THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
Department of Physics
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PHY 2231 - Properties of Matter and Thermodynamics Notes

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CHAPTER 4: HEAT TRANSFER

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There are three modes of heat transfer. These are conduction, convection and radiation. Conduction is the transfer of heat between substabces that are in direct contact with each other. The better the conductor, the more rapidly heat will be transferred. Metals are good conductors of heat. Convection is the transfer of heat in fluids (liquids and gases). Radiation on the other hand is the mode of heat transfer that does not necessarily require a material media.

THERMAL CONDUCTION

Consider a material heated at one end (**A**). After sometime you will be burnt by touching the other end (**B**). This mode of heat transfer in solids e.g metals is called conduction. Heat conduction (transfer) in solids is different from material to material (This means conduction depends on a material). Transfer is due to vibration of particles about their mean position.

$$KE \propto T \tag{0.1}$$

$$\frac{1}{2}mv^2 \propto T \tag{0.2}$$

When heated, molecules in solids absorb energy they vibrate more and more colliding with its neighbours hence transfering energy from one molecule to another until heat energy reaches (**B**). Transfer of heat by this way is very slow and hence energy transfer by vibration of molecules is minimal. In conductors, there are free electrons which move from one point to another and conduction by electrons is prominent. Hence conduction is transfer of heat by vibrating molecules (or atoms) and to a larger extent by movement of mobile electrons.

VARIABLE AND STEADY STATE

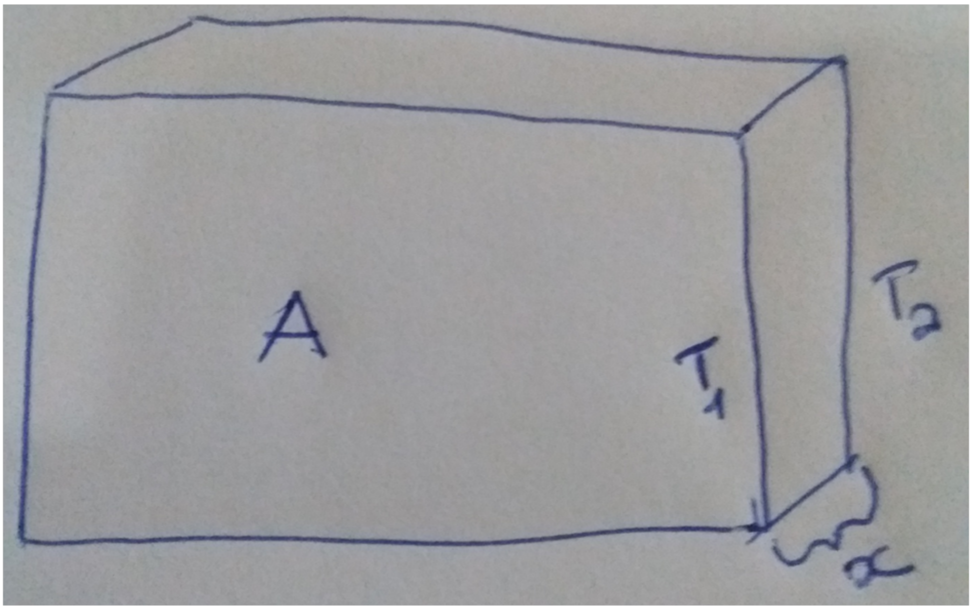
Consider a solid of uniform cross section. The amount of heat supplied to any section is not all transmitted to the next cross section but part of it is retained by that cross section to raise its temperature, part of it is transferred to the next section and the rest to the surrounding. This is what is called variable state. Hence the temperature at one end will be different from the temperature of the other end. Once the first section attains enough temperature, then all the heat supplied will be transmitted to the next section if there is no radiation and this is called steady state. Under steady state, heat transfer is the same through out the whole section.
Consider the slab with $T_1 > T_2$. Heat flows from the surface with T_1 to the surface with temperature T_2.

$$Q \propto A \tag{0.3}$$

$$Q \propto T_1 - T_2 \tag{0.4}$$

$$Q \propto t \tag{0.5}$$

$$Q \propto \frac{1}{x} \tag{0.6}$$



$$\therefore Q \propto \frac{A(T_1 - T_2)t}{x} \quad (0.7)$$

$$\therefore Q = \frac{KA(T_1 - T_2)t}{x} \quad (0.8)$$

where K(constant) is the thermal conductivity of a material and Q is the amount of heat supplied to a material.

$$K = \frac{Qx}{A(T_1 - T_2)t} \quad (0.9)$$

Units $\frac{Jm}{m^2Csec} = \frac{J}{mCsec}$
Thermal resisitivity $= \frac{1}{K}$. $\frac{T_1 - T_2}{x}$ is the temperature gradient between two points. $\frac{dT}{dx}$ is the temperature gradient at a given point.

COMPOSITE SLABS

If two materials with thermal conductivity K_1 and K_2 respectively, are joined together with the same surface area, under steady state we have $Q_1 = Q_2$. But

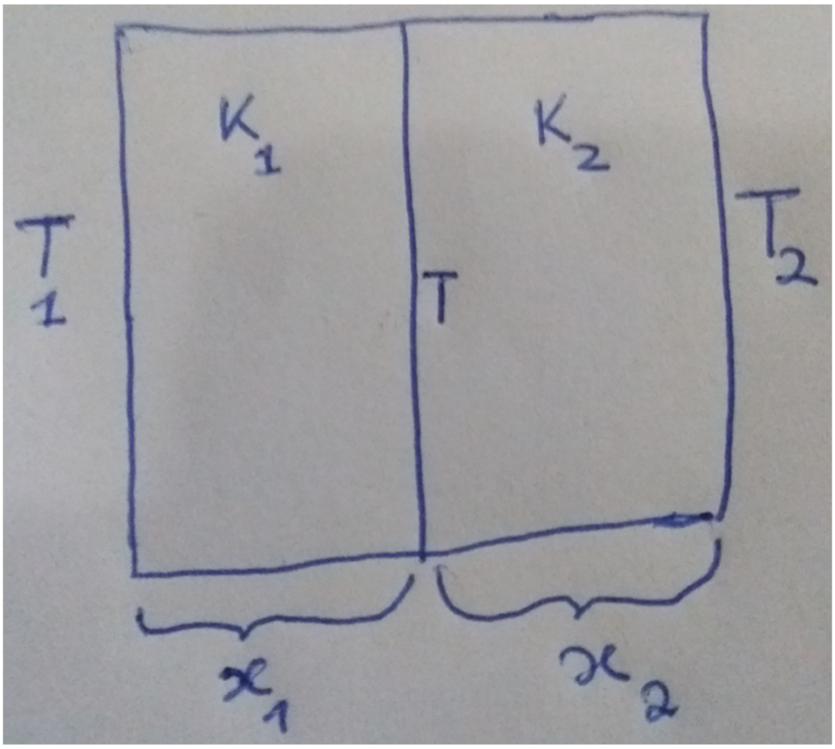
$$Q_1 = \frac{K_1A(T_1 - T)}{x_1} \quad (0.10)$$

$$Q_2 = \frac{K_2A(T - T_2)}{x_2} \quad (0.11)$$

$$\therefore \frac{K_1A(T_1-T)}{x_1} = \frac{K_2A(T-T_2)}{x_2} \tag{0.12}$$

$$\frac{K_1T_1}{x_1} - \frac{K_1T}{x_1} = \frac{K_2T}{x_2} - \frac{K_2T_2}{x_2} \tag{0.13}$$

$$\frac{K_1T_1}{x_1} + \frac{K_2T_2}{x_2} = \frac{K_2T}{x_2} + \frac{K_1T}{x_1} \tag{0.14}$$



$$\frac{K_1T_1}{x_1} + \frac{K_2T_2}{x_2} = \left(\frac{K_2}{x_2} + \frac{K_1}{x_1}\right)T \tag{0.15}$$

$$\frac{K_1T_1x_2 + K_2T_2x_1}{x_1x_2} = \left(\frac{K_2x_1 + K_1x_2}{x_1x_2}\right)T \tag{0.16}$$

$$K_1T_1x_2 + K_2T_2x_1 = (K_2x_1 + K_1x_2)T \tag{0.17}$$

$$T = \frac{K_1T_1x_2 + K_2T_2x_1}{K_2x_1 + K_1x_2} \tag{0.18}$$

Substituting in equation 0.10 we have

$$Q_1 = \frac{K_1 A(T_1 - T)}{x_1} \tag{0.19}$$

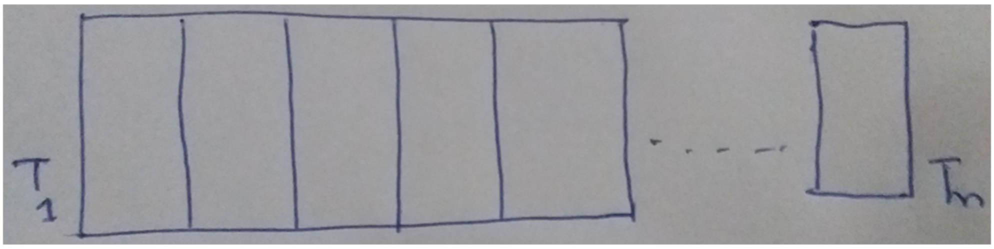
$$Q_1 = \frac{K_1 A}{x_1}\left(T_1 - \frac{K_1 T_1 x_2 + K_2 T_2 x_1}{K_2 x_1 + K_1 x_2}\right) \tag{0.20}$$

$$Q_1 = \frac{K_1 A}{x_1}\left(\frac{T_1(K_2 x_1 + K_1 x_2) - (K_1 T_1 x_2 + K_2 T_2 x_1)}{K_2 x_1 + K_1 x_2}\right) \tag{0.21}$$

$$Q_1 = \frac{K_1 A}{x_1}\left(\frac{K_2 x_1 T_1 - K_2 x_1 T_2}{K_2 x_1 + K_1 x_2}\right) \tag{0.22}$$

$$Q_1 = \frac{A(T_1 - T_2)}{\frac{x_1}{K_1} + \frac{x_2}{K_2}} \tag{0.23}$$

If there are n slabs



$$Q_1 = \frac{A(T_1 - T_n)}{\frac{x_1}{K_1} + \frac{x_2}{K_2} + \frac{x_3}{K_3} + ... + \frac{x_n}{K_n}} \tag{0.24}$$

Generalised equation

$$Q_1 = \frac{A(T_1 - T_n)}{\sum \frac{x}{K}} \tag{0.25}$$

Specific heat capacity

Specific heat capacity is the amount of heat required to increase the heat of a unit mass by 1 °C.

$$Q \propto m\Delta T \tag{0.26}$$

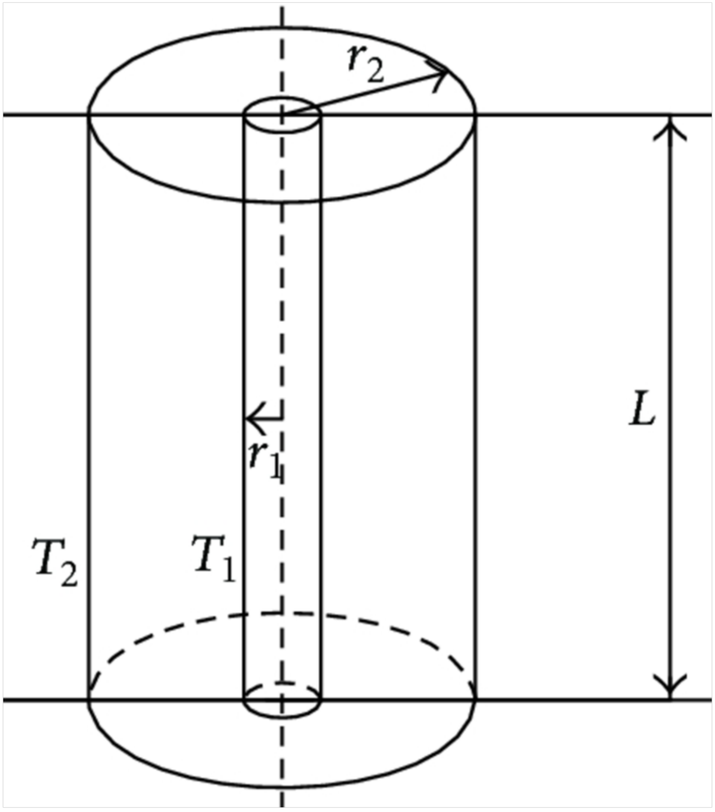
$$Q = mc\Delta T \tag{0.27}$$

Latent heat

Latent heat is the amount of heat needed to convert a solid into a liquid or a liquid into a vapour without any change in temperature.

CYLINDRICAL FLOW OF HEAT

Consider a cylinder with inner radius r_1 and outer radius r_2. Under steady condition T_1 is the temperature of the inner side and T_2 is the temperature of the outer surface.



$$Q = \frac{KA(T_1 - T_2)}{x}. \tag{0.28}$$

But $A = 2\pi rL$ and temperature gradient $\frac{(T_1-T_2)}{x} = \frac{T-(T+dT)}{dr} = -\frac{dT}{dr}$.

$$\therefore Q = K2\pi rL\left(-\frac{dT}{dr}\right). \tag{0.29}$$

$$Q = -2K\pi rL\left(\frac{dT}{dr}\right). \tag{0.30}$$

$$Q\frac{dr}{r} = -2K\pi LdT. \tag{0.31}$$

Integrating we have,

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = -2K\pi L\int_{T_1}^{T_2} dT. \tag{0.32}$$

$$Qlog_e\frac{r_2}{r_1} = 2K\pi L(T_1 - T_2). \tag{0.33}$$

$$Q = \frac{2K\pi L(T_1 - T_2)}{log_e\frac{r_2}{r_1}}. \tag{0.34}$$

Equating equation 0.29 and 0.34, we have

$$K2\pi rL\left(-\frac{dT}{dr}\right) = \frac{2K\pi L(T_1 - T_2)}{log_e\frac{r_2}{r_1}}. \tag{0.35}$$

$$rdTlog_e\frac{r_2}{r_1} = -dr(T_1 - T_2). \tag{0.36}$$

$$\int dT = -\frac{(T_1 - T_2)}{log_e\frac{r_2}{r_1}}\int\frac{dr}{r}. \tag{0.37}$$

$$T = -\frac{(T_1 - T_2)}{log_e\frac{r_2}{r_1}}log_e r + C. \tag{0.38}$$

at $r = r_1$, $T = T_1$

$$T_1 = -\frac{(T_1 - T_2)}{log_e\frac{r_2}{r_1}}log_e r_1 + C. \tag{0.39}$$

$$C = T_1 + \frac{(T_1 - T_2)}{log_e\frac{r_2}{r_1}}log_e r_1. \tag{0.40}$$

$$T = -\frac{(T_1 - T_2)}{log_e\frac{r_2}{r_1}}log_e r + T_1 + \frac{(T_1 - T_2)}{log_e\frac{r_2}{r_1}}log_e r_1. \tag{0.41}$$

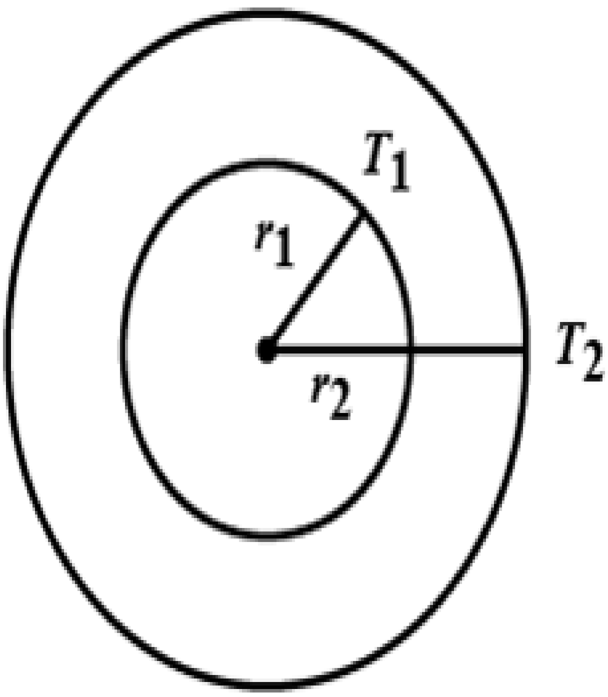
$$T = \frac{1}{log_e\frac{r_2}{r_1}}[-(T_1 - T_2)log_e r + T_1 log_e\frac{r_2}{r_1} + (T_1 - T_2)log_e r_1]. \tag{0.42}$$

$$T = \frac{1}{log_e\frac{r_2}{r_1}}[-T_1 log_e r + T_2 log_e r + T_1 log_e r_2 - T_1 log_e r_1 + T_1 log_e r_1 - T_2 log_e r_1]. \tag{0.43}$$

$$\therefore T = \frac{1}{log_e \frac{r_2}{r_1}}[-T_1 log_e r + T_2 log_e r + T_1 log_e r_2 - T_2 log_e r_1]. \quad (0.44)$$

RADIAL FLOW OF HEAT

In order to find heat flow in sand, clay and charcoal, this method is used because such samples are in most cases spherical. Consider a sphere of radius r_1 and r_2. Consider also a small sphere with radius r and thickness dr. Temperature gradient $= -\frac{dT}{dr}$



$$Q = KA\left(-\frac{dT}{dr}\right) \quad (0.45)$$

But $A = 4\pi r^2$

$$\therefore Q = -K4\pi r^2 \frac{dT}{dr} \quad (0.46)$$

$$Q\frac{dr}{r^2} = -K4\pi dT \quad (0.47)$$

Integrating, we have

$$Q\int_{r_1}^{r_2} \frac{dr}{r^2} = -K4\pi \int_{T_1}^{T_2} dT \quad (0.48)$$

$$Q\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=K4\pi(T_1-T_2) \tag{0.49}$$

$$Q=\frac{K4\pi(T_1-T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)} \tag{0.50}$$

$$Q=\frac{K4\pi(T_1-T_2)r_1r_2}{r_2-r_1} \tag{0.51}$$

Equation 0.46 and 0.51

$$-K4\pi r^2\frac{dT}{dr}=\frac{K4\pi(T_1-T_2)r_1r_2}{r_2-r_1} \tag{0.52}$$

$$dT=-\frac{(T_1-T_2)r_1r_2}{r_2-r_1}\frac{dr}{r^2} \tag{0.53}$$

$$\int dT=-\frac{(T_1-T_2)r_1r_2}{r_2-r_1}\int\frac{dr}{r^2} \tag{0.54}$$

$$T=-\frac{(T_1-T_2)r_1r_2}{r_2-r_1}\frac{-1}{r}+C \tag{0.55}$$

At $r=r_1, T=T_1$

$$T_1=\frac{(T_1-T_2)r_1r_2}{r_2-r_1}\frac{1}{r_1}+C \tag{0.56}$$

$$C=T_1-\left(\frac{(T_1-T_2)r_1r_2}{r_2-r_1}\frac{1}{r_1}\right) \tag{0.57}$$

$$T=-\frac{(T_1-T_2)r_1r_2}{r_2-r_1}\frac{-1}{r}+T_1-\left(\frac{(T_1-T_2)r_1r_2}{r_2-r_1}\frac{1}{r_1}\right) \tag{0.58}$$

$$T=\frac{1}{r_2-r_1}\left[r_1r_2\frac{T_1-T_2}{r}+r_2T_2-r_1T_1\right] \tag{0.59}$$