The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

MAT 2100 - Analytic Geometry and Calculus

Test 1 - September 19, 2021

Time allowed : One (1) hour

Instructions:

- Indicate your name and **computer number** on your answer script.
- There are **four** (4) questions in this paper. Attempt all questions.

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1. (a) The function

$$f(x) = 5^{ax} - 2b(5^{x+1}) + 125$$

satisfies the conditions of the Rolle's theorem in the interval $[1, 2]$.

(i) Find the values of a and b. [6]

(ii) Hence, find the real number $c \in (1, 2)$ that satisfies the conclusion of the theorem. [3]

(b) (i) Find the 3rd order ~~Maclaurin~~ 2nd Taylor polynomial of the function

$$f(x) = \frac{x}{1 + \ln x}$$

about $x = 1$. [6]

(ii) Use part (i) to approximate

$$\int_0^1 \frac{x}{e^x + 1}\, dx$$ [2]

Turn Over/...

2. (a) L'Hopital's rule can be used to evaluate the limit

$$\lim_{x\to\frac{\pi}{4}}\left[\frac{-1+\cot(mx)}{4x-\pi}\right].$$

(i) Find the value of m. [2]

(ii) Hence, or otherwise, evaluate the limit. [2]

(b) (i) Find the radius of curvature for the curve $x=\frac{y^2}{2}$ at the point where $y=\sqrt{3}$. [3]

(ii) A particular curve $y=f(x)$, $x\geq 0$, has the derivative given by

$$\frac{dy}{dx}=\sqrt{e^x-1}.$$

Find an intrinsic equation in the form $s=g(\psi)$, where s is measured from the point $(0,0)$ and ψ is the angle the tangent to the curve makes with the positive $x-$axis. [5]

3. (a) Evaluate the following integrals:

(i) $\int \sin\theta\cos^2(2\theta)\,d\theta$ [3]

(ii) $\int_{-0.5}^{0}\frac{1}{(2x+1)^3}\,dx$ [3]

(b) Use the Riemann sum to evaluate

$$\int_a^b f(x)\,dx$$

given that $f(x)=x^2+1$, $a=-1$, $b=2$,
$a=-1<-0.5<0<0.5<1<1.5<2=b$ and c_i is the midpoint of the i^{th} subinterval. [6]

4. (a) The region bounded by

$$y=-\frac{1}{2}x,\quad y=1,\quad x=0$$

is rotated about the $y-$axis. Find the volume of the resulting solid. [4]

(b) Find the moment of inertia about the $y-$axis for the region under

$$y=\sin x,\quad 0\leq x\leq\pi.$$

[5]

END OF TEST