THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT 2100–Analytic Geometry and Calculus
Tutorial Sheet 1

---

1. Discuss the following parabolas:
   (a) $y^2 = -6x$ **(b)** $x^2 = 4y$ **(c)** $7x = \dfrac{y^2}{28}$

2. A parabola has focus at $(3, -4)$ and the $x-$axis is the equation of the directrix. Find the equation of the parabola and sketch it, clearly showing the vertex and the $y-$intercepts.

3. Show that the line $4x + 12y = -21$ is a focal chord of the parabola $x^2 = -7y$. Hence, find the coordinates at which this line intersects the parabola.

4. Find the length of latus rectum for each of the following parabolas:
   (i) $2y^2 = 3x$ (ii) $4x^2 = 6y$ (iii) $5x^2 = -2y$

5. Discuss each of the following ellipses and sketch the graph:
   (a) $\frac{2x^2}{9} + \frac{4y^2}{25} = 1$ (b) $\frac{x^2}{25} + 4y^2 = 1$ (c) $100x^2 + 5100y^2 = 1$

6. Find the equation of the hyperbola that goes through the points $(2, 3)$ and has foci at $(\pm 2, 0)$.

7. A nuclear cooling tower's cross section is a hyperbola of diameter 60 metres at the centre. The distance between the two foci is 100 metres.

8. Find the standard equation of the following conic sections:
   (a) Foci : $(0, \pm 3); e = 0.5$
   (b) $e = 1.25$; Foci : $(0, \pm 5)$
   (c) $e = 3$; Vertices : $(0, \pm 1)$
   (d) Vertices : $(\pm 10, 0); e = 0.24$

9. Find the standard equation of the conic section described below:
   (a) minor axis is 10 and foci at $(\pm 4, 0)$
   (b) vertical transverse axis is 12 and $e = 2$
   (c) distance between the foci is 32, major axis is along x-axis and $e = \frac{1}{3}$
   (d) vertices are at $(\pm 2, 0)$ and foci at $(\pm 6, 0)$

10. Discuss the graph of each of the following conic sections:
    (a) $2x^2 - y^2 + 6y = 3$
    (b) $9x^2 + 4y^2 + 36x - 24y = -36$
    (c) $2x^2 + 2y^2 - 28x + 12y = -114$
    (d) $-x^2 + 2x + 4y = 2917$
    (e) $-4x^2 + 8x + y^2 + 4y = 0$
    (f) $y^2 - 6y - 16 = 0$

11. Find a Cartesian equation for the hyperbola centred at $(3, 0)$ with $x = 1$ as one equation of the directrix and distance from vertex to the directrix equal to $\frac{3}{2}$.

12. Discuss the graph of each of the following conics:

(a) $3x^2 + 4\sqrt{3}xy - y^2 = 7$
(b) $x^2 + xy + y^2 = 1$
(c) $3x^2 + 2\sqrt{3}xy + y^2 - 8x + 8\sqrt{3}y = 0$
(d) $x^2 - \sqrt{3}xy + 2y^2 = 1$
(e) $x^2 - 4xy + 4y^2 - 5 = 0$
(f) $4x^2 + 12xy + 9y^2 = 52$

13. Use the discriminant test to identify the following conic sections:
    (a) $16x^2 - 24xy + 9y^2 - 30x - 40y = 0$
    (b) $x^2 - 4xy - 2y^2 - 6 = 0$
    (c) $13x^2 - 8xy + 7y^2 - 45 = 0$
    (d) $2x^2 + 4xy + 5y^2 + 3x - 4y - 20 = 0$

14. For each of the following given polar equations, find the equivalent Cartesian equation:

    (a) $r^2 = \dfrac{3 - 2r\cos\theta}{2 - \cos^2\theta}$
    (b) $r = 4\tan\theta\sec\theta$

    (c) $r\sin\theta = \ln r + \ln\cos\theta$
    (d) $r = \dfrac{7}{1 - \cos\theta}$

15. Replace the following Cartesian equations by their equivalent polar equations:

    (a) $x^2 - y^2 = 1$
    (b) $xy = 2$

    (c) $\frac{(x-1)^2}{4} + \frac{y^2}{2} = 1$
    (d) $x^2 + xy + y^2 = 1$

16. Identify and describe the following conics:

    (a) $r = \dfrac{6}{1 + \cos\theta}$
    (b) $r(2 - 3\cos\theta) = 6$

    (c) $r = \dfrac{400}{16 + 8\sin\theta}$
    (d) $r = \dfrac{6\sec\theta}{2\sec\theta - 2}$

17. Halley's Comet has an elliptical orbit with the sun at one focus. Its orbit is approximately given by

$$r = \frac{1.069}{1 + 0.967\sin\theta}.$$

    Find the distance from Halley's Comet to the sun at its shortest and greatest distance from the sun.