# The University of Zambia
# School of Natural Sciences
# Department of Physics
# PHY 1010
# Lecture 4
# Work, Energy and Power

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## Introduction

This lecture introduces the concepts of **work**, **energy**, the **work-energy theorem**, and the **conservation of energy** and **power**.

## Learning Outcomes

By the end of this lecture, the student should be able to:

1. understand the concept of work in terms of the product of a force and displacement in the direction of the force;
2. calculate the work done in a number of simple situations;
3. recall and apply the formula for kinetic energy $KE = \frac{1}{2}mv^2$
4. understand and recall that the gravitational potential energy GPE changes near the earth's surface is equal to the work done by the weight of the object i.e. $GPE = mgh$.
5. recall and apply the principle of conservation of energy to a number of simple situations.

6. define power as work done per unit time and derive power as the product of force and velocity i.e. $P = Fv$

7. solve problems using the relationships $P = \dfrac{W}{t}$ and $P = Fv$.

# Work

In Physics, the **work done** by a force $F$ is defined as the product of the force and the displacement $s$ in the direction of the force. The force that is of interest here is the component of the force $F_x$ in the direction of the displacement $s$. Consider a situation where a force $F$ acts on a body and the body undergoes a displacement $s$. The component of the force $F$ in the direction of displacement $s$ is given by $F_x = F\cos\theta$ where $\theta$ is the angle between the Force $F$ and the displacement $s$. Therefore, the work done by the force is given as

$$W = F_x s = (F\cos\theta)(s)$$

$$W = Fs\cos\theta \tag{1}$$

where

$F$ is the external force applied on an object

$s$ is the displacement

$\theta$ is the angle between the direction of the force $F$ and the displacement $s$

The work done by external force $F$ is a **scalar quantity** i.e. it only has magnitude and has no direction. The SI unit for work is the **Joule** denoted $J$. The **Joule** is also equivalent to the **newton-meter**. 1 Joule is defined as the the work done by a external force of 1 N when it moves or displaces an object by 1 m in the direction of the external force.

If an external force $F$ applied on the object does not cause the object to cover a displacement i.e. $s = 0$ m, then the work done by the external force $F$ is zero.

The next important point is that work done is also dependent on $\theta$.

If the angle between the external force $F$ and the displacement covered by the object $s$ is $\theta = 0°$ i.e. the external force $F$ and displacement $s$ are pointing in the same direction, then the work done is given by

$$W = Fs\cos 0° = Fs \tag{2}$$

The work done on the object in this case is positive. **Positive work** results in the transfer of energy from the object applying the external force $F$ to the object that is moved by the external force $F$ through a displacement $s$.

If the external force $F$ applied on an object is acting in a direction that is perpendicular to the displacement of the object $s$, then in this case $\theta = 90°$, and we get the work done on the object as zero.

$$W = Fs\cos 90^\circ = 0 \tag{3}$$

The work done in this case is zero. **Zero work** means that there is no energy transfered by the external force $F$ to the object that is displaced through $s$.

If the external force $F$ acting on an object is acting in a direction that is opposite the motion of the object, then the angle between the external force $F$ and the displacement $s$ will be $\theta = 180^\circ$. In this case, we get the work done by the external force $F$ as

$$W = Fs\cos 180^\circ = -Fs \tag{4}$$

Thw work done in this case is negative. **Negative work** means that energy is not transferred to the object but instead is removed.

**Example 1**

> **A force of 3.0 N acts through a distance of 12 m in the same direction of the force. Find the work done.**
>
> The work done $W$ is given by
>
> $$W = Fs\cos\theta$$
>
> where $F$ is applied force, $s$ is the displacement and $\theta$ is the angle between $F$ and $s$.
>
> $$W = (3.0 \text{ N})(12 \text{ m})\cos 0 = 36 \text{ J}$$

**Example 2**

> **An object is being pulled along the ground by a 75 N force directed at** $28^\circ$ **above the horizontal. How much work does the force do in pulling the object 8.0 m.**
>
> The work done $W$ is given by
>
> $$W = Fs\cos\theta$$
>
> where $F$ is applied force, $s$ is the displacement and $\theta$ is the angle between $F$ and $s$.
>
> $$W = (75 \text{ N})(8.0 \text{ m})\cos 28^\circ = 530 \text{ J}$$

**Example 3**

**A 0.3 kg object slides 0.80 m along the horizontal table. How much work is done overcoming friction between the object and the table if the coefficient of friction is 0.20.**

The displacement $s = 0.80$ m, mass of object $m = 0.3$ kg, coefficient of kinetic friction $\mu_k = 0.20$

Kinetic friction force $F_f$ is given by

$$F_f = \mu_k F_N$$

The normal force $F_N$ is given by

$$F_N = mg$$

$$F_N = (0.3 \text{ kg})(9.8 \text{ m/s}^2)$$

$$F_N = 2.94 \text{ N}$$

Therefore, we get friction force as

$$F_f = \mu_k F_N$$

$$F_f = (0.20)(2.94 \text{ N})$$

$$F_f = 0.588 \text{ N}$$

The kinetic friction force $F_f$ always acts in a direction that is opposite the motion. The angle between the friction force $F_f$ and the displacement $s$ is 180°.

The work done $W$ is given by

$$W = F_f s \cos\theta$$

where $F_f$ is kinetic friction force, $s$ is the displacement and $\theta$ is the angle between $F$ and $s$.

$$W = (0.588 \text{ N})(0.8 \text{ m}) \cos 180^\circ$$

$$W = -0.47 \text{ J}$$

**Example 4**

**How much work is done against gravity in lifting a 3.0 kg object at constant velocity through a distance of 0.4 m**

To lift an object, an external force $F$ has to be applied on the object being lifted. If we assume that the object is lifted at a constant velocity i.e the object is not accelerating, then the lifting force $F$ will be equal to the weight of the object $F_W$.

$$F_{\text{lift}} = F_W = mg$$

$$F_{\text{lift}} = (3 \text{ kg})(9.8 \text{ m/s}^2)$$
$$F_{\text{lift}} = 29.4 \text{ N}$$

The lifting force $F_{\text{lift}}$ and the displacement $s = 0.4$ m are in the same direction. Therefore, $\theta = 0°$.

The work done $W$ is given by

$$W = F_{\text{lift}} s \cos\theta$$

where $F_{\text{lift}}$ is lifting force, $s$ is the displacement and $\theta$ is the angle between $F_{\text{lift}}$ and $s$.

$$W = (29.4 \text{ N})(0.4 \text{ m}) \cos 0°$$

$$W = 11.8 \text{ J}$$

## Energy

Life as we know it on Earth is not possible without the energy we receive from the Sun, our nearest star. The Earth receives 174 petawatts (PW) of incoming solar radiation at the upper atmosphere. Approximately 30% is reflected back to space while the rest is absorbed by clouds, oceans and land masses. A fraction of the solar radiation that reaches the Earth's landmass and oceans is captured by plants and algea containing chroloplasts via Photosynthesis. The captured solar radiation is stored in plants in form of chemical compounds such as carbohydrates, proteins, fats etc in a form called **chemical energy**.

When we eat food for breakfast, lunch or supper, we are simplying trying to access the chemical energy stored in the food. This chemical energy is what makes it possible for us as humans to go about our daily activities. When the chemical energy that we accessed through the food we eat gets depleted, we get a sensation of hunger and our bodies will tell us to look for new food to eat. The chemical energy we get from food is used for motion, heat generation to keep our body temeperature at the right temperature and for many other functions in our body.

# Forms of Energy

## Kinetic Energy

Kinetic Energy (K.E) is the energy possessed by an object because of its motion. If an object of mass $m$ is moving with a speed $v$, it has translational energy KE given by

$$KE = \frac{1}{2}mv^2 \tag{5}$$

where $m$ is in kg and $v$ in m/s, the units of K.E are Joules.

# Gravitational Potential Energy (GPE)

**Gravitational Potential Energy** (GPE) is the energy possessed by an object because of its position in a gravitational field. A gravitational field is a region in which the force of attraction due to gravity is felt. If an object with mass $m$ falls through a vertical distance $h$, it does an amount of work equal to $mgh$. The GPE is always given relative to the Earth's surface. If an object is at a height $h$ above the surface of the Earth, then its GPE is given by

$$\text{GPE} = mgh \tag{6}$$

where $g$ is the accleration due to gravity, $m$ is mass in kg and $h$ is height in $m$. Notice that $mg$ is the weight of the object $F_W$. The units of GPE are Joules.

### Example 5

**A 2.0 kg mass falls through a distance of 4.0 m. (a) How much work is done on it by gravitational force (b) How much gravitational potential energy did it lose?**

The object has mass $m = 2.0$ kg, thus has a weight $F_W$ given by

$$F_W = mg$$

$$F_W = (2.0\ \text{kg})(9.8\ \text{m/s}^2)$$

$$F_W = 19.6\ \text{N}$$

The gravitation force is the weight of the object. The work done $W$ is given by

$$W = F_W s \cos\theta$$

where $F_W$ is the gravitaional force, $s$ is the height and $\theta$ is the angle between $F$ and $s$.

$$W = (19.6\ \text{N})(4\ \text{m})\cos 0°$$

$$W = 78.4 \text{ J}$$

The gravitational potential energy GPE lost is given by

$$\text{GPE} = mgh = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) = 78.4 \text{ J}$$

$$\text{GPE} = mgh$$

$$\text{GPE} = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m})$$

$$\text{GPE} = 78.4 \text{ J}$$

## The Work-Energy Theorem

The **work-energy theorem** is also known as the **principle of work and kinetic energy** states that when work is done on object of mass $m$ and there is no change in GPE, the energy transferred to the object can only appear as kinetic energy.

$$\text{Work done} = \text{change in KE}$$

$$W = \Delta KE \tag{7}$$

where

$W$ is the work done

$\Delta KE$ is the change in the kinetic energy of the object

The change in kinetic energy $\Delta KE$ of the object is given by

$$\Delta KE = KE_f - KE_i \tag{8}$$

where

$KE_f$ is the final kinetic energy of the object

$KE_i$ is the initial kinetic energy of the object

The initial and final kinetic energies are given by

$$KE_f = \frac{1}{2}mv^2 \qquad , \qquad KE_i = \frac{1}{2}mu^2 \tag{9}$$

where

$m$ is the mass of the object

$v$ is the final velocity of object at time $t$

$u$ is the initial velocity of the object at time $t = 0$

Therefore, the change in kinetic energy $\Delta KE$ can alternatively be written as

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \tag{10}$$

If the velocity of an object increases after work in done on it, the change it in its kinetic energy will be positive. This means that **positive work** was done on the object, resulting in the object gaining kinetic energy.

On the other hand, if the velocity of the object reduces after work is done on it, the change in kinetic energy of the object will be negative. This means that **negative work** was done on the object, resulting in the object losing its kinetic energy.

**Example 6**

**A 0.5 kg block slides across a table with an initial velocity of 0.20 m/s and comes to rest at a distance of 0.70 m. Find the average frictional force that retards its motion**

Mass of block, $m = 0.5$ kg, displacement of block $s = 0.70$ m.

The initial velocity of block $u = 0.20$ m/s. Therefore the block had initial Kinetic energy denoted $KE_i$ given by

$$KE_i = \frac{1}{2}mu^2 = \frac{1}{2}(0.5 \text{ kg})(0.20 \text{ m/s})^2 = 0.01 \text{ J}$$

When the block comes to rest, its final kinetic energy $v = 0$ m/s. Therefore block has final kenetic energy denoted $KE_f$ given by

$$KE_f = \frac{1}{2}mv^2 = \frac{1}{2}(0.5 \text{ kg})(0 \text{ m/s})^2 = 0 \text{ J}$$

The change in kinetic energy , $\Delta KE$, is given by

$$\Delta KE = KE_f - KE_i = 0 \text{ J} - 0.01 \text{ J} = -0.01 \text{ J}$$

The minus sign shows that the block lost kinetic energy due to negative work done on the block.

We now use the work-energy theorem, which states that the work done is equal to the change in kinetic energy.

$$W = \Delta KE$$

Therefore, we obtain the work done as

$$W = \Delta KE = -0.01 \text{ J}$$

But friction force $F_f$ is always acts in a direction that is opposite the direction of motion. Therefore, the angle between the friction force $F_f$ and the displacement of the object $s$ is $\theta = 180°$. We get the work done by the friction force $F_f$ in bring the object to a stop as

$$W = F_f s \cos\theta$$

$$W = F_f (0.70 \text{ m}) \cos 180°$$

$$F_f (0.70 \text{ m}) \cos 180° = -0.01 \text{ J}$$

$$-F_f (0.70 \text{ m}) = -0.01 \text{ J}$$

Dividing both side by -0.70 m, we obtain

$$F_f = \frac{-0.01 \text{ J}}{-0.7 \text{ m}}$$

$$F_f = 0.014 \text{ N}$$

The friction force $F_f$ is found to be 0.014 N.

**Example 7**

**The coefficient of kinetic friction between a 900 kg car and the pavement is $\mu_k$ =0.80. If a car is moving at 25 m/s along the level pavement when it begins to skid to a stop, how far will it go before stopping?**

The mass of the car $m = 900$ kg, the coefficient of kinetic friction $\mu_k = 0.80$. The initial velocity of the car is $u = 25$ m/s and since the car eventually comes to a stop, its final velocity is $v = 0$ m/s. The change in kinetic energy of the car $\Delta KE$ is given by

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Delta KE = \frac{1}{2}(900 \text{ kg})(0 \text{ m/s})^2 - \frac{1}{2}(900 \text{ kg})(25 \text{ m/s})^2$$

$$\Delta KE = -281\ 250 \text{ J}$$

The friction force the car experiences as its skids is given by

$$F_f = \mu_k F_N$$

Since the normal force $F_N = mg$, we get the kinetic friction force as

$$F_f = \mu_k mg$$

$$F_f = 0.80(900 \text{ kg})(9.8 \text{ m/s}^2)$$

$$F_f = 7\ 056 \text{ N}$$

The change in kinetic energy is equal to work done by friction force in bring the car to a stop. The friction force is always directed opposite the direction of motion.

The work done by kinetic friction force is given

$$W = F_f s \cos\theta$$

$$W = (7\ 056 \text{ N})(s)\cos 180^\circ$$

$$W = -(7\ 056 \text{ N})(s)$$

Applying the work - energy theorem, we get

$$W = \Delta KE$$

$$-(7\ 056 \text{ N})(s) = -281\ 250 \text{ J}$$

We obtain the stopping distance $s$ as follows

$$s = \frac{-281\ 250 \text{ J}}{-7\ 056 \text{ N}}$$

$$s \approx 40 \text{ m.}$$

The car comes to a stop in about 40 m.

## The principle of conservation of energy

The **principle of conservation of energy** states that energy can neither be created or destroyed, but is only converted from one form to another. In other words, the total energy of an object remains constant and is said to be conserved over time. For an object with KE and GPE, the sum of the change it its KE and GPE is always constant.

$$\Delta \text{K.E} + \Delta \text{GPE} = \text{constant}$$

**Example 8**

**A 1 kg projectile is shot upwards from earth with a speed of 20 m/s. How high is it when the speed is 8.0 m/s .**

Mass of projectile, $m = 1$ kg. The initial velocity of projectile $u = 20$ m/s and the final velocity $v = 8.0$ m/s. The change in the kinetic energy of the projectile $\Delta KE$ is given by

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Delta KE = \frac{1}{2}(1\ \text{kg})(8.0\ \text{m/s})^2 - \frac{1}{2}(1\ \text{kg})(20\ \text{m/s})^2$$

$$\Delta KE = -168\ \text{J}$$

The negative sign in $\Delta KE$ shows that the projectile lost kinetic energy. This lost kinetic energy according to the principle of conservation of energy cannot be destroyed. Instead, it is only changed to a different from of energy. In this case, the lost kinetic energy is changed to GPE. Thus, we have

$$\Delta GPE + \Delta KE = 0$$

$$\Delta GPE + (-168\ \text{J}) = 0$$

$$\Delta GPE - 168\ \text{J} = 0$$

$$\Delta GPE = 168\ \text{J}$$

The change in GPE is given by

$$\Delta GPE = mgh$$

$$\Delta GPE = (1 \text{ kg})(9.8 \text{ m/s}^2)h$$
$$(1 \text{ kg})(9.8 \text{ m/s}^2)h = 168 \text{ J}$$

we obtain the height $h$ the projectile rises to as

$$h = \frac{168 \text{ J}}{(1 \text{ kg})(9.8 \text{ m/s}^2)}$$

$$h = 17.1 \text{ m}$$

The projectile rises to a height of 17.1 m.

# Power

**Power** denoted $P$ is defined as the rate of doing work. More, generally, Power is how measure of how fast energy is transferred to an object.

$$\text{Average Power} = \frac{\text{Work done by force}}{\text{time taken to do this work}}$$

$$P = \frac{W}{t} \tag{11}$$

$$P = \frac{Fs\cos\theta}{t} \tag{12}$$

If the external applied force $F$ and and the displacement of the object $s$ are in the same direction, then $\theta = 0°$, then we get the power as

$$P = \frac{Fs}{t}$$

$$P = F\left(\frac{s}{t}\right)$$

$$P = Fv \tag{13}$$

where

$F$ is the external applied force acting on the object

$v$ is the average velocity of the object in the direction of the external force.

$$P = \frac{W}{t} = Fv \tag{14}$$

The SI unit for Power is the **Watt** denoted W. 1 W is defined as work done at a rate of 1 Joule per second or 1 N m/s. Another useful unit of Power is the

**kilowatt-hour**(kW·h). If a force is doing work at a rate of 1 kW ( i.e. 1000 J/s), then in 1 hour, it will do 1 kW·h of work. The kW·h is a measure of how much work a force does in 1 hour.

$$1 \text{ kW} \cdot \text{h} = 1000 \text{ J/s} \times 1 \text{ h}$$

$$1 \text{ kW} \cdot \text{h} = 1000 \text{ J/s} \times 3\ 600 \text{ s}$$

$$1 \text{ kW} \cdot \text{h} = 3\ 600\ 000 \text{ J}$$

$$1 \text{ kW} \cdot \text{h} = 3.6 \text{ MJ}$$

**Example 9**

**How large a force is required to accelerate a 1 300 kg car from rest to a speed of 20 m/s in distance of 80 m**

The car has a mass $m = 1300$ kg and increases speed from rest i.e. $u =$ 0 m/s to final velocity i.e. $v = 20$ m/s in a distance $s = 80$ m.

The change in kinetic energy, $\Delta KE$, of the car is given by

$$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Delta KE = \frac{1}{2}(1300 \text{ kg})(20 \text{ m/s})^2 - \frac{1}{2}(1300 \text{ kg})(0 \text{ m/s})^2$$
$$\Delta KE = 260\ 000 \text{ J}$$

Using the work-energy theorem, the work done by is equal to the change in kinetic energy

$$W = \Delta KE$$

$$W = Fs\cos\theta = 260\ 000 \text{ J}$$

$$F(80 \text{ m})\cos 0^\circ = 260\ 000 \text{ J}$$

$$F(80 \text{ m}) = 260\ 000 \text{ J}$$

The force provided by the car engine causing the acceleration is given by

$$F = \frac{260\ 000 \text{ J}}{80 \text{ m}}$$

$$F = 3\ 250 \text{ N}$$

**Example 10**

**Calculate the average power required to raise a 150 kg drum to a height of 20 m in a time of 60 s.**

The weight of the drum, $F_W$ is given by

$$F_W = mg$$

$$F_W = (150 \text{ kg})(9.8 \text{ m/s}^2)$$

$$F_W = 1\ 470 \text{ N}$$

If we assume, that there is no acceleration when lifting the drum, i.e. lifting force is equal to the weight of the drum, then our lifting force $F$ becomes

$$F = F_W = 1\ 470 \text{ N}$$

The work done in lifting the drum through a height of 20 m i.e. $s = 20$ m is given by

$$W = Fs\cos\theta$$

$$W = (1\ 470 \text{ N})(20 \text{ m})\cos 0^\circ$$

$$W = 29\ 400 \text{ J}$$

The average power, $P$, is defined as the amount of work,$W$, divided by the time to do the said work, $t$. We obtain the average power $P$ as

$$P = \frac{W}{t}$$

$$P = \frac{29\ 400 \text{ J}}{60 \text{ s}} = 490 \text{ W}$$

$$P = 490 \text{ W}$$

The average power required is 490 W.