THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT2100 Tutorial Sheet 2

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1. Find equations for (i) the tangent lines and (ii) the normal lines to the hyperbola for the given values of x.
   (a) $\frac{x^2}{9} - y^2 = 1, \quad x = 6$
   (b) $\frac{y^2}{4} - \frac{x^2}{2} = 1, \quad x = 4$

2. Find an equation of the tangent line and normal at the point given.
   (a) $xy - 2y = 1, (3, 1)$
   (b) $x^2y^3 + 2y = 3x, (2, 1)$
   (c) $x^{\frac{1}{2}} + y^{-\frac{1}{2}} = 2xy, (1, 1)$
   (d) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2, (1, 1)$

3. Find all points on the graph of $3x^2+4y^2+3xy = 24$, where the tangent line is horizontal.

4. Find the linearization approximation of the following at a.
   (a) $f(x) = x^4 + 3x^2, a = -1$
   (b) $f(x) = \cos x, a = \frac{\pi}{2}$.
   (c) $f(x) = \ln x, a = 1$
   (d) $f(x) = (x - 1)^2, a = 0$
   (e) $f(x) = e^{-2x}, a = 0$.
   (f) $f(x) = 1 + \ln(1 - 2x), a = 0$.

5. Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $x = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$.

6. Verify that the function satisfies the three hypotheses of Rolle's theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's theorem.
   (a) $f(x) = x^2 - 4x + 1, [0, 4]$
   (b) $f(x) = x^3 - 3x + 2x + 5, [0, 2]$
   (c) $f(x) = \sin 2\pi x, [-1, 1]$
   (d) $f(x) = x\sqrt{x + 6}, [-6, 0]$
   (e) $f(x) = 4x - \tan \pi x, [-\frac{1}{4}, \frac{1}{4}]$
   (f) $f(x) = \frac{6x}{\pi} - 4\sin^2 x, [0, \frac{\pi}{6}]$

7. Evaluate the following limits:
   (a) $\lim_{x\to 1} \frac{x^4+3x^3-13x^2-27x+36}{x^2+3x-4}$
   (b) $\lim_{x\to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$
   (c) $\lim_{x\to 2} \left(\frac{1}{x-2} - \frac{4}{x+2}\right)$
   (d) $\lim_{x\to 3} \frac{1}{(x-3)^2}$
   (e) $\lim_{x\to +\infty} \frac{2x+5}{x^2-7x+3}$
   (f) $\lim_{x\to -\infty} (3x^4 - x^2 + x - 7)$
   (g) $\lim_{x\to\infty} \frac{4x-1}{\sqrt{x^2+2}}$
   (h) $\lim_{x\to -\infty} \frac{3x^3+2}{\sqrt{x^4-2}}$
   (i) $\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{x^4-2}$
   (j) $\lim_{x\to\infty} (\sqrt{x^2+x} - x)$
   (k) $\lim_{x\to 2} \frac{\sqrt{x^2+5} - 3}{x^2 - 2x}$
   (l) $\lim_{x\to\infty} \frac{3x-4}{\sqrt[3]{x^3-2}}$

8. Evaluate the following:

(a) $\lim_{x\to 0}\frac{\sin x}{x}$
(b) $\lim_{x\to 0}\frac{1-\cos x}{x}$
(c) $\lim_{x\to\infty}\frac{x^2}{(\ln x)^3}$
(d) $\lim_{x\to 0^+} x\ln x$
(e) $\lim_{x\to 0^+} x^x$
(f) $\lim_{x\to 0}\frac{3^x-2^x}{x}$
(g) $\lim_{x\to\infty}(\cos\frac{1}{x})^x$
(h) $\lim_{x\to 0^+}(\sin x)^{\tan x}$
(i) $\lim_{x\to\infty}\frac{2^x}{x^{10}}$
(j) $\lim_{x\to 1}\frac{\ln x}{\tan\pi x}$
(k) $\lim_{x\to\frac{\pi}{2}^+}\frac{x\cos x}{1-\sin x}$
(l) $\lim_{x\to 1}\frac{x^3-1}{x+1}$
(m) $\lim_{x\to 0^+} x^2\ln x$
(n) $\lim_{x\to\infty} x^{\frac{1}{x}}$
(o) $\lim_{x\to 0^+}\frac{\sin x}{\sqrt{x}}$
(p) $\lim_{x\to\infty}\frac{(\ln x)^n}{x}$

9. Use the definition of the limit to show that

(a) $\lim_{x\to 4}(2x-5)=3$
(b) $\lim_{x\to 3}(2+5x)=17$
(c) If $\lim_{x\to a} f(x)=L$ and $\lim_{x\to a} g(x)=K$, then $\lim_{x\to a}[f(x)+g(x)]=L+K$.

10. Find the Maclaurin series of the following functions:

(a) $f(x)=e^x$
(b) $f(x)=\sin x$
(c) $f(x)=\ln(1-x)$
(d) $f(x)=e^{\cos x}$
(e) $f(x)=\sec x$
(f) $f(x)=\sin^{-1}x$
(g) $f(x)=\tan x$
(h) $f(x)=\frac{1}{1+x}$
(i) $f(x)=\cos^2 x$

11. Find the Taylor series of the following functions centered at the given value of c:

(a) $f(x)=\cos x; c=\frac{\pi}{3}$
(b) $f(x)=\sin x; c=\frac{\pi}{4}$
(c) $f(x)=\frac{1}{x}; x=1$
(d) $f(x)=\ln x; c=2$
(e) $f(x)=e^x; c=1$
(f) $f(x)=x^4; c=-3$
(g) $f(x)=\cos x; c=\frac{\pi}{2}$
(h) $f(x)=\sqrt{x}; c=2$
(i) $f(x)=2^x; c=1$
(j) $f(x)=\ln(x^2+1); c=0$

12. Find the first 3 nonzero terms in the Maclaurin series for each function.

(a) $y=e^{-x^2}\cos x$
(b) $y=\sec x$
(c) e^x+e^{2x}
(d) $\cos(x^2)$
(e) xe^{-x}
(f) e^x+2e^{-x}
(c) $y=\frac{x}{\sin x}$
(d) $y=e^x\ln(1+x)$