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F_c is the friction

$$1470 = \mu mg \quad \therefore \quad \mu = \frac{1470}{mg} = 0.125$$

Generally

$$F_c = \mu m g$$

$$\frac{mv^2}{r} = \mu m g$$

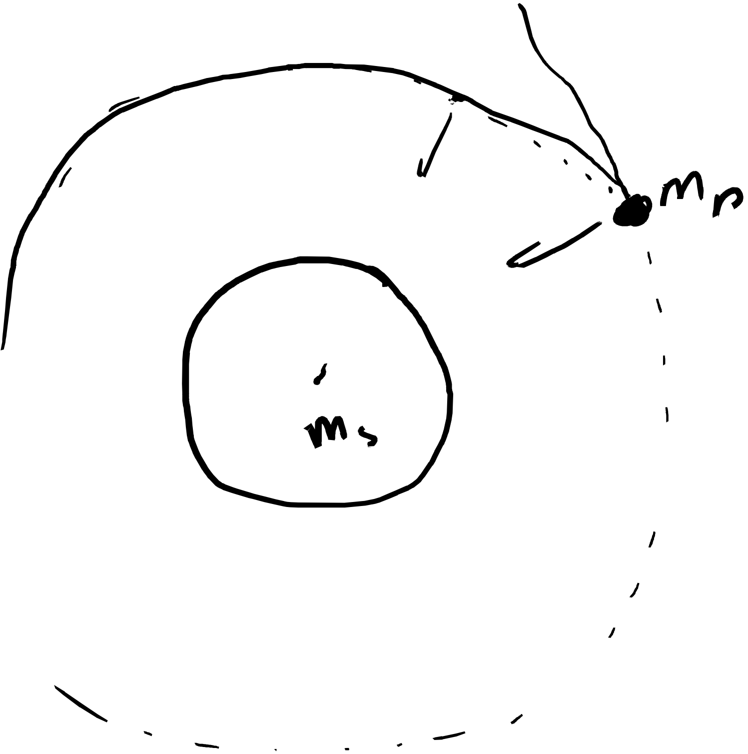
$$\mu_s = v^2/gr$$

- μ does not depend on mass
- Because the required value depends on v^2, high speed turns on a level ground can be very dangerous!

Newton's laws of gravitation

- This is one of the most interesting examples of circular motion — the motion of planets around the Sun.
- laws indicated that planetary orbits were almost circular and that the square of the time taken by the planet to orbit the Sun $(T) \propto R^3$ (distance from the Sun).

$$T^2 \propto R^3 \equiv \text{Kepler's third law}$$



From Newton's laws of motion, the attractive force between the Sun & the planet is provided by the centripetal force which enables the planet to move in a circular path

$$F_g = ma_c = m_p \frac{v^2}{R} \quad - G$$

$m_p \equiv$ planet mass

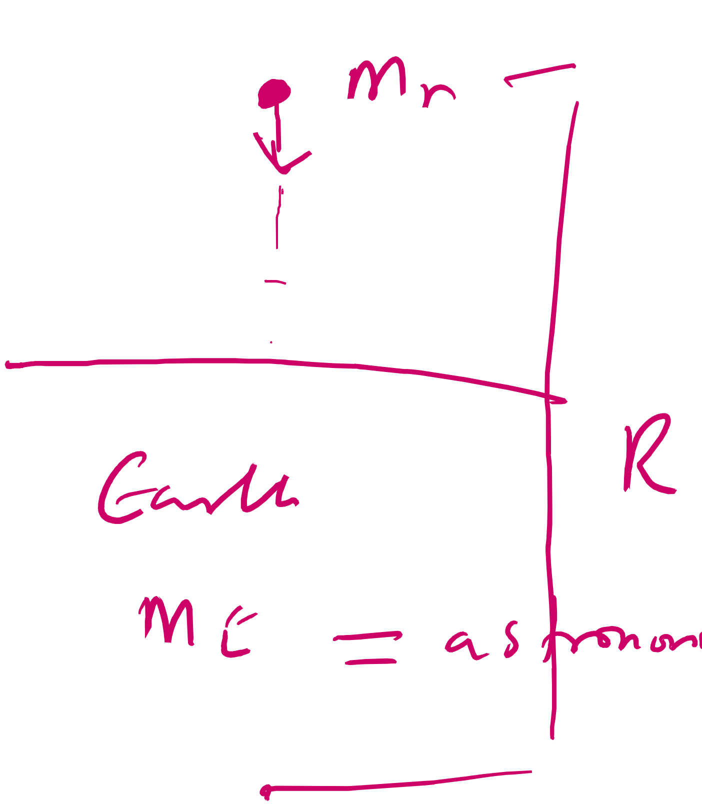
Law of gravitation

$$F_g = G\frac{m_1 m_2}{R^2}$$

The objects attract each other with this force

G = Gravitational Constant
$= 6.672 \times 10^{-11}\ N \cdot m^2/kg^2$

- This force is only significant if one of the masses is astronomical.



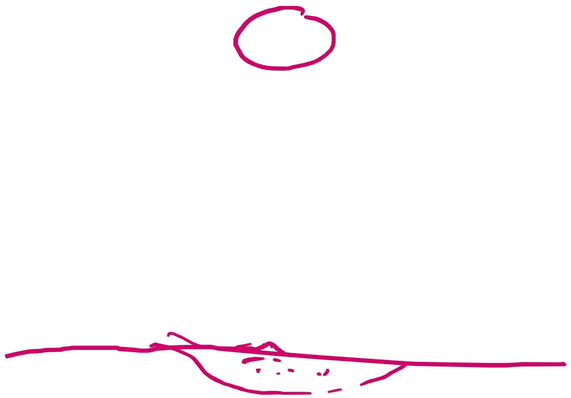
m_E = astronomical mass

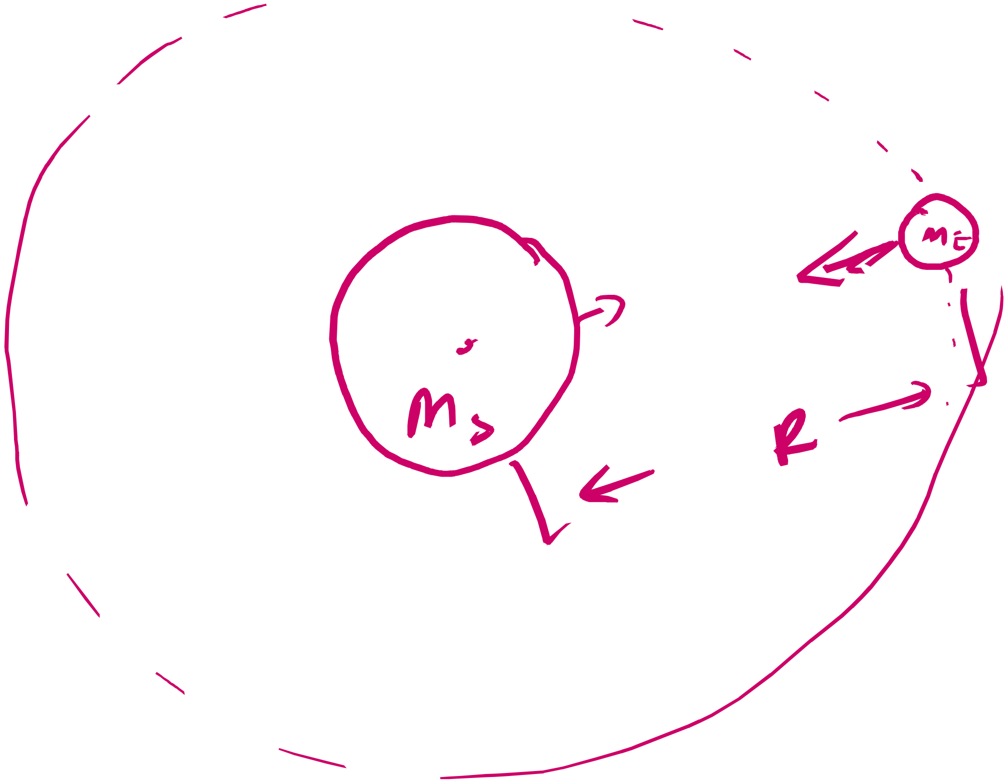
$$F_g = \frac{G m_r m_E}{R^2} = \underbrace{m_r g}_{\text{weight}}$$

$$\frac{G m_E}{R^2} = \underbrace{g}_{\text{acceleration due to grav}}$$

$$F = ma$$

$$= m_r g = m_r \frac{G m_E}{R^2}$$





Consider the Earth's orbit.

The force that attracts these objects

$$\frac{G M_s M_E}{R^2} = \frac{M_E v^2}{R}$$

$$G\frac{\cancel{M_E} M_s}{R^2} = \frac{\cancel{M_E} v^2}{R}$$

$$\frac{G M_s}{R^2} = \frac{v^2}{R} \;;\quad \frac{G M_s}{R} = v^2$$

$$M_s = \frac{v^2 R}{G} = \text{mass of the Sun}$$

Assuming the orbit of the earth about the Sun is circular with radius 1.5×10^{11} m. Find the mass of the Sun.

$$\frac{G M_s M_E}{R^2} = \frac{m_E v^2}{R}$$

$R = 1.5 \times 10^{11}$ m

$G = 6.672 \times 10^{-11}$

$$M_s = \frac{v^2 R}{G} = \underline{\underline{2.00 \times 10^{30} \text{ kg}}}$$

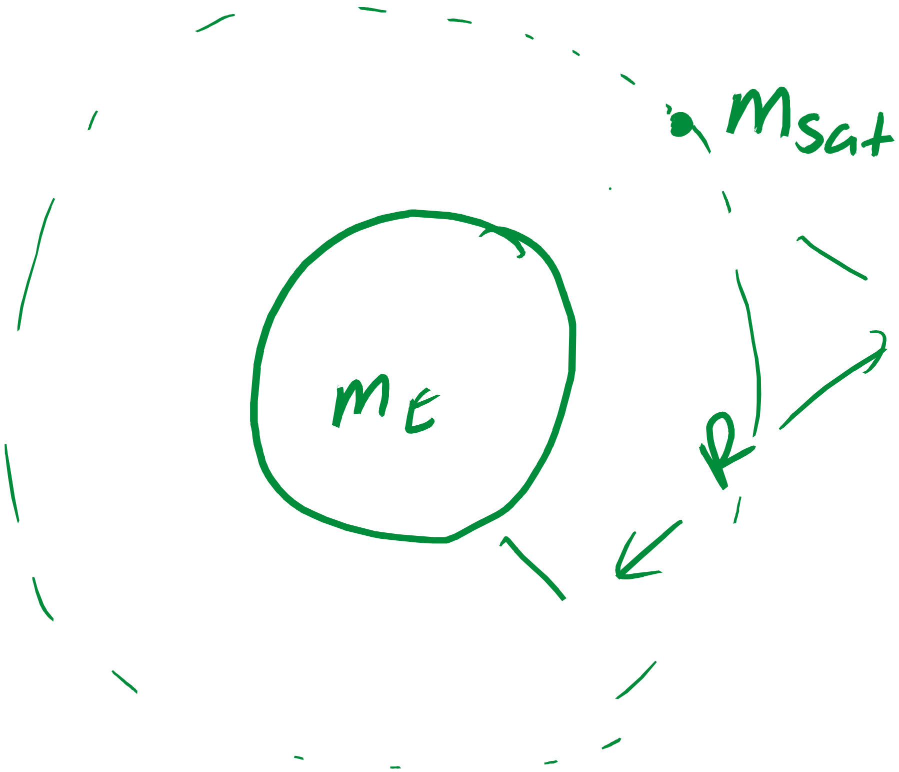
How do we find v?

$$v = \frac{2\pi R}{T}$$

$\therefore T = 365.25$ days

$= 3.16 \times 10^7$ s

$$= \frac{2\pi (1.5 \times 10^{11})}{3.16 \times 10^7} = 2.98 \times 10^4 \text{ m/s}$$



$$\frac{G M_E M_{sat}}{R^2} = \frac{M_{sat} v^2}{R}$$

$$\frac{G M_E}{R} = v^2$$

$$M_E = \frac{v^2 R}{G}$$

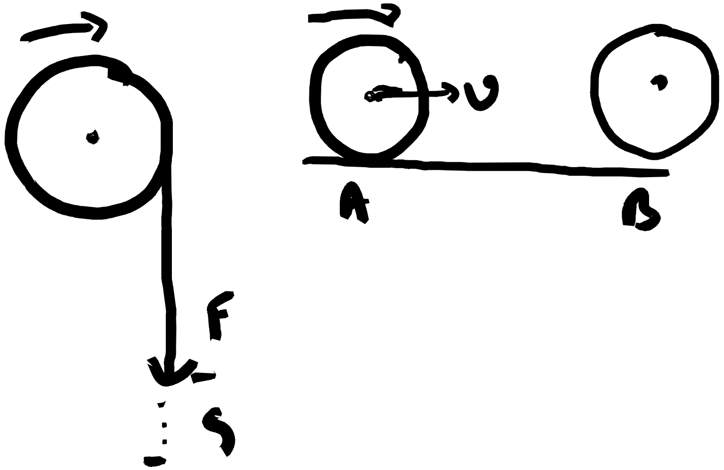
M_{sat} = moon

30 days

$$v = \frac{2\pi R}{T} = \frac{2\pi R}{30 \times 24 \times 3600\,s}$$

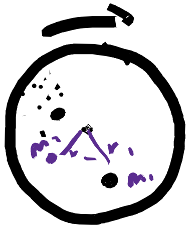
Rotational Work, Energy & Momentum

- According to the net work energy theorem the work done on the wheel by the net force appears as KE



The KE of ~~as ob~~ the wheel due to its linear motion is

$$KE_{trans} = \frac{1}{2} m v^2$$



The KE of small pieces on the rotating wheel is

$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \dots + \frac{1}{2}m_N v_N^2$$

$v = \omega r$

$$= \frac{1}{2}m_1(\omega r_1)^2 + \frac{1}{2}m_2(\omega r_2)^2 + \dots + \frac{1}{2}m_N(\omega r_N)^2$$

$$= \frac{1}{2}\omega^2\left[m_1 r_1^2 + m_2 r_2^2 + \dots + m_N r_N^2\right]$$

$$\boxed{KE_r = \frac{1}{2}\omega^2 I}$$

I = moment of Inertia

$$= m_1 r_1^2 + m_2 r_2^2 + \dots + m_N r_2^2$$

$$I = \sum_i m_i r_i^2$$

it has units of $kg\,m^2$

Already we can see a connection with linear motion

$a \equiv \alpha$

$v \equiv \omega$

$m \equiv I$

$F = \tau$

$KE_T = \frac{1}{2} m v^2$

$KE_R = \frac{1}{2} I \omega^2$

- whenever a wheel is rotating a moving from point to point B,

it has two KEs :

$KE_T + KE_R$

$\frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

Newton's Second law of motion

- For an object to move, there must be a net force. This net force causes acceleration
- Similarly for a wheel to rotate there must be a 'force'

$$F = ma$$

-
- multiply by r on both sides

$$rF = mra \qquad a = \alpha r$$

$$rF = mr^2\alpha$$

$$rF = mr^2\alpha$$

$$\boxed{\tau = I\alpha}$$

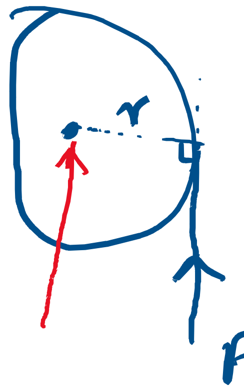
$y = x$

$100y = \cancel{100}x$

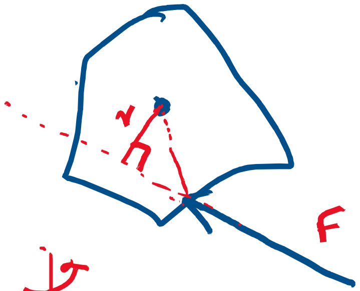
$y = x$

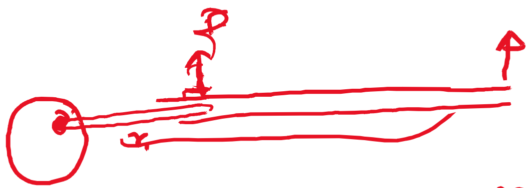
$\frac{4}{5} \frac{\times 100}{\times 100}$

Torque = turning effect = Force × lever arm → perpendicular distance from the axis ~~to~~ of rotation to the line of force



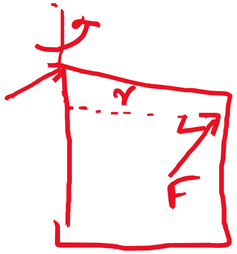
$\tau = rF$

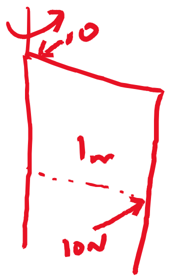




$\tau = r_1 F$

$r_2 F = \tau \uparrow$

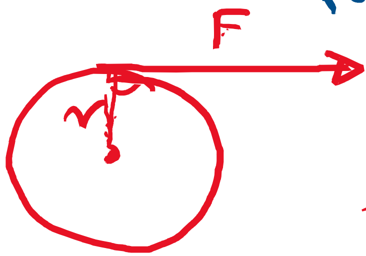




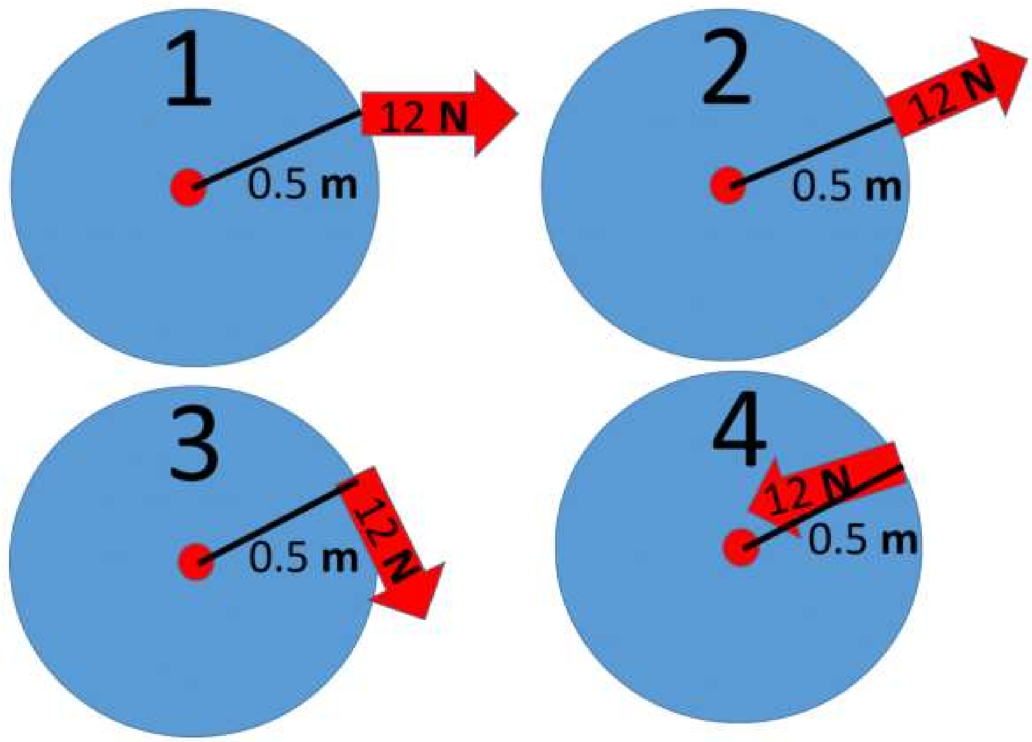
$r = 0.01m$

$\tau = 10(0.01)$
$= 0.1 Nm$

$\tau = F \times r$
10×1
$= 10 Nm$



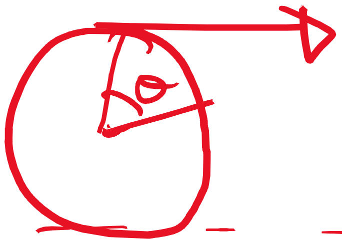
τ causes the object to rotate



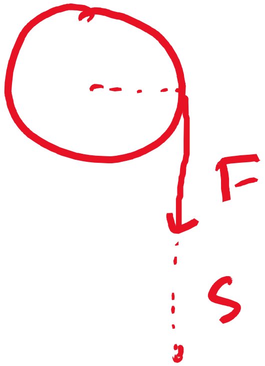
1
12 N
0.5 m
2
12 N
0.5 m
3
12 N
0.5 m
4
12 N
0.5 m

Workdone by a force in rotating an object

$$W = F\,S$$



Rotational work $= \tau\theta$: $\boxed{W_R = \tau\theta}$



$\theta = s/r$: $s = \theta r$

$$W = F\,S$$

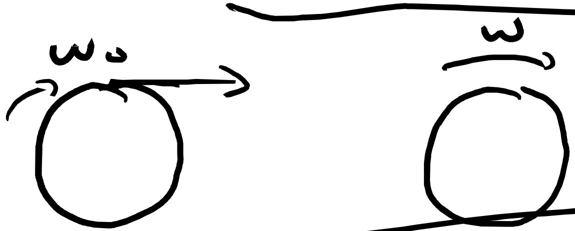
$$= F r \theta$$

$$\boxed{W = \tau\theta}$$

Work - Energy theorem

- Net force causes change in v

$$\boxed{F_{net} \cdot S = \frac{1}{2} m v^2 - \frac{1}{2} m u^2}$$



$$\boxed{\tau \theta = \frac{1}{2} I \omega^2 - \frac{1}{2} I \omega_0^2}$$

Moment of Inertia of different objects

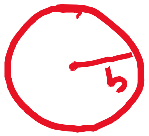
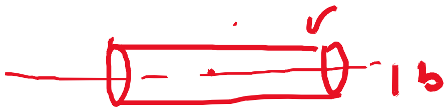
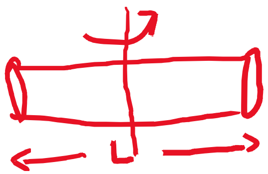
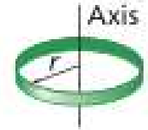
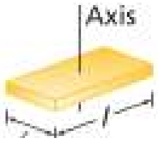
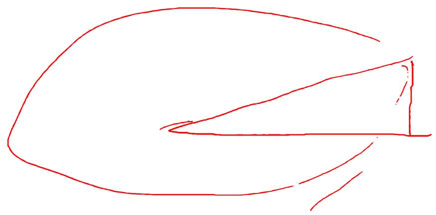
Object	axis	I
Point mass		mr^2
 hoop		mb^2
Solid disk		$\frac{1}{2}mb^2$
Solid Sphere		$\frac{2}{5}mb^2$
Solid cylinder		$\frac{1}{2}mb^2$
thin Solid cylinder		$\frac{1}{12}mL^2$

Table 8-2			
Moments of Inertia for Various Objects			
Object	**Location of Axis**	**Diagram**	**Moment of Inertia**
Thin hoop of radius r	Through central diameter		mr^2
Solid, uniform cylinder of radius r	Through center		$\frac{1}{2}mr^2$
Uniform sphere of radius r	Through center		$\frac{2}{5}mr^2$
Long, uniform rod of length l	Through center		$\frac{1}{12}ml^2$
Long, uniform rod of length l	Through end		$\frac{1}{3}ml^2$
Thin, rectangular plate of length	Through center		$\frac{1}{12}m(l^2 + w^2)$

- In general

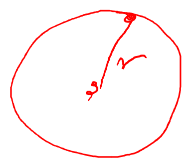
$$\boxed{I = mk^2}$$

k = radius of gyration



where

- point mass $\quad k = r$
- Hoop $\quad k = b$
- Solid disk $\quad k = b/\sqrt{2}$



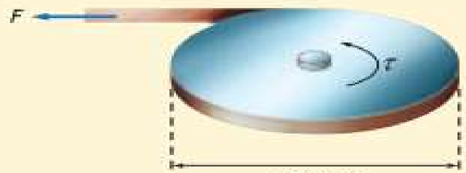
- Solid sphere $\quad k = b\sqrt{2/5}$

- Cylinder $\quad k = b/\sqrt{2}$

Torque A solid steel wheel has a mass of 15 kg and a diameter of 0.44 m. It starts at rest. You want to make it rotate at 8.0 rev/s in 15 s.

a. What torque must be applied to the wheel?

b. If you apply the torque by wrapping a strap around the outside of the wheel, how much force should you exert on the strap?



1 Analyze and Sketch the Problem

- Sketch the situation. The torque must be applied in a counterclockwise direction; force must be exerted as shown.

Known:	Unknown:
$m = 15$ kg	$\alpha = ?$
$r = \frac{1}{2}(0.44\text{ m}) = 0.22$ m	$I = ?$
$\omega_i = 0.0$ rad/s	$\tau = ?$
$\omega_f = 2\pi(8.0\text{ rev/s})$	$F = ?$
$t = 15$ s	

ω_f $\quad \tau = \alpha I$

$\tau = rF$

2 Solve for the Unknown

a. Solve for angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{2\pi(8.0\text{ rev/s}) - (0.0\text{ rad/s})}{15\text{ s}}$$ **Substitute $\omega_f = 2\pi(8.0$ rev/s$)$, $\omega_i = 0.0$ rad/s**

$$= 3.4\text{ rad/s}^2$$

Solve for the moment of inertia.

$$I = \frac{1}{2}mr^2$$

$$= \frac{1}{2}(15\text{ kg})(0.22\text{ m})^2$$ **Substitute $m = 15$ kg, $r = 0.22$ m**

$$= 0.36\text{ kg·m}^2$$

Solve for torque.

$$\tau = I\alpha$$

$$= (0.36\text{ kg·m}^2)(3.4\text{ rad/s}^2)$$ **Substitute $I = 0.36$ kg·m², $\alpha = 3.4$ rad/s²**

$$= 1.2\text{ kg·m}^2/\text{s}^2$$

$$= 1.2\text{ N·m}$$

b. Solve for force.

$$\tau = Fr$$

$$F = \frac{\tau}{r}$$

$$= \frac{1.2\text{ N·m}}{0.22\text{ m}}$$ **Substitute $\tau = 1.2$ N·m, $r = 0.22$ m**

$$= 5.5\text{ N}$$

$F = \frac{\tau}{r}$

$= \frac{1.2}{0.22}$

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Operations with Significant Digits
pages 835–836

$\omega_f = 8$ rev/s

$= 8(2\pi)$ rad/s

$=$

$\alpha = \frac{\omega_f - \omega_0}{t}$

$= \frac{8(2\pi) - 0}{15} =$

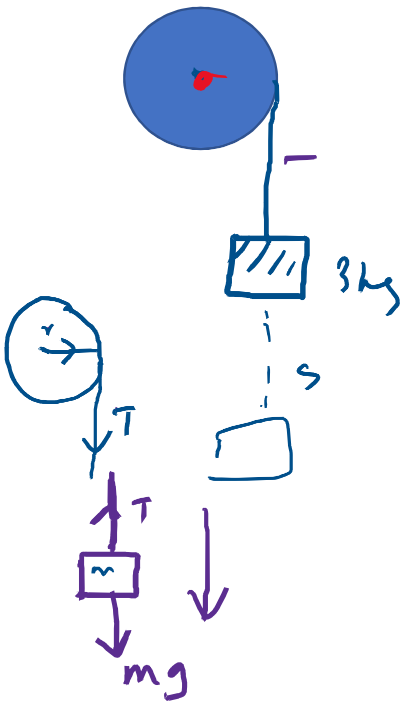
$I = \frac{1}{2} m b^2$

$= \frac{1}{2} 15 (0.\cancel{44}\,22)^2$

A 3.00 kg (m) block hangs from a cord wound on a 40.0 kg (M) wheel. The wheel has a radius (r) of 0.750 m and a radius of gyration of 0.600 m (k)

Find a) the angular acc of the wheel

b) the distance the block falls in the first 10 sec after it is released



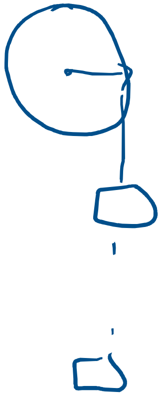
$$I = mk^2 = Mk^2 \quad ; \quad \tau = rT$$

$$\tau = I\alpha \qquad \alpha = \tau/I = \frac{rT}{I} \quad ; \quad \cancel{\alpha = a} \quad a = \alpha r$$

$$\alpha = \frac{rT}{Mk^2} \quad (1) \quad \Rightarrow \frac{Mk^2\alpha}{r} = T$$

$$\rightarrow \quad mg - T = ma$$

$$mg - \frac{Mk^2\alpha}{r} = m\alpha r \rightarrow \; \alpha = \frac{mgr}{Mk^2 + mr^2} = 1.37 \text{ rad/s}^2$$



$$a = r\alpha$$

$$= (0.75)(1.37) = 1.03\ \mathrm{m/s^2}$$

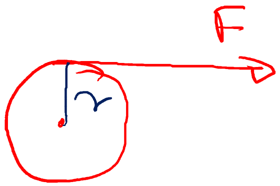
$$y = ut + \frac{1}{2} a t^2$$

$$= 0 + \frac{1}{2}(1.03)(10)^2$$

$$= 57.5\ \mathrm{m}$$

A wheel of radius 40 cm has a mass of 30 kg and radius of gyration of 25 cm. A cord wound around its rim supplies a tangential force of 1.8 N to the wheel which turns freely on the axle through its centre. Find the angular acceleration of the wheel.

$$\tau = I\alpha \quad ; \quad \alpha = \tau / I$$



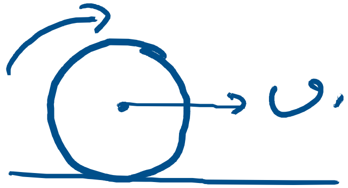
$$I = mk^2 = 30(0.25)^2 = 1.9 \text{ kgm}^2$$

$$\tau = rF = 0.4(1.8) = 0.72 \text{ Nm}$$

$$\alpha = \frac{0.72}{1.9} = \cancel{0.72}\ 0.38 \text{ rad/s}^2$$

Combined Rotation and Translational Motion.

- Consider a rolling wheel (without slipping)

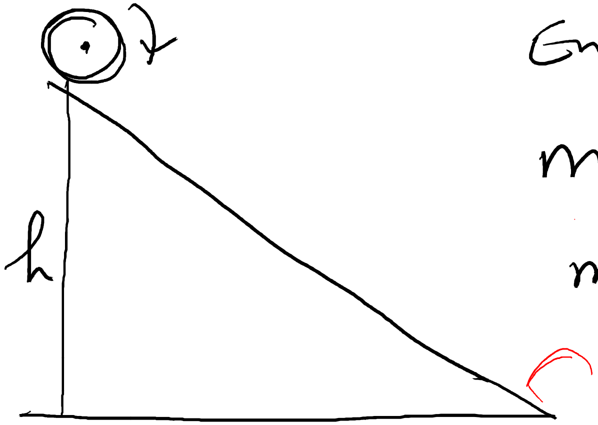


- Since it moves from point A to B, it ~~contains two~~ possesses both KE_T and KE_R

$$\text{Total } KE = KE_T + KE_R$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

A uniform sphere of radius r and mass m starts from rest at the top of the incline at height h and rolls down. How fast is the sphere moving when it reaches the bottom. [neglect friction).



$I_{sphere} = \frac{2}{5} m r^2$

Energy at the top:

$v = r\omega$

$\omega = v/r$

$$mgh = KE$$

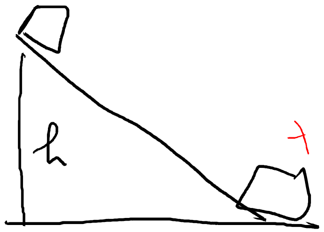
$$mgh = KE_T + KE_R$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} I \left(\frac{v}{r}\right)^2$$

$$\cancel{m} g h = \frac{1}{2} \cancel{m} v^2 + \frac{1}{\cancel{2}} \left(\frac{\cancel{2}}{5} \cancel{m} \cancel{r^2}\right) \frac{v^2}{\cancel{r^2}}$$

$$gh = \frac{1}{2} v^2 + \frac{1}{5} v^2$$



$\cancel{m} g h = \frac{1}{2} \cancel{m} v^2$

$\cancel{v^2 = 2gh}$

$v = \sqrt{2gh}$

$$v^2 = \frac{10}{7} g h$$

$$v = \sqrt{\frac{10}{7} g h}$$