THE UNIVERSITY OF ZAMBIA
SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS & STATISTICS
MAT 2100 – Analytic Geometry and Calculus

Tutorial Sheet 8

---

1. Classify each of the following differential equations as ODE or PDE, state the order and degree of each equation and determine whether the equation under consideration is linear or non-linear

   (a) $\frac{d^2y}{dx^2} + y \sin x = 0$ **(b)** $\frac{d^6x}{dt^6} + \left(\frac{d^4x}{dt^4}\right)\left(\frac{d^3x}{dt^3}\right) + x = t$ **(c)** $\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + u = 0$

2. Separate the variables in each of the following differential equations and find the general solution

   (a) $(xy^4 - 4y^4)\, dx - (x^3y^2 - 3x^3)dy = 0$ **(b)** $x \sin y dx + (x^2 \cos y + \cos y)dy = 0$
   (c) $(3x+8)(y^2+4)dx - 4y(x^2+5x+6)dy = 0$

3. Show that for each given differential equation below, $M(x, y)$ and $N(x, y)$ are homogeneous of the same degree and find the general solution

   (a) $y' = \frac{y+x}{x}$ **(b)** $\left(\sqrt{x^2 - y^2} - 2y\right) dx + 2xdy = 0$
   (c) $\left(y + x \tan\left(\frac{y-x}{x}\right)\right) dx - xdy = 0$

4. Determine the most general function $M(x, y)$ or $N(x, y)$ such that the following differential equations are exact:

   (a) $(x^{-2}y^{-2} + xy^{-3})\, dx + N(x, y)dy = 0$ **(b)** $M(x, y)dx + (2ye^x + y^2e^{3x})dy = 0$

5. Determine whether the following differential equations are exact; if not exact find the integrating factor and then solve them:

   (a) $(2x \sin y + y^3e^x)dx + (x^2 \cos y + 3y^2e^x)dy = 0$ **(b)** $(x^2+y^2+1)dx - (xy+y)dy = 0$
   (c) $(x^2 + 2y)dx - xdy = 0$ **(d)** $\cosh 2x \cosh 2y dx + \sinh 2x \sinh 2y = 0$

6. Solve the following linear first-order differential equations:

   (a) $x^4 \frac{dy}{dx} + 2x^3y = 1$ **(b)** $y^2dx + (3xy - 1)dy = 0$
   (c) $\cos\theta dr + (r \sin\theta - \cos^4\theta)d\theta = 0$ **(d)** $\frac{dr}{d\theta} + r \tan\theta = \cos\theta$

7. Find the general solution to each of the following:

   (a) $y' + xy = xy^2$ **(b)** $\frac{dx}{dt} + \frac{t+1}{2t}x = \frac{t+1}{xt}$ **(c)** $\frac{dy}{dx} - \frac{y}{x} = -\frac{y^2}{x}$

8. Solve the following:

   (a) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 16y = 0$ **(b)** $y'' + 16y' = 0$ **(c)** $\frac{d^2s}{dt^2} - 2\frac{ds}{dt} + 5s = 0$ **(d)** $y'' - \pi y' = 0$

9. Use the method of undetermined coefficients to solve the following differential equations:

   (a) $y'' + 4y = x \sin x$ **(b)** $y'' + 9y = e^{3x} + e^{-3x} + e^{3x} \sin 3x$

   (c) $y'' + 6y' + 13y = xe^{-3x} \sin 2x + x^2 e^{-2x} \sin 3x$

10. Use the method of variation of parameters to solve the following diffrential equations:

    (a) $y'' - 4y' + 4y = \frac{e^{2x}}{1+x}$ **(b)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \ln x,\ x > 0$

    (b) $(x+1)^2 \frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + 2y = 1$, given that $y = x + 1$ and $y = (x+1)^2$ are linearly independent solutions of the corresponding homogeneous equation.

11. Use power series to find the solution to the following differential equations:

    (a) $x\frac{dy}{dx} = 2y$ **(b)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$ **(c)** $\frac{dy}{dx} + y - e^x$

12. Solve the following IVPs and BVPs:

    (a) $\frac{d^2y}{dx^2} + y = 0,\ \ y(0) = 0,\ \ y'\left(\frac{\pi}{2}\right) = 1$ (b) $\frac{d^2y}{dx^2} + y = 0,\ \ y(0) = 0,\ \ y'(\pi) = 1$

    (c) $x^3y''' - 3x^2y'' + 6xy' - 6y = 0,\ \ y(2) = 0,\ \ y'(2) = 2,\ \ y''(2) = 6$

    (d) $(x+2)\frac{dy}{dx} + y = f(x),\ \ y(0) = 4,$ where $f(x) = \begin{cases} 2x, & 0 \le x < 2 \\ 4, & x \ge 2 \end{cases}$.