THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
Department of Physics
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PHY 2231 - Optics

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# CHAPTER 1: GEOMETRIC OPTICS

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## INTRODUCTION

All known properties of light are described in terms of experiments by which they were discovered and demonstrations by which they are frequently illustrated. These demonstrations/properties can be grouped under three main headings:

- Geometric optics
- Wave Optics
- Quantum Optics

**Geometric Optics** is easily described in terms of straight lines and plane geometry. It deals with rectilinear propagation of light, finite speed and dispersion.
**Wave Optics** deals with wave nature of light e.g diffraction of electromagnetic waves.
**Quantum Optics** assumes that light is made up of tiny bundles of energy called quanta. Here we deal with atomic orbits, probability, densities, energy levels, quanta, lasers etc.

## RECTILINEAR PROPAGATION OF LIGHT

This is a term applied to the principle that light travels in a straight line. This can be described by the observation of shadows and is used in photography to make images of objects either using cameras etc.

## THE SPEED OF LIGHT

The speed of light is finite with and approximate value of 300 000 km/s $= 3.0\times10^8$ m/s = 186400 miles/s.

Light is an electromagnetic wave and all EM waves from X-rays at one end of the spectrum to the longest radio waves are believed to travel with exactly the same speed in vacuum. This is a universal constant with an accepted value of 299797.5 km/s $= 2.997975\times10^8$ m/s.

## REFRACTIVE INDEX

The index of refraction or simply the refractive index ($n$) of any optical medium (a medium through which light can travel) is defined as the ratio of speed of light in a vacuum ($c$) and speed of light in a medium ($v$).

$$n = \frac{c}{v}$$

For one type of glass $n = 1.520$.
For water, $n = 1.333$.
For air $n = 1.000$.
At 0 °C and atmospheric pressure $n$ for air is 1.000292.
Different kinds of glass and plastic have different refractive indices. The most commonly used optical glasses have indices ranging from 1.52 to 1.72.

**Optical density** of a transparent material is a measure of its refractive index. A medium with a relatively higher refractive index is said to have high optical density.

## OPTICAL PATH

This is defined as the path $d$ of a ray of light in any medium and is given by the product of velocity and time i.e. $d = vt$.
By definition $n = \frac{c}{v} \Rightarrow v = \frac{c}{n}$.
$\therefore d = \frac{c}{n}.t$ or $nd = ct$.
The product $nd = \Delta =$ Lop is called optical path. This quantity $\Delta$ represents the distance light travel in the vacuum in the same time it travels a distance $d$ in the medium.
If a light ray travels through a series of optical media of thickness

$d, d', d''$.... with refractive indices $n, n', n''$, ...., then the optical path:

$$\Delta = nd + n'd' + n''d'' + ..... \quad (0.1)$$

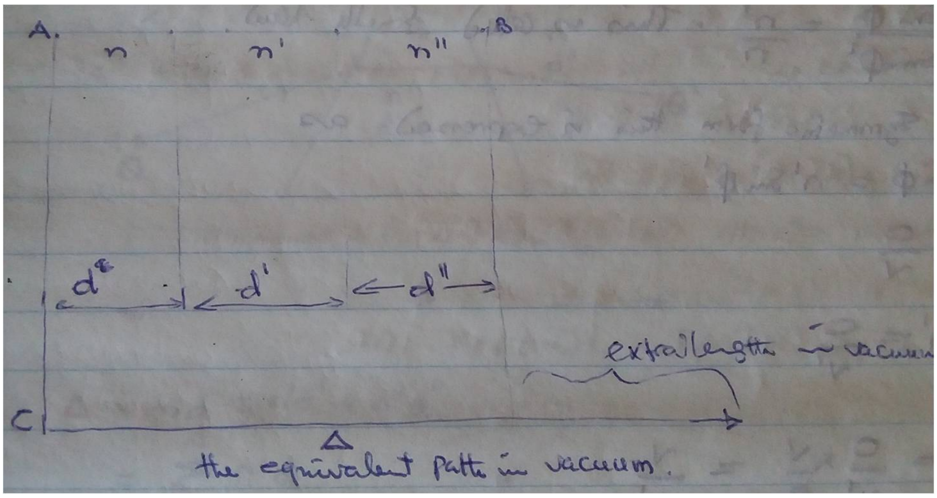


Figure 0.1

## LAWS OF REFLECTION AND REFRACTION

When a ray of light is incident on a boundary separating two different media, part is reflected back in the first media and the remainder is is refracted (bent in its path) as it enters the second media. The directions taken by these rays can be described by two well established laws.
1. The angle at which the incident ray strikes the interface MM' is exactly equal to the angle the reflected ray makes with the same interface. In other words, the angle of incidence is equal to the angle of reflection.
2. For all angles, sine of angle of incidence divided by sine of angle of refraction is equal to a constant i.e $\frac{sin\phi}{sin\phi'}$ = constant. This law ($\frac{sin\phi}{sin\phi'} = \frac{n'}{n}$ = constant) is called Snells law.
In symmetric form, this is expressed as $nsin\phi = n'sin\phi'$
$n = \frac{c}{v}$, so $n' = \frac{c}{v'}$. $\therefore$, $\frac{n'}{n} = \frac{c}{v} \times \frac{v'}{c} = \frac{v'}{v} \longrightarrow \frac{sin\phi}{sin\phi'} = \frac{v'}{v}$.
If one or both of the indices is different from unity, then $\frac{n'}{n}$ is called the relative index of refraction. If the first medium is a vacuum i.e $n = 1$, the refractive index is just the value of the second medium i.e $\frac{sin\phi}{sin\phi'} = n'$

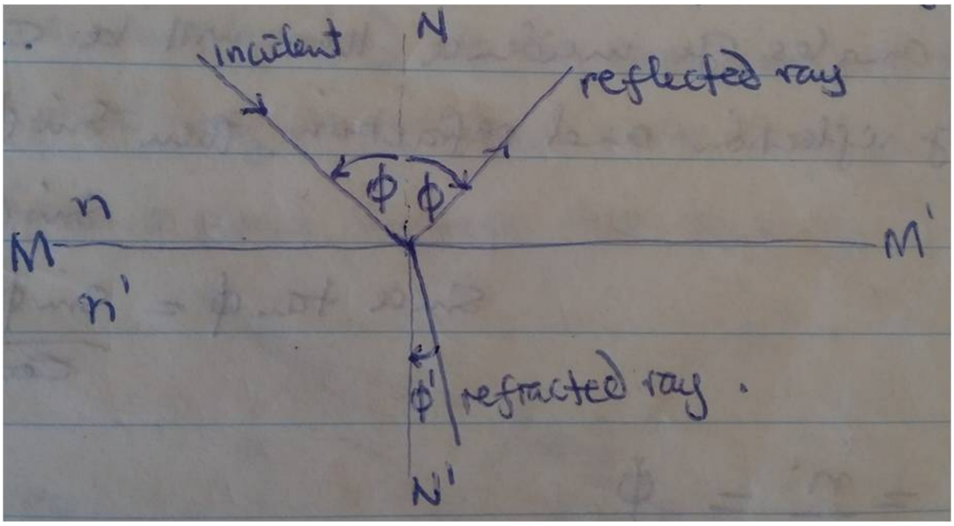


Figure 0.2

For very small angles of incidence, there will be corresponding small angles of reflection and refraction. Then $sin\phi \approx \phi \approx tan\phi$. Also $sin\phi^{'} \approx \phi^{'} \approx tan\phi^{'}$. This is so because $tan\phi = \frac{sin\phi}{cos\phi} \approx sin\phi \approx \phi$.

$\frac{sin\phi}{sin\phi^{'}} = \frac{n'}{n} = \frac{\phi}{\phi'}$

**FERMAT'S PRINCIPLE**

The term optical path has been defined as the distance a light ray would travel in the vacuum in the same time it travels from one point to another, a specified distane, through one or more optical media.

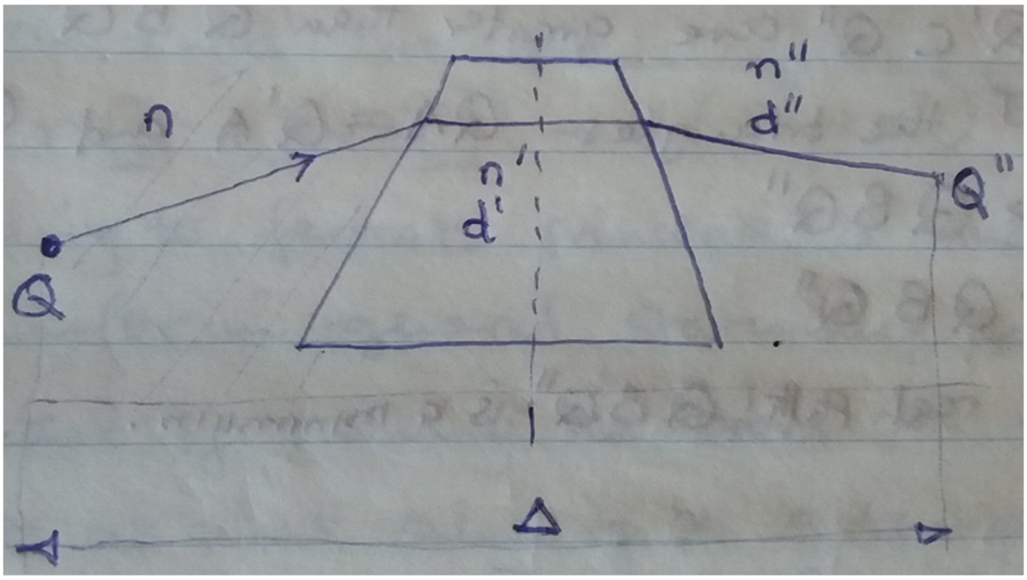


Figure 0.3

$\Delta = nd + n'd' + n''d'' + ..... = \sum_i^N n_i d_i$
For a varying refractive index
$\Delta = \int n_i d_i$.
In this case Snells law is invalid.
Fermat stated that the time required by light to traverse a path is a minimum and the optical path is the measure of this time. However, there are many cases in which the optical path is a maximum or neither a maximum or minimum but merely a stationary.
Consider a ray that must pass through a point $Q$ and then after reflection from a plane surface pass through a second point $Q''$.

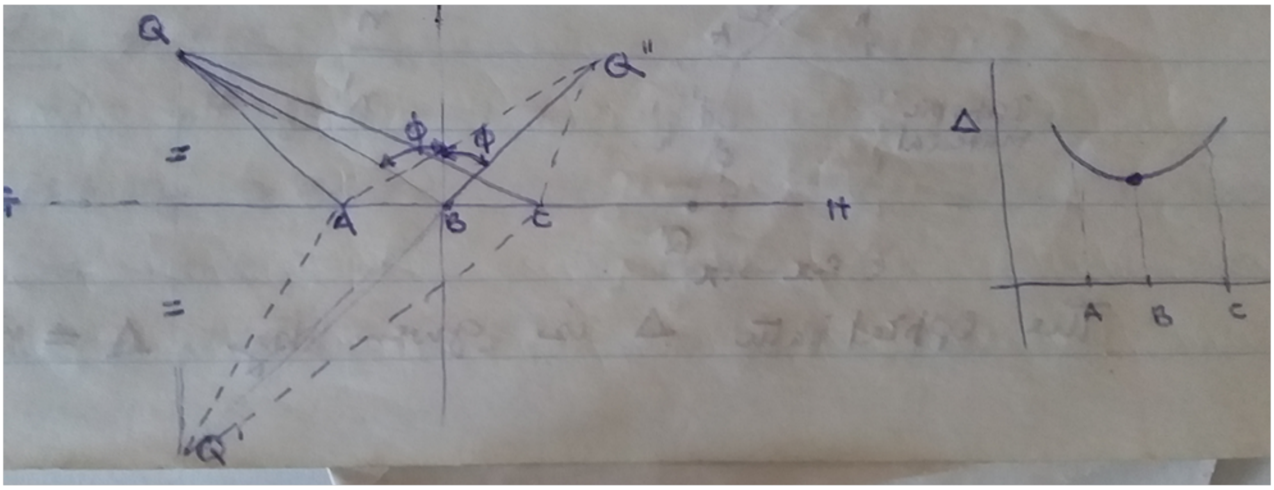


Figure 0.4

To find the real path, first drop a perpendicular to GH and extend it to the other side of $Q'$. $Q'Q''$ is a straight line through the point of intersection. The real light path is $QBQ''$ and obeys the law of reflection. The line $QBQ''$ is the shortest path. $Q'AQ''$ and $Q'CQ''$ are greater than $QBQ''$. By considering the lines $QA = Q'A$ and $QC = Q'C$, so that
$QAQ'' > QBQ''$
$QCQ'' > QBQ''$
This implies that the real path $QBQ''$ is a minimum.

**ELLIPSOIDAL REFLECTOR**

All the rays emanating from a point $Q$ at one focus are reflected according to the law of reflection and come together at the other focus $Q'$. Ellipsoidal reflectors find application in solar energy concentrators.

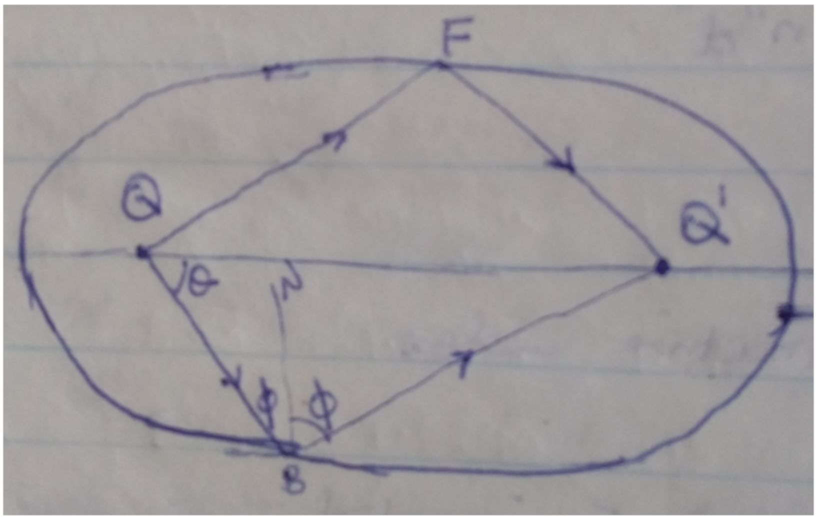


Figure 0.5: *Ellipsoidal reflector*

**ISOTROPIC MATERIAL**

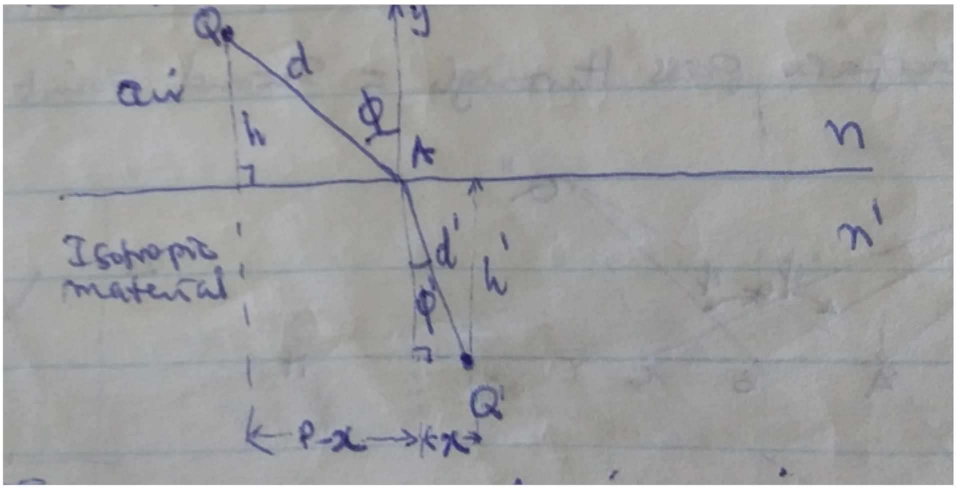


Figure 0.6

The optical path $\Delta$ is given by $\Delta = nd + n'd'$, where $d$ represents the distance $QA$ and $d'$ represents $Q'A$. Use of Pythagoras theorem for the triangles to get

$$d^2 = h^2 + (p - x)^2 \tag{0.2}$$

$$d'^2 = h'^2 + x^2 \tag{0.3}$$

Substituting the values of $d$ and $d'$ in the optical path equation $\Delta = nd + n'd'$, we get

$$\Delta = n[h^2 + (p - x)^2]^{1/2} + n'[h'^2 + x^2]^{1/2} \tag{0.4}$$

According to Fermats principle $\Delta$ must be a minimum or a maximum (or in general stationery)for the actual path.
One method of showing the above, we get a plot of $\Delta$ against $x$ and find out at what value of $x$ a tanget to the curve is parallel to the x-axis.

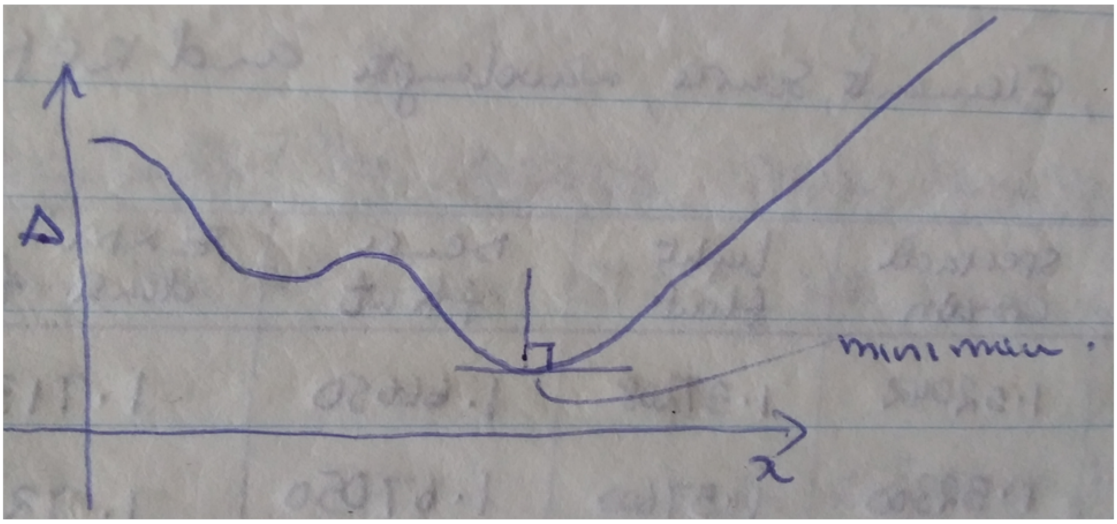


Figure 0.7

Another method is to differentiate $\Delta$ with respect to $x$ obtaing an equation for the slope. Differentiate $\Delta$ with respect to $x$ to obtain

$$\frac{d\Delta}{dx} = \frac{\frac{1}{2}n(-2p+2x)}{[h^2+(p-x)^2]^{1/2}} + \frac{\frac{1}{2}n'2x}{[h'^2+x^2]^{1/2}}. \tag{0.5}$$

Equate this to obtain

$$\frac{n(p-x)}{[h^2+(p-x)^2]^{1/2}} = \frac{n'x}{[h'^2+x^2]^{1/2}}. \tag{0.6}$$

$$\frac{n(p-x)}{d} = \frac{n'x}{d'}. \tag{0.7}$$

The multipliers of $n$ and $n'$ are just the sines of corresponding angles, so we have

$$nsin\phi = n'sin\phi' \tag{0.8}$$

which is Snells law.