THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
Department of Physics
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PHY 2231 - Optics

CHAPTER 2: PHYSICAL OPTICS

WAVE OPTICS

INTERFERENCE; DIFFRACTION AND POLARIZATION OF LIGHT

In order to understand interference, diffraction, and polarization of light, one has to know the properties of light. For any type of wave motion, we talk of

- Time period of vibration (τ)
- frequency (ν)
- Wavelength (λ)
- Amplitude (A_o or E_o)

$$y = asin\frac{2\pi}{\lambda}(x - vt) \tag{0.1}$$

We know that $v = \nu\lambda = \frac{\lambda}{T}$. Sometimes $v = f\lambda$. Then

$$y = asin2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \tag{0.2}$$

or

$$y = asin\frac{2\pi}{T}\left(t - \frac{x}{v}\right) \tag{0.3}$$

Sometimes written as

$$y = asin2\pi\nu\left(t - \frac{x}{v}\right) \tag{0.4}$$

where ν is the frequency.
A simple way of expressing the equation for simple harmonic motion (SHM) wave in terms of the angular frequency $\omega = 2\pi\nu = 2\pi f$ and the propagation constant $K = \frac{2\pi}{\lambda}$. So we may simply write

$$y = asin(Kx - \omega t) = asin(\omega t - Kx + \pi) = acos(\omega t - Kx + \frac{\pi}{x}) \tag{0.5}$$

$y = acos(\omega t - Kx)$ and $y = asin(\omega t - Kx)$. This imly $t = \frac{T}{4}$ and $t = \frac{T}{2}$, respectively instead of $t = 0$. And for any arbitrary phase factor ϕ, the equations are valid.

INTERFERENCE

Superposition of Waves

Consider the effect of superposing two sine waves of the same frequency:

$$y_1 = a_1 sin(\omega t - \alpha_1) \tag{0.6}$$

and

$$y_2 = a_2 sin(\omega t - \alpha_2) \tag{0.7}$$

The resultant displacement y is merely the sum of y_1 and y_2 i.e.
$y = y_1 + y_2$, giving

$$y = a_1 sin(\omega t - \alpha_1) + a_2 sin(\omega t - \alpha_2) \tag{0.8}$$

This can be written as

$$y = a_1 sin\omega t cos\alpha_1 - a_1 cos\omega t sin\alpha_1 + a_2 sin\omega t cos\alpha_2 - a_2 cos\omega t sin\alpha_2 \tag{0.9}$$

$$\therefore y = (a_1 cos\alpha_1 + a_2 cos\alpha_2)sin\omega t - (a_1 sin\alpha_1 + a_2 sin\alpha_2)cos\omega t \tag{0.10}$$

Since $a's$ and $\alpha's$ are constants we are justified in setting

$$a_1 cos\alpha_1 + a_2 cos\alpha_2 = Acos\theta \tag{0.11}$$

and

$$a_1 sin\alpha_1 + a_2 sin\alpha_2 = Asin\theta \tag{0.12}$$

provided that the constants A and θ can be found which satisfy these equations.
Squaring and adding both sides of equation 0.11 and equation 0.12 we have

$$A^2(cos^2\theta + sin^2\theta) = a_1^2(cos^2\alpha_1 + sin^2\alpha_1) + a_2^2(cos^2\alpha_2 + sin^2\alpha_2)+$$

$$2a_1a_2(cos\alpha_1 cos\alpha_2 + sin\alpha_1 sin\alpha_2) \tag{0.13}$$

or

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2cos(\alpha_1 - \alpha_2) \tag{0.14}$$

Dividing equation 0.11 by equation 0.12, we get

$$tan\theta = \frac{a_1 sin\alpha_1 + a_2 sin\alpha_2}{a_1 cos\alpha_1 + a_1 cos\alpha_1}. \tag{0.15}$$

Equation 0.14 and 0.15 show that values of A and θ exists and must both satisfy equations 0.11 and 0.12.
We may rewrite equation 0.11 and 0.12 by substituting the right handside members to give

$$y = Acos\theta sin\omega t - Asin\theta cos\omega t \tag{0.16}$$

which has the same form of the sine of the difference of two angles and can be expressed as

$$y = Asin(\omega t - \theta). \tag{0.17}$$

If we add two waves with different amplitudes and different phase, we get a new wave with different amplitude and new phase. The new amplitude A and the new phase angle θ but of the same form as the original separate equations.
According to equation 0.14, the amplitude depends on the separate amplitudes a_1 and a_2. The phase difference is $\alpha_1 - \alpha_2 = \delta$.

For $\alpha_1 = \alpha_2$ in dealing with light waves, the intensity of the light at any point will be proportional to the square of the resultant amplitude.

$$I = A^2 = 2a^2(1 + cos\delta) = 4a^2cos^2\frac{\delta}{2}. \tag{0.18}$$

where I is the intensity.
If the phase difference is such that $\delta = 0$, 2π, 4π etc., then $I = 4a^2$ or 4 times the intensity of either beam (constructive interference). If $\delta = \pi$, 3π, 5π, the intensity is zero. This is called destructive interference.

COMPLEX REPRESENTATION

Often it is more convenient to use complex representation in which displacement $x_1 = a_1cos(\omega t + \theta_1)$ is written as $x_1 = a_1e^{i(\omega t+\theta_1)}$ where it is implied that the actual displacement is the real part of x_1 that is
$x_1 = a_1cos(\omega t + \theta_1)$ +i$a_1sin(\omega t + \theta_1)$
If $x_2 = a_2e^{i(\omega t+\theta_2)}$
Then $x_1 + x_2 = (a_1e^{I\theta_1}a_2e^{i\theta_2})e^{i\omega t} = ae^{i(\omega t+\theta)}$, where $ae^{i\theta} = a_1e^{i\theta_1} + a_2e^{i\theta_2}$
As an illustration, consider N displacements with each amplitude a and increment angle θ_0.

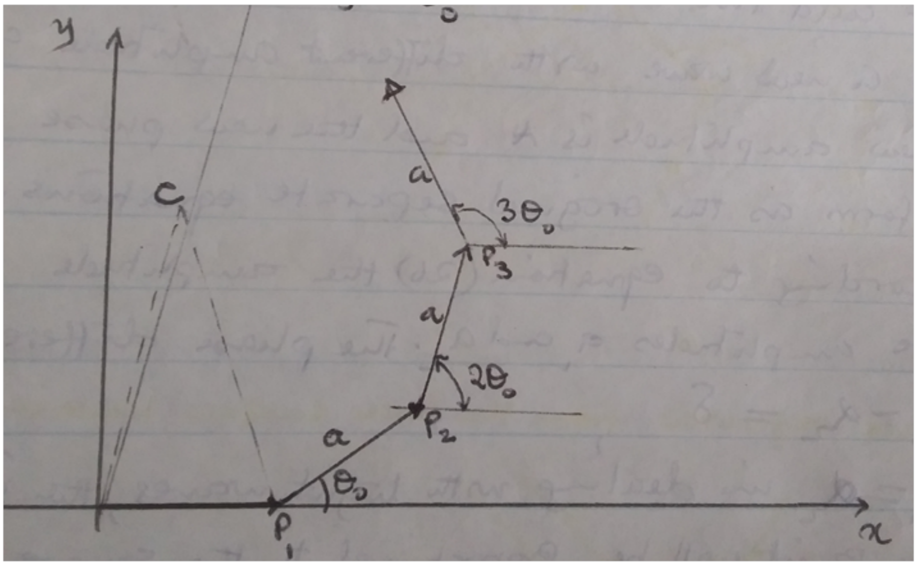

Figure 0.1

$$x_1 = ae^{i\omega t} \tag{0.19}$$

$$x_2 = ae^{i(\omega t+\theta_0)} \tag{0.20}$$

$$x_3 = ae^{i(\omega t + 2\theta_0)} \tag{0.21}$$

$$x = x_1 + x_2 + x_3 + ... + x_n \tag{0.22}$$

$$x = ae^{i\omega t} + ae^{i(\omega t + \theta_0)} + ae^{i(\omega t + 2\theta_0)} + ... \tag{0.23}$$

$$\therefore x = ae^{i\omega t}(1 + e^{i\theta_0} + e^{2i\theta_0} + ... + e^{i(N-1)\theta_0}) \tag{0.24}$$

$$x = ae^{i\omega t}\left(\frac{1 - e^{Ni\theta_0}}{1 - e^{Ni\theta_0}}\right) = ae^{i\omega t}\left[\frac{e^{\frac{iN\theta_0}{2}}}{e^{\frac{i\theta_0}{2}}} \times \frac{e^{\frac{iN\theta_0}{2}} - e^{\frac{-iN\theta_0}{2}}}{e^{\frac{i\theta_0}{2}} - e^{\frac{-i\theta_0}{2}}}\right] \tag{0.25}$$

$$\therefore x = \frac{asin\frac{N\theta_0}{2}}{\frac{sin\theta_0}{2}} exp\left[i(\omega t + (N-1)\frac{\theta_o}{2}\right] \tag{0.26}$$

SPECIAL CASES OF INTERFERENCE

Interference from thin films

The colours of soap bubbles, oil slicks, and other films are the result of interference. We consider the oblique incidence of plane wave on a thin film as shown.

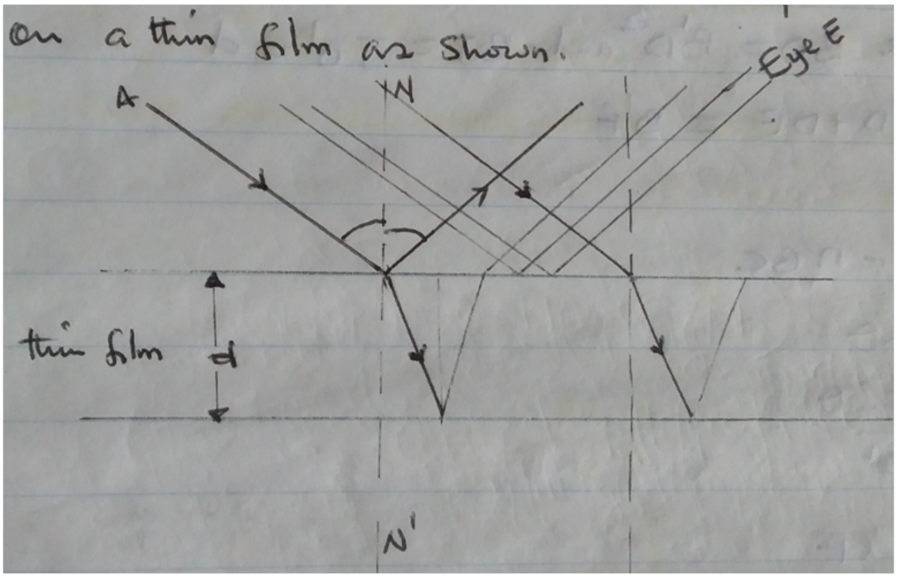

Figure 0.2

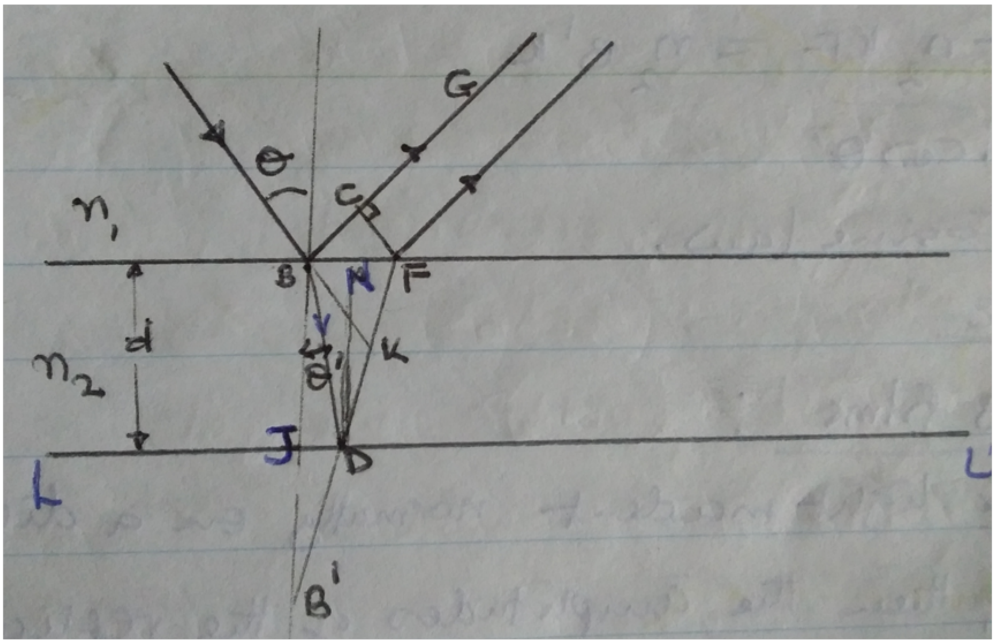

Figure 0.3

The wave reflected from the lower surface traverses an additional path Δ.
$\Delta = n_2(BD + DF) - n_1BC$ where C is the foot of the perpendicular from F.
$\Delta = 2n_2dcos\theta^{'}$
Consider a film placed in air; a phase change of π will occur when reflection takes place at point B and as such the conditions of distructive and constructive interference would be given by
$\Delta = 2n_2dcos\theta^{'} = m\lambda$ (Minima) and
$\Delta = 2n_2dcos\theta^{'} = (m + \frac{1}{2})\lambda$ (Maxima)
$\langle JBD = \langle BDN = \langle NDF = \theta^{'}$
$\langle BDJ = \frac{\pi}{2} - \theta^{'}$
$\langle B^{'}DJ = \pi - [(\frac{\pi}{2} - \theta^{'}) + \theta^{'} + \theta^{'}] = \frac{\pi}{2} - \theta^{'}$
Therefore, we conclude $BD = B^{'}D$ and $BJ = JB^{'} = d$.
$BD + DF = B^{'}D + DF = B^{'}F$
Hence

$$\Delta = n_2B'F - n_1BC \tag{0.27}$$

$$\therefore BC = BFsin\theta = \frac{KFsin\theta}{sin\theta'} = \frac{n_2}{n_1}KF \tag{0.28}$$

$$\Delta = n_2B'F - n_2KF = n_2B'K = 2n_2dcos\theta' \tag{0.29}$$

This is the cosine law.

POINTS TO NOTE

Non reflective films

If we consider light incident normally on a dielectric of refractive index n_2 from n_1 then the amplitudes of the reflected and transimitted beams are related by the following relations:

$$a_r = \frac{n_2 - n_1}{n_2 + n_1} a_i \tag{0.30}$$

and

$$a_t = \frac{2n_2}{n_2 + n_1} a_i \tag{0.31}$$

where a_i is the incident amplitude, a_r is the reflected amplitude and a_t is the transmitted amplitude.

When $n_2 > n_1$, a_r is negative showing that the reflection occurs at a denser medium with phase change of π. In this case the reflection and transimission coefficients r and t are therefore given by

$$r = \frac{n_1 - n_2}{n_1 + n_2} \tag{0.32}$$

and

$$t = \frac{2n_1}{n_1 + n_2} \tag{0.33}$$

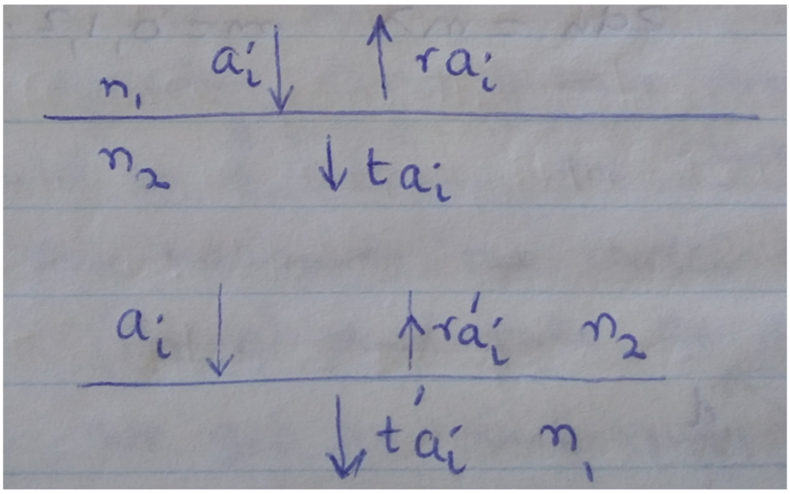

Figure 0.4

We note that r' and t' are the reflection and transimission coefficients when the light propagating in a medium of refractive index n_2 is incident on that of refractive index n_1, then

$$r = \frac{n_2 - n_1}{n_1 + n_2} \tag{0.34}$$

and

$$t = \frac{2n_2}{n_1 + n_2} \tag{0.35}$$

From the above two equations we can say that

$$1 - tt' = i - \frac{4n_1 n_2}{(n_1 + n_2)^2} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = r^2 \tag{0.36}$$

Equations (0.34),(0.35) and (0.36) represents what are known as Stokes relations.

According to the figure below, to get a maximum intensity assuming normal incidence we must have $2d = (m + \frac{1}{2})\lambda_n$, m = 0, 1, 2, 3.....

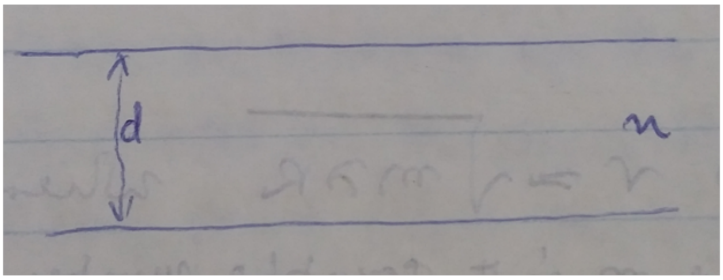

Figure 0.5

The term $\frac{1}{2}\lambda_n$ is introduced because of the phase change on reflection. A phase change of 180° is equal to half wavelength. $\lambda_n = \frac{\lambda}{n}$ just like $v_n = \frac{c}{n}$.

$$\therefore 2d = (m + \frac{1}{2})\frac{\lambda}{n} \tag{0.37}$$

or

$$2nd = (m + \frac{1}{2})\lambda \tag{0.38}$$

for Maxima, and

$$2nd = m\lambda \tag{0.39}$$

for Minima, with m= 0, 1, 2, 3,.....

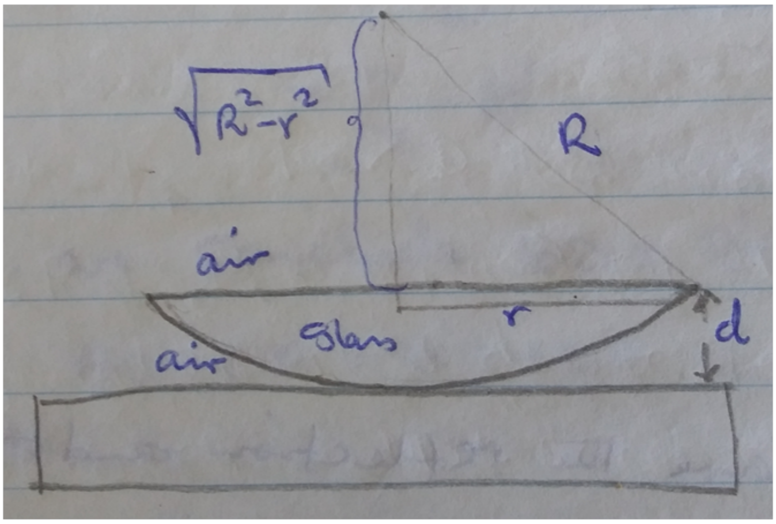

Figure 0.6

Consider the figure above

$$d = R - \sqrt{R^2 - r^2} = R - R\left[1 - \left(\frac{r}{R}\right)^2\right]^{\frac{1}{2}} \tag{0.40}$$

The condition of maxima is still

$$2d = (m + \frac{1}{2})\lambda \tag{0.41}$$

because n= 1 (for air)
If $\frac{1}{R} \leq 1$ or $\frac{r}{R} \leq 1$ we can expand the square bracket by binomial theorem keeping only two terms, then

$$d = R - R\left[1 - \frac{1}{2}\left(\frac{r}{R}\right)^2 + ...\right] = \frac{r^2}{2R} \tag{0.42}$$

Combining this with $2d = (m + \frac{1}{2})\lambda$, we get
$2\left(\frac{r^2}{2R}\right) = (m + \frac{1}{2})\lambda \rightarrow \left(\frac{r^2}{R}\right) = (m + \frac{1}{2})\lambda$ or $r = \sqrt{(m + \frac{1}{2})\lambda R}$ where m $= 0, 1, 2, 3, ...$ for bright rings.
For dark rings, $2d = m\lambda$. Therefore, $2\left(\frac{r^2}{2R}\right) = m\lambda \rightarrow \left(\frac{r^2}{R}\right) = m\lambda$ or $r = \sqrt{m\lambda R}$ where $m = 0, 1, 2, 3, ...\infty$
Since in $r = \sqrt{(m + \frac{1}{2})\lambda R}$ and $r = \sqrt{m\lambda R}$, $\frac{1}{2}$ and λR are constants, then it means r varies with m.

Between two dark rings, there will be a bright ring whose radius is given by $\sqrt{(m+\frac{1}{2})\lambda R}$.
The equation $2nd = m\lambda$ predicts that the central spot should be dark.
If one measures the radii of the m^{th} and the $(m+p)^{th}$ ring, ($p \approx 10$) and take the difference in the squares of the radii, we get for dark rings

$$r_{m+p}^2 - r_m^2 = p\lambda R \tag{0.43}$$

$$\left(\frac{D_{m+p}}{2}\right)^2 - \left(\frac{D_m}{2}\right)^2 = p\lambda R \rightarrow \lambda = \frac{D_{m+p}^2 - D_m^2}{4pR} \tag{0.44}$$

THE INTERFERENCE PATTERN (YOUNGS INTERFERENCE EXPERIMENT)

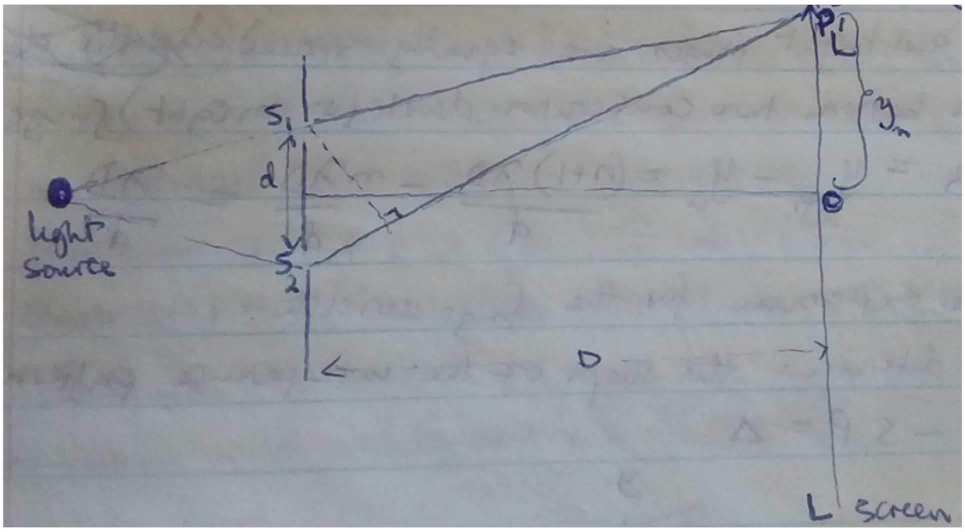

Figure 0.7

Let S_1 and S_2 represent the two pin holes of the Youngs interference experiment. We want to determine the position of the maxima or minima on the line LL'. LL' is parrallel the y-axis and lies in the same plane containing S, S_1 and S_2. For an arbitrary point P (on the line LL' to correspond to a maximum we must have $S_2P = S_1P = n\lambda$ where $n = 0, 1, 2, 3, ...$

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y_n - \frac{d}{2}\right)^2\right] \tag{0.45}$$

$$(S_2P)^2 - (S_1P)^2 = 2y_nd \tag{0.46}$$

where $S_1S_2 = d$ and $OP = y_n$.

$$S_2P - S_1P = \frac{2y_nd}{S_2P + S_1P} \tag{0.47}$$

If $y_n, d \leq D$, then negligible error will be introduced if $S_2P + S_1P$ is replaced by 2D.
Typical values of d are for example, $d = 0.02\ cm$, and $D = 50\ cm$. $OP = 0.5\ cm$, $2D{=}100cm$. Then $S_2P - S_1P \approx 100.005\ cm$
$\rightarrow S_2P - S_1P = \frac{2y_nd}{2D} = n\lambda$
Using $S_2P - S_1P = \frac{2y_nd}{2D} = n\lambda$, where $n = 0, 1, 2, 3.....$ we obtain

$$y_n = \frac{n\lambda D}{d} \tag{0.48}$$

The dark and bright fringes are equally spaced and the distance between two consecutive dark (or bright) fringes is given by

$$\beta = y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d} \tag{0.49}$$

which is the expression for fringe width.
In order to determine the shape of the interference pattern, we note that $S_2P - S_1P = \Delta$

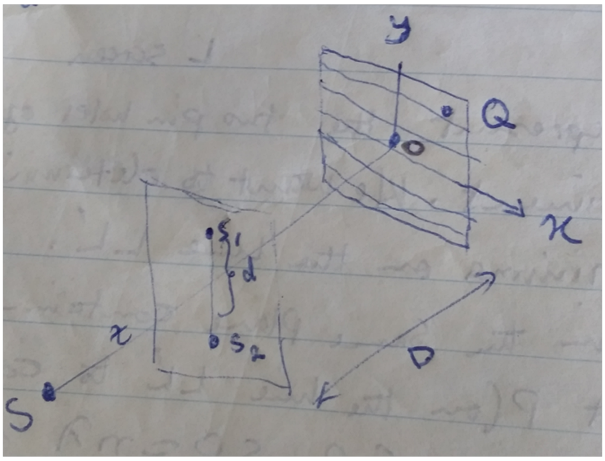

Figure 0.8

Assume y-axis is parallel to S_1S_2 for any arbitrary point Q on the screen(z=0).

With point O as the origin, the coordinates of Q is (x,y,0). The coordinates of the point S_1 and S_2 are $\left(0, \frac{d}{2}, D\right)$ and $\left(0, -\frac{d}{2}, D\right)$, respectively. Therefore,

$$S_2P - S_1P = \left[x^2 + \left(y + \frac{d}{2}\right)^2 + D^2\right] - \left[x^2 + \left(y - \frac{d}{2}\right)^2 + D^2\right] = \Delta \tag{0.50}$$

Solving for y

$$y = \pm\left(\frac{\Delta^2}{d^2 - \Delta^2}\right)^{\frac{1}{2}}\left[x^2 + D^2 + \frac{1}{4}(d^2 - \Delta^2)\right]^{\frac{1}{2}} \tag{0.51}$$

which is the equation of a hyperbola. Therefore, the shape of fringes ia hyperbolic.
For $x^2 \leq D^2$, the loci are straight lines parallel to the axis giving straight line fringes on the screen.

THE INTENSITY DISTRIBUTION

Let E_1 and E_2 be the electric fields produced at a point P by S_1 and S_2, respectively. The electric fields E_1 and E_2 will in general have different directions and different magnitudes. However, if S_1P and S_2P are very large in comparison to the distance S_1S_2, the fields will almost be in the same direction. Then, we can write

$$E_1 = iE_{01}cos\left(\frac{2\pi}{\lambda}S_1P - \omega t\right) \tag{0.52}$$

$$E_2 = iE_{02}cos\left(\frac{2\pi}{\lambda}S_2P - \omega t\right) \tag{0.53}$$

where i represents the unit vector along a direction of either of the electric fields.
The resultant field is

$$E = E_1 + E_2 = i\left[E_{01}cos\left(\frac{2\pi}{\lambda}S_1P - \omega t\right) + E_{02}cos\left(\frac{2\pi}{\lambda}S_2P - \omega t\right)\right] \tag{0.54}$$

The intensity will be proportional to the square of the electric field and will be given by

$$I = KE^2 \tag{0.55}$$

where K is a constant.

$$\begin{aligned}\therefore I = K[E_{01}^2 cos^2(\frac{2\pi}{\lambda}S_1P - \omega t) + E_{02}^2 cos^2(\frac{2\pi}{\lambda}S_2P - \omega t) + \\ E_{01}E_{02}cos(\frac{2\pi}{\lambda}(S_2P - S_2P) + cos(2\omega - \frac{2\pi}{\lambda}(S_2P + S_1P)))]\end{aligned} \tag{0.56}$$

For an optical beamthe frequency is very large($\omega \approx 10^{15}s$) and the terms depending on ωt will vary extremely rapdily(10^{15} times in a second). therefore a detector will only record an average value of quantities.
The average value of A is written as

$$\langle A \rangle = \frac{1}{2t}\int_{t_1}^{t_n} f(A)dt = \frac{1}{2\tau}\int_{-\tau}^{\tau} f(A)dt \tag{0.57}$$

Using the idea we write

$$\left\langle cos^2(\omega t - \theta)\right\rangle = \frac{1}{2\tau}\int_{-\tau}^{\tau}\frac{1 + cos2(\omega t - \theta)}{2}dt = \frac{1}{2} + \frac{1}{16\pi}\frac{T}{\tau}[sin2(\omega t - \theta)]_{-\tau}^{\tau} \tag{0.58}$$

where $T = \frac{2\pi}{\omega} \approx 2\pi \times 10^{-15}$ for an optical beam.
For any practical detector $\frac{T}{\tau} << 1$

$$\therefore \left\langle cos^2(\omega t - \theta)\right\rangle \approx \frac{1}{2} \tag{0.59}$$

The intensity that the detector will record will be given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}cos\delta \tag{0.60}$$

where $\delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$ represents the phase difference between displacements reaching the point P.
Further $I_1 = \frac{1}{2}KE_{01}^2$ .... intensity from S_1
$I_2 = \frac{1}{2}KE_{02}^2$ .... intensity from S_2
The maximum and minimum of I are given by

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 \tag{0.61}$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 \tag{0.62}$$

The maximum intensity occurs when $\delta = 2n\pi$, n = 0, 1, 2,..... or $S_2P - S_1P = n\lambda$.
For minimum intensity, $\delta = (2n+1)\pi$, n = 0, 1, 2,.....

ANOTHER APPROACH TO YOUNGS INTERFERENCE EXPERIMENT

Two waves having traversed different paths S_2P and S_1P arrive at P. They are superimposed with a phase difference of $\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(S_2P - S_1P)$. The waves are assumed to start at S_1 and S_2 in the same phase since the slits are equidistant from S. If the waves are sinusoidal with same frequency and amplitude but a phase difference δ, then the intensity

$$I^2 \approx A^2 = 4a^2cos^2\delta \tag{0.63}$$

where a is the amplitude of the separate waves and A is that of their resultant.

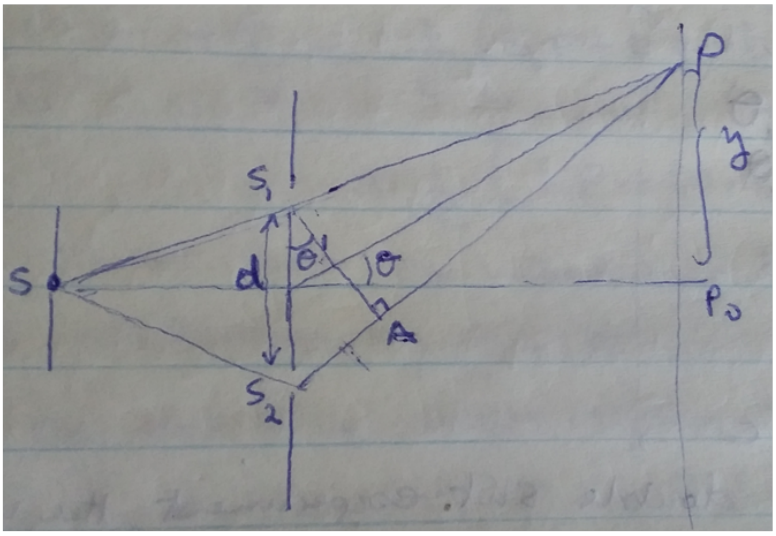

Figure 0.9

In the diagram above, the path difference is S_2A. In Youngs experiment$D >> d$ or y. Hence θ' and θ are very small and practically equal to each other. Under these conditions S_1AS_2 may be regarded as a right angled triangle and the path difference becomes $dsin\theta' = dsin\theta$.
To the same approximation, we may get $sin\theta \approx \theta = \frac{y}{D} = tan\theta$.
$\Delta = dsin\theta = \frac{dy}{D}$

$\frac{dy}{D} = (m + \frac{1}{2})\lambda$ for constructive interference and $\frac{dy}{D} = m + \lambda$ for distructive interference.
Δ is directly proportional to d and y and inversely proportional to D.

FRESNEL BIPRISM

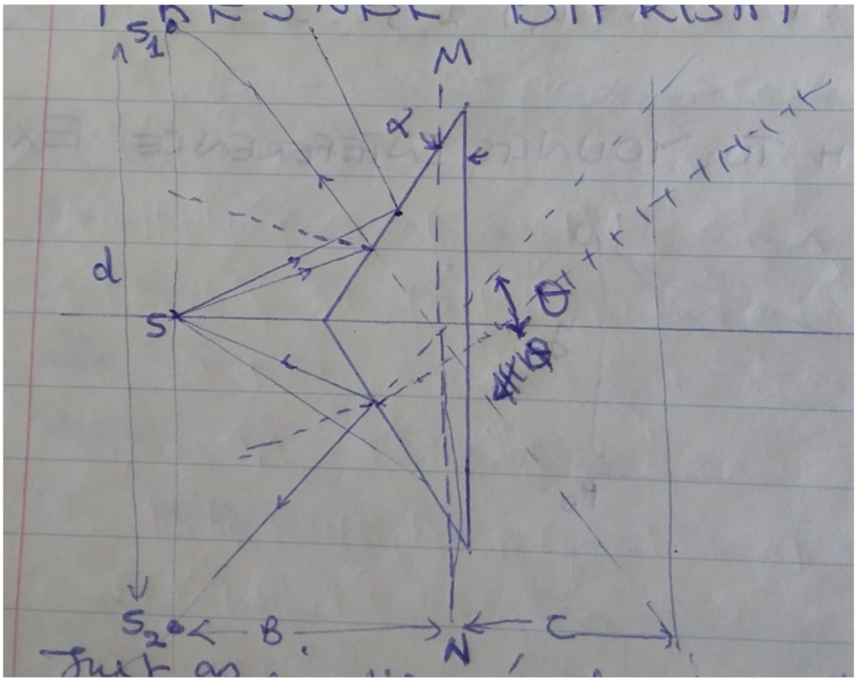

Figure 0.10

Just as in Youngs double slit experiment, the wavelength of light can be determined from the measurement of interference fringes from the biprism. The distance between successive bright fringes on the screen will be given by $\Delta y = m\lambda\frac{d}{D}$. The wavelength of the light is given by $\lambda = \frac{yd}{B+C} = \frac{yd}{D}$, because $B + C = D$

DISPLACEMENT OF FRINGES

Let t be the thickness of the plate and let n be its refractive index. The light reaching point P from S_1 has to traverse a distance t in the plate and a distance $S_1P - t$ in air.
Time taken by light from S_1 to reach P is given by

$$\frac{S_1P - t}{c} + \frac{t}{v} = \frac{S_1P}{c} - \frac{t}{c} + \frac{t}{v}. \tag{0.64}$$

But $\frac{c}{n} = v$, hence

$$\frac{S_1P - t}{c} + \frac{t}{v} = \frac{S_1P}{c} - \frac{t}{c} + \frac{nt}{c}. \tag{0.65}$$

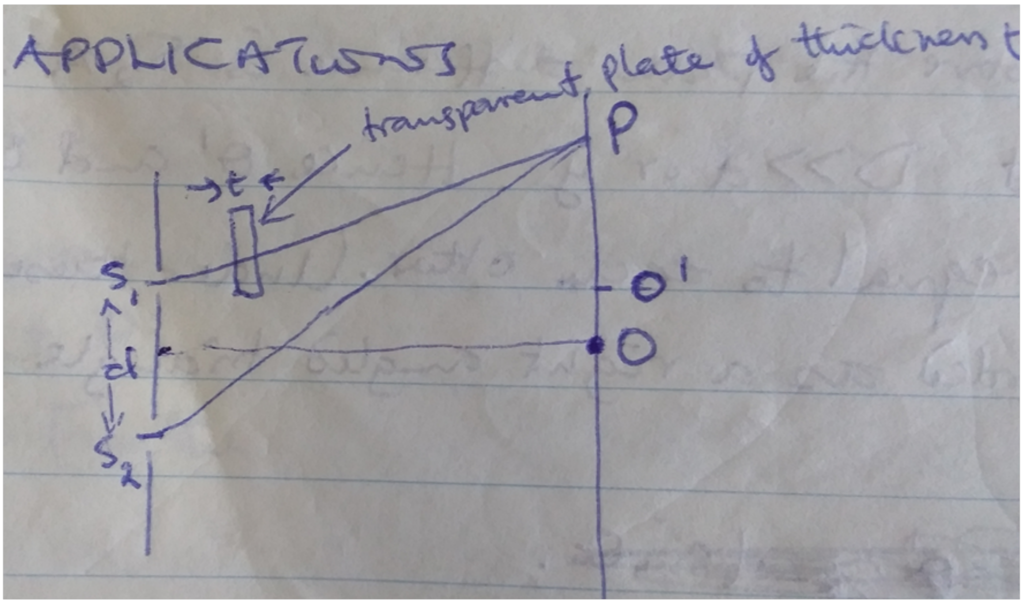

Figure 0.11

$$\frac{S_1P - t}{c} + \frac{t}{v} = \frac{1}{c}[S_1P - t + nt] = \frac{1}{c}[S_1P + (n-1)t]. \tag{0.66}$$

This shows that by introducing the plate, the effective optical path decreases by $(n-1)t$.
The central fringe which corresponds to equal optical paths from S_1 and S_2 is formed at point $O^{'}$.

$$S_1O^{'} + (n-1)t = S_2O^{'} \tag{0.67}$$

Without the thin plate we know that $S_2P - S_1P \approx \frac{y_n d}{D}$
Here we have $S_2O^{'} - S_1O^{'} \approx \frac{d}{D}OO^{'}$. Therefore, $(n-1)t = \frac{d}{D}OO^{'}$. This means that the fringe pattern is shifted by $OO^{'} = \Delta = \frac{D(n-1)t)t}{d}$.

ADDITION OF SIMPLE HARMONIC MOTIONS AT RIGHT ANGLES- POLARISATION

We consider the effect of adding two sine waves of the same frequency but having displacements in the two perpendicular directions and acting simultaneously at a point. Choosing the directions as y and z we may express the to component motions as

$$y = a_1 sin(\omega t - \alpha_1) \tag{0.68}$$

and

$$z = a_2 sin(\omega t - \alpha_2) \tag{0.69}$$

We add these equations according to the principle of superposition to find the path of the resultant motion. This is done by eliminating t from these equations. We find

$$\frac{y}{a_1} = sin\omega t cos\alpha_1 - cos\omega t sin\alpha_1 \tag{0.70}$$

and

$$\frac{z}{a_2} = sin\omega t cos\alpha_2 - cos\omega t sin\alpha_2 \tag{0.71}$$

Multiplying equation 0.70 by $sin\alpha_2$ and equation 0.71 by $sin\alpha_1$ and subtracting the first equation from the second we have

$$-\frac{y}{a_1} sin\alpha_2 + \frac{z}{a_2} sin\alpha_1 = sin\omega t(cos\alpha_2 sin\alpha_1 - cos\alpha_1 sin\alpha_2 \tag{0.72}$$

Similarly, multiplying equation 0.70 by $cos\alpha_2$ and equation 0.71 by $cos\alpha_1$ and subtracting the second equation from the first we have

$$\frac{y}{a_1} cos\alpha_2 + \frac{z}{a_2} cos\alpha_1 = cos\omega t(cos\alpha_2 sin\alpha_1 - cos\alpha_1 sin\alpha_2 \tag{0.73}$$

We now eliminate t from equations **??** and **??** by squaring and adding the two equations. The result is

$$sin^2(\alpha_1 - \alpha_2) = \frac{y^2}{a_1^2} + \frac{z^2}{a_2^2} - \frac{2yz}{a_1 a_2} cos(\alpha_1 - \alpha_2) \tag{0.74}$$

This is the equation of the resultant path. The phase difference $\delta = \alpha_1 - \alpha_2$. If $\delta = 0$, we have

$$\frac{y^2}{a_1^2} + \frac{z^2}{a_2^2} - \frac{2yz}{a_1 a_2} = 0 \tag{0.75}$$

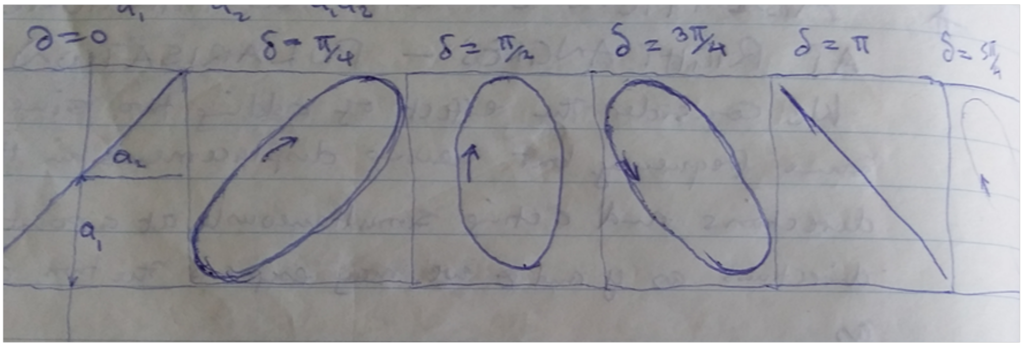

Figure 0.12

Except for special cases where they degenerate into straight lines, the principle axis of the ellipse are in general inclined to the y and z axis but coincide with them when $\delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ t etc.
In these axes

$$\frac{y^2}{a_1^2} + \frac{z^2}{a_2^2} = 1 \tag{0.76}$$

which is the equation of the ellipse with semi axis a_1 and a_2 coinciding with y and z axes respectively.
When $\delta = 0, 2\pi, 4\pi.....$, we have $y = \frac{a_1 z}{a_2}$, representing a straight line passing through the origin with the slope of $\frac{a_1}{a_2}$.
If $\delta = \pi, 3\pi, 5\pi.....$, we have $y = \frac{-a_1 z}{a_2}$, representing a straight line passing through the origin with the slope of $\frac{-a_1}{a_2}$.

APPLICATIONS OF POLARISATION

- Photography
- Spectacles- correct eye problems
- X-ray photography
- Non destuctive testing
- Radio communication
- Night vision technology
- Radar - Remote sensing

- Mineral and oil prospecting
- Medical applications - Lasers
- Holography
- Missible guidance using laser

Light can be produced for which the form of vibration is an ellipse of any definite essentricity. The so called polarised light approximates a sine wave lying in a plane say x-y plane.

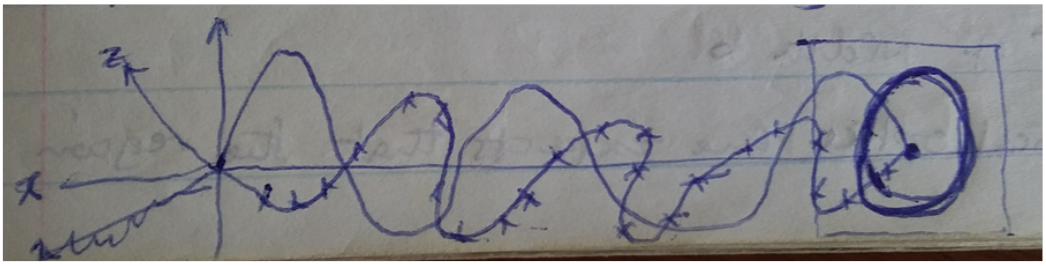

Figure 0.13

The displacements are linear in the y-direction.
If this light is combined with another plane polarised wave lying in the z-x plane (the dotted), and having a constant phase difference with the first, the resultant motion at any value of x will be an ellipse in the y-z plane. Such light is said to be elliptically polarised. when the amplitudes a_1 and a_2 are equal, the two waves are equal and the phase difference is an odd multiple of $\frac{\pi}{2}$. The vibrations form a circle and the light is said to be circularly polarised. When the directions of rotation is clockwise ($\delta = \frac{\pi}{2}, \frac{5\pi}{2}, ...$) looking to the direction in which light is travelling, the light is called RIGHT circlarly polarised. While if the rotation is counterclockwise ($\delta = \frac{3\pi}{2}, \frac{7\pi}{2}, ...$), it is called LEFT circularly polarised wave.

DIFFRACTION

If we look at the shadow cast by an opaque object, we would find that it is very complicated. The shadow would consist of dark and bright regions which are not expected from everyday geometrical optics. This is known as diffraction, and was first shown in 1600's to be a general characteristic of wave phenomenon which occurs whenever a portion of a wavefront is obstructed in someway. In particular, if a wave encounters an obstacle, the diffraction occurs when a region of the wave front is altered in amplitude and phase.

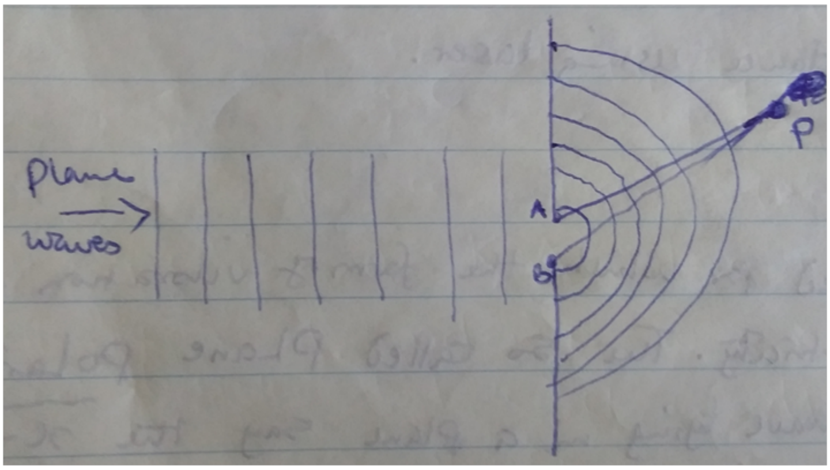

Figure 0.14

Consider narrow slit width b.

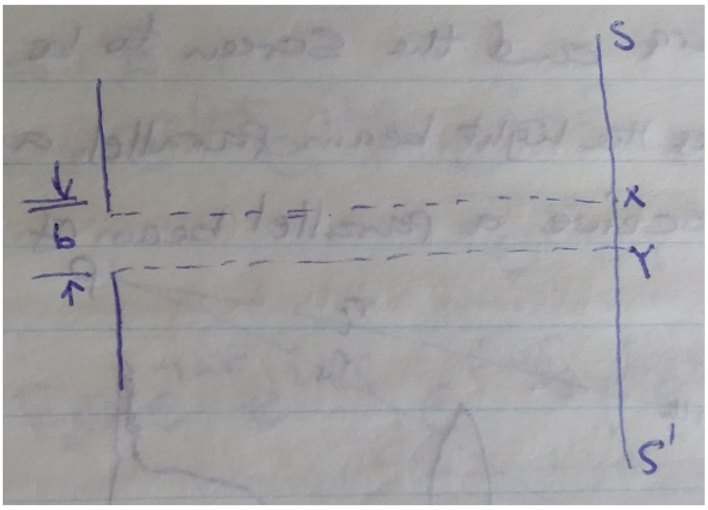

Figure 0.15

According to geometrical optics, one expects the region XY on SS$'$ to be illuminated and the remaining portion (known as geometrical shadow) to be absolutely dark. This spreading out of light is called diffraction. The spreading out decreases with the decrease in wavelength. Since the light wavelengths are very small, ($\lambda \approx 5 \times 10^{-5} cm$) the effect due to diffraction are not readily observed.

There is no much difference between interference phenomenon and diffraction. Interference corresponds to the superposition of waves coming out from a number of point (or line) sources and diffraction corresponds to the situation when we consider waves coming out from an area source like circular or rectangular arpeture or even a number of rectangular arpertures (like diffraction grating).
Diffraction phenomena are usually divided into two categories

- Fresnel diffraction
- Frauhofer diffraction

In Fresnel diffraction, the source of light and thje screen are at finite distance from the diffracting aperture

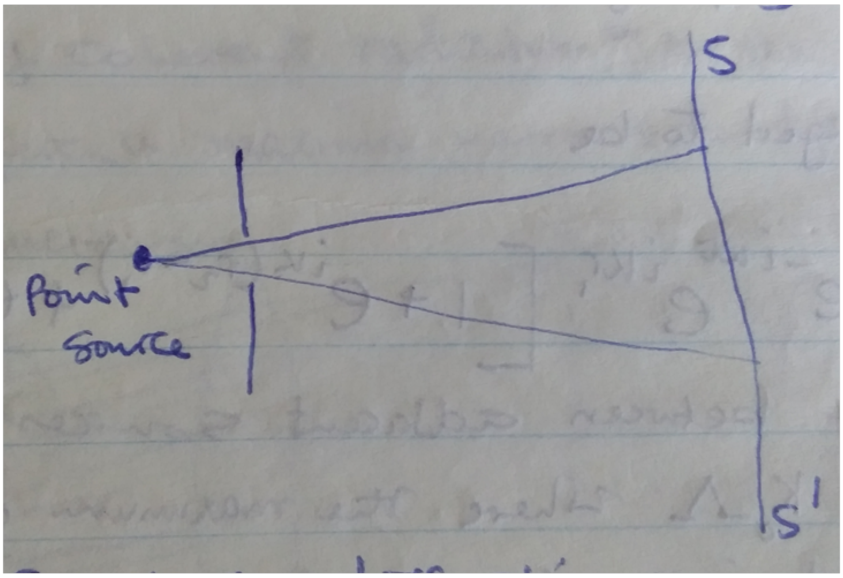

Figure 0.16

In Fraunhofer diffraction, the source and screen are at finite distances from the arpeture, this is achieved by placing the source on the focal plane of a convex lens and placing the screen on the focal plane of another conex lens. The two lenses effectively make the light source and the screen to be at infinity because the first lens makes the light beam parallel and the second lens make the screen receive a parallel beam of light.

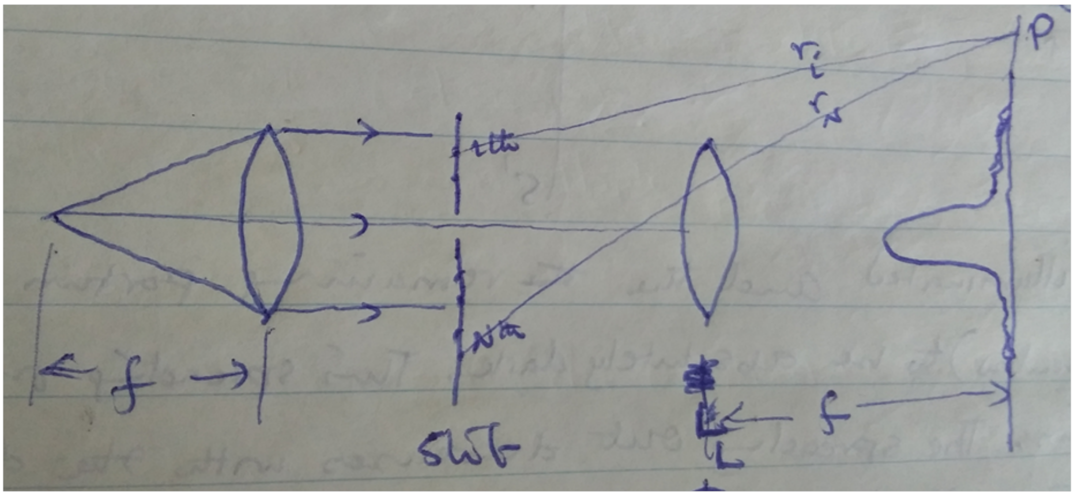

Figure 0.17

Coherent Oscillators

Consider an arrangement made up of a linear array f N coherent oscillators which are identical. Assume that the oscillators have no intrinsic phase difference, then the rays emmitted by the oscillators will be practically parallel. The spartial extent of the oscillator array is small compared to the wavelength of the radiation then the amplitude of the separate waves arriving at some observation point P will be essentially equal. So $E_o(r_1) = E_o(r_2) = E_o(r_3) = ... = E_o(r_N) = E_o(r)$, where r_i is the distance from the i^{th} oscillator to the obsevation point P. The sum of interfering wavelets yield a composite (superposition) electric field at P that is the real part

$$E = E_o(r)e^{i(kr_1-\omega t)} + E_o(r)e^{i(kr_2-\omega t)} + ... + E_o(r)e^{i(kr_N-\omega t)} \qquad (0.77)$$

This can be rearranged to be

$$E = E_o(r)e^{-i\omega t}e^{ikr_1}[1 + e^{ik(r_2-r_1)} + e^{ik(r_3-r_1)} + ... + e^{ik(r_N-r_1)}] \qquad (0.78)$$

The phase difference between adjacent sources is obtained from the expression $\delta = k_o\Lambda$ where the maximum optical path length difference is $\Lambda = ndsin\theta$ in a medium with refractive index n. But since d is the distance between adjacent oscillators, therefore $dsin\theta = r_2 - r_1$ so that the field at P becomes

$$E = E_o(r)e^{-i\omega t}e^{ikr_1}[1 + e^{i\delta} + e^{(i\delta)^2} + e^{(i\delta)^3} + ... + e^{(i\delta)^{N-1}}] \qquad (0.79)$$

This simplifies to

$$E = E_o(r)e^{-i\omega t}e^{ikr_1}\frac{e^{iN\delta}-1}{e^{i\delta}-1} = E_o(r)e^{-i\omega t}e^{ikr_1}\frac{e^{iN\frac{\delta}{2}}(e^{iN\frac{\delta}{2}}-e^{-iN\frac{\delta}{2}})}{e^{i\frac{\delta}{2}}(e^{i\frac{\delta}{2}}-e^{-i\frac{\delta}{2}})} \tag{0.80}$$

$$\rightarrow E = E_o(r)e^{-i\omega t}e^{ikr_1}e^{i(N-1)\frac{\delta}{2}}\frac{sin(N\frac{\delta}{2})}{sin(\frac{\delta}{2})} \tag{0.81}$$

If $R = r_1 + \frac{1}{2}(N-1)dsin\theta$ is the distance from the center of the line of oscillators to P. Once we know E, the flux density can be determined using the field

$$I = I_o\frac{sin^2(N\frac{\delta}{2})}{sin^2(\frac{\delta}{2})} \tag{0.82}$$

DIFFRACTION GRATING

Any periodic arrangement of diffracting bodies may be called a diffraction grating.

Gratings usually consists of equdistant diamond rulings on a plate or mirror. The grating elements are narrow slits which act as sources of coherent radiation. As the number of slits are increased, the interference maxima and sharpness of the principal maxima increases.

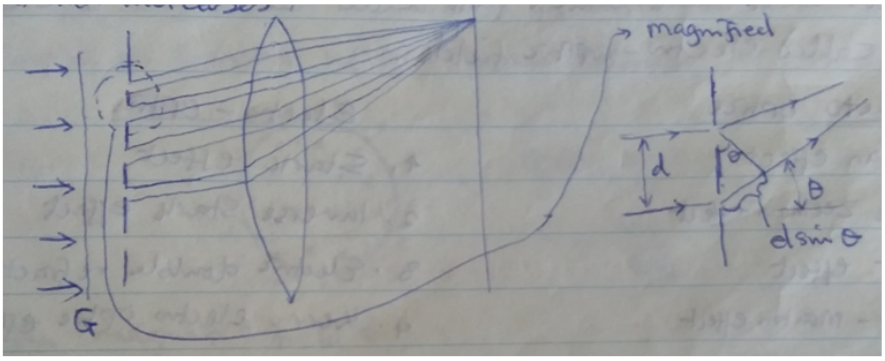

Figure 0.18

Slits of the grating consists N slits a distance d apart will interfere at point P. The phase difference at P between the two disturbances from adjacent slits is given by $\delta = \frac{2\pi}{\lambda}(dsin\theta)$.

When this is a multiple of 2π, there is constructive interference at P. maxima.

Therefore $\delta = \frac{2\pi}{\lambda}(dsin\theta)$ must be $m2\pi$ where m is an integer. OR $dsin\theta = m\lambda$ for maxima.
Only for these values of θ will disturbances from all slits be exactly in phase at P. For these values of θ we obtain principal maxima. Our equation $dsin\theta = m\lambda$ does not contain N and for a given slit separation, the positions of principal maxima are independent of the number of slits in the grating.

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CHAPTER 3: MAGNTO OPTICS AND ELECTRO OPTICS

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In further support of electromagnetic character of light, there is a group of experiments which demonstrate the interaction between light and matter when the matter is subjected to strong external magnetic or electric field. Those which fall under the action of magnetic field are called magneto optic effects and those which fall under the action of electric fields are called electro-optic fields.
Magneto optics include

- Zeeman effect
- Inverse Zeeman effect
- Voigt effect
- Cotton-Moaton effect
- Faraday effect, and
- Kerr-Magneto optic effect.

Whiie **Electro optics** include

- Stark effect
- Inverse Stark effect
- electric double refraction, and
- Kerr- electro optic effect.

ZEEMAN EFFECT

In 1892, Zeeman discovered that when sodium flame S is placed between the poles of a strong magnet, two yellow lines are considerably broadened.

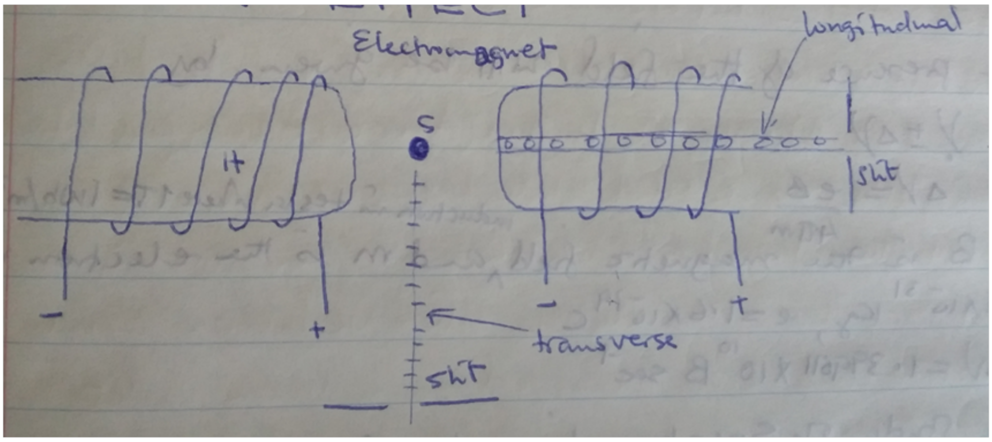

Figure 0.19

Later on Lorentz presented a simple theory for the observation based upon the electro theory of matter. Lorentz predicted that each spectrum line when produced in such a field should split into two components when viewed parallel to the field and into three components when viewed perpendicular to the field.

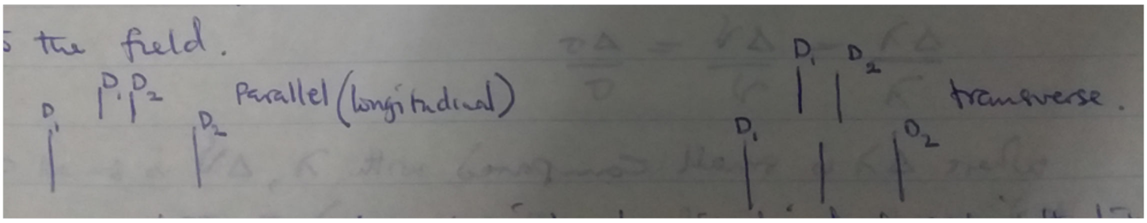

Figure 0.20

Lorenz also predicted that in the longitudinal direction, the lines should be circularly polarised and in the transverse direction, plane polarised. Lorentz assumed the electrons in matter are responsible for the origin of light waves and they are charged particles whose motion are modified by external magnetic field.

In the special case, of an electron moving in a circular orbit, the plane of which is normal to the direction B, the electron should be speeded up or slowed down by an amount proportional to the magnetc field B.

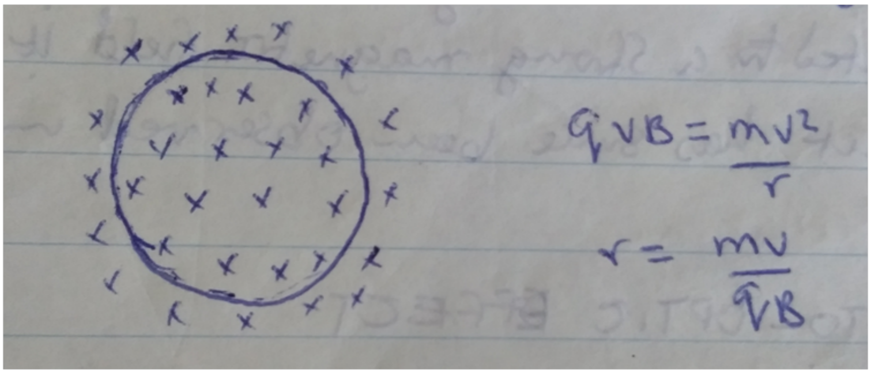

Figure 0.21

If the problem is treated classically, then the frequency in the presence of the field will be given by $\nu \pm \Delta\nu$, where $\Delta\nu = \frac{eB}{4\pi m}$, where B is the magnetic field induction in teslas ($1T = 1Wb/m^2 = 10\ 000$ Gauss) and m is the electron mass

In the study of spectrum lines the frequency difference ν is most conviniently in wave numbers by dividing the frequency by the speed of light in cm/s.

Wave number $\Delta\sigma = \frac{\Delta\nu}{c} = 0.46686B\ cm^{-1}$.

From this useful relationship between wavelength and frequency in hertz wave number follows from the equation $c = \nu\lambda$.

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu} = \frac{\Delta\sigma}{\sigma} \tag{0.83}$$

where $\Delta\lambda$ is small compared to λ, $\Delta\nu$ is small compared to ν and $\Delta\sigma$ is also small compared to σ

INVERSE ZEEMAN EFFECT

The Zeeman effect obtained in absorption is called the Inverse Zeeman effect. This phenomenon is observed by sending white light through an absorbing vapour when the later is subjected to a uniform magnetic field.

FARADAY EFFECT

In 1845, Michael Farady discovered that when a block of glass is subjected to a strong magnetic field it becomes optically active. This effect has since been observed in many solids, liquids and gases.

KERR MAGNETO-OPTIC EFFECT

In 1888, Kerr made the discovery that when plane polarised light is reflected at normal incidence on a polished pole of an electromagnet, it becomes elliptically polarised to a slight degree with the major axis of ellipse rotatd with respect to the incident vibrations.

STARK EFFECT

In 1913, Stark observed that when hydrogen spectrum is excited in a strong electric field of 100KV/cm, each line splits into a symmetrical pattern.

LASERS

The light emmitted from a conventional light source (like sodium lamp) is said to be incoherent because the radiation emmitted from different atoms do not in general , bear any definite relationship with each other.

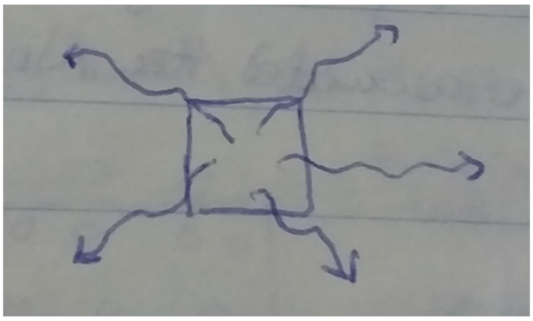

Figure 0.22

On the other hand, the light emitted from a laser has a very high degree of coherence.

The name **LASER** is an acronym of **L**ight **A**mplification by **S**timulated **E**mmission of **R**adiation. A laser beam is so parallel that 10 cm beam radius from a laser source, on the surface of the moon (384,000 km) the width of the laser beam is not more than 5 km.

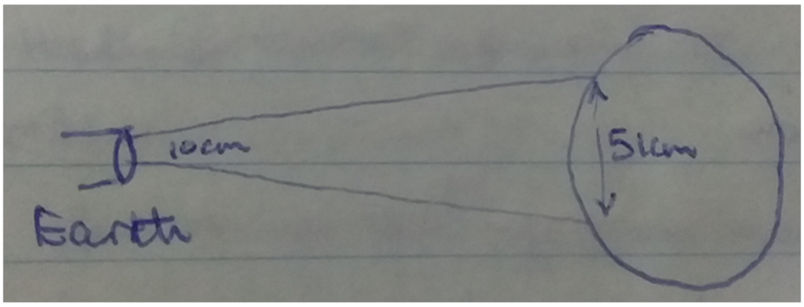

Figure 0.23

Historically, the laser is the outgrowth of MASER, a similar device using radio microwaves instead of visible light waves. Maser stands for

Microwave Amplification by Stimulated Emmission of Radiation. The first Maser was built by C.H. Towness and his associates at Columbia University between 1951 and 1954.
In 1958 A.H. Schalow and C.H. Towness set forth the principle of optical masers or lasers. The first laser was built by T.H. Maiman at Hughes Aircraft company laboratories in 1960. This was a solid state laser using ruby crystal. Within a few months, Jarven and his associates constructed and operated the first gas laser namely He-Ne laser. Since then, laser action has been observed in a large variety of minerals, gases, dyes, semiconductors and so forth.
The three main components of any laser device are

- active medium
- the pumping source, and
- the optical resonator

The active media consist of a collection of molecules, atoms or ions (in solid, liquid, gas) which is capable of amplifying light.
Towness, Basov and Prochorov were awarded the Nobel prize in Physics in 1964 for their invention of the laser.

STIMULATED EMMISSION

There are atleast 10 basic principles involved in the operation of most lasers.

- The active media must have a metastable state
- The media must be optically pumped
- The media must be able to cause flourescence
- Must provide population inversion
- Resonance
- stimulated emmission
- coherence
- Use Fabry-Perot interferometer

- Polarisation
- Cavity oscillation

Stimulated emmission was predicted by Albert Einstein in 1917. Einstein argued that when an atom is in an excited state, it can make a transition to a lower energy state through emmission of electromagnetic radiation, however in contrast to the absorption process, the emmission can occur in two different ways.
1.

- The first is referred to as spontaneous emission in which an atom in the excited state emits radiation even in the absence of incident radiation. This transion is not stimulated by an incident signal but occurs spontaneously. The rate of sponteneous emmission is proportional to the number of atoms in the excited state.
- Stimulated emmission. In stimulated emmission, an incident signal of appropriate frequency trigger in the excited state to emit radiation. The rate of stimulated emission depends on the number of atoms both in the upper and lower states. Consider a gas enclosed in a container and the gas has free atoms having a number of energy levels, at least one of which is METASTABLE.

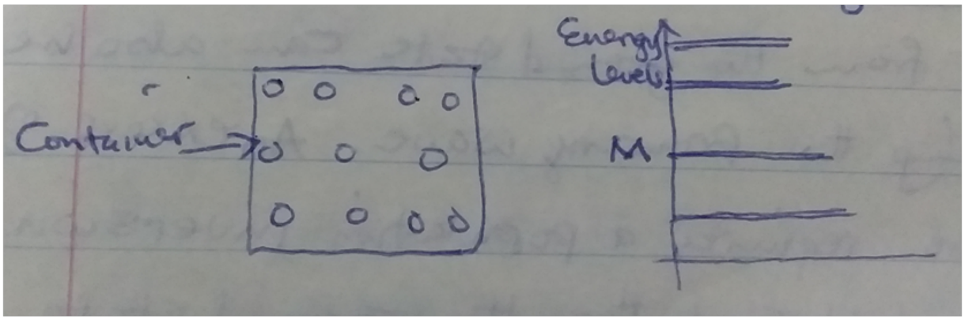

Figure 0.24

By shinning white light into this gas many atoms are raised through resonance from ground state to excited state. As the electrons drop back many of them become trapped in the METASTABLE state. If the pumping light is intense enough we may obtain population inversion, that is more atoms in the metastable state than in the ground state.

When an electron in one of these metastable states spontaneously jumps to the ground state as it eventually falls, it emits a photon (light) of energy $h\nu$. This process is called flourescence or Phosphorescent radiation. As the photon passes by another nearby atom in the same metastable state, it can by principle of resonance imediately stimulate that atom to radiate a photon of the exactly same frequency and return to the ground state. Amazing enough this stimulated photon has the same frequency, direction and polarisation as the primary photon (This is called Spartial coherence) and has exactly the same phase and speed (temporal coherence).

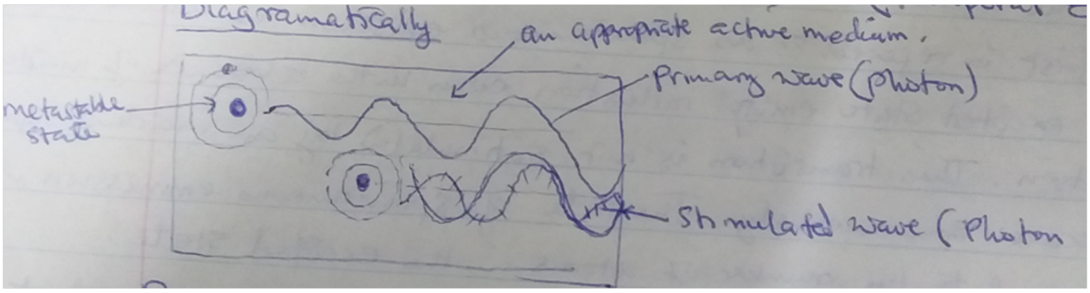

Figure 0.25: *Principle of stimulated emission of light from an atom.*

Both waves have the same wavelength λ and the same phase (vibrating in parallel planes. Both photons may now be considered primary waves, and upon passing close to other atoms in their metastable state, they may stimulate them to emit in the same direction and phase.
However,the transition from the ground state can also be stimulated there by absorbing the primary wave. An excess of stimulated emission therefore requires a population inversion-that is more atoms in the metastable state than the ground state.
We require to know how a laser is designed.
In order to produce a laser, one must collimate the stimulated emsission and this is done by designing a CAVITY in which the waves can be used over and over again - the principles of Fabry-Perot interferometer (similar to Michelson interferometer). The ends of the cavity has mirrors of high reflectivity. The mirrors have to be parallel and both are half silvered.

RUBY LASER

The first laser developed by Maiman in 1960 used a single crstal of synthetic pink ruby as its active medium. The ruby is primarily a transparent crystal of Al_2O_3 doped with approximately o.o5% of trivalent chromium ions in the form Cr_2O_3. Al_2O_3 is the host and Cr_2O_3 contain the activator atoms

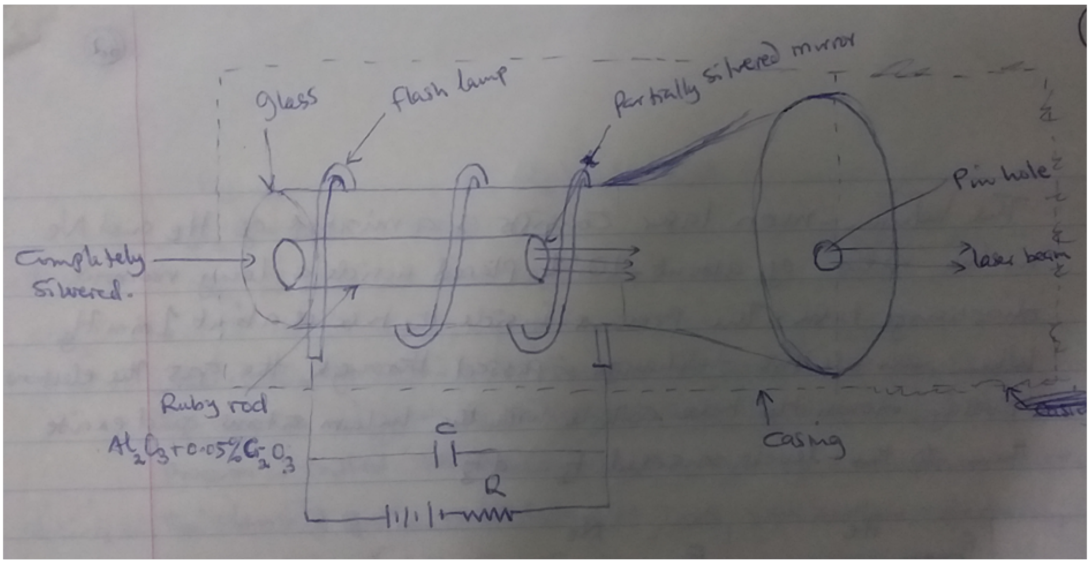

Figure 0.26

The energy states of chromium ions are shown below. E_1 and E_2 are energy bands.

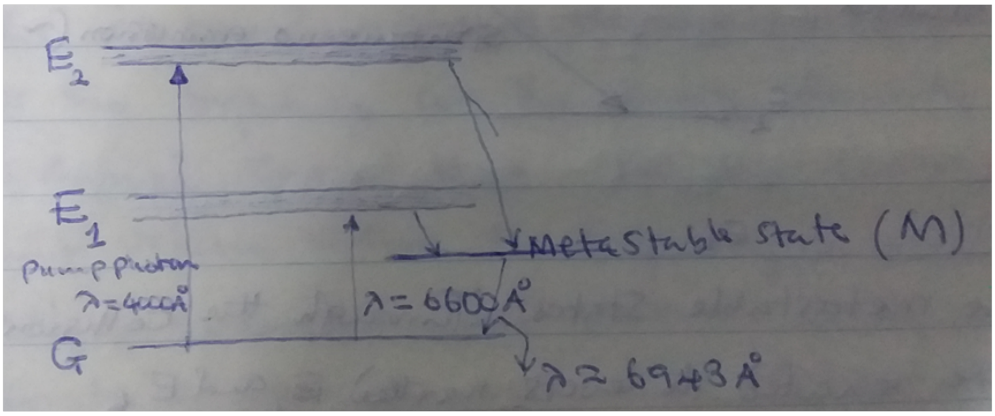

Figure 0.27

The chief characteristic of energy levels of chromium ion is the fact that the bands labelled E_1 and E_2 have a life time of $\leq 10^{-6}$ seconds, while the marked M state has a life time of $\approx 3 \times 10^{-3}$ seconds.
The chromium atom in its ground state can absorb a photon (whose wavelength is around 6600 Å) and make a transition to one of the states in the band E_1. It could also absorb a photon of $\lambda \approx 4000$ and make a

transion to one of the states E_2. In either case, it imediately makes a non radiative transion (in a time $\leq 10^{-8}$ seconds) to the metastable state M.

THE He-Ne LASER

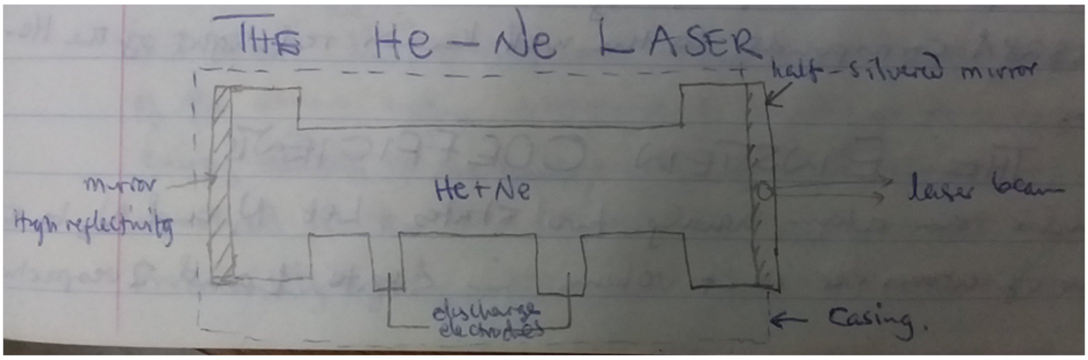

Figure 0.28

The helium -neon laser consists of a mixture of He and Ne in the ration of about 19:1 placed inside a long narrow discharge tube. The pressure inside the tube is about 1mm Hg. When an electric field is passed through the gas, the electrons travelling down the tube collide with helium atoms and excite them to the levels marked F_2 and F_3

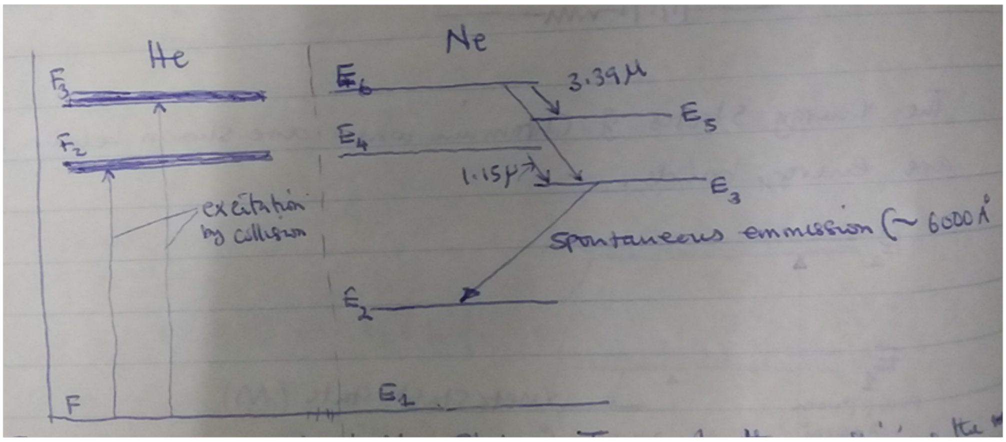

Figure 0.29

F_2 and F_3 are metastable states. Trough the collisions the Ne-atoms are excited to levels marked E_4 and E_6.
F_2 and F_3 have almost the same energy level as E_4 and E_6, respectively. Trough these collisions, there will be a sizeable population of the levels E_4 and E_6. The population of these levels happen to be much more than those in the lower levels E_3 and E_5.

The Ne-atoms can drop down to lower levels E_2 through spontaneous emission. From level E_2 the Ne atoms are brought back to the ground state E_1 through collisions with the walls of the cavity. The transion from E_6 to E_5, E_4 to E_3, and E_6 to E_3 result in the emission of radiation having wavelength 3.39 μm, 1.5 μm and 6328 Å., respectively. The 6328 Åcorresponds to the well known He-Ne Laser.

THE EINSTEIN COEFFICIENTS

Consider an atom having two states. Let N_1 and N_2 be the number of atoms per unit volume in state 1 and 2, respectively.

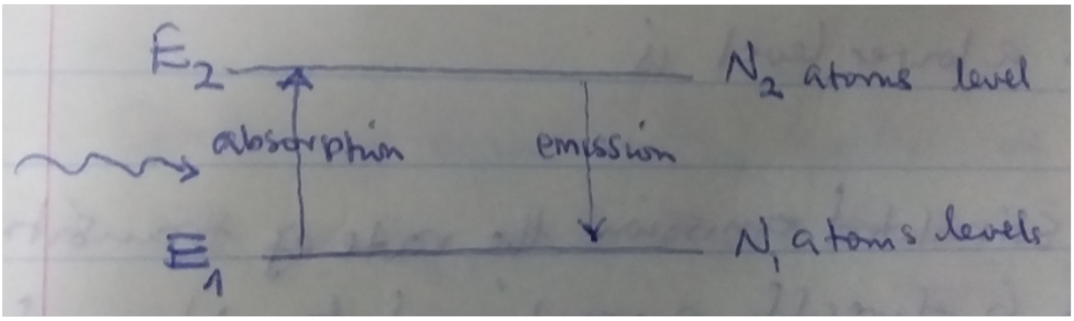

Figure 0.30

The levels correspond to energies E_1 and E_2. The atom in level E_1 can be excited by radiation to E_2 by absorbing radiation. The process is called **absorption**. The rate of absorption depends on the density of radiation at a particular frequency which corresponds to $\left(\omega = \frac{E_2 - E_1}{\hbar}\right)$, where $\hbar = \frac{h}{2\pi}$ and h is the plancks constant.

The absorption process depends on the energy density of the radiation at the frequency ω. The energy density is denoted by $U(\omega)$ and is defined such that $U(\omega)d\omega$ represents the radiation per unit volume within the frequency interval ω and $\omega + d\omega$. The rate of absorption does not depend on N_2 but on N_1 and $U(\omega)$. In other words, the rate of absorption is proportional to N_1 and also to $U(\omega)$. The number of absorption per unit volume can be written as $N_1 B_{12} U(\omega)$ = number of absorptions per unit volume per unit time, where B_{12} is the coefficient of proportionality and is a characteristic of energy levels.

Consider the reverse process, namely the emission of radiation at a frequency ω, when the atom de-excites from E_2 to E_1 level. An atom in the excited state can make radiative transition to a lower energy state either by spontaneous emission or stimulated emission.

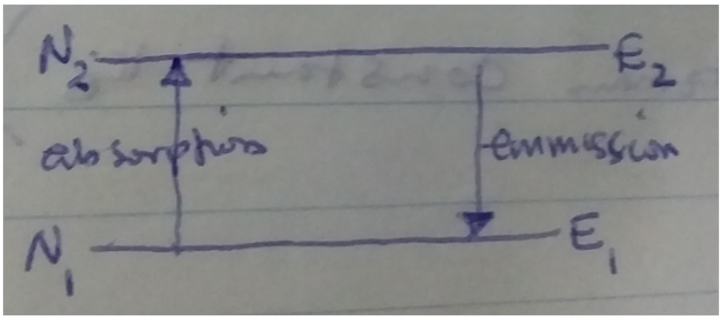

Figure 0.31

In spontaneous emission, the probability per unit time of the atom making downward transition is independent of the energy density of the radiation field and depends only on the levels involved in the transition. If we represent the coefficient of probability by A_{21} then the rate of emission per unit volume per unit time to a lower level is N_2A_{21}.
In the case of stimulated emission, the rate of transition to the lower energy levels is directly proportional to N_2 as wellas the energy density at the frequency ω. The rate of stimulated emission would be given by $N_2B_{21}U(\omega)$.
The quantities A_{21}, B_{12} and B_{21} are known as Einstein coefficients, and are deter´mined by the system. at thermal equilibrium, the number of upward transitions must be equal to the number of of downward transitions. This is expressed as

$$N_1B_{12}U(\omega) = N_2A_{21} + N_2B_{21}U(\omega) \tag{0.84}$$

or

$$N_1B_{12}U(\omega) - N_2B_{21}U(\omega) = N_2A_{21} \rightarrow U(\omega) = \frac{N_2A_{21}}{N_1B_{12} - N_2B_{21}} = \frac{A_{21}}{\frac{N_1}{N_2}B_{12} - B_{21}} \tag{0.85}$$

According to Boltzman's law, we have the following expression for the ratio of the population of the two levels at temperature T.

$$\frac{N_1}{N_2} = exp\left(\frac{E_2 - E_1}{k_BT}\right) = exp\left(\frac{\hbar\omega}{k_BT}\right) \tag{0.86}$$

where $k_B = 1.38\ \times 10^{-23}$J/kg is the Boltzmann constant.

We may therefore write $U(\omega)$ as

$$U(\omega) = \frac{A_{21}}{B_{12} exp\left(\frac{\hbar\omega}{k_B T}\right) - B_{21}} \tag{0.87}$$

According to Planck's law, the energy density of radiation is given by

$$U(\omega) = \frac{\hbar\omega^3 n_o^3}{\pi^2 c^3} \frac{1}{exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} \tag{0.88}$$

where n_o represents the refractive index of the medium.
Comparing equation 0.87 and 0.88, we obtain $B_{12} = B_{21} = B$(say) and $\frac{A_{21}}{B_{21}} = \frac{\hbar\omega^3 n_o^3}{\pi^2 c^3}$. We note that if we had not assumed the presence of stimulated emission we would not have been able to arrive at the expression for $U(\omega)$ which is similar to Planck's law.
It may also be noted that at thermal equilibrium the ratio of the number of spontaneous to stimulated emissions is given by
$\frac{A_{21}}{B_{12}U(\omega)} = exp\left(\frac{\hbar\omega}{k_B T}\right) - 1$.
For normal optical sources, $T = 10^3$ K with $\omega \approx 3 \times 10^{15} s^{-1}$, corresponding to $\lambda = 6000$. Then $\frac{\hbar\omega}{k_B T}$ ≈23, giving $\frac{A_{21}}{B_{12}U(\omega)} \approx 10^{10}$.

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END OF PHY 2712

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