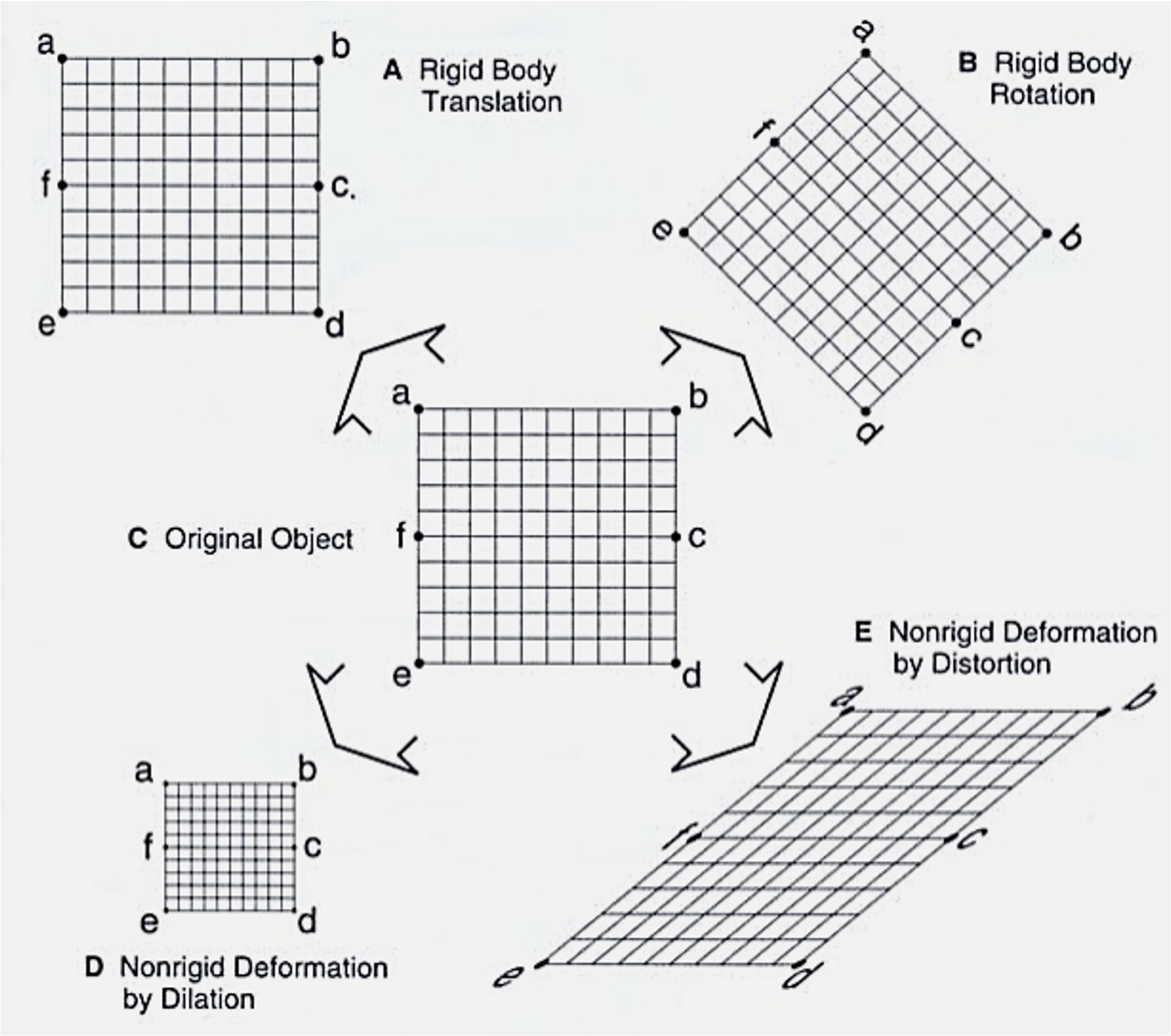
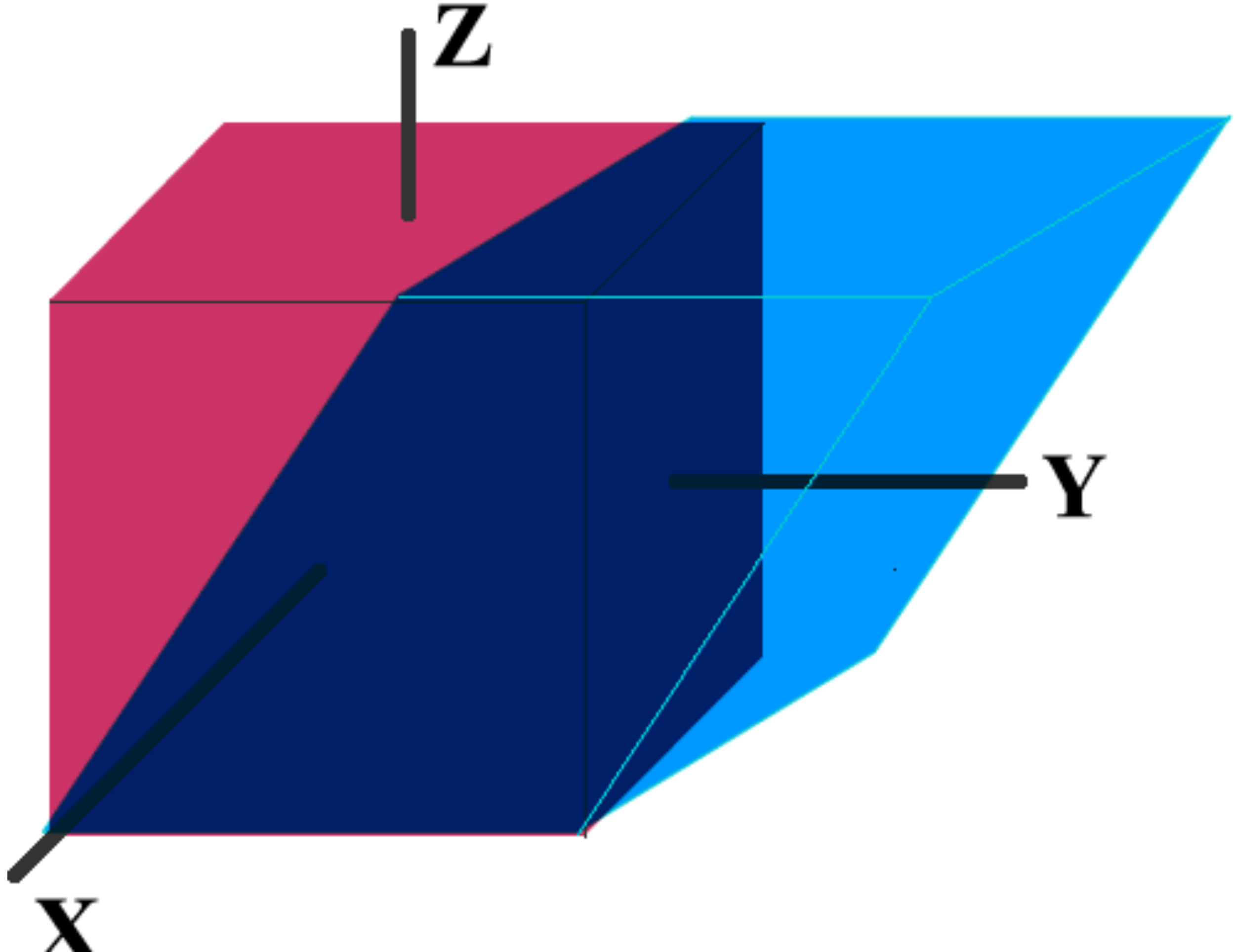


Fig. 2.2. Originally undeformed body (C) in centre of diagram (i.e. square *abcd*) is deformed by (A) rigid body translation, (B) rigid body rotation, (D) non-rigid body dilation, and (E) non-rigid body distortion.

Non-rigid body deformation (Strain) cont'd

Distortional or Strain cont'd

Strain or Distortion



Non-rigid body deformation (Strain) cont'd

Distortional or Strain cont'd

- When a non-rigid body deformation results in the systematic distortion of what we normally regard as **solid rock**, the results can be unusually interesting.
- A **circle** changes **into ellipse** through non-rigid body distortion.
- However, in rocks we deal with processes that lead to both movement and distortion.

Distortional strain

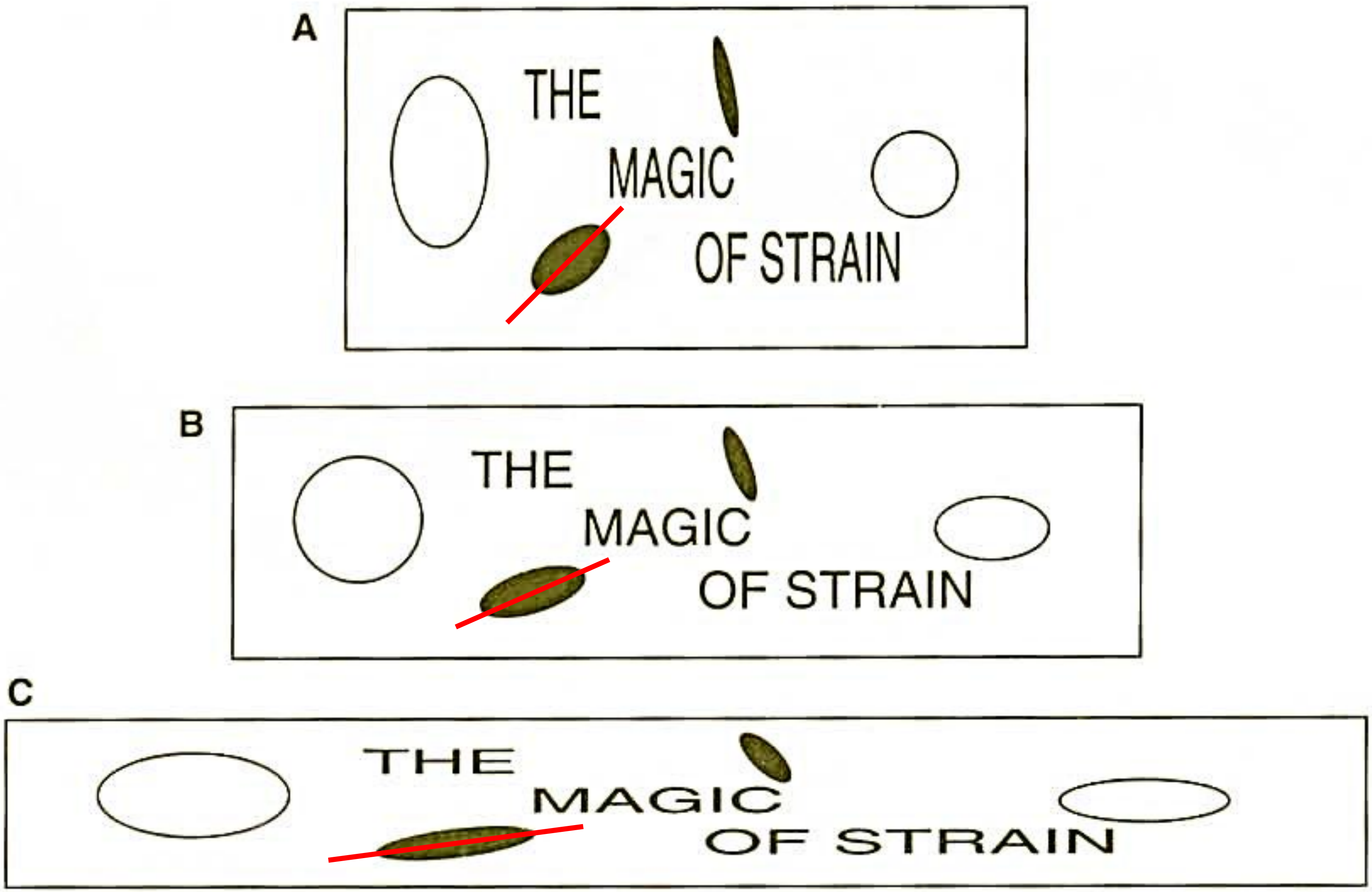


Fig. 2.26. (A) A deformable block is imprinted with a circle, a vertically oriented ellipse, two black ellipses, and words. (B) When flattened and extended, changes in the shapes and orientations of the reference objects on the front face of the block record the nature of the internal strain. The amount of flattening and stretching is just enough to transform the original vertical ellipse into a perfect circle. **(C)** With even more flattening tighter and tighter; the two black ellipses continuously rotate toward the direction of stretching; and the letters of the words continuously change font.

Non-rigid body deformation (Strain) cont’d

B. Dilation

- Is an operation involving a change **in volume or size.**
- **Pure dilation** is a **change in size without a change in shape.**
- **Fig 2.2D** pictures a decrease in size **(negative dilation):**
- The spacing of points within the original body is cut in half.

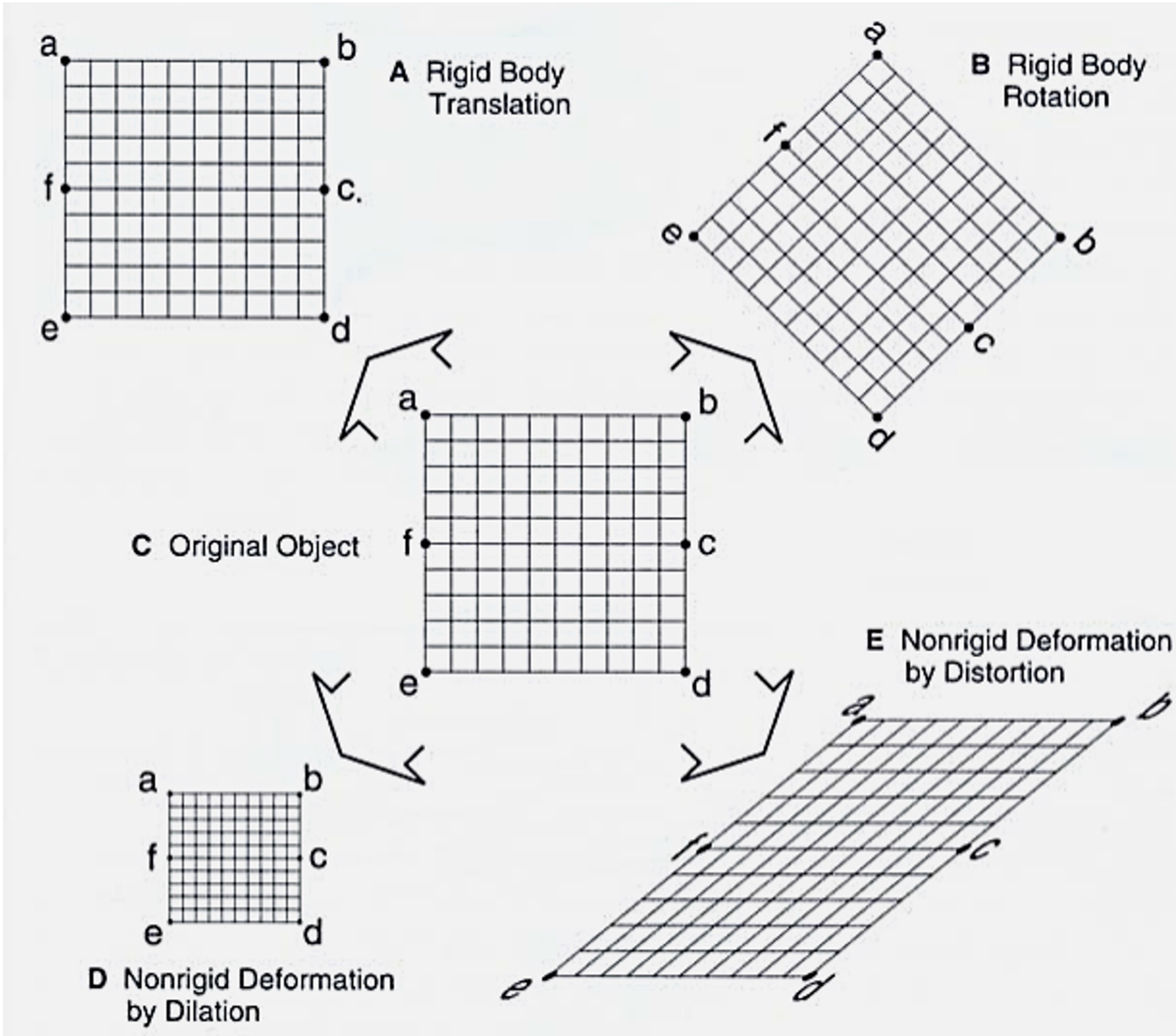
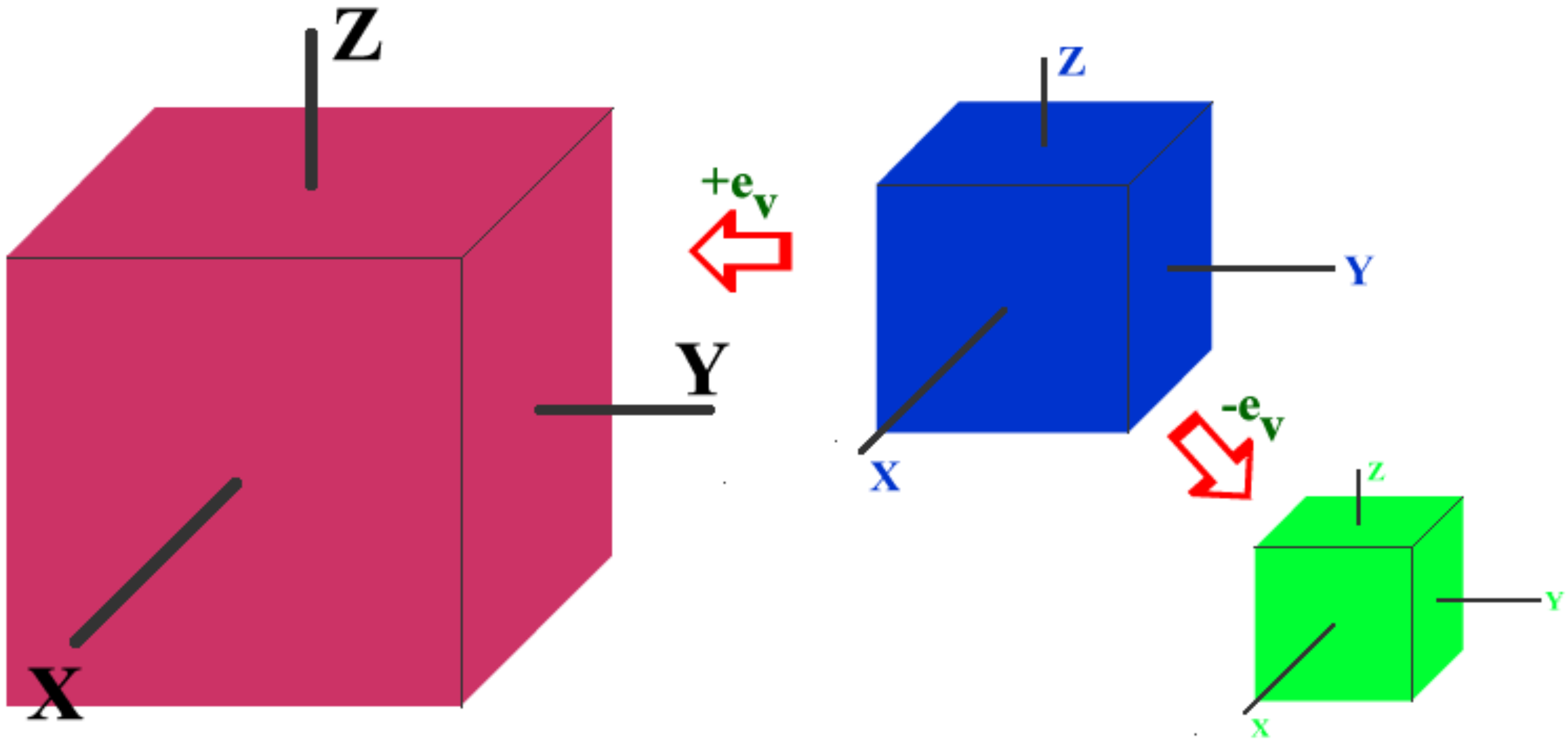


Fig 2.2D pictures negative dilation

Non-rigid body deformation (Strain) cont'd

Pure Dilation



Non-rigid body deformation (Strain) cont'd

In pure dilation:

- The overall shape remains the same
- Internal points of reference spread apart or pack closer together
- Line lengths between points become uniformly longer or shorter

Non-rigid body deformation (Strain) cont'd

Dilation

- Significant dilation accompanies such non-rigid structural processes as:

- The shrinkage of mud to produce mud cracks,
- The compaction of sediments during burial to produce thinning,
- The cooling of basalt to produce columnar joints

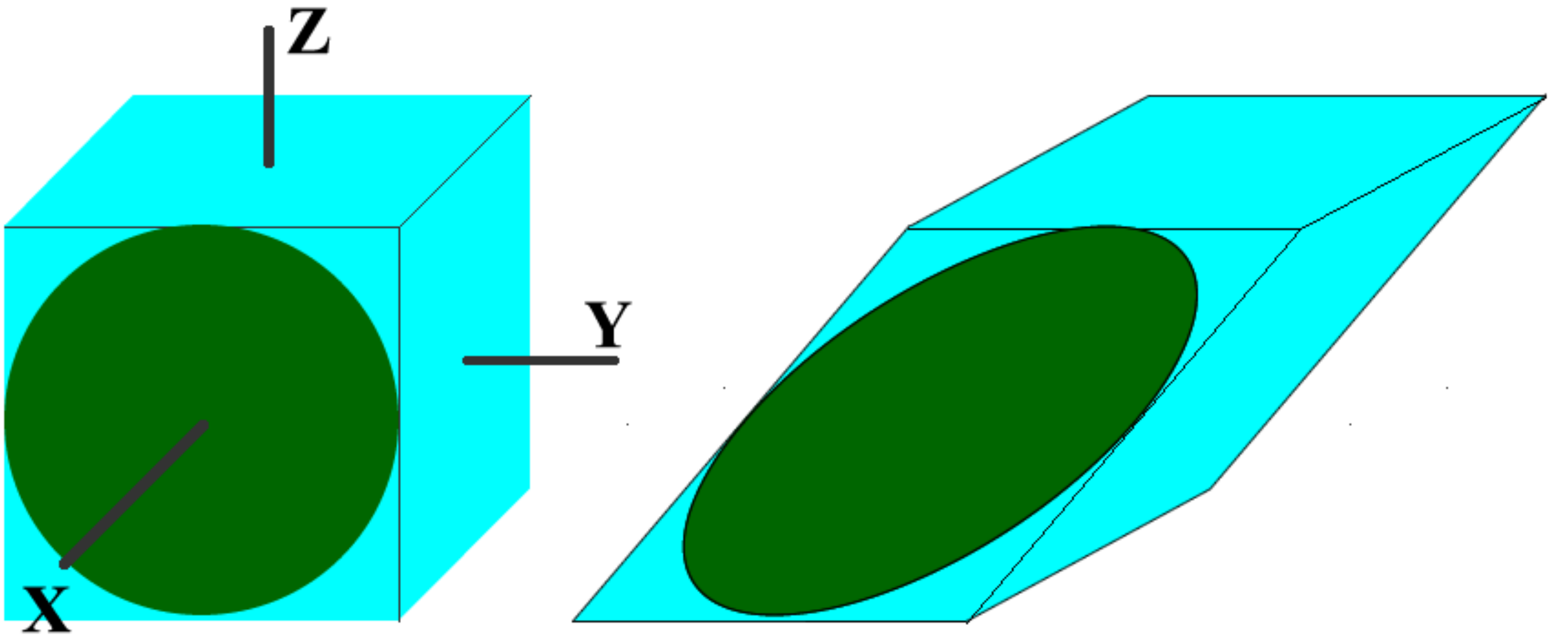
Homogeneous
and
Inhomogeneous strains

The chief constraints that the **“homogeneous deformation” clause** imposes are:

1. Straight lines that exist in the non-rigid body before deformation remain straight after deformation.
2. Lines that are parallel in the body before deformation remain parallel after deformation.

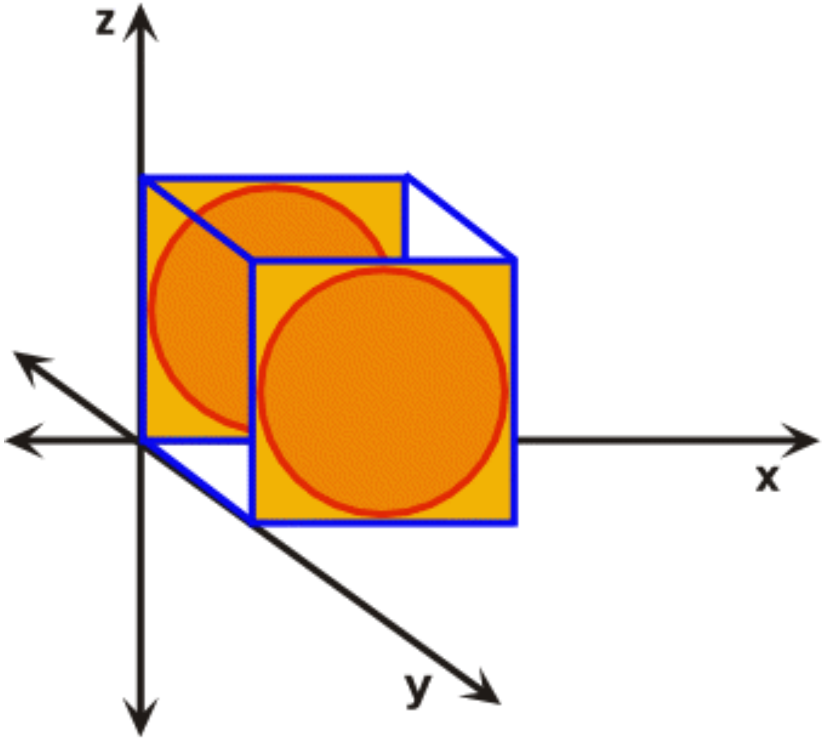
- For these conditions to hold, the strain must be **systematic** and **uniform** across the body that has been deformed.
- A simple **test for homogeneity** will become obvious: **Homogeneous deformation** transforms;
  - Perfect **circles** into perfect **ellipses**, and
  - Perfect **spheres** into perfect **ellipsoids**.
- **Strained rocks** that we find in nature **typically** **depart** from the rules of **homogeneous deformation.**

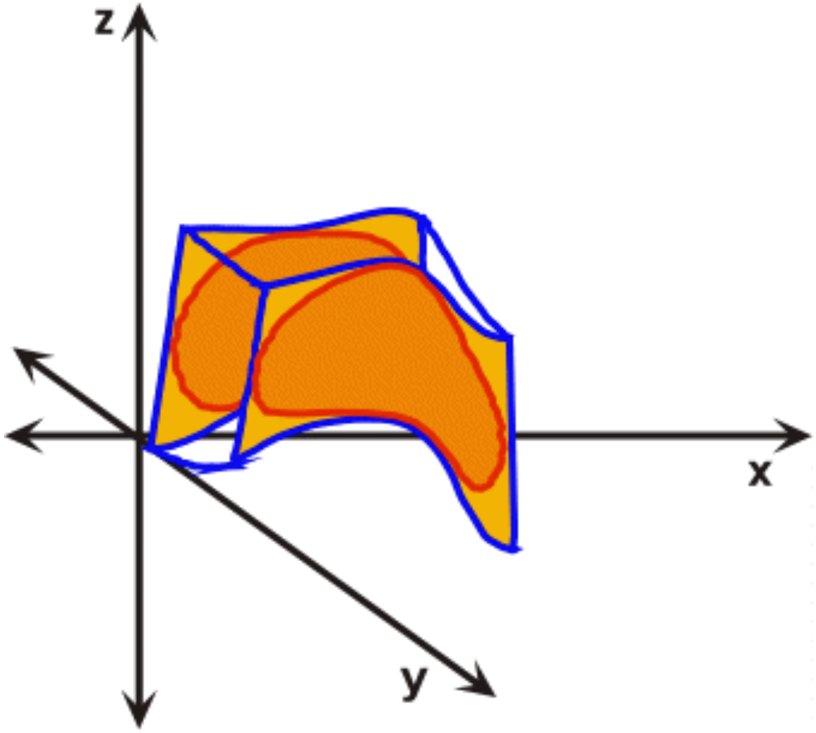
Homogeneous Strain



Heterogeneous or Inhomogeneous strain

Leads to distorted complex forms





Measurement of strain

Measurement of strain:

General concept

1. Change of length (extension, ***e*****,** and stretch, ***s***)
2. Angular shear (***Psi*****,** ***ψ***),
3. Shear strain (gamma, γ)

Strain produces **non-rigid body deformation:**

- Dilation, a change in size, and
- Distortion, a change in shape.

- Points within strained bodies do not retain their original spacing and configuration relative to one another.
- The original spacing of points within the body is changed.

- Where **pure dilation** takes place without change in shape, internal points of reference **spread apart or pack closer together** in such a way that line lengths between points become **uniformly longer** or **shorter.**
- The overall shape remains the same.
- On the other hand, during **distortion**, the changes in spacing of points in a body are such that the overall **shape of the body is altered**, with or without a change in size/volume **(Fig. 2.2).**

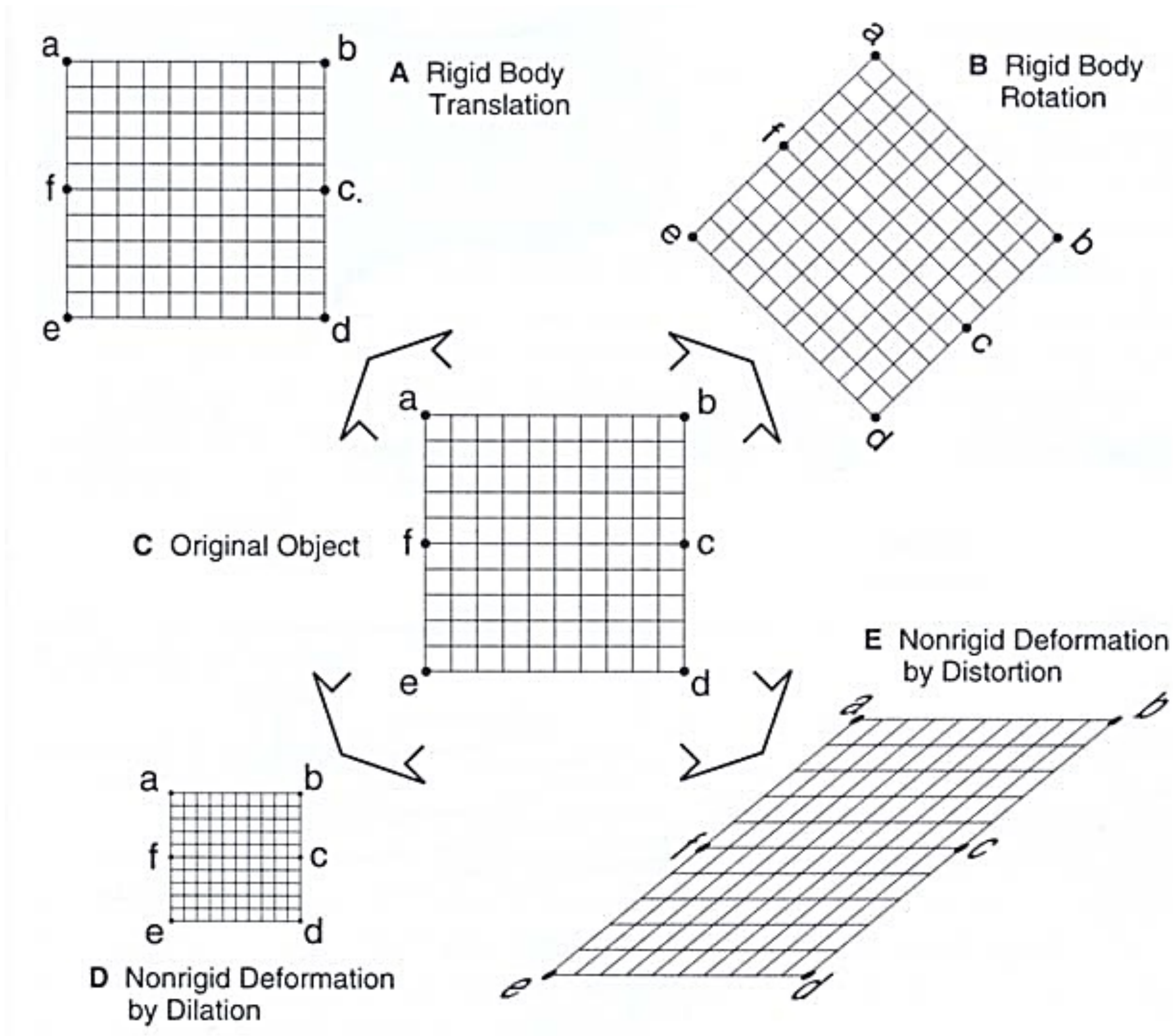


Fig. 2.2. Originally undeformed body **(C)** in centre of diagram (i.e. square ***abcd***) is deformed by **(A)** rigid body translation, **(B)** rigid body rotation, **(D)** non-rigid body dilation, and **(E)** non-rigid body distortion.

The ground rules cont'd:

- We simplify our **work in strain analysis** by studying the theory in **two,** and **not three**, **dimensions.**

- A **customary simplification** is to **restrict strain analysis** to the description of ***homogeneous deformation***.

The ground rules cont'd:

- It is almost impossible to apply mathematical theory to unwieldy, irregularly deformed, *heterogeneous* structural systems.

- Instead, strain analysis focuses on the **properties of bodies**, or **parts of bodies**, that have **deformed in a regular, uniform manner.**

The ground rules cont'd:

The chief constraints that the **"homogeneous deformation" clause** imposes are:

1. Straight lines that exist in the non-rigid body before deformation remain straight after deformation
2. Lines that are parallel in the body before deformation remain parallel after deformation.

The ground rules:

- For these conditions to hold, the strain must be **systematic** and **uniform** across the body that has been deformed.
- A simple test for homogeneity will become obvious; homogeneous deformation transforms:
  - Perfect circles into perfect ellipses, and
  - Perfect spheres into perfect ellipsoids.

The ground rules:

- Strained rocks that we find in nature typically depart from the **rules of homogeneous deformation.**

The strain ellipse:

- Given the perfection of ellipses derived from **homogeneous deformation** (distortion) of circles and ellipses,

- It is no wonder that the strain within geologic bodies is conventionally described through the image of a strain ellipse.

The strain ellipse cont'd:

- A strain ellipse pictures the distortion accommodated by a geologic body.

- It pictures how the shape of an imaginary circular reference object, or perhaps a not-so-imaginary circular geologic object, would be changed as a result of distortion.

The strain ellipse cont'd:

- We distinguish **two kinds of strain ellipses**:
  - The ***instantaneous*** **ellipse** and
  - The ***finite strain*** **ellipse**.
- An **instantaneous strain ellipse**:
  - Is used to portray how a circle is affected by a tiny increment of deformation.
  - The **ellipse** should be nearly circular because it represents infinitesimally small amounts of strain.

The strain ellipse cont'd:

The **finite strain ellipse;**

- Represents the total strain experienced by a circle that has been deformed.
- It is the **final result of deformation**, which we normally see as geologists.
- It is the **summation** of all the incremental components.

Looking at Lines inside an Ellipse:

- The shape and orientation of an ellipse can be constructed on the basis of orientations and lengths of lines that pass through the centre of the ellipse and connect with the perimeter.

- Let us take an ellipse of our choice **(Fig. 2.27A),** and construct within it a set of perpendicular lines that intersect at the “centre” of the ellipse (**Fig. 2.27B**).

Looking at Lines inside an Ellipse cont'd:

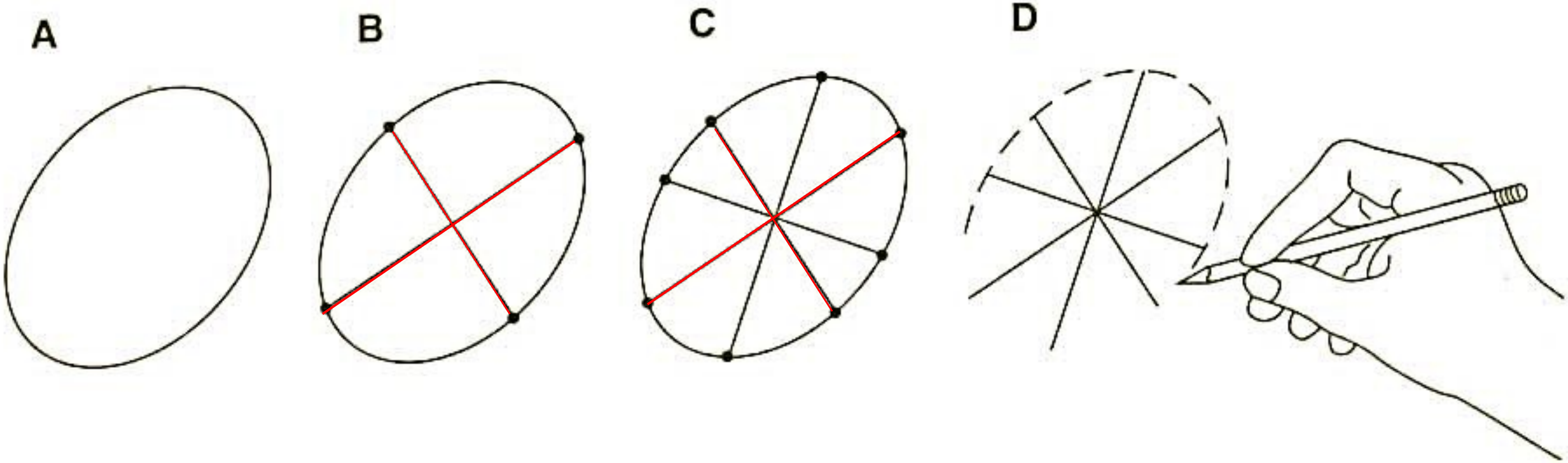


Fig. 2.27. **(A)** A perfect ellipse "contains" **(B)** sets of mutually perpendicular lines of just the right lengths and orientations. **(C)** If we knew the lengths and orientations of just two such sets of mutually perpendicular lines, **(D)** we could reasonably draw the ellipse.

Looking at Lines inside an Ellipse:

- Of course, each of these two lines has the perfect length just to touch, but not extend beyond, the perimeter of the ellipse.

- If we were given the two sets of mutually perpendicular lines **(Fig. 2.27C),** we could construct the size, shape and orientation of the ellipse within which they lie **(Fig. 2.27D).**

- The mastery of strain analysis requires **keeping track of changes in the orientations** and **length of lines** and keeping track of the **angles between lines.**

- If we learn to do this, we do not have to rely exclusively on the relatively uncommon occurrence of perfect circular or spherical objects being transformed and preserved in the geologic record as perfect ellipses or ellipsoids.

- Instead, we can describe the **state of strain** in a deformed body based on a surprisingly small amount of information bearing on changes in lengths in lines and changes in the angles between lines.

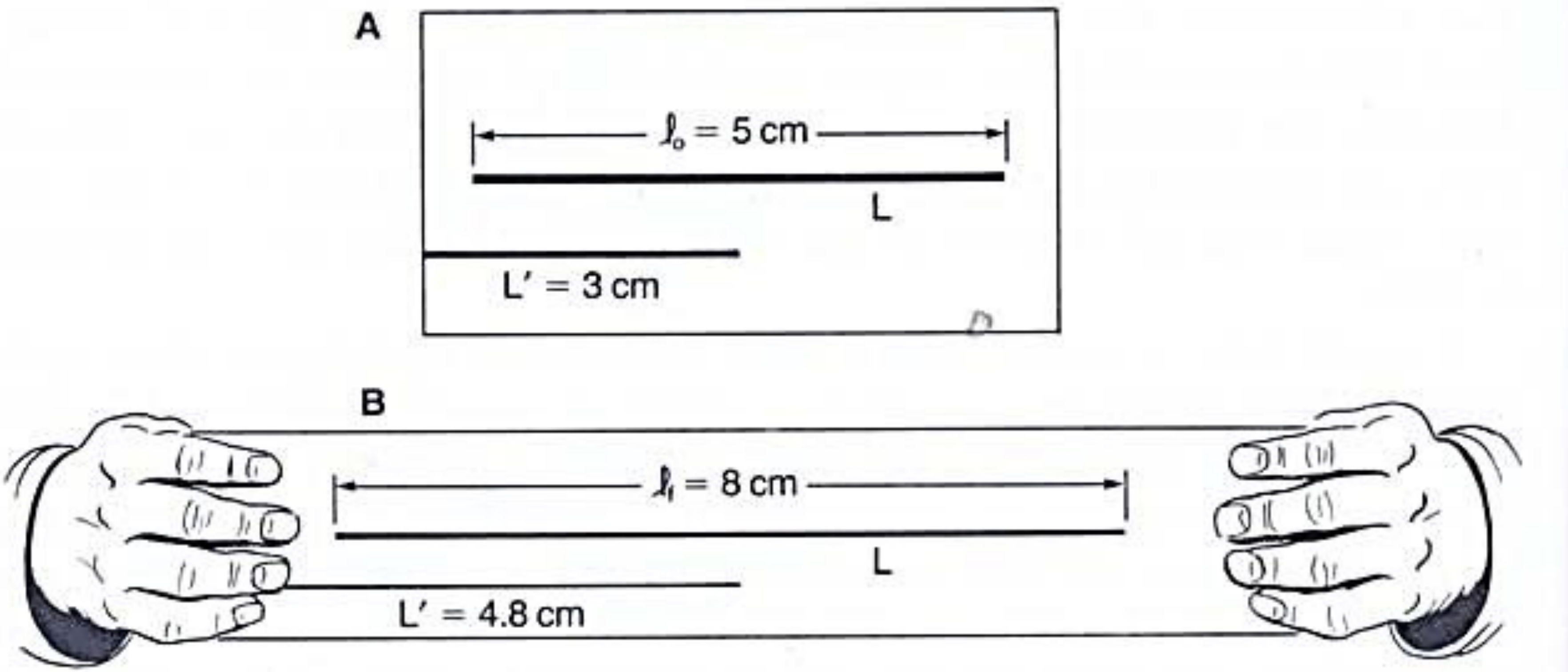
- In fact, the **changes in lengths of lines**, and the **change in angles between lines** that were originally perpendicular, are sufficient to **convey the magnitudes and directions of greatest shortening or stretching** in the rock body as a whole!

Describing changes in lengths of lines:

There are two parameters that permit changes in lengths of lines to be described easily:

1. Extension, symbolized by (***e***);
2. Stretch, symbolized by (***s***).

Consider line ***L*** whose original length (l_o) is **5 cm** (**Fig. 2.29A**).



$$e = \frac{8\ \text{cm} - 5\ \text{cm}}{5\ \text{cm}} = 0.6$$

Fig. 2.29. **(A)** Lines ***L*** and ***L'*** before stretching.

(B) Lines ***L*** and ***L'*** after stretching.

- During deformation, the non-rigid body in which ***L*** is contained changes shape and or size such that the line stretches to a final length **(l_f)** of **8 cm** **(Fig. 2.29B).**

- The change in length **(ΔL)** is **3 cm**.

- The **magnitude of extension (*e*)** in the direction of lengthening is the **change in unit length** of the line.

$$e = \frac{l_f - l_o}{l_o} \qquad (2.1)$$

$$e = \frac{8\ cm - 5\ cm}{5\ cm} = 0.6$$

- A **0.6** value for extension corresponds to a **60%** lengthening of the line,
- Percent lengthening (or percent shortening) is determined by multiplying ***e*** *by* ***100%.***

- A second way to describe the magnitude of the change in length of line ***L*** is in terms of the **stretch *(S).***
- **Stretch** is equal to **final length (l_f)** divided by original **length (l_o),** which is also equal to the value of extension plus one (**i.e., 1+ e**).
- **Stretch** tells us the final length of a line originally of unit length.
- A stretch of **3** means that a line was lengthened **3 x**.

- For the example we are considering,

$$S = \frac{8 \text{ cm}}{5 \text{ cm}} = 1.6$$

- Here is how this relationship is derived:

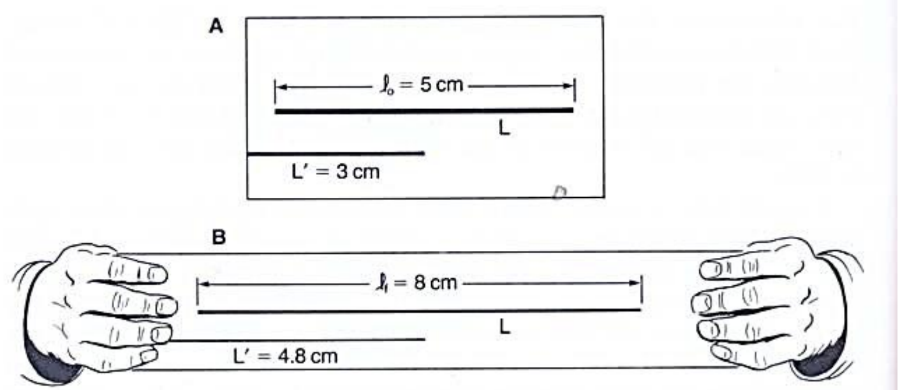
$$e = \frac{l_f - l_o}{l_o} \qquad (2.1)$$

$$e = \frac{l_f}{l_o} - 1$$

$$e + 1 = \frac{l_f}{l_o}$$

$$S = \frac{l_f}{l_o} = 1 + e \qquad (2.2)$$

- If line ***L*** in **Fig.2.29B** lies within a body that has undergone homogeneous deformation, the values of **e = 0.6** and ***S*** **= 1.6** must hold for all the lines in the body that are parallel to ***L***. Line ***L'*** is such a line (**Fig. 2.29A).**



$$e = \frac{8\text{ cm} - 5\text{ cm}}{5\text{ cm}} = 0.6$$

Fig. 2.29. (A) Lines **L** and **L'** before stretching.

(B) Lines **L** and **L'** after stretching

- If the length of ***L'*** before deformation is **3 cm**, its length after deformation can be determined by using **Equation 2.1**:

$$\boldsymbol{e = \frac{l_f - l_o}{l_o}}$$

$$\boldsymbol{0.6 = \frac{l_f - 3\ cm}{3\ cm}}$$

$$\boldsymbol{l_f = (0.6)\ (3\ cm) + 3\ cm}$$

$$\boldsymbol{l_f = 4.8\ cm}$$

(Fig. 2.29B).

- An even simpler way to compute l_f for this line is to multiply l_0 by the stretch **(*S*):**

l_f = 1.6 x 3 cm = 4.8 cm **(Fig. 2.29B).**

- Percent lengthening or shortening is determined by multiplying **100%** times **(*S* - 1.0*).***

Angular Shear

- Although **strain parameters** of extension and stretch effectively describe changes in length of lines in deformed bodies,
- They provide **no information regarding changes that take place in the angles between lines.**
- A parameter known as **angular shear**, symbolized by the Greek letter **psi *(ψ),*** comes to the rescue.

- To determine the **angular shear** along a given line, ***L*,** in a strained body, it is essential to identify a line that was originally perpendicular to ***L***.

- Angular shear describes the departure of this line from its perpendicular relation with ***L*** (**Fig. 2.35**).
- The full description requires a **sign** and a **magnitude** expressed in degrees:
  - Positive equals clockwise;
  - Negative equals counterclockwise.

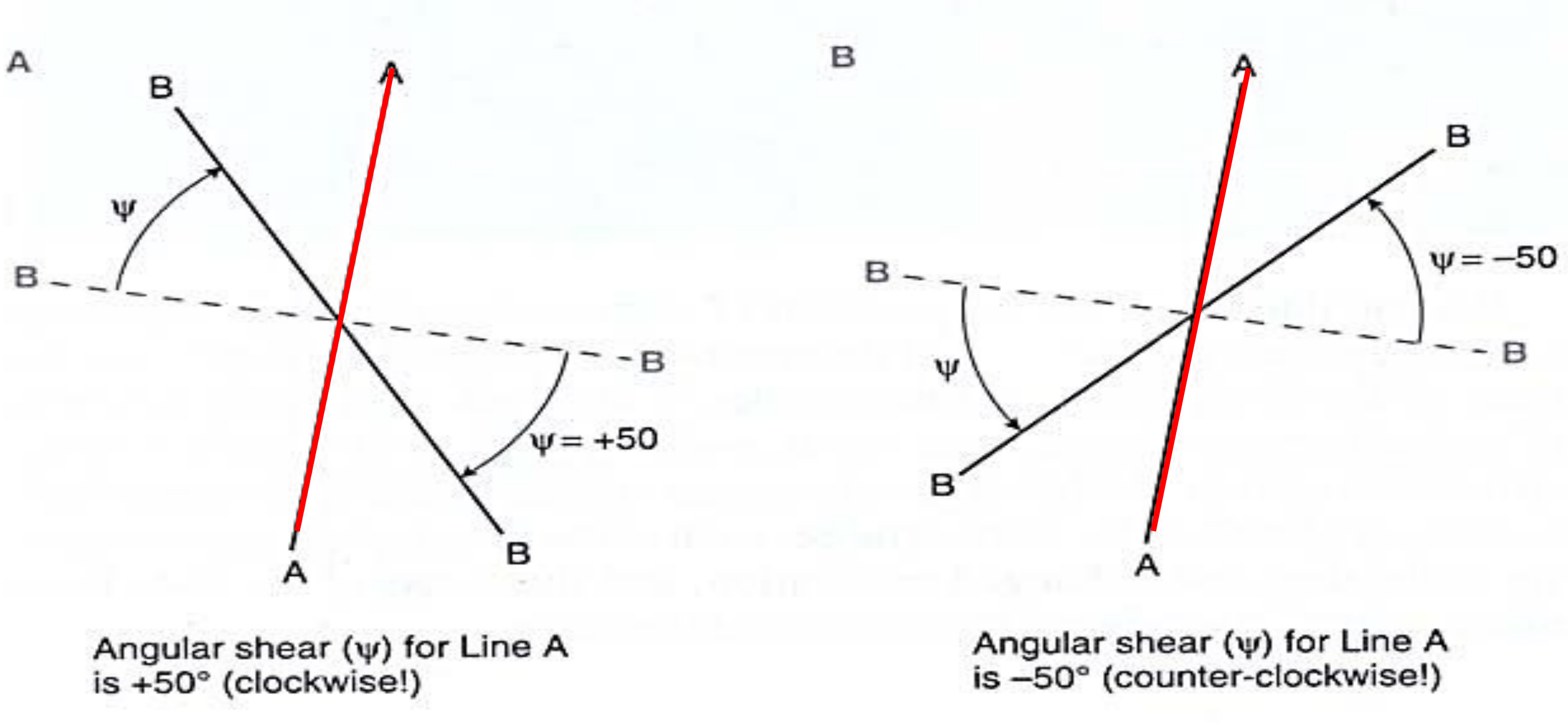


Fig. 2.35. Sign conventions for angular shear.

(A) Determination of the angular shear of line A requires identifying a line, in this case B, that was originally perpendicular to A. The original orientation of line B relative to line A is shown by the dash line. Angular shear is the shift in angle of B original versus B final. Because the shift is clockwise, the angular shear is positive (+).

(B) In this example, the angular shear of line A is -50°. A counterclockwise shift is denoted by a negative (-) sign.

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Fig. 2.35. Sign conventions for angular shear:

- **(A)** Determination of the angular shear of line **A** requires identifying a line, in this case **B,** that was originally perpendicular to **A**.
- The original orientation of line **B** relative to line **A** is shown by the dash line.
- **Angular shear is the shift in angle of B original versus B final.** Because the shift is clockwise, the angular shear is **positive (+).**
- **(B)** In this example, the angular shear of line **Anegative (** is **-50°**.
- A counterclockwise shift is denoted by a (**-)** sign.

We can illustrate the measurement of angular shear by:

- ❖ First fashioning a block of material that we will deform by flattening, and
- ❖ "paint" on the front of the block four reference circles (1-4),
- ❖ Each circle containing two sets of mutually perpendicular lines **(a-b, c-d, e-f, g-h)** **(Fig. 2.36A).**
- ❖ After deformation, the block has shortened vertically and lengthened horizontally **(Fig. 2.36B).**

Fig. 2.36. (A) Block containing reference circles and lines before deformation.

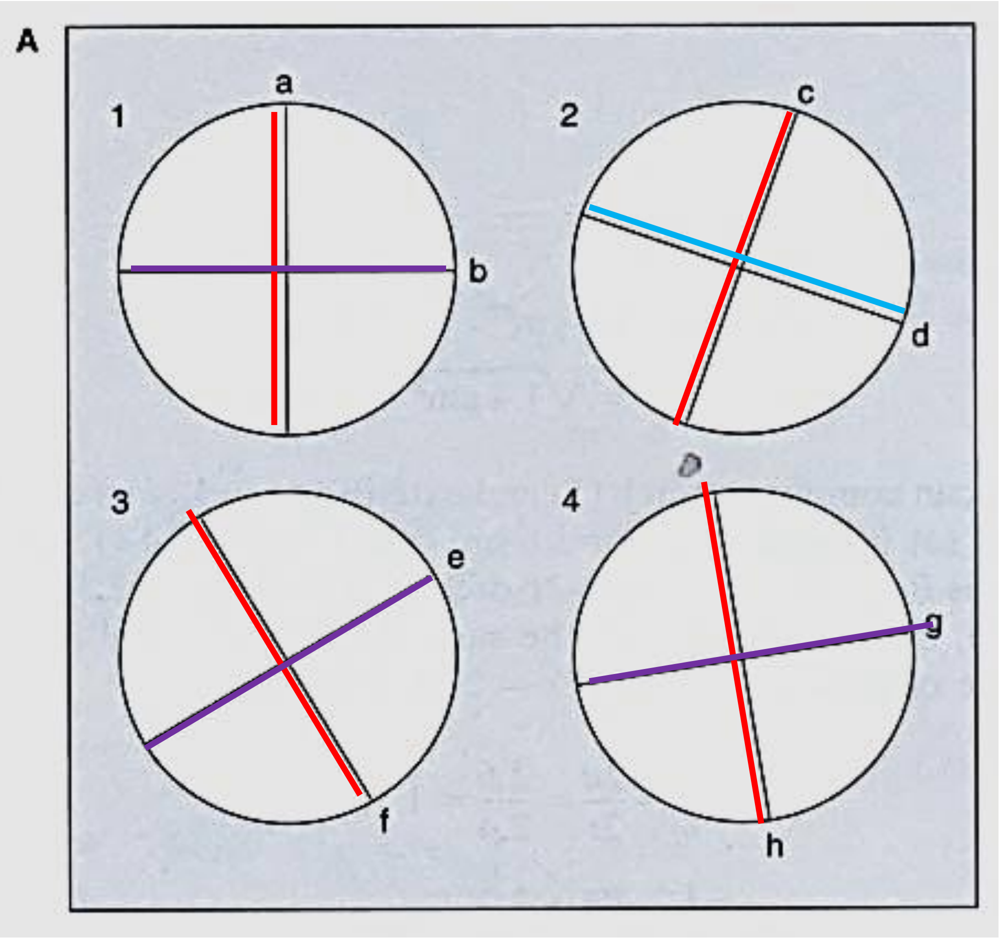


Fig. 2.36. (B) Shape of the block after deformation.

- Original reference circles now are ellipses.
- The originally mutually perpendicular reference lines have all changed length, and most have changed orientation as well.

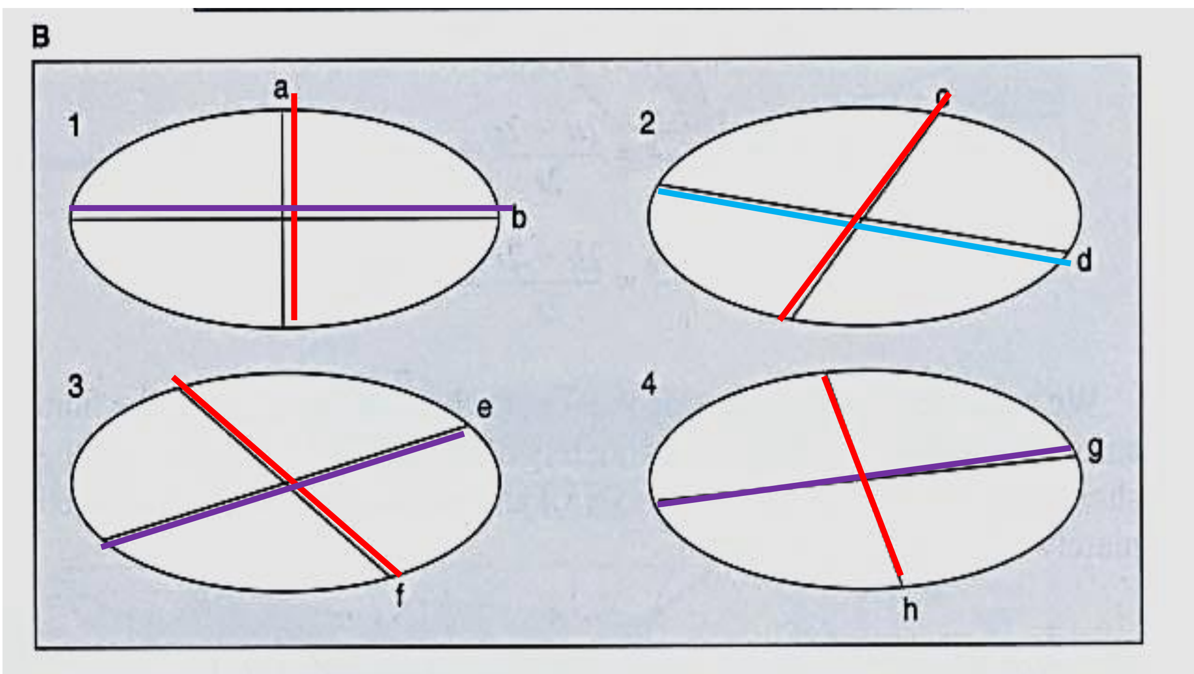
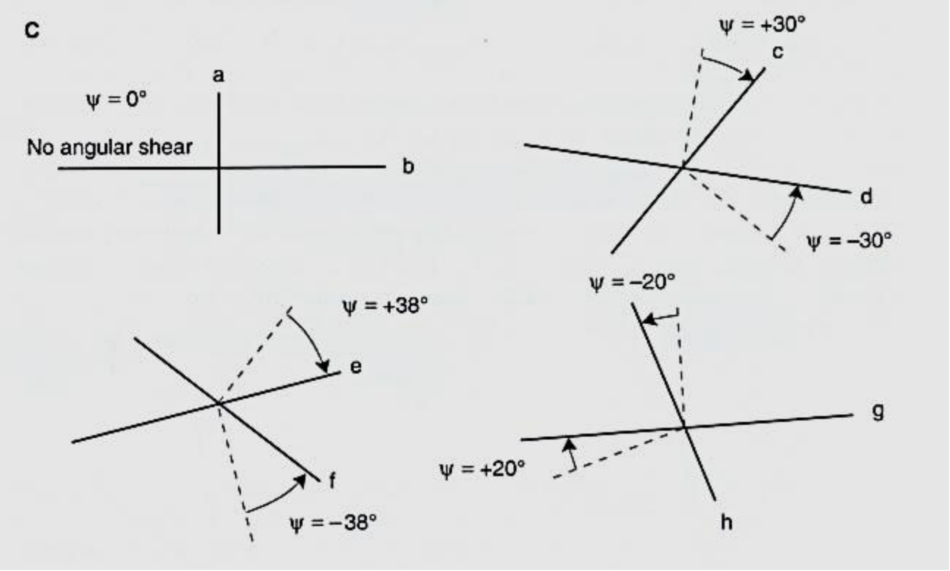
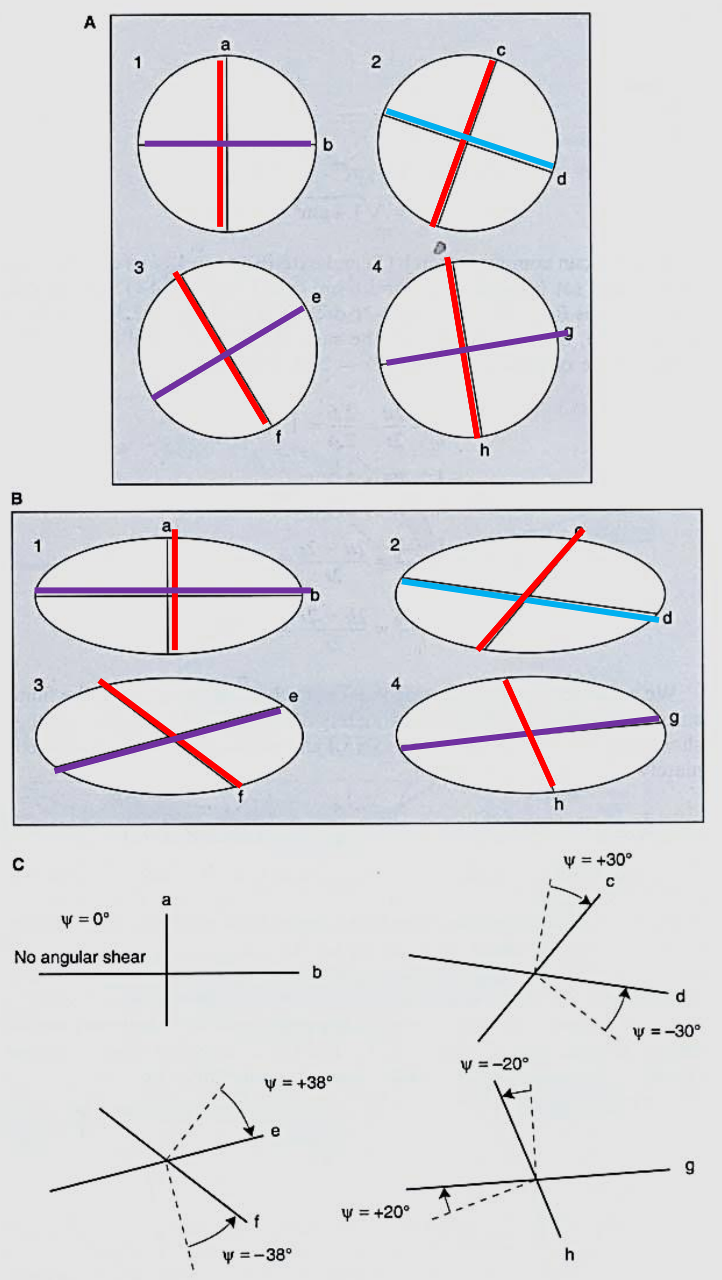


Fig. 2.36: (C) Angular shear along any line can be determined by first identifying a line originally perpendicular to it, and then measuring the angular shift. Remember, clockwise shifts are **positive (+): counterclockwise shifts are negative (-).**



- In ellipse **2**, the **angular shear** along **c** is **-30°** and **angular shear** along **d** is **+30° (Fig. 2.36C).**
- For ellipse **3**, the **angular shear** along ***e*** is **-38**° and **angular shear** along ***f*** is +**38° (Fig. 2.36C).**
- Finally, for ellipse **4**, the **angular shear** along ***g*** is **-20°** and angular shear along **h** is +20°.


A
1
a
b
2
c
d
3
e
f
4
g
h
B
1
a
b
2
c
d
3
e
f
4
g
h
C
ψ = 0°
No angular shear
a
b
ψ = +30°
c
d
ψ = –30°
ψ = +38°
e
f
ψ = –38°
ψ = –20°
g
h
ψ = +20°

Furthermore,

- Each of the lines has changed length,
- Six of the lines have changed orientation, and
- Three sets of the lines have moved out of the original right-angle relationship.

❖ It would not be difficult to calculate the extension and stretch values for each of the lines in the flattened block.

❖ We already know how to do it.

- We can describe the **angular shear (*ψ*),** for any given line by identifying a line that was originally perpendicular to it, then
- Measuring the angle through which the perpendicular line moved during deformation.
- Within **ellipse 1**, there is **no angular shear** along the line **a**, nor is there any along line **b**, for the original perpendicular relationship remains after deformation (Fig. 2.36C).

- In **ellipse 2**, the angular shear along **c** is **-30°** and the angular shear along ***d*** is **+30° (Fig. 2.36C).**
- For **ellipse 3,** the angular shear along **e** is **+38°** and the angular shear along ***f*** is **-38°** (**Fig. 2.36C).**
- Finally, for **ellipse 4**, the angular shear along **g** is **-20°** and the angular shear along ***h*** is **+20°.**

- If we keep our eyes open, we will see expressions of angular shear (**Fig. 2.37 A,** B).
- Some fossils contain perpendicular lines in the makeup of their shells.
- The modification of such originally perpendicular lines by distortion can be used as a means of determining **angular shear.**

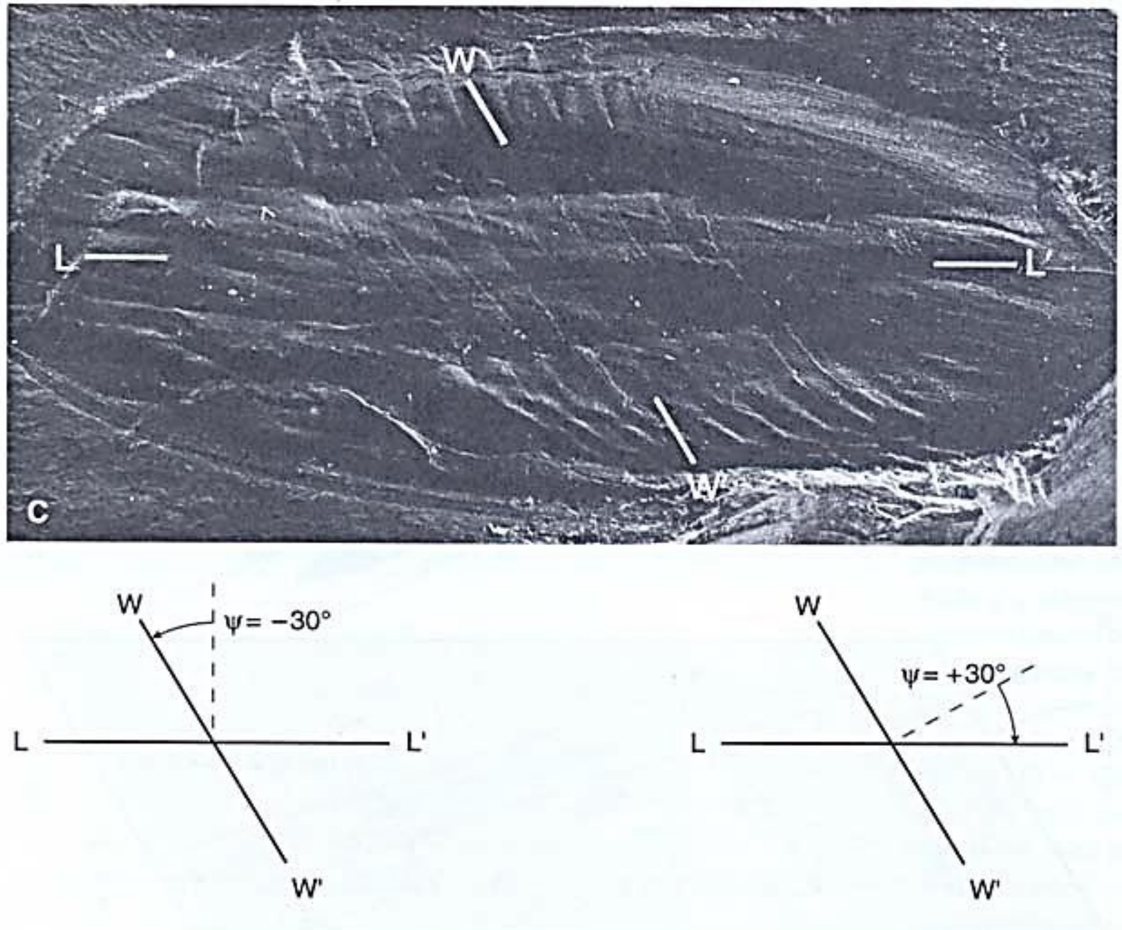


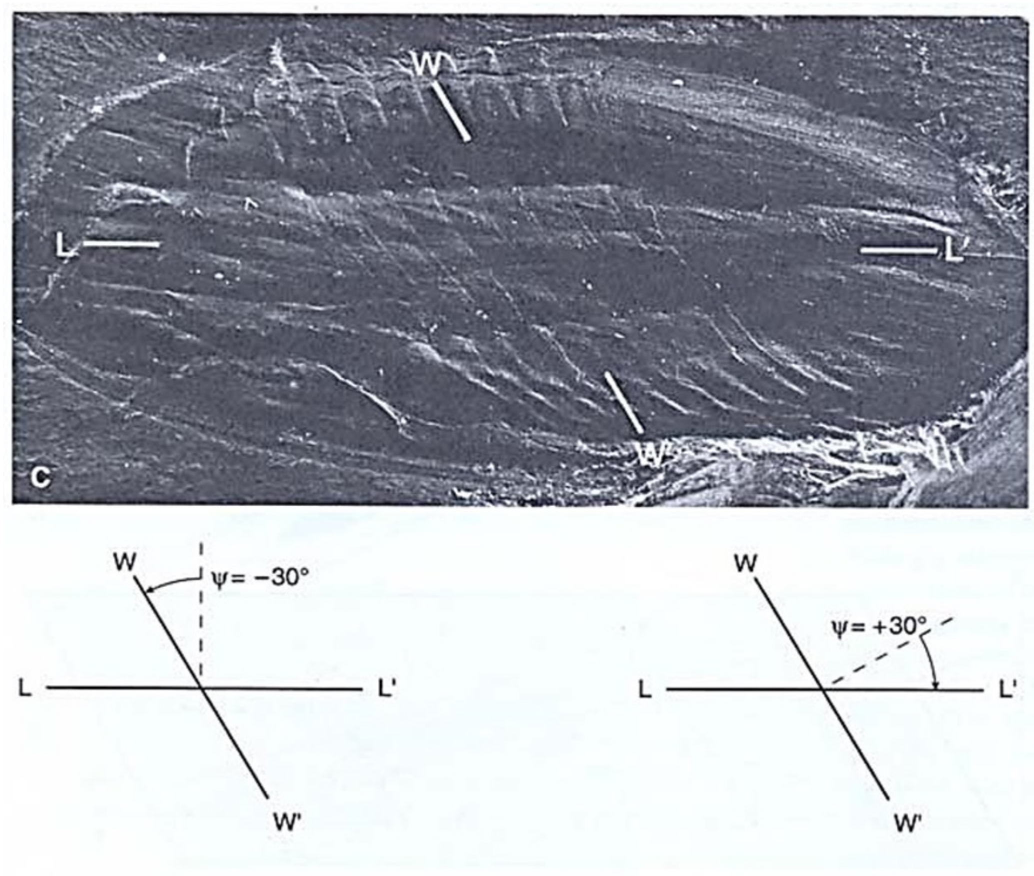
Fig. 2.37: Cambrian shale, Caernarvonshire, Wales. Angular shear of rock within which this fossil is found can be determined by measuring the angular relationship between lines ***L-L'*** and ***W –W',*** Lines that were perpendicular before the deformation. *(from The Minor Structures of Deformed Rocks: A petrographic Atlas by L.E. Weiss. Published with permission of Springer-Verlag. New York, copyright* © *1972*). **Angular shear along L –L' is -30^0**. Angular shear along **W –W; is +30^0**

- The distorted trilobite featured in **Fig. 2.37 C** readily lends itself to appraisal of angular shear.
- Lines parallel to the original length (**line *L-L'***) and to the original width (**line *W-W'***) of the trilobite are assumed to have been perpendicular before deformation.
- **Now they intersect at 60°.**

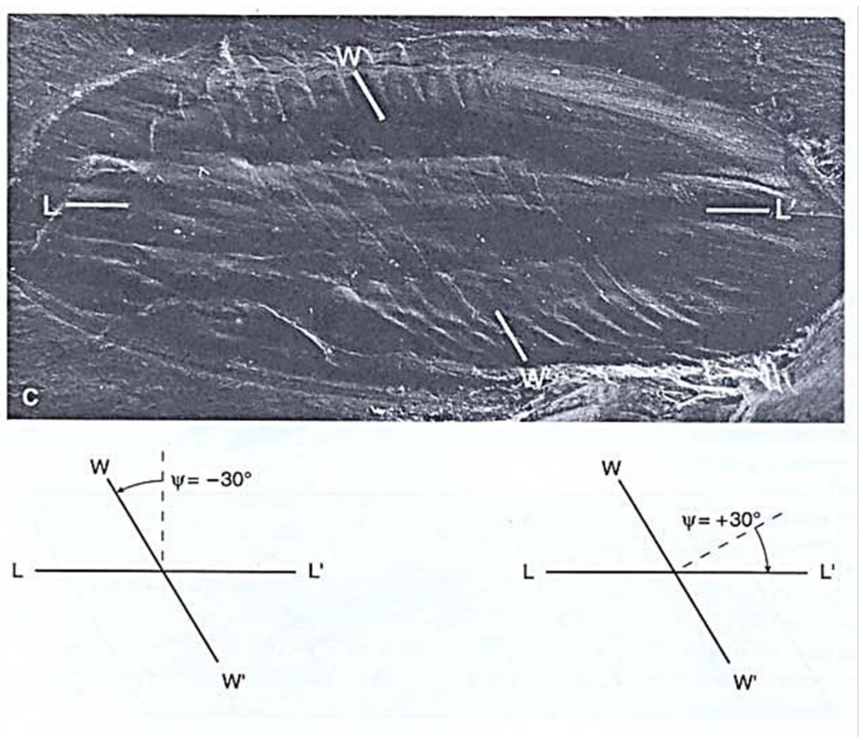
Fig. 2.37: Cambrian shale, Caernarvonshire, Wales.

- Angular shear of rock within which this fossil is found can be determined by measuring the angular relationship between lines ***L-L'*** and ***W–W',*** Lines that were perpendicular before the deformation.
- Angular shear along ***L–L'*** is **- 30°**.
- Angular shear along ***W–W***; is **+30°**

- The angular shear along line ***L-L'*** is **-30° (Fig 2.37 C).**
- This value is determined by focusing on the line ***W-W'***, which was originally perpendicular to ***L-L'***, and describe the sense and amount of deflection of that line.



- In the same fashion, the angular shear along **W-W**’ is found to be **+30° (Fig. 2.37C).** In this case, we focus on line ***L-L’***, which was originally perpendicular to ***W-W’***, and we measure the magnitude and sense of rotation of ***L-L’*** with respect to **W-W’**.



Shear Strain (γ):

- Let us consider how points on a line move as a response to angular shear.
- Points 1 to 4 on line $\boldsymbol{A_0}$ in **Fig. 2.38A** are translated by various distances as a result of the rotation of the line on which they reside.
- Line $\boldsymbol{A_0}$ is the locus of points **1- 4**.
- Line $\boldsymbol{A_f}$ is the locus of the same points in their deformed locations **(Fig. 2.38B)**.

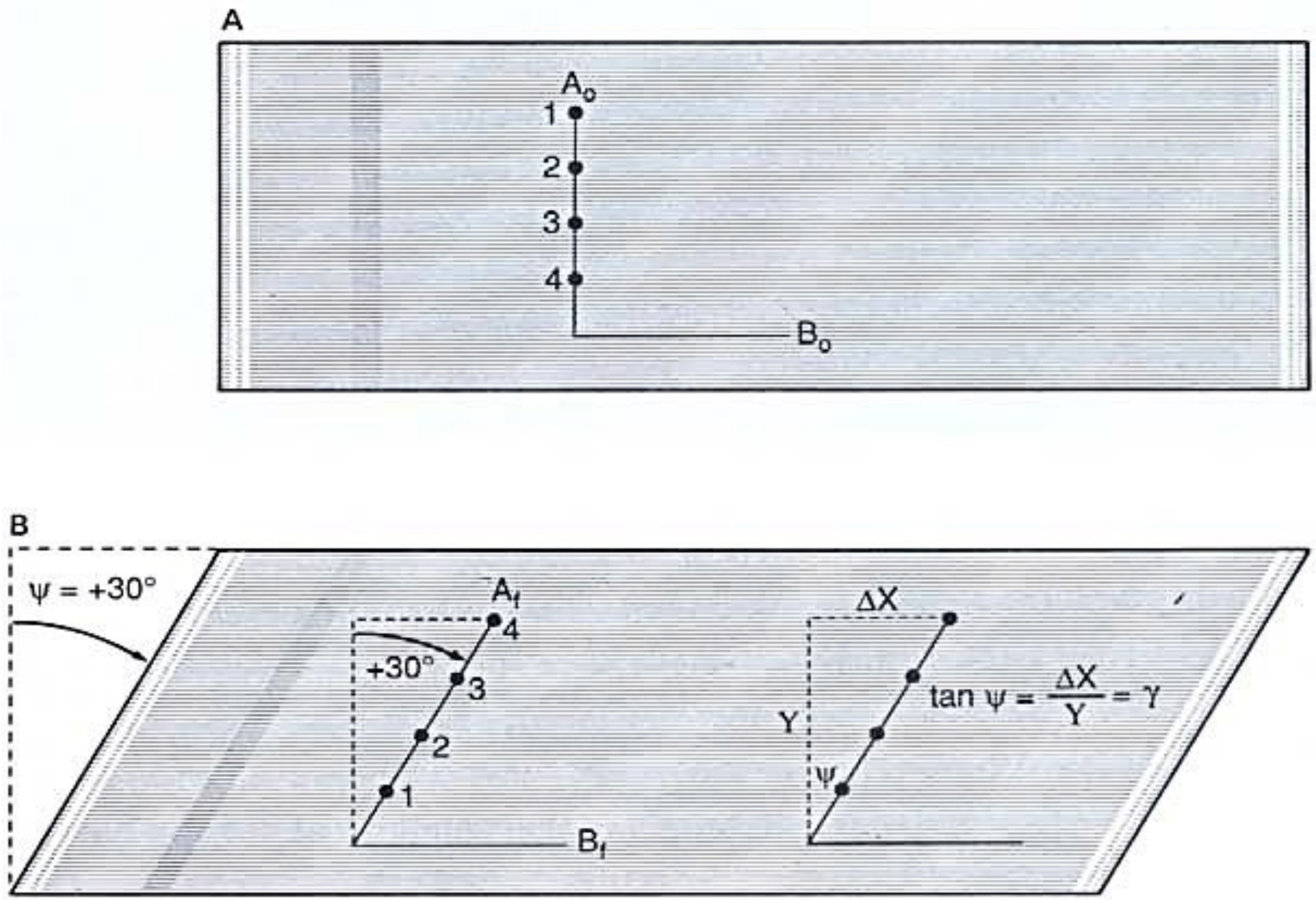


Fig. 2.38. Simulation of the shearing of a computer card deck.

A. Deck embossed with lines $\mathbf{A_0}$ and $\mathbf{B_0}$ and points **1** to **4** before deformation.

B. Configuration of the deck, including the reference lines and points, after shearing.

- Since angular shear was systematic and deformation was homogeneous, lines $\boldsymbol{A_0}$ *and* $\boldsymbol{A_f}$ are straight.

- Points **1-4** move a distance that is directly related to the angular shear and to the distance of each point above the point of intersection with the complementary line.

- If the distance of each point above the intersection is denoted as ***y*** **(Fig. 2.38B),**
- The horizontal distance of translation can be found as follows (Ramsay, 1967):

$$\tan \psi = \frac{\Delta x}{y}$$

$$\Delta x = y \tan \psi$$

- Thus **tan *ψ*** is another way of describing relative shifts in orientations of lines that were originally perpendicular.
- **It is called shear strain**, symbolized by the Greek letter **gamma** (γ),

  γ **(gamma) = tan *ψ*** = **Δ*x/y*.**
- If we know the value of **Δx** and **y**, we can find the value of **shear strain**).

- Shear strain along a line may be **positive** or **negative**, depending on the **sense of rotation** (deflection) of the line originally perpendicular to it.
- The range of shear strain is zero to infinity.
- For example shown in **Fig. 2.38B,** the shear strain of line $\mathbf{B_f}$ **is + tan 30° or +0.58**.

- The shear strain of line A_f is **– tan 30°** or **-0.58.**
- For the example of distorted trilobite shown in **Fig. 2.37C**, the **shear strain of line *L- L'* is**
- **- tan 30°** or **- 0.58°;** and the shear strain of line ***W-W'*** is **+ tan 30°** or **+ 0.58°.**

02/04/2024

Thank you

End of Lecture-4