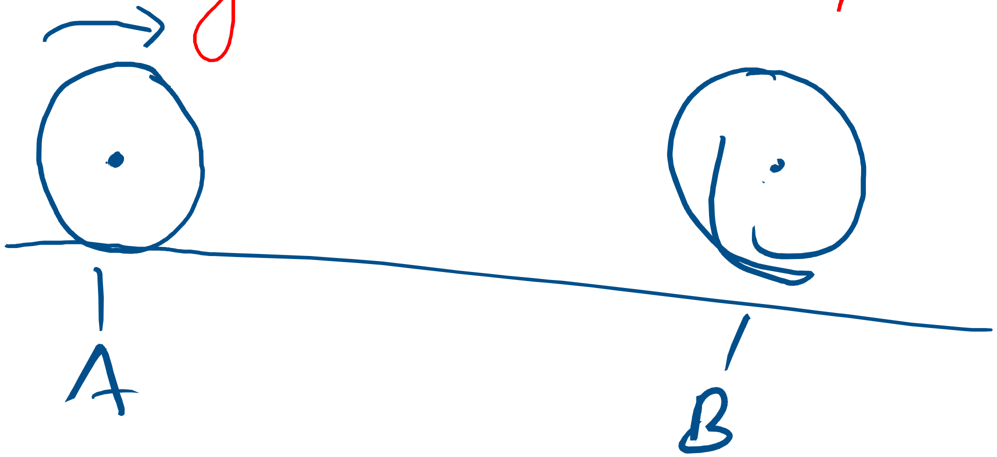
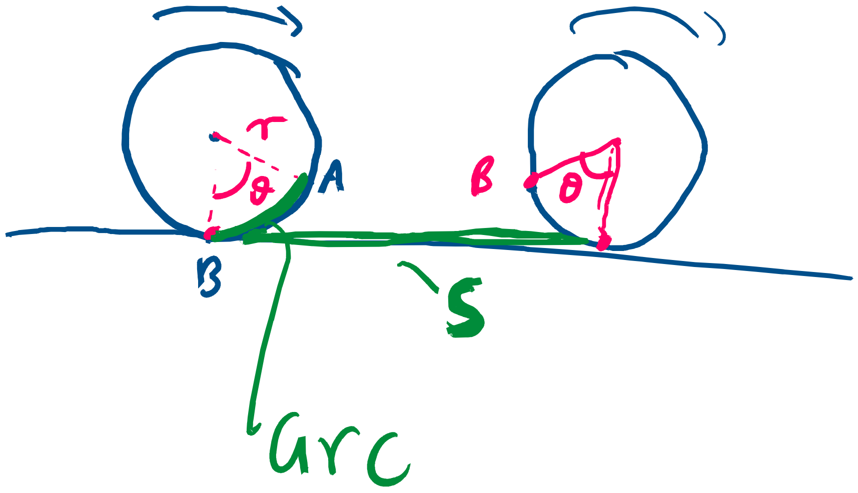
Rotational Motion

Angular Displacement



- Rotational
- Translational

Consider a wheel

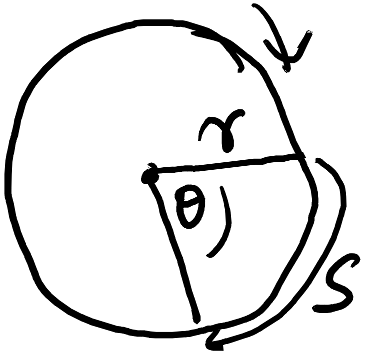


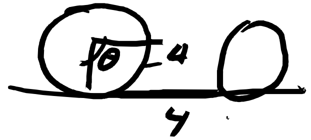
- when the wheel turns from A to B, it has turned through an angle θ

- There are three ways in which θ can be measured

1. Degrees : 1 full circle $\equiv 360^\circ$
2. Revolutions : 1 full circle $\equiv$ 1 revolution
3. Radian (rad) :

What is a radian





$$\theta(\text{rad}) = \frac{\text{arc}}{\text{radius}}$$

$$\theta(\text{rad}) = S/r$$

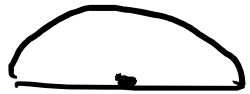
1 full circle = $360^\circ \Rightarrow \theta(\text{rad}) = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi$ rad

1 full circle = 2π rad = 360°

1 full circle = 1 rev = 2π rad = 360°

$\frac{1}{2}$ rev = π rad = 180°

$1/4$ rev = $\frac{\pi}{2}$ rad = 90



$$\theta = \frac{1}{2}\,\frac{2\pi \cancel{r}}{\cancel{r}}$$

Convert 70° into rad & rev

$360° = 2\pi \text{ rad}$

$70° = x$

$x = \frac{70}{360} \times 2\pi \text{ rad} = 1.22 \text{ rad}$

$360° = 1 \text{ rev}$

$70° = x$

$x = 70/360 \times 1 \text{ rev} = 0.194 \text{ rev}$

Convert ~~50° in~~ 1.75 rad into degrees & revolutions.

$1.75\,\text{rad} = 100^\circ = 0.3278\,\text{rev}$

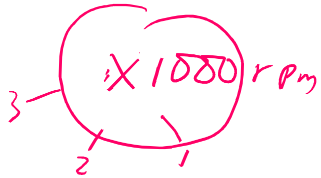
$2\pi\,\text{rad} = 360^\circ$

$1.75\,\text{rad} = x$

$1\,\text{rev} = 2\pi\,\text{rad}$

$x = 1.75\,\text{rad}$

Angular Velocity, ω



- This measures how fast something is rotating. Just as as linear velocity is linear displacement / time, angular velocity is angular displ / time

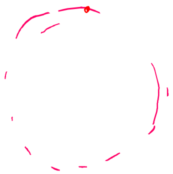
$$\omega = \theta / t \quad (\text{rad/s}) \text{ or } (\text{rev/s}) \text{ or } (\text{deg/sec})$$

Revolution per minute (RPM or rpm)

An object completing a full circle (2π rad) in a time period T has angular velocity

$$\omega = \frac{2\pi}{T}$$

$$f = 1/T = \text{frequency}$$



$$= 2\pi \times \frac{1}{T}$$

$$\boxed{\omega = 2\pi f}$$ → Angular velocity = angular frequency

1.8K RPM   1800 RPM

A wheel turns through 1800 rev in 1.0 min. Find its average angular velocity in rad/s.

$$\omega = \theta / t = \frac{1800\text{ rev}}{60\text{ s}} = \frac{180\cancel{0}(\cancel{2}\pi\text{ rad})}{6\cancel{0}\text{ s}}$$

$$= \cancel{60\pi\text{ rad}}/s$$

$$= 60\pi\text{ rad/s}$$

$$= 188.5\text{ rad/s}$$

$$= \underline{\underline{190\text{ rad/s}}}$$

Angular Acceleration, α

There is a lot of similarity btn linear & angular motion

linearly, $a = \dfrac{v - u}{t}$

Angularly, $\alpha = \dfrac{\omega_f - \omega_o}{t}$ $\left(rad/s^2\right)\left(rev/s^2\right)$

Similarly, average angular velocity $\bar{\omega}$

$\bar{\omega} = \dfrac{\omega_o + \omega_f}{2}$

A wheel starts from rest and attains a rotational velocity of 240 rev/s in 2.0 min. What is its average angular acceleration

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{240 - 0}{120\,s} = 2\ rev/s^2$$

$= 2.0\ rev/s^2$

$= 2(2\pi\ rad)/s^2$

$= 4\pi\ rad/s^2$

What is the wheels' angular speed 130 s after starting from rest

$$\alpha = \frac{\omega_f - \omega_0}{t}$$

$$\omega_f = \omega_0 + \alpha t$$

$$= 0 + 2\,rev/s^2 \times 130\,s$$

$$= \underline{\underline{260\ rev/s}}$$

	Linear Motion	Rotation Motion
Displacement	s = linear displacement	θ = Angular displacement $\theta(\text{rad}) = s/r$
Velocity	v = linear velocity $v = s/t$	ω = angular velocity $\omega = \theta/t$; $\omega = v/r$
acceleration	a = linear acceleration $a = \frac{v-u}{t}$	α = angular acceleration $\alpha = \frac{\omega_f - \omega_0}{t}$, $\alpha = a/r$

Analogy of Linear & Rotational Motion

Linear	Rotational Motion
$S = \bar{v} t$	$\theta = \bar{\omega} t$
$v = u + a t$	$\omega_f = \omega_0 + \alpha t$
$\bar{v} = \frac{v + u}{2}$	$\bar{\omega} = \frac{\omega_f + \omega_0}{2}$
$v^2 = u^2 + 2 a s$	$\omega_f^2 = \omega_0^2 + 2 \alpha \theta$
$S = u t + \frac{1}{2} a t^2$	$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

Ex: A wheel turning at 3.00 rev/s coasts to rest uniformly in 18.0 seconds. What is its deceleration? How many revolutions does it turn through while coming to rest?

$\omega_0 = 3.00$ rev/s

$\omega_f = 0$

$t = 18.0$ s

$\theta = \bar{\omega} t$

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{0 - 3.00}{18} = -0.167 \text{ rev/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 3(18) + \frac{1}{2}(-0.167)18^2$$

$$= 27 \text{ rev}$$

Ex: A wheel turning at 3.00 rev/s coasts to rest uniformly in 18.0 seconds. What is its deceleration? How many revolutions does it turn through while coming to rest?

$\omega_0 = 3.00$ rev/s

$\omega_f = 0$

$t = 18.0$ s

$\theta = \bar{\omega} t$

$= \left(\frac{\omega_f + \omega_0}{2}\right) t = \left(\frac{0+3}{2}\right) 18 = 27$ rev

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{0 - 3.00}{18} = -0.167 \text{ rev/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

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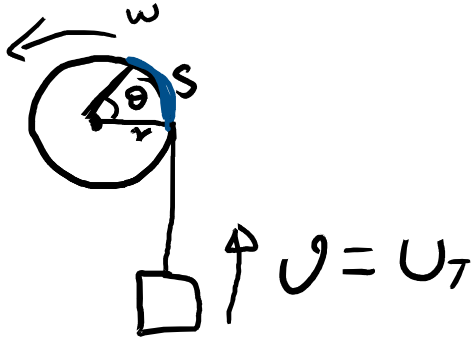
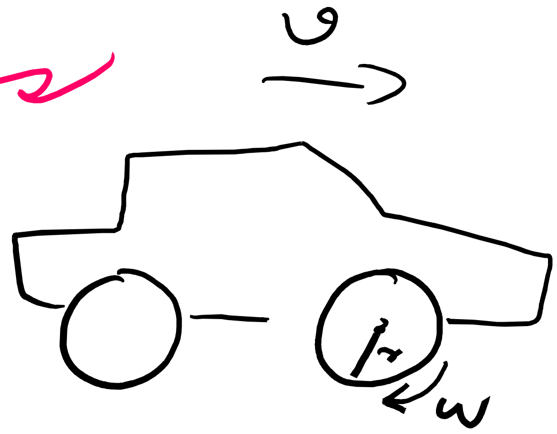
$$= 27 \text{ rev}$$

Tangential Quantities

$\Rightarrow$





We know that

$$\omega = \frac{\theta}{t} = \frac{S/r}{t} = \frac{S}{t}\cdot\frac{1}{r} = \frac{v}{r}$$

$$\omega = v/r \qquad v = \omega r$$

Tangential Quantities

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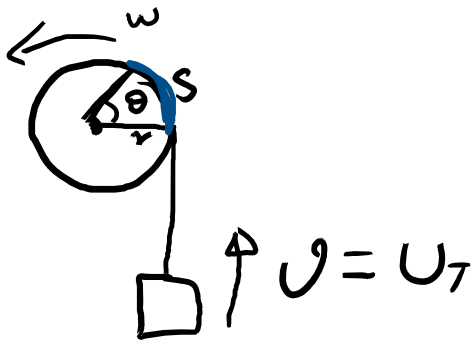

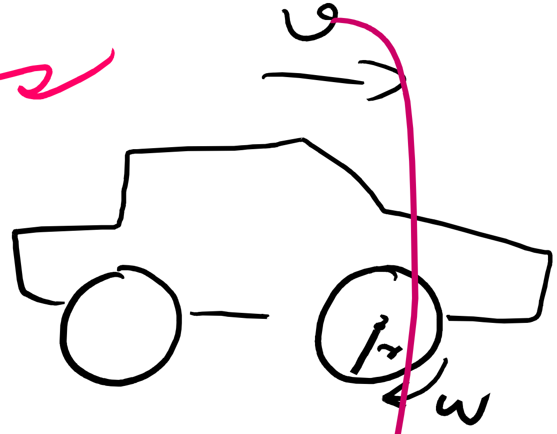

We know that

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$$\omega = v/r$$

$$v = \omega r$$

- The tangential speed is simply the translational speed

$\theta \rightarrow$

a

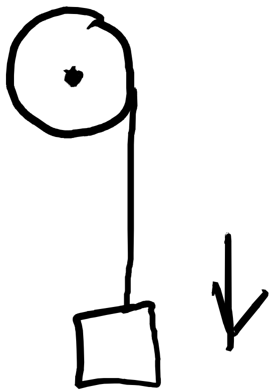
- Similarly, for angular acc

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{\frac{v_f}{r} - \frac{v}{r}}{t} = \frac{1}{r}\left[\frac{v - v}{t}\right]$$

$$\boxed{\alpha = a/r}$$

$a = a_T$ = tangential acceleration

Suppose the mass starts from rest and accelerates downward at $8.6\ m/s^2$. If the radius of the spool is 20cm, what is the rotation rate after 3.0 s?



$$\alpha = \frac{a}{r} = \frac{8.6}{0.2} = 43\ rad/s^2$$

$$W_f = W_0 + \alpha t$$
$$= 0 + 43(3)$$
$$= 130\ rad/s$$

$$v = u + at$$
$$= 0 + 8.6(3)$$
$$= 25.8\ m/s$$

$$W_f = v/r$$
$$= \frac{25.8}{0.2} = 130\ rad/s$$

Centripetal Acceleration

* A car is moving at 10 m/s. After 3s, it is still moving at 10 m/s. Calculate the acc.

In a straight line

$$a = \frac{v - u}{t}$$

$$= \frac{10 - 10}{3} = 0$$

Moving at constant speed of 10 m/s

$a = 0$.

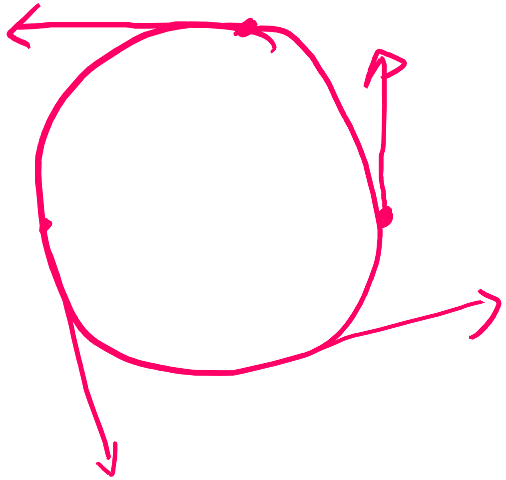
Centripetal Acceleration

A car is moving around a curve at constant speed of 10 m/s.

is this car accelerating?

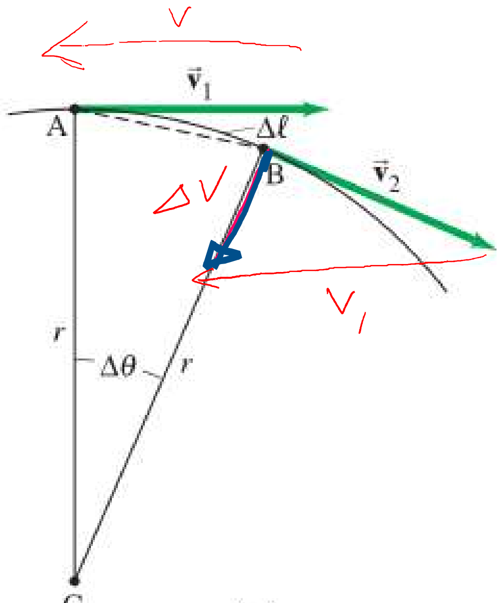
v is a vector $\equiv$ [size , dir]

to change Vel $\rightarrow$ change
- size
- dir



If a car is moving along a circular path with constant speed v, that fact that the direction is change, it means vel is changing. That means it is accelerating.

$$\vec{a} = \frac{\text{change in vel}}{t}$$



(a)

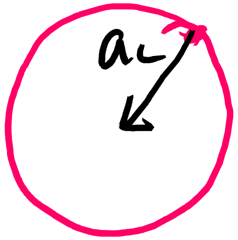
$\Delta \quad a = \Delta V / t$

$$\Delta V = V_2 - V_1$$
$$= V_2 + (-V_1)$$

$$\vec{a} = \frac{\Delta \vec{V}}{t} \qquad \frac{\vec{A}}{2} = \text{Vccl}$$

a = points towards the centre of the circle $\equiv$ Centripetal acceleration

- An object moving with constant speed v along a circular path of radius r undergoes acceleration directed towards the centre of the circle.



. This acceleration is called the centripetal acceleration a_c.

$$a_c = \frac{\overset{\swarrow \omega r}{v}{}^2}{r} = \omega^2 r$$

This denotes the rate of turning or rate of change of motion.

EXAMPLE 5–2 Moon's centripetal acceleration. The Moon's nearly circular orbit around the Earth has a radius of about 384,000 km and a period T of 27.3 days. Determine the acceleration of the Moon toward the Earth.

APPROACH Again we need to find the velocity v in order to find a_R.

SOLUTION In one orbit around the Earth, the Moon travels a distance $2\pi r$, where $r = 3.84 \times 10^8$ m is the radius of its circular path. The time required for one complete orbit is the Moon's period of 27.3 d. The speed of the Moon in its orbit about the Earth is $v = 2\pi r/T$. The period T in seconds is $T = (27.3\text{ d})(24.0\text{ h/d})(3600\text{ s/h}) = 2.36 \times 10^6$ s. Therefore,

$$a_c = \frac{v^2}{r}$$

$$v = d/t = \frac{2\pi r}{T} = \frac{2\pi R}{T}$$

$$v^2 = \frac{4\pi^2 R^2}{T^2}$$

$$a_c = \frac{4\pi^2 R}{T^2}$$

$$a_R = \frac{v^2}{r} = \frac{(2\pi r)^2}{T^2 r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(3.84 \times 10^8\text{ m})}{(2.36 \times 10^6\text{ s})^2}$$

$$= 0.00272\text{ m/s}^2 = 2.72 \times 10^{-3}\text{ m/s}^2.$$

We can write this acceleration in terms of $g = 9.80\text{ m/s}^2$ (the acceleration of gravity at the Earth's surface) as

$$a_R = 2.72 \times 10^{-3}\text{ m/s}^2\left(\frac{g}{9.80\text{ m/s}^2}\right) = 2.78 \times 10^{-4}\, g$$

$$\approx 0.0003\, g.$$

Therefore having known the existence of a_c, we can thus determine the centripetal force

$$F = ma$$

$$\boxed{F_c = ma_c = m\frac{v^2}{r} = m\omega^2 r}$$

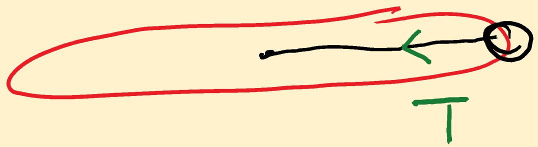
Directed towards the centre of the circle.

Therefore the force required to hold a object of mass m and moving at speed v in a circular path of radius r is called <u>centripetal force</u>.

- No work is done by the F_c. To do work there must be a component of force in the dir of motion

$$W = Fs\cos\theta$$
$$= F_c s \cos 90 = 0$$

Uniform Circular Motion A 13-g rubber stopper is attached to a 0.93-m string. The stopper is swung in a horizontal circle, making one revolution in 1.18 s. Find the tension force exerted by the string on the stopper.

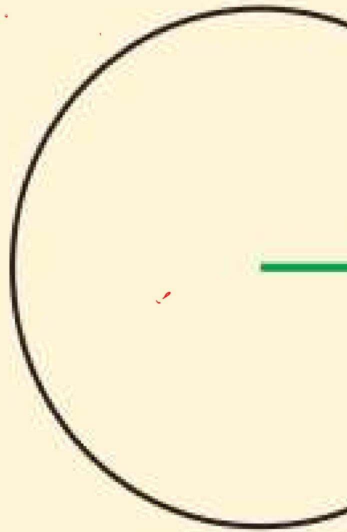


$$F_c = T = \frac{m v^2}{r} = m\omega^2 r$$

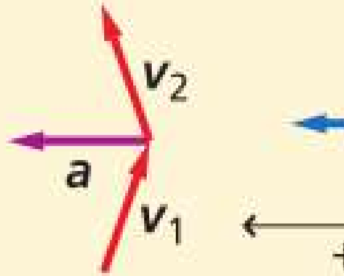
$$v = \frac{2\pi r}{t}, \quad v^2 = \frac{4\pi^2 r^2}{t^2}$$

$$T = m\frac{4\pi^2 r^2}{r\,t^2} = \frac{4m\pi^2 r}{t^2}$$

$$= \frac{4 \times 0.013 \times \pi^2}{1.18} \times 0.93 =$$



$\bar{a}_c = v^2/r$ or



A 1200 kg car is turning a corner at 8.00 m/s and it travels along an arc of a circle as shown

If the radius of the circle is 9.00 m

a) how large a horizontal force must the pavement exert on the tyres to hold the ~~circle~~ car in the circular path?

b) What minimum coefficient of friction must exist to hold the car in the circular path?

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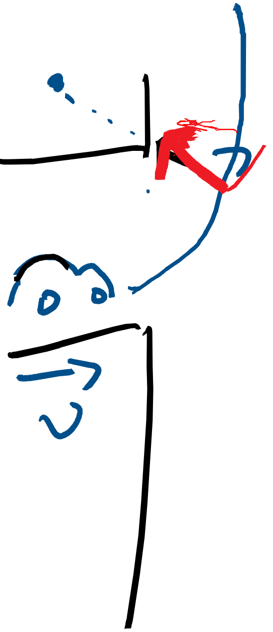

If the radius of the circle is 9.00 m

a) how large a horizontal force must the pavement exert on the tyres to hold the circle in the circular path?

$$\underbrace{F=\frac{mv^2}{r}}_{F_c} = \frac{1200(8)^2}{9} = 8530N$$

$= f_s$

b) What minimum coefficient of friction must exist to hold the car in the circular path?

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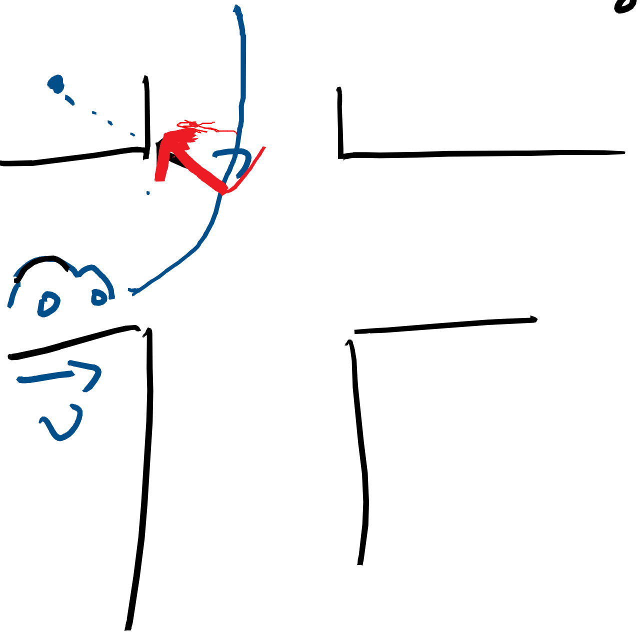

a) If the radius of the circle is 9.00 m how large a horizontal force must the pavement exert on the tyres to hold the car in the circular path?

b) What minimum coefficient of friction must exist to hold the car in the circular path?

$$\underbrace{F = \frac{mv^2}{r}}_{F_c} = \frac{1200(8)^2}{9} = \underline{8530\,N} = f_s$$

$$f_s = \mu_s F_N = \mu_s mg = 8530$$

$$\mu_s = \underline{\underline{0.725}}$$

A car racing on a flat track travels at 22 m/s around a curve with a 56-m radius. Find the car's centripetal acceleration. What minimum coefficient of static friction between the tires and road is necessary for the car to round the curve without slipping?

An airplane traveling at 201 m/s makes a turn. What is the smallest radius of the circular path (in km) that the pilot can make and keep the centripetal acceleration under 5.0 m/s^2 ?

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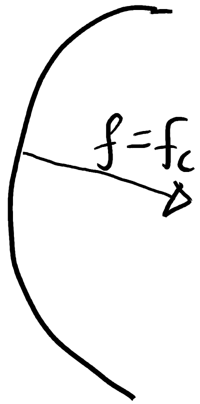

$$F_c = \frac{mv^2}{r} = \frac{m\,22^2}{56}$$

$$F_c = f = \mu m g = \frac{m\,22^2}{56}$$

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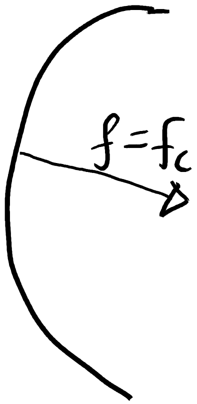


$$F_c = \frac{m v^2}{r} = \frac{m\, 22^2}{56}$$

$$F_c = f = \mu \cancel{m} g = \frac{\cancel{m}\, 22^2}{56}$$

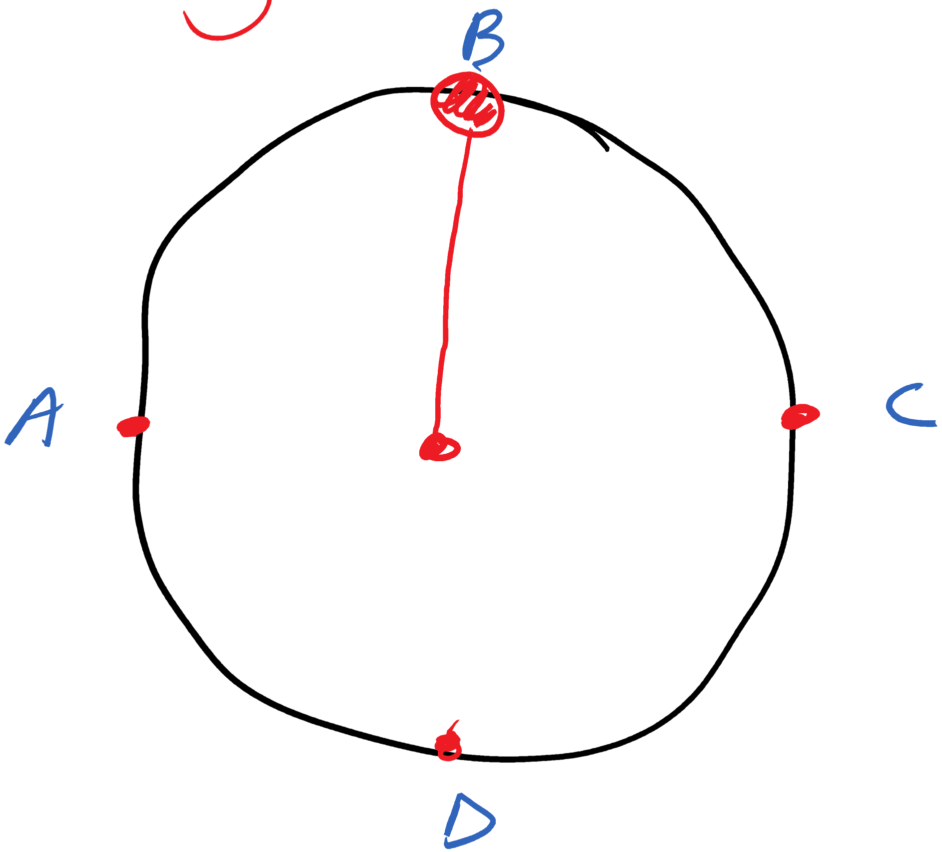
$$\mu = \frac{22^2}{56 \times 9.8}$$

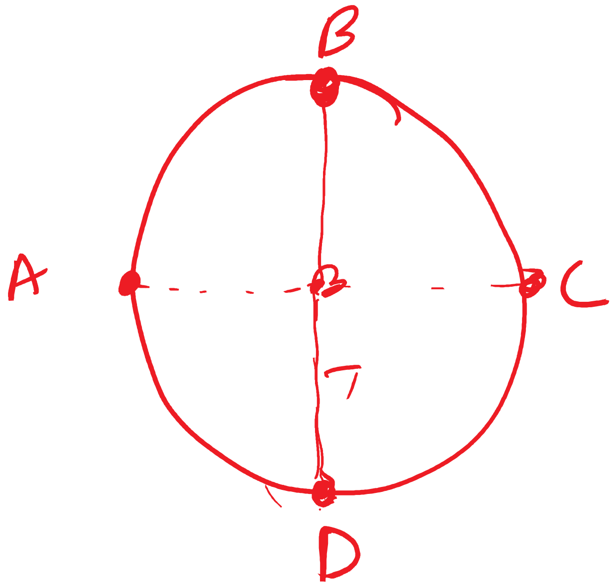
An airplane traveling at 201 m/s makes a turn. What is the smallest radius of the circular path (in km) that the pilot can make and keep the centripetal acceleration under 5.0 m/s^2?

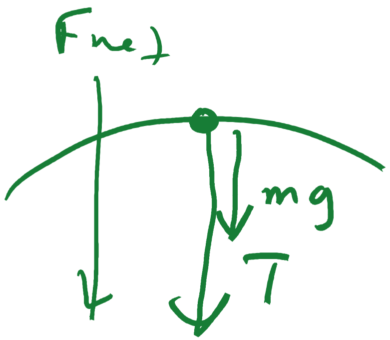
$$a = \frac{v^2}{r} \Rightarrow r = \frac{(201)^2}{5} \text{ m}$$

Convert to km

What are the tensions in the Cord if you Swirl the mass Vertically







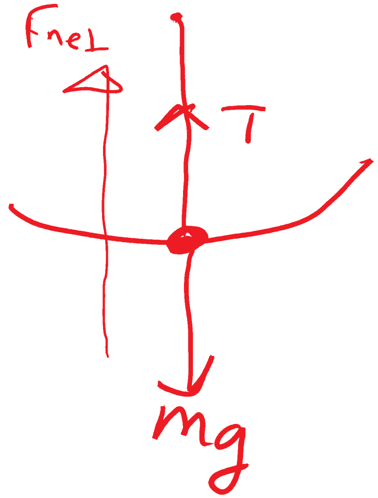
$$F_{net} = F_c = T + mg$$

$$T = F_c - mg$$

$$T = \frac{mv^2}{r} - mg$$

least tension

Point D



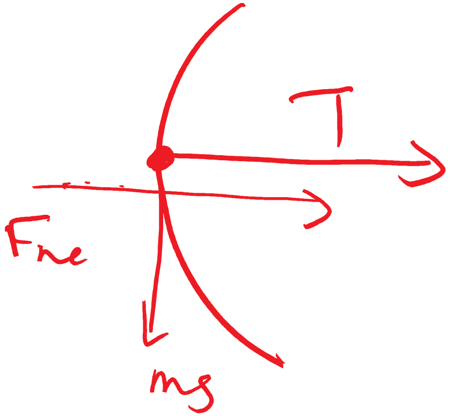
$$F_{net} = T - mg$$

$$F_c = T - mg$$

$$\frac{mv^2}{r} = T - mg$$

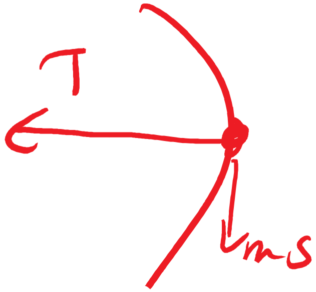
$$\boxed{T = \frac{mv^2}{r^2} + mg}$$

max tension



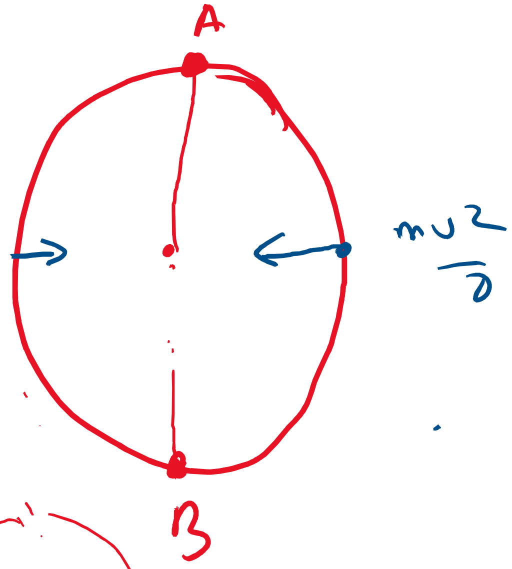
$$F_{net} = F_c = T$$

$$F_c = \frac{mv^2}{r}$$



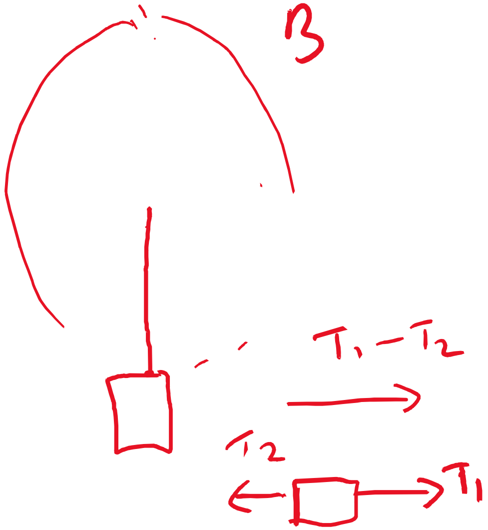
$$F_{net} = F_c = T = \frac{mv^2}{r}$$

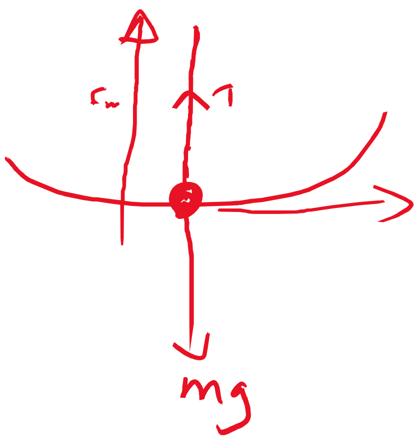
What are the tensions at A & B



$$T + mg = \frac{mv^2}{r}$$

$$\boxed{T_A = \frac{mv^2}{r} - mg}$$

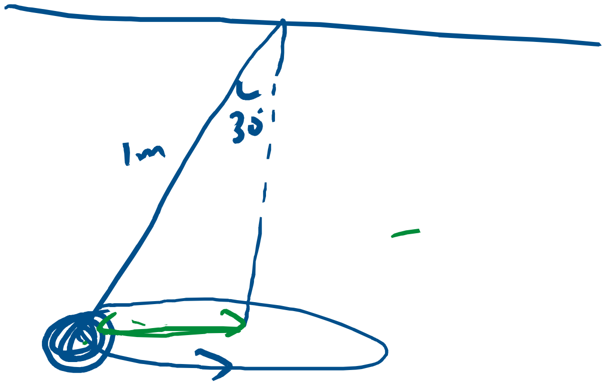




$$T - mg = \frac{mv^2}{r}$$

$$\boxed{T_B = \frac{mv^2}{r} + mg}$$

A ball is swirled on a 1m string so that it makes an angle of 30° with the vertical as shown. What is the minimum velocity required to make this angle.



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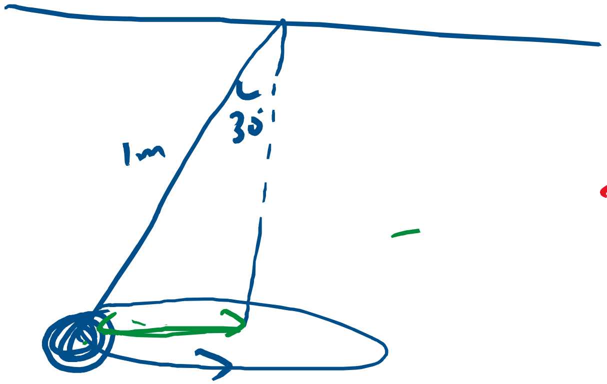

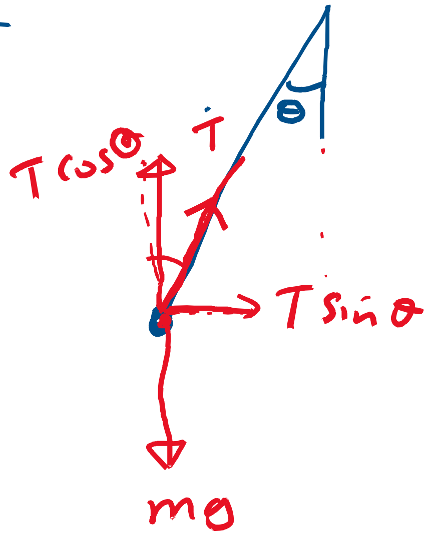

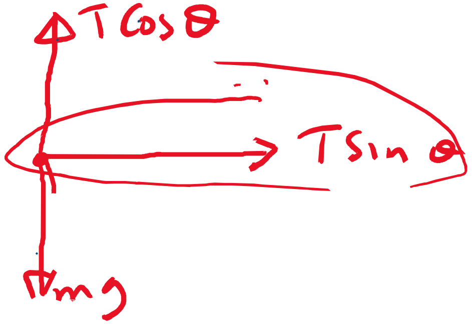

$$T \sin\theta = \frac{m v^2}{r} \quad (1)$$

$$T \cos\theta = mg \quad (2)$$

Divide eqn 1 & eq 2

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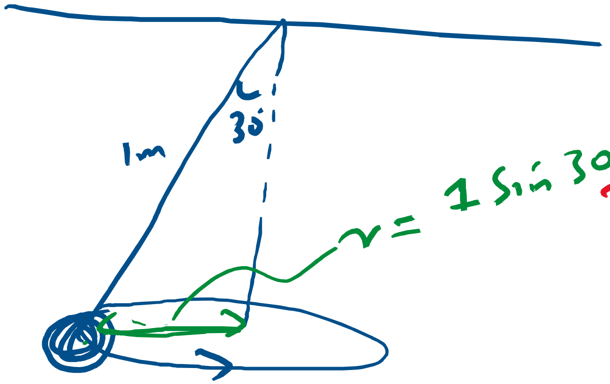

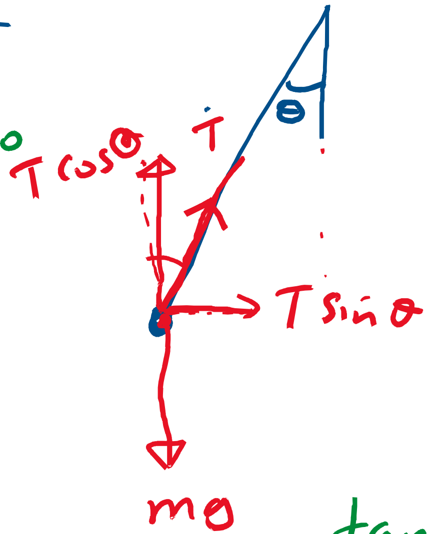

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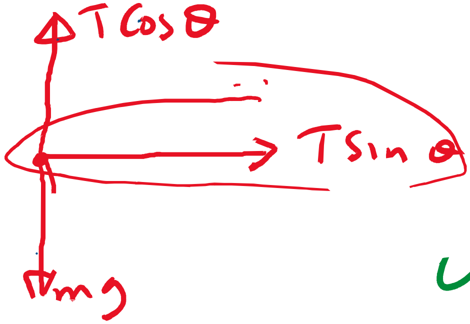

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

$$v = \sqrt{rg\tan 30} =$$