The University of Zambia
School of Natural Sciences
Department of Physics
PHY 1010
Lecture 1
Vectors

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1 Learning Outcomes

By the end of this lecture, the student should be able to:

1. Define a scalar quantity and give examples of scalars;
2. Define a vector quantity and give examples of vectors;
3. Decompose or resolve a vector into two vector components;
4. Carry out addition of vectors by components.

2 Scalars and Vectors

All physical quantities used in Physics can be categorised into **scalars** and **vectors**. **Scalars** are physical quantities that only have magnitude (or size). **Vectors** are physical quantities that have both magnitude and direction.

Scalars may be added or subtracted together by simple arithmetics as follows:

$$5 \text{ kg} + 3 \text{ kg} = 8 \text{ kg}$$

$$25 \text{ s} - 13 \text{ s} = 12 \text{ s}$$

$$25 \text{ m} + 50 \text{ m} = 75 \text{ m}$$

Mathematically, a vector B maybe denoted as $\vec{B}$ or **B**. As shown in Figure 1, a vector may also be represented pictorially by a line with an arrow at the end of the line. The length of the line represents the magnitude of the vector while the arrow indicates the direction of the vector.

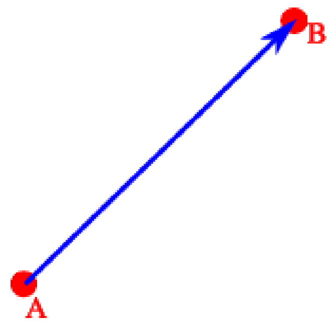

Figure 1: A pictorial representation of a vector

3 Vectors Addition

When adding or subtracting vectors, the **direction** of the vector must be considered. The sum of adding vectors is referred to as the **resultant**. In this lecture, we will only consider **vector addition** because vector subtraction is a form of vector addition.

3.1 Adding vectors acting in the same line

The resultant of adding two vectors acting along the **same line** is the sum of the magnitudes of the two vectors. The direction of the resultant vector remains unchanged. If one of the two vectors acting along the same line is acting in the opposite direction, then its magnitude is given a negative sign.

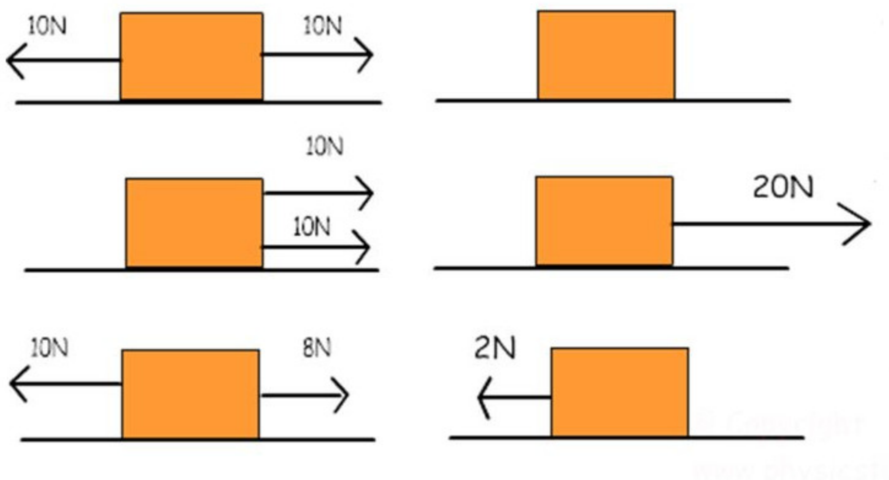

Figure 2: Adding two vectors acting along the same line of action

3.2 Adding vectors acting along different lines of action

If two vectors are acting in different directions are added, the **resultant vector** is found using a **triangle of vectors** as shown in Figure 3.

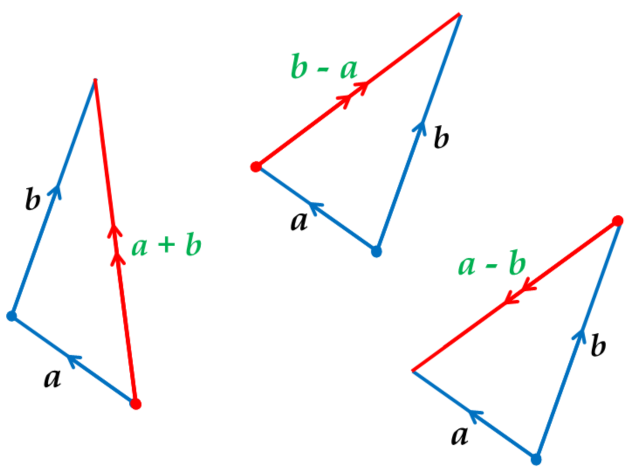

Figure 3: Adding two vectors acting along different directions with the aid of a triangle of vectors

4 Components of Vectors

It is often necessary to find the **components of a vector**, usually in two directions at right angles to each other. A **component of a vector** is the effective value of a vector along a particular direction. The directions along which the components are found is generally chosen to be the x and y-axes as shown in Figure 4. This process of finding the components of a vector is called **vector resolution** or **vector decomposition**.

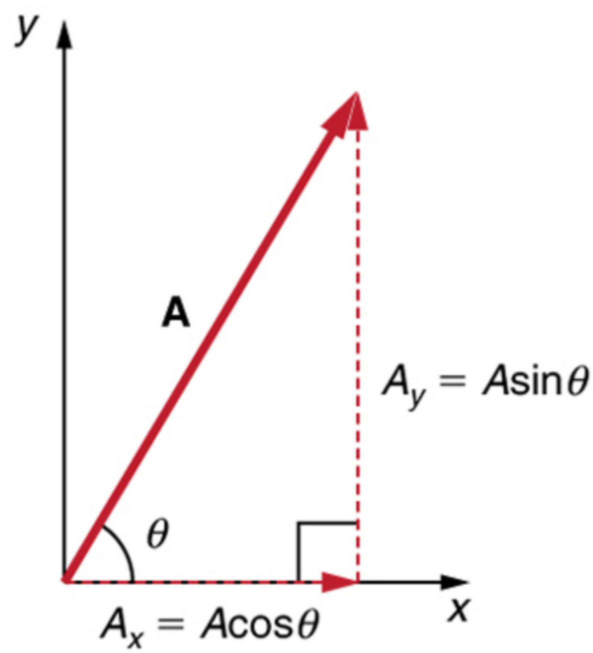

Figure 4: Decomposing vector **A** into its x and y-components

Mathematically, the component of a vector is defined as the magnitude of the vector multiplied by the cosine of the angle between the direction of the component and the direction of the vector. Usually, we decompose components of any vector along the x and y-axis direction to obtain the x-component and y-component of a vector. The x-component of vector $\vec{A}$ is denoted as A_x. Similarly, the y-component of vector $\vec{A}$ is denoted as A_y.

Table 1 shows the decomposition of vector $\vec{A}$ into its components x and y-components.

Table 1: Vector decomposition

Vector	x-component	y-component
$\vec{A}$	$A_x = A\cos\theta$	$A_y = A\cos(90-\theta) = A\sin\theta$

If the x-component and y-component of vector $\vec{A}$ are known, then the magnitude of vector $\vec{A}$ denoted $|A|$ is given by

$$|A| = \sqrt{A_x^2 + A_y^2} \tag{1}$$

Similarly, if the x-component and y-component of vector $\vec{A}$ are known, then the direction of vector $\vec{A}$ denoted by θ is given by

$$\tan\ \theta = \frac{A_y}{A_x} \tag{2}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \tag{3}$$

The angle θ by convention is measured from the positive x-axis in an anticlockwise direction.

5 Vector Addition using table of components

Consider vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ which have magnitudes 20 m, 30 m and 10 m respectively. The vector $\vec{A}$ makes 45° angle anticlockwise with the positive $x-$ axis. The vector $\vec{B}$ makes a 120° angle anticlockwise with the positive $x-$ axis. The vector $\vec{C}$ lies along the negative $x-$axis.

These vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ can be added or subtracted as shown in Table 2.

The magnitude of $\vec{A}+\vec{B}$ denoted $|\vec{A}+\vec{B}|$ is given by

$$|\vec{A}+\vec{B}| = \sqrt{(A_x+B_x)^2 + (A_y+B_y)^2}$$

$$|\vec{A}+\vec{B}| = \sqrt{(-1\text{ m})^2 + (40\text{ m})^2}$$

$$|\vec{A}+\vec{B}| = \sqrt{1601\text{ m}^2}$$

$$|\vec{A}+\vec{B}| \simeq 40\text{ m}$$

The direction of the vector $\vec{A}+\vec{B}$ is given by

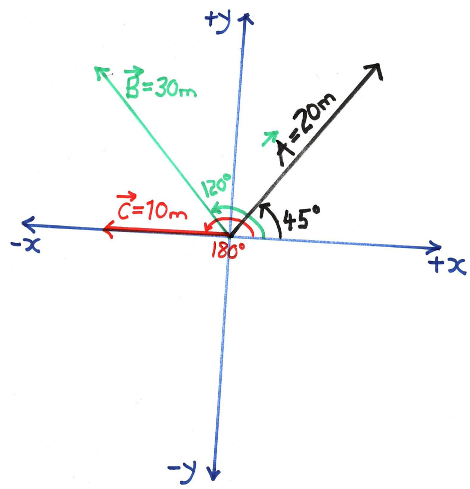

Figure 5: The coplanar vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ on the xy-plane.

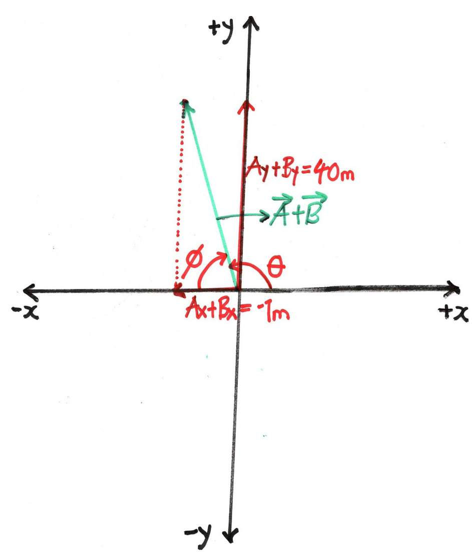

Figure 6: The resultant vector $\vec{A} + \vec{B}$

Table 2: Vector Addition using table of components

Vector	x-component	y-component
$\vec{A}$	A_x	A_y
$\vec{B}$	B_x	B_y
$\vec{C}$	C_x	C_y
$\vec{A}+\vec{B}$	$A_x + B_x$	$A_y + B_y$
$\vec{A}-\vec{B}$	$A_x - B_x$	$A_y - B_y$
$\vec{B}-\vec{A}$	$B_x - A_x$	$B_y - A_y$
$\vec{A}+\vec{B}+\vec{C}$	$A_x + B_x + C_x$	$A_y + B_y + C_y$
$\vec{A}+\vec{B}-\vec{C}$	$A_x + B_x - C_x$	$A_y + B_y - C_y$

$$\tan\phi = \frac{A_y + B_y}{A_x + B_x}$$

$$\tan\phi = \frac{40 \text{ m}}{-1 \text{ m}} = -40$$

$$\phi = \tan^{-1}(-40)$$

$$\phi \simeq -89^\circ$$

$$\theta = 180^\circ + \phi = 180^\circ + (-89^\circ)$$

$$\theta = 91^\circ$$

The magnitude of $\vec{A}-\vec{B}$ denoted $|\vec{A}-\vec{B}|$ is given by

$$|\vec{A}-\vec{B}| = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$$

$$|\vec{A}-\vec{B}| = \sqrt{(29 \text{ m})^2 + (-12 \text{ m})^2}$$

$$|\vec{A}-\vec{B}| = \sqrt{985 \text{ m}^2}$$

$$|\vec{A}-\vec{B}| \simeq 31.4 \text{ m}$$

The direction of the vector $\vec{A}-\vec{B}$ is given by

Table 3: Addition of vectors acting in different directions

vector	x-component	y-component
$\vec{A}$	$A_x = (20\text{ m})\cos 45° \simeq 14\text{ m}$	$A_y = (20\text{ m})\sin 45° \simeq 14\text{ m}$
$\vec{B}$	$B_x = (30\text{ m})\cos 120° = -15\text{ m}$	$B_y = (30\text{ m})\sin 120° \simeq 26\text{ m}$
$\vec{C}$	$C_x = (10\text{ m})\cos 180° = -10\text{ m}$	$C_y = (10\text{ m})\sin 180° = 0$
$\vec{A}+\vec{B}$	$A_x + B_x = -1\text{ m}$	$A_y + B_y = 40\text{ m}$
$\vec{A}-\vec{B}$	$A_x - B_x = 29\text{ m}$	$A_y - B_y = -12\text{ m}$
$\vec{B}-\vec{A}$	$B_x - A_x = -29\text{ m}$	$B_y - A_y = 12\text{ m}$
$\vec{A}+\vec{B}+\vec{C}$	$A_x + B_x + C_x = -11\text{ m}$	$A_y + B_y + C_y = 40\text{ m}$
$\vec{A}+\vec{B}-\vec{C}$	$A_x + B_x - C_x = 9\text{ m}$	$A_y + B_y - C_y = 40\text{ m}$

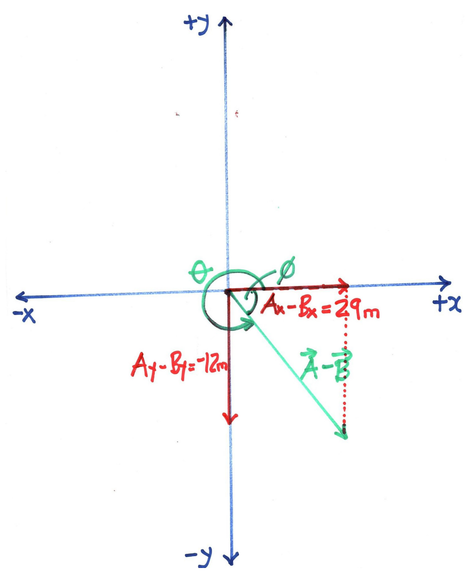

Figure 7: The resultant vector $\vec{A}-\vec{B}$

$$\tan\phi = \frac{A_y - B_y}{A_x - B_x}$$

$$\tan\phi = \frac{-12\text{ m}}{29\text{ m}} = -0.41$$

$$\phi = \tan^{-1}(-0.41)$$

$$\phi \simeq -22.3^\circ$$

$$\theta = 360^\circ + \phi = 360^\circ + (-22.3^\circ)$$

$$\theta = 337.7^\circ$$

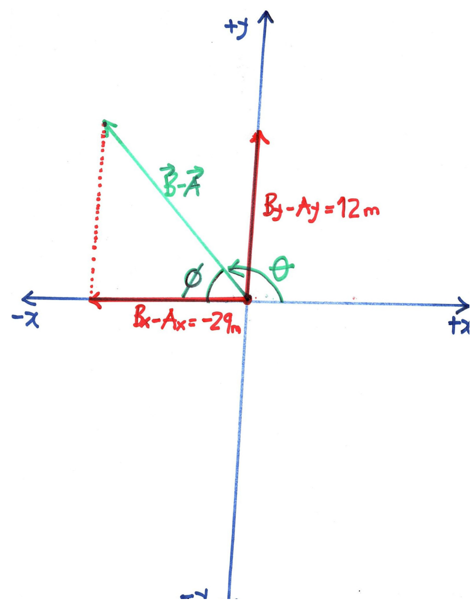

Figure 8: The resultant vector $\vec{B} - \vec{A}$

The magnitude of $\vec{B} - \vec{A}$ denoted $|\vec{B} - \vec{A}|$ is given by

$$|\vec{B} - \vec{A}| = \sqrt{(B_x - A_x)^2 + (B_y - A_y)^2}$$

$$|\vec{B} - \vec{A}| = \sqrt{(-29 \text{ m})^2 + (12 \text{ m})^2}$$

$$|\vec{B} - \vec{A}| = \sqrt{985 \text{ m}^2}$$

$$|\vec{B} - \vec{A}| \simeq 31.4 \text{ m}$$

The direction of the vector $\vec{B} - \vec{A}$ is given by

$$\tan\phi = \frac{B_y - A_y}{B_x - A_x}$$

$$\tan\phi = \frac{12\text{ m}}{-29\text{ m}} = -0.41$$

$$\phi = \tan^{-1}(-0.41)$$

$$\phi \simeq -22.3°$$

$$\theta = 180° + \phi = 180° + (-22.3°)$$

$$\theta = 157.7°$$

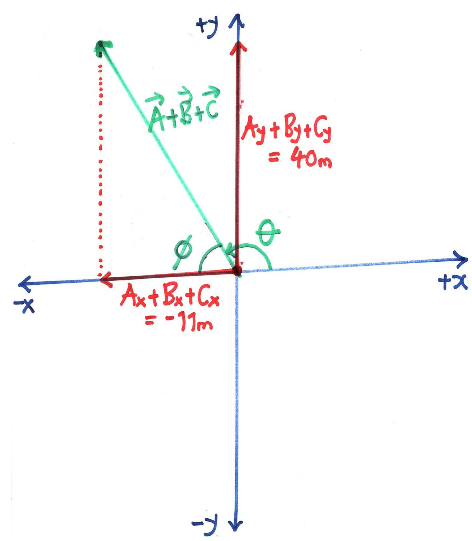

Figure 9: The resultant vector $\vec{A}+\vec{B}+\vec{C}$

The magnitude of $\vec{A}+\vec{B}+\vec{C}$ denoted $|\vec{A}+\vec{B}+\vec{C}|$ is given by

$$|\vec{A}+\vec{B}+\vec{C}| = \sqrt{(A_x+B_x+C_x)^2+(A_y+B_y+C_y)^2}$$

$$|\vec{A}+\vec{B}+\vec{C}| = \sqrt{(-11\text{ m})^2+(40\text{ m})^2}$$

$$|\vec{A}+\vec{B}+\vec{C}| = \sqrt{1\ 721\text{ m}^2}$$

$$|\vec{A}+\vec{B}+\vec{C}| \simeq 41.5\text{ m}$$

The direction of the vector $\vec{A}+\vec{B}+\vec{C}$ is given by

$$\tan \phi = \frac{A_y + B_y + C_y}{A_x + B_x + C_x}$$

$$\tan \phi = \frac{40 \text{ m}}{-11 \text{ m}} = -3.64$$

$$\phi = \tan^{-1}(-3.64)$$

$$\phi \simeq -74.6^\circ$$

$$\theta = 180^\circ + \phi = 180^\circ + (-74.6^\circ)$$

$$\theta = 105.4^\circ$$

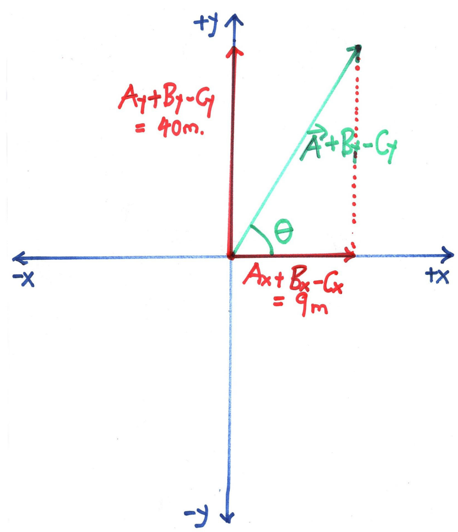

Figure 10: The resultant vector $\vec{A} + \vec{B} - \vec{C}$

The magnitude of $\vec{A} + \vec{B} - \vec{C}$ denoted $|\vec{A} + \vec{B} - \vec{C}|$ is given by

$$|\vec{A} + \vec{B} - \vec{C}| = \sqrt{(A_x + B_x - C_x)^2 + (A_y + B_y - C_y)^2}$$

$$|\vec{A} + \vec{B} - \vec{C}| = \sqrt{(9 \text{ m})^2 + (40 \text{ m})^2}$$

$$|\vec{A} + \vec{B} - \vec{C}| = \sqrt{1\ 681 \text{ m}^2}$$

$$|\vec{A} + \vec{B} - \vec{C}| \simeq 41 \text{ m}$$

The direction of the vector $\vec{A}+\vec{B}-\vec{C}$ is given by

$$\tan\theta = \frac{A_y + B_y - C_y}{A_x + B_x - C_x}$$

$$\tan\theta = \frac{40 \text{ m}}{9 \text{ m}} = 4.44$$

$$\theta = \tan^{-1}(4.44)$$

$$\theta \simeq 77.3^\circ$$