



THE UNIVERSITY OF ZAMBIA  
SCHOOL OF NATURAL SCIENCES  
Department of Physics  
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PHY 2231 - Properties of Matter and Thermodynamics Notes

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CHAPTER 3: THERMAL EXPANSION  
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When you heat a substance it increases in size. In solids there is expansion in length, area and volume.

**EXPANSION IN SOLIDS**

Expansion in solids can be in length, area and volume.

If only length increases on application of heat, then it is linear expansion.

Expansion in solids is mainly due to the distance between molecules increasing due to increase in velocity of vibrations.

If area increases then it is superficial expansion.

If volume increases then it is cubical expansion.

Materials do not expand at the same rate hence we use coefficient of thermal expansion.

**LINEAR EXPANSION**

Change in length is directly proportional to original length and temperature change.

$L_2 - L_1 \propto L_1$  and also  $L_2 - L_1 \propto (T_2 - T_1)$ .

Therefore,  $L_2 - L_1 \propto L_1(T_2 - T_1)$

$$L_2 - L_1 = \alpha L_1(T_2 - T_1) \quad (0.1)$$

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where  $\alpha$  is the coefficient of linear expansion.

$$L_2 = \alpha L_1(T_2 - T_1) + L_1 \quad (0.2)$$

$$\begin{aligned} L_0 &\rightarrow 0 \text{ }^\circ\text{C} \\ L_T &\rightarrow T \text{ }^\circ\text{C} \end{aligned}$$

$$L_T - L_0 = \alpha L_0(T) \quad (0.3)$$

$$L_T = \alpha L_0(T) + L_0 = L_0(1 + \alpha T) \quad (0.4)$$

### **SUPERFICIAL EXPANSION**

$$\begin{aligned} A_1 &\rightarrow T_1 \text{ }^\circ\text{C} \\ A_2 &\rightarrow T_2 \text{ }^\circ\text{C} \end{aligned}$$

$$A_2 - A_1 \propto A_1(T_2 - T_1) \quad (0.5)$$

$$A_2 - A_1 = \beta A_1(T_2 - T_1) \quad (0.6)$$

where  $\beta$  is the coefficient of superficial expansion

$$\begin{aligned} A_0 &\rightarrow 0 \text{ }^\circ\text{C} \\ A_T &\rightarrow T \text{ }^\circ\text{C} \end{aligned}$$

$$A_T - A_0 = \beta A_0 T \quad (0.7)$$

$$A_T = \beta A_0 T + A_0 = A_0(1 + \beta T) \quad (0.8)$$

**relationship between  $\alpha$  and  $\beta$**

$$L_T = L_0(1 + \alpha T) \quad (0.9)$$

and

$$A_T = A_0(1 + \beta T) \quad (0.10)$$

Considering a square plate  $A_0 = L_0^2$

$$A_T = L_T^2$$

Therefore  $A_T = A_0(1 + \beta T)$  becomes  $L_T^2 = L_0^2(1 + \beta T)$

$$\therefore, [L_0(1 + \alpha T)]^2 = L_0^2(1 + \beta T) \quad (0.11)$$

$$L_0^2(1 + 2\alpha T + \alpha^2 T^2) = L_0^2(1 + \beta T) \quad (0.12)$$

$$1 + 2\alpha T + \alpha^2 T^2 = 1 + \beta T \quad (0.13)$$

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Since change in temperature is small,  $\alpha^2 T^2$  is very small and hence it can be neglected, and we have

$$1 + 2\alpha T = 1 + \beta T \rightarrow 2\alpha = \beta \quad (0.14)$$

$$\alpha = \frac{\beta}{2} \quad (0.15)$$

approximately.

### CUBICAL EXPANSION

$V_1 \rightarrow T_1$  °C

$V_2 \rightarrow T_2$  °C

$$V_2 - V_1 = \gamma V_1 (T_2 - T_1) \quad (0.16)$$

$$V_2 = \gamma V_1 (T_2 - T_1) + V_1 \quad (0.17)$$

where  $\beta$  is the coefficient of superficial expansion.

$V_0 \rightarrow 0$  °C

$V_T \rightarrow T$  °C

$$V_T - V_0 = \gamma V_0 T \quad (0.18)$$

$$V_T = \gamma V_0 T + V_0 = V_0 (1 + \gamma T) \quad (0.19)$$

### FORCE DUE TO CONTRACTION OR EXPANSION

The force due to expansion is very large. Let  $L$  be the length at  $T_1$  °C and if temperature increases to  $T_2$  °C length becomes  $L_2$ .

$$L_2 - L_1 = \alpha L_1 (T_2 - T_1) \quad (0.20)$$

Strain = increase in length/original length =  $\frac{\alpha L_1 (T_2 - T_1)}{L_1}$ .

Strain =  $\alpha (T_2 - T_1)$ . But  $Y = \frac{\text{Stress}}{\text{Strain}}$  where  $Y$  is the Young's modulus.

Stress =  $Y \times \text{Strain} = Y [\alpha (T_2 - T_1)]$ .

But Stress =  $\frac{F}{A} \rightarrow F = \text{Stress} \times A$

$$\therefore F = Y [\alpha (T_2 - T_1) A]. \quad (0.21)$$

### EXPANSION OF LIQUIDS

Expansion of liquids is cubical expansion because volume increases.

$V_1 \rightarrow T_1$  °C

$V_2 \rightarrow T_2$  °C

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There are two types of expansion in liquids. These are **apparent** and **real** expansion.

Real expansion = apparent expansion + cubical expansion, i.e

$$\gamma_r = \gamma_a + \gamma_c \quad (0.22)$$

Expansion of water is abnormal (anomalous). The advantage of abnormal expansion of water is that aquatic life can survive even under very cold conditions.

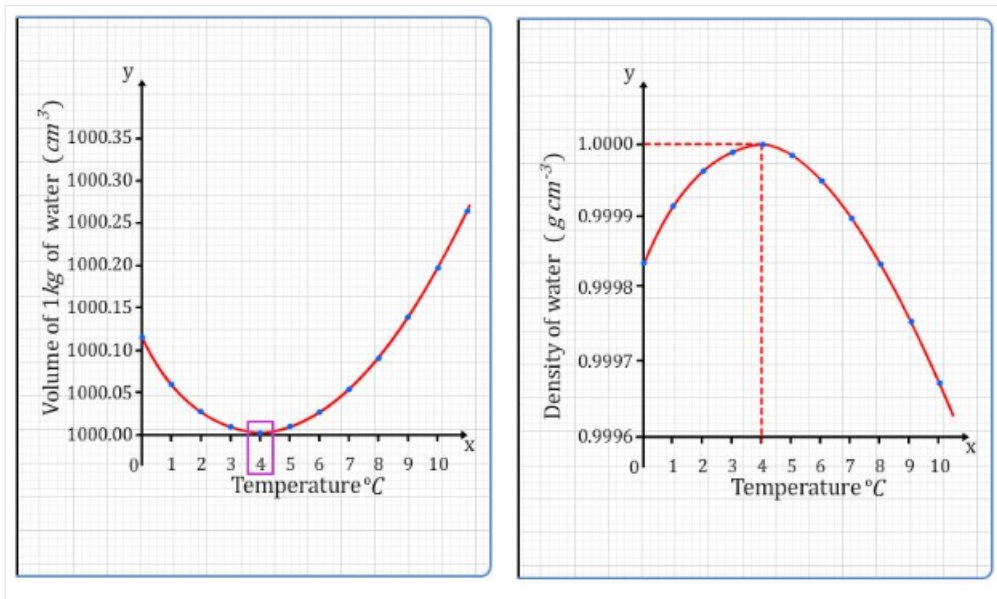


Figure 0.1: *Volume and density of water vs temperature diagram*