Mechanics

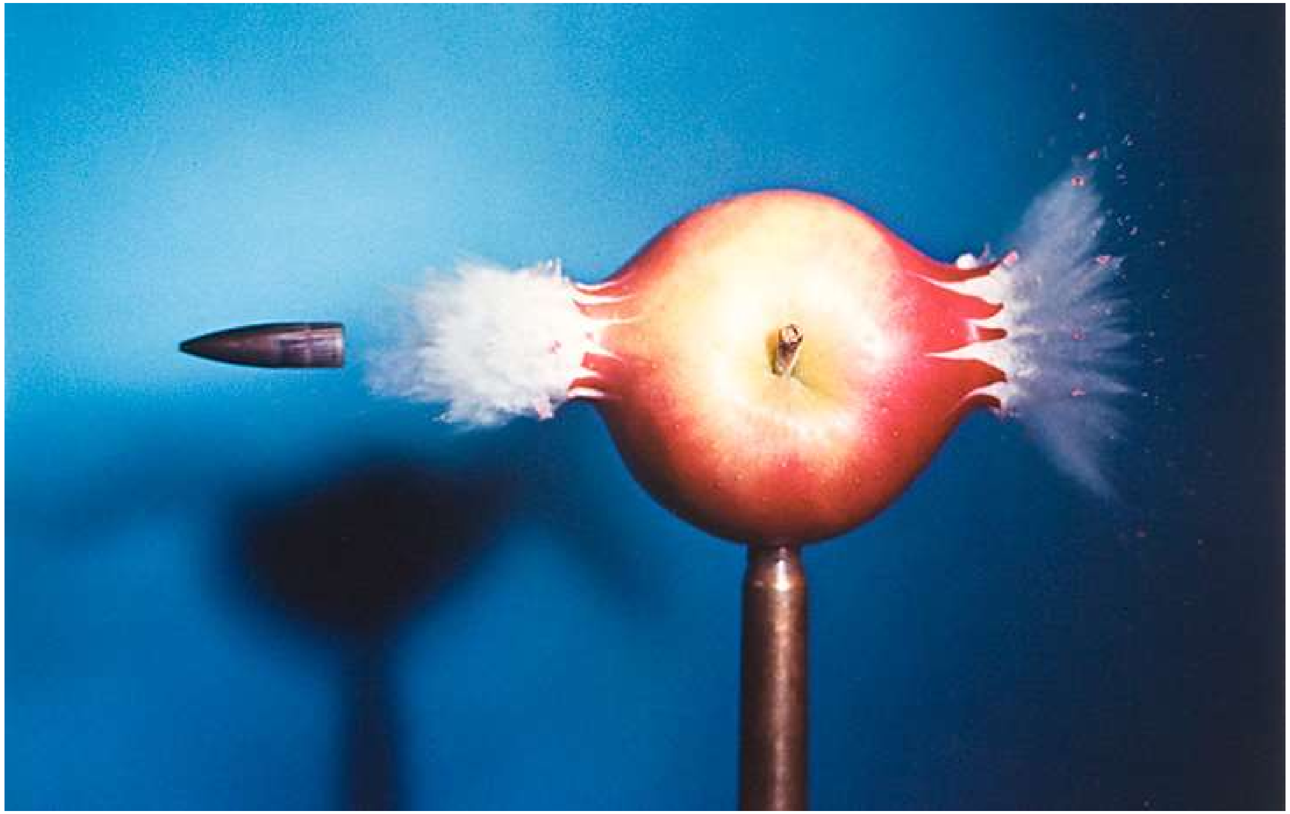
Momentum

1. Define Momentum
2. Impulse
3. Applications
4. Collissions

- An experience common to us is that a moving object possesses a property that causes it exert a force on anyone or anything trying to stop it.

- The faster the object is moving, the harder it is to stop.

eg bullet. [m↓ v↑]



- The more massive the object the harder it is to stop it

[$m\uparrow v\downarrow$]

- This property is a function of velocity & mass

- This property is called <u>momentum</u>.

- In this topic we shall discuss <u>linear momentum</u>.

- Momentum (P) = mass × Velocity

$$\vec{P} = m\vec{v}$$

mass ≡ kg , Velocity ≡ m/s

Units for momentum = kg m/s

momentum is a vector, in the direction of vel.

Ex. An athletics hammer has mass of 7.26 kg and can be released at speeds in excess of 25 m/s. What is its momentum

$$p = mv$$

$$= 7.26 \times 25$$

$$= 182 \text{ kg m/s}$$

Connection between KE & P

- We know that KE is also a fn of mass & velocity.

-
$$E_k = \frac{1}{2} m v^2$$

$$E_k = \frac{1}{2} m v^2 \cdot \frac{m}{m}$$

$$E_k = \frac{1}{2} \frac{m^2 v^2}{m}$$

$$E_k = \frac{1}{2} \frac{(mv)^2}{m}$$

$$E_k = \frac{1}{2} \frac{p^2}{m} = \frac{p^2}{2m}$$

But $p = mv$

Connection between KE & P.

$$\boxed{E_k = \frac{p^2}{2m}}$$

What happens to kinetic energy when momentum is doubled.

What happens to E_k when momentum is doubled.

$$E_k = \frac{p^2}{2m}$$

$$E_k = \frac{(2p)^2}{2m} = \frac{4p^2}{2m}$$

$$E_k = 4 \times \frac{p^2}{2m} \Rightarrow \text{it quadrupples.}$$

What happens to E_k when momentum is trebbled.

$$E_k = \frac{p^2}{2m}$$

$$E_k = \frac{(3p)^2}{2m}$$

$$E_k = 9 \times \frac{p^2}{2m}$$

$\Rightarrow$ Increases by a factor of 9.

Newton's Second law re-stated.

- We discuss the relationship between net force applied to an object and the change in linear momentum.

$$F_{net} = m a$$

$$= m \left[\frac{v - u}{\Delta t} \right]$$

$$= \frac{mv - mu}{\Delta t}$$

$$F = \frac{P_f - P_i}{\Delta t} = \frac{\Delta P}{\Delta t}$$

$P_f = mv$ = final

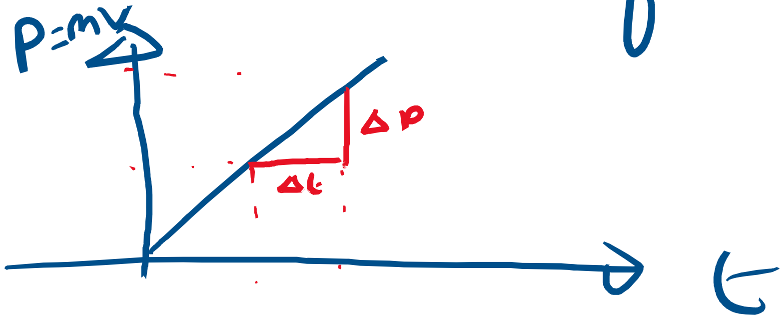
$P_i = mu$ = Initial

$$F = \frac{\Delta P}{\Delta t}$$

In calculus

$$F = \frac{dP}{dt}$$

Force is the rate of change of momentum



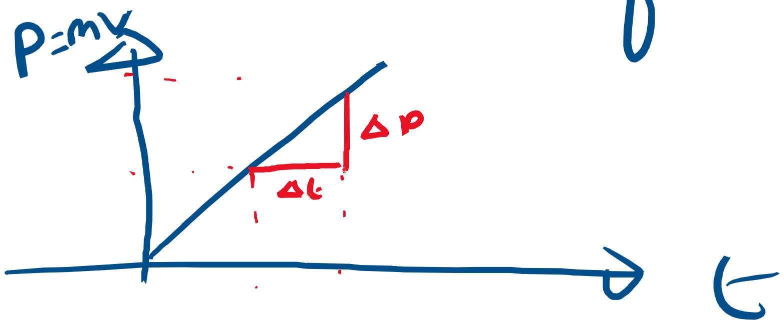
slope $= \frac{\Delta P}{\Delta t}$ = force

$$F = \frac{\Delta p}{\Delta t}$$

In calculus

$$F = \frac{dp}{dt}$$

Force is the rate of change of momentum



slope $= \frac{\Delta p}{\Delta t} =$ force

Impulse

$$I = F\,\Delta t$$

product of force and duration.

$$I = \Delta p = F\,\Delta t$$

↳ Change in mom.

$$y = \frac{\alpha}{\beta} = \frac{\text{numerator}}{\text{denominator}}$$

$$\Rightarrow \frac{1}{1} = 1$$

$$\Rightarrow \frac{1}{10} = 0.1$$

$$\Rightarrow \frac{1}{100} = 0.01$$

Increase the β:

$\alpha = 1$

$\beta = 1, 10, 100, 1000$

$$y = \frac{\alpha}{\beta} = \frac{\text{numerator}}{\text{denominator}}$$

$$\Rightarrow \frac{1}{1} = 1$$

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$$\Rightarrow \frac{1}{100} = 0.01$$

Increase the β.

$\alpha = 1$

$\beta = 1, 10, 100, 1000$

as $\beta \uparrow$, $y \downarrow$

Consequences of N- 2nd law

$$\therefore F = \frac{\Delta P}{\Delta t} = \frac{dP}{dt}$$

To reduce the force, you reduce the change in momentum

$$F\downarrow = \frac{\Delta P\downarrow}{\Delta t}$$

$m = 1\,kg$, $10\,m/s = u$

$0 = v$

$\Delta P = P_f - P_i$

$= mv - mu$

$= 1(0) - 1(10)$

$= -10\,kg\,m/s^2$

$10\,m/s \longrightarrow 0$, $F\uparrow$

Consequences of N- 2nd law

$$\therefore F = \frac{\Delta P}{\Delta t} = \frac{dP}{dt}$$

To reduce the force, you reduce the change in momentum

$$F\downarrow = \frac{\Delta P\downarrow}{\Delta t}$$

$m = 1\,kg$, $10\,m/s = u$

$9\,m/s = v$

$$\Delta P = P_f - P_i$$
$$= mv - mu$$
$$= 1(9) - 1(10)$$
$$= -1\ kg\,m/s\ 2$$

$10 \rightarrow 9 \rightarrow 0$, $F\downarrow$

Consequences of N- 2nd law

$$\therefore F = \frac{\Delta P}{\Delta t} = \frac{dP}{dt}$$

To reduce the force, you increase the duration in which mom. changes.

$$F\downarrow = \frac{\Delta P}{\Delta t\uparrow}$$

$$\frac{n\,P}{d\uparrow} = b$$

$\frac{1}{10} = 0.1$

$\frac{1}{100} = 0.01$

$\frac{1}{1000} = 0.001$

The opposite is true

$$F\uparrow = \frac{\Delta p \uparrow}{\Delta t}$$

OR

$$F\uparrow = \frac{\Delta p}{\Delta t \downarrow}$$

reducing the duration

$$\frac{1}{10} = 0.1$$

$$\frac{1}{1} = 1$$

$$\frac{1}{0.1} = 10$$

Activity

If you find someone willing to play 'catch with eggs', what is the best way to move your hands so that the egg does not break when you change its momentum to zero?

EXAMPLE 7–2 **Washing a car: momentum change and force.** Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it, Fig. 7–2. (That is, we ignore any splashing back.) What is the force exerted by the water on the car?

Rate 1.5 kg/s :

$m = 1.5\ kg, \quad 1s = t$

$u = 20 m/s$

$v = 0$

$$F = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{t}$$

$$= \frac{mv - mu}{t}$$

$$= \frac{0 - 1.5(20)}{1}$$

$$\underline{\underline{F = -30\,N}}$$

Ex: A 1200 kg car initially moving at 20 m/s strikes a tree and comes to rest in a distance of $s = 1.5\,m$. Estimate the average stopping force.

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$$F = \frac{\Delta p}{\Delta t} = \frac{m v_f - m u}{t}$$

$$m v_f = m(0) = 0$$

$$m u = 1200(20) = 24\,000 \text{ kg m/s}$$

what about t?

Ex: A 1200 kg car initially moving at 20 m/s strikes a tree and comes to rest in a distance of $s = 1.5$ m. Estimate the average stopping force.

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_f - mu}{t}$$

$$mv_f = m(0) = 0$$

$$mu = 1200(20) = 24\,000 \text{ kg m/s}$$

what about t?

$$s = \bar{v}t \therefore t = \frac{s}{\bar{v}} = \frac{1.5}{10} = 0.15\,s$$

Ex: A 1200 kg car initially moving at 20 m/s strikes a tree and comes to rest in a distance of $s = 1.5\,m$. Estimate the average stopping force.

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_f - mu}{t}$$

$$\bar{v} = \frac{v+u}{2} = \frac{0+20}{2} = 10\,m/s$$

$$mv_f = m(0) = 0$$

$$mu = 1200(20) = 24\,000\ kg\,m/s$$

what about t?

$$s = \bar{v}t \therefore t = \frac{s}{\bar{v}} = \frac{1.5}{10} = 0.15\,s$$

$$\therefore \quad F = \frac{P_f - P_i}{t} = \frac{0 - 24\,000}{0.15}$$

$$= -1.6 \times 10^5 \, N$$

The negative indicates that the force is in the opposite direction.

Applications

1. Using ideas of N-2nd law, hitting an airbag will cause less injury than if a passenger hits the dash board.

Very Injurious | Less Injurious

$$F_{\downarrow} = \frac{\Delta p}{\Delta t_{\uparrow}}$$

Air bags have saved millions of people.

In the absence of an air bag the passenger hits the ~~[illegible]~~ dashboard and momentum changes in an instant

The person comes to stop immediately

$\therefore$ Δt is small : $F\uparrow = \Delta P / \Delta t \downarrow$

This causes serious injuries

2. Jumping from a height

- If you jump from a height and lock your knees, the time for momentum change is very small

$$\uparrow F = \frac{\Delta p}{\Delta t \downarrow}$$

- This can damage your knee or break your leg

$= \Delta t \uparrow$

- The best way to jump is when you land, you bend your knees until you are in a squatting position.
- In so doing you are simply prolonging the time in which momentum is changing.

- The best way to jump is when you land, you bend your knees until you are in a squatting position.

- In so doing you are simply prolonging the time in which momentum is changing

$$F_{\downarrow} = \frac{\Delta P}{\Delta t \uparrow}$$

This will exert less force on your knees or legs.

- After the Impact the body is at rest or momentum is zero :

$$\therefore \Delta p = p_f - p_i = 0 - W\sqrt{2h/g}$$

$$= -W\sqrt{2h/g}$$

- The Impact force $= \dfrac{\Delta p}{\Delta t} = -\dfrac{W}{\Delta t}\sqrt{2h/g}$

• hard impact, with knees locked $\Delta t \simeq 10^{-2}\,s$

• If the person bends or squats, $\Delta t \simeq$ longer

Fracture due to fall.

- Falling on soft sand is less injurious than falling on a hard a concrete surface.
- This is because the duration of the collision is lengthened. On concrete it is short.
- When a person falls from a height h, his or her velocity on impact is, neglecting air friction

$$v = \sqrt{2gh}$$

The momentum on impact is

$$mv = m\sqrt{2gh}$$
$$= W\sqrt{2h/g}$$

$$mgh = \frac{1}{2}mv^2$$
$$v = \sqrt{2gh}$$

- Using the same principle it is less painful if you fall on a matress than on a hard floor.

- Footballers are able to trap balls by simply prolonging the duration of momentum change

Why are cars made with bumpers that can be pushed in during a crash?

Can a bullet have the same momentum as a truck? Explain

You are sitting at a baseball game when a foul ball comes in your direction. You prepare to catch it bare-handed. To catch it safely, should you move your hands toward the ball, hold them still, or move them in the same direction as the moving ball? Explain.

Why are cars made with bumpers that can be pushed in during a crash?

Conservation of Momentum

- When objects collide, the laws of physics allows us to predict where they will go after the collision.
- The principle of conservation of momentum can be used to predict the motion of objects after collision.

- Just as energy is conserved, linear momentum is also conserved

- If a stationary object explodes, the total momentum of all the fragments even if they are moving must add up to zero because the object had zero momentum at start.

What is the principle of conservation of momentum?

Law of cons. of linear momentum.

For any Isolated System, the total linear momentum is constant.

An isolated system is a group of objects on which there is no external influence of force.

Mathematically, Conservation of momentum is

Total Momentum before = Total Momentum After

$$\sum P_i = \sum P_f$$

$$\boxed{\sum P_i = \sum P_f}$$