DESCRIPTION OF MOTION

Units of Length & Time

Basic SI units:

length : meter
= distance light travels in vacuum in 1/299,792,458 second

1 second = time taken for 9,192,631,770 cycles of a certain wavelength of light to be emitted by a ceasium atom.

SPEED

Distance travelled = speed × time taken

Solving for speed

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}.$$

Same equation applies for average speed, i.e. speed not constant.

Example. Car moving at 100km/h in 1hr will cover 100 km at constant speed
in 0.5hr will go 0.5×100 = 50 km
in 2 hrs will go 2×100 = 200 km

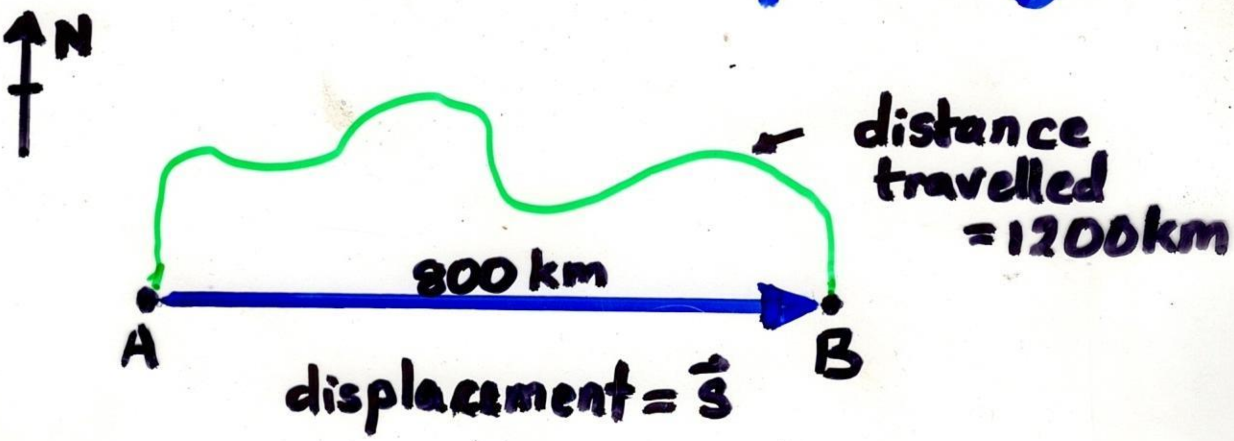
Suppose a car goes 200 km in 4 hrs
its average speed $= \dfrac{200\,\text{km}}{4\,\text{hr}} = 50$ kph

Speed is a scalar quantity, it has no DIRECTION

In general terms speed and velocity are used interchangeably.

In science these quantities mean different things.

VELOCITY is a vector quantity
SPEED is a scalar quantity



Displacement is direct path from A to B blue line.

average speed for green path

$$= \frac{1200\,\text{km}}{\text{time taken}}$$

but average velocity $= \dfrac{800\,\text{km}}{\text{time taken}} = \dfrac{\text{Displacement vector}}{\text{time taken}}$

$$= \frac{\vec{S}}{t}$$

Symbolically

Av. velocity $\bar{v} = \frac{\vec{s}}{t}$ $\therefore$ $\vec{s} = \bar{v}t$

= average vel. x time

Suppose car took 20hrs between A & B using green (longer) route

since $\vec{s}$ = 800 km

Av. vel. $\bar{v} = \frac{800\text{km east}}{20\text{hrs}}$ = 40 km/hr E

but average speed = $\frac{\text{distance travelled}}{\text{time taken}}$

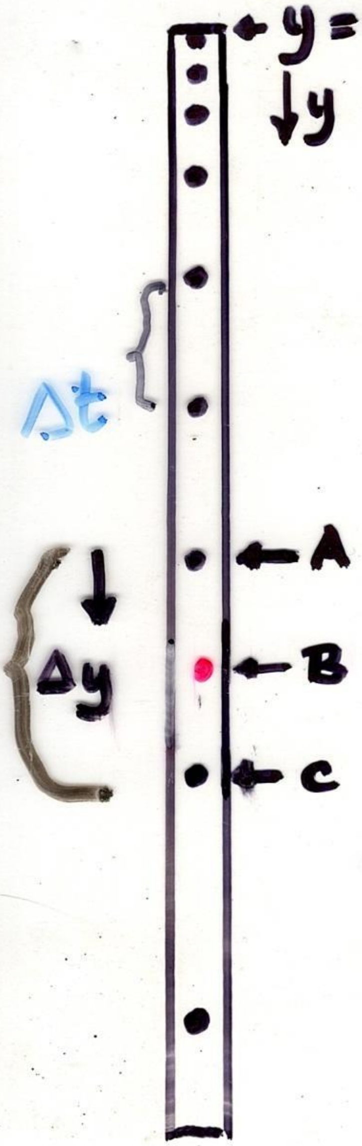
$= \frac{1200\text{ km}}{20\text{hrs}}$

= 60 km/hr

N.B. Object speed need not equal to magnitude of its average velocity

Average speed does not tell whether the car was moving at same speed ~~for~~ 20hrs or it was changing.

INSTANTANEOUS VELOCITY



Position of ball uniformly timed
Let time interval be Δt

Δy for successive intervals gets larger.

What is the instantaneous velocity at pt B

Estimate is ave. velocity between A & C
Displacement $= \Delta y$
Time interval $= \Delta t$

$\therefore$ Av. vel $\bar{v} = \dfrac{\text{displ.}}{\text{time taken}} = \dfrac{\Delta y}{\Delta t} \to 0$

If we made Δt very small, the $\bar{v}$ will approach true velocity at B

If Δt is so small as to approach 0, then $\bar{v}$ will approach true velocity at B

i.e. Instantaneous velocity $= \vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta y}{\Delta t}$

Since Δt is very small, assume no directional change

$\therefore$ Distance travelled = displacement

Instantaneous velocity at B = speed at B

1-D

One-dimensional Motion

Motion along a straight line can be shown by a graph.

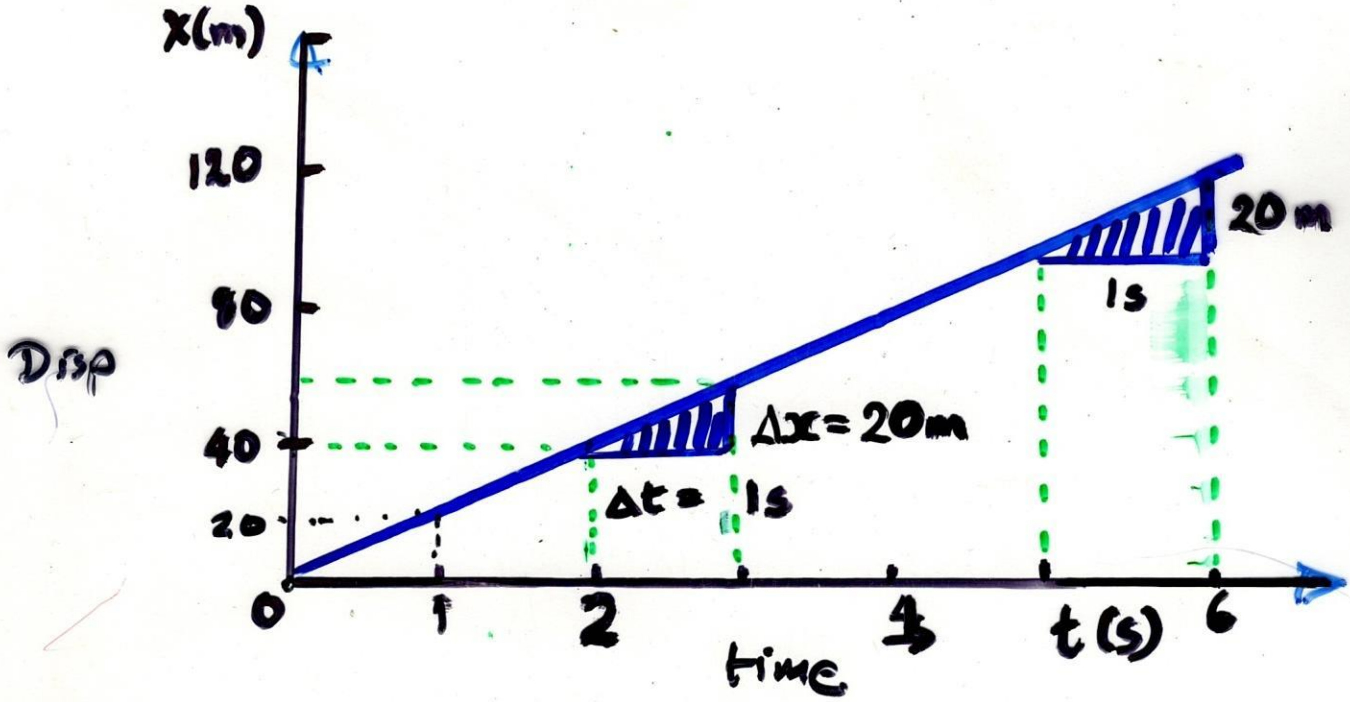
If a car is moving at $20\,m\,s^{-1}$, and we observe its position every second, starting at $x=0$ when $t=0$

Data obtained

t (s)	0	1	2	3	4	5	6
x (m)	0	20	40	60	80	100	120

Thus displacement increases by 20 m each s.

Plot of data gives a graph of x as a function of t.



Δ's in fig. tell us that car goes 20 m in +ve direction each sec.

Δx is displacement during Δt

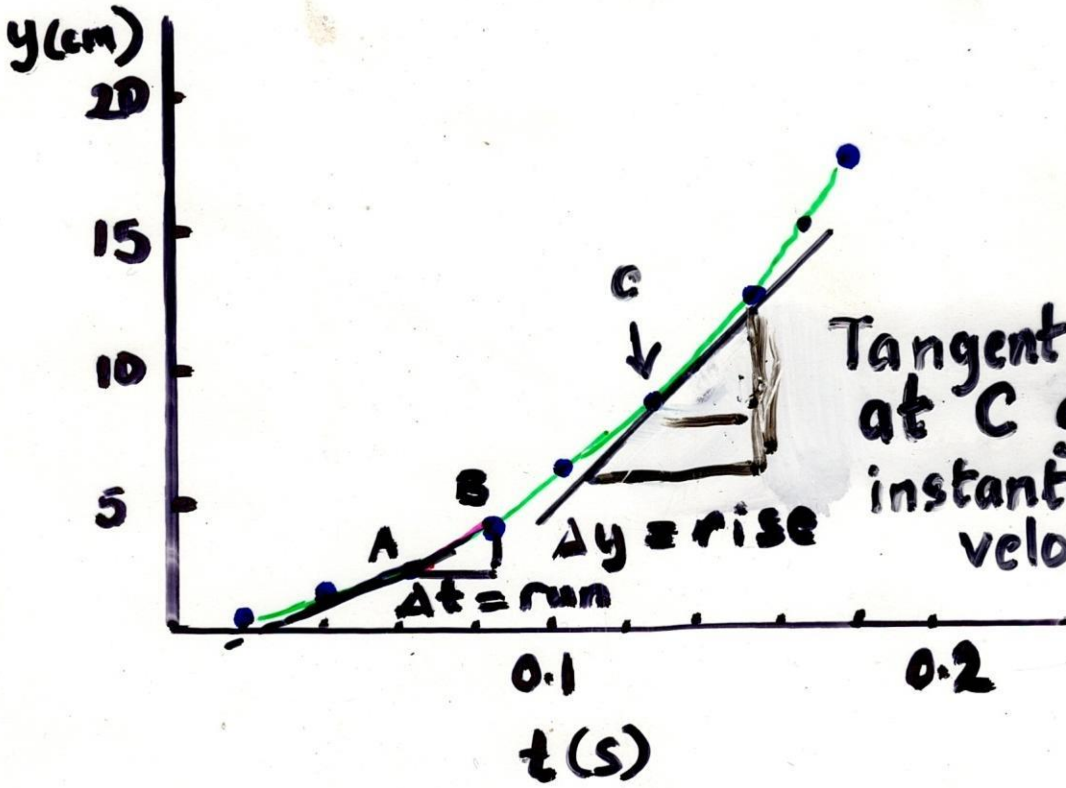
$\boxed{\text{Average velocity is slope of plot of displacement versus time}}$

i.e. $\bar{v} = \dfrac{\text{displacement}}{\text{time taken}} = \dfrac{\Delta x}{\Delta t}$ = SLOPE

This a case of constant velocity

In the case of a falling ball velocity is continuously increasing.

Plot of this case yields a curve.



Tangent drawn at C gives instantaneous velocity

If A & B are very close, line connecting them becomes a tangent.

Tangent gives slope at any pt on the curve. Eg at point C.

However average velocity between A & B is:

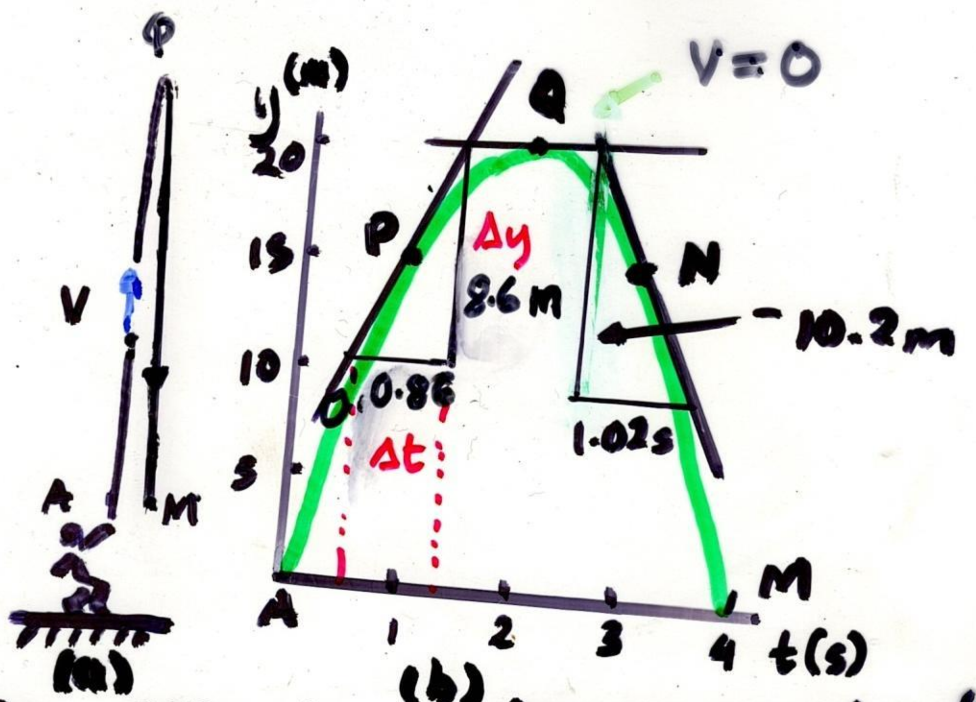
$$\bar{v}_{AB} = \frac{\Delta y}{\Delta t} = \frac{\text{rise}}{\text{run}} = \frac{y_B - y_A}{t_B - t_A}$$

For falling stone downward was taken as positive.

$\therefore$ $\bar{v}_{AB}$ is +ve, downward.

Slope of graph of displacement vs time at any point is equal to instantaneous velocity at that point.

THUS SLOPE OF DISPLACEMENT VERSUS TIME GRAPH GIVES US THE INSTANTANEOUS VELOCITY OF A MOVING OBJECT



Motion in (a) is shown graphically in (b)

To find instantaneous velocity at any point, we draw a tangent at that point.

Point P

$\therefore \quad V_p = \text{slope at } P = \dfrac{8.6\,m}{0.86\,s} = 10\,m/s$

at Point Q $\quad V_Q = \text{slope at } Q = 0$

at N ball is falling $\therefore$

$V_N = \text{slope at } N = -\dfrac{10.2\,m}{1.02\,s} = -10.0\,m/s$

Slope gives both magnitude & direction -ve slope implies velocity is -ve y-direction.

Acceleration

Object at certain instant has velocity V_0, then later time t it is V_f

Average acceleration $\bar{a}$ in time interval t is defined as:

$$\bar{a} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$= \frac{V_f - V_0}{\Delta t}$$

Acceleration is change in velocity per unit time interval.

Units : units of length divided by time squared.

S.I. (ms^{-2}) or (m/s^2)

Example:

Car starts from rest and attains a speed of $20\,ms^{-1}$ in $12\,s$ as it goes in the +ve x-direction. What is its acceleration?

$V_0 = 0$, $V_f = 20\,ms^{-1}$

$$\therefore \bar{a} = \frac{V_f - V_0}{t} = \frac{(20-0)\,ms^{-1}}{12\,s} = 1.7\,ms^{-2}$$

Suppose it continues same direction but slows down from $20\,ms^{-1}$ to $0\,ms^{-1}$ in 12s. What is its acceleration?

$$\bar{a} = \frac{v_f - v_0}{t} = \frac{0 - 20\,ms^{-1}}{12s} = -1.7\,ms^{-2}$$

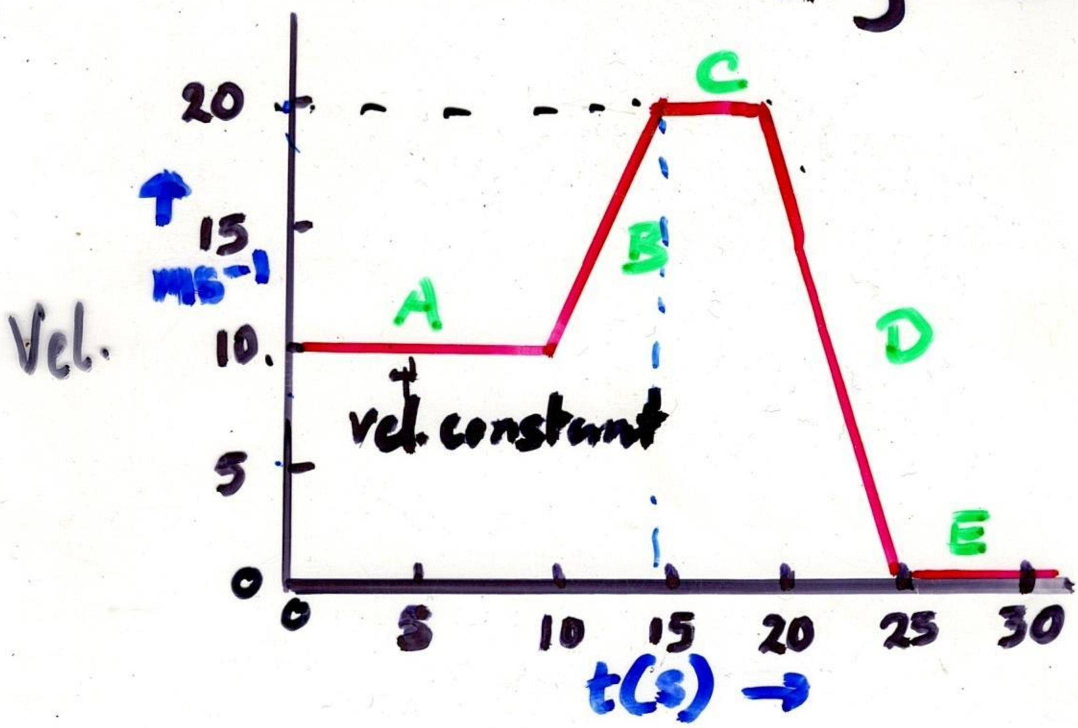
-ve sign denotes $\vec{a}$ is toward -ve x-direction, opposite direction of motion.

Car slowing down denotes negative acceleration or deceleration.

For 1-Dimensional vectors (1-D) can choose any of two possible directions to represent +ve vectors

N.B. Use correct sign for all vectors that enter into a calculation.

Instantaneous velocity vs time



A - region of 0 acceleration
B - " of +ve acceleration
C - " of 0 acceleration
D - " of -ve acceleration (i.e. slowing down)
E - 0 vel, 0 acc.

B : acc. $= \frac{(20 - 10)\,ms^{-1}}{5\,s} = 2\,ms^{-2}$

D : acc $= \frac{(0 - 20)\,ms^{-1}}{5\,s} = -4\,ms^{-2}$

horizontal line ⇒ 0 acceleration
sloping upward ⇒ +ve "
slopping downward ⇒ -ve "

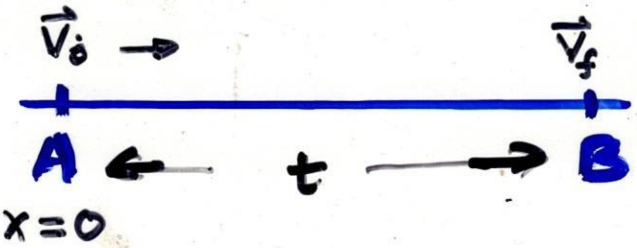
UNIFORMLY ACCELERATED MOTION

- Changing acceleration is difficult to analyse mathematically.

- Usually motion is in a straight line.
  $\therefore$ simply indicate direction by $+$ or $-$ sign.

- vector displacement is represented by $\vec{x}$
- x-axis directed velocity by $\vec{v}$

e.g.
for an object moving with **constant acceleration** '**a**' in direction along x-axis



It passes point A with velocity $\vec{v}_0$ and after time t passes point B with velocity $\vec{v}_f$

1) Average velocity $\bar{v}$ for A to B is:

$$\bar{v} = \frac{\text{displacement}}{\text{time}} = \frac{\vec{x}}{t} \qquad \bar{v} = \frac{\vec{x}}{t}$$

from which we can say: $\vec{x} = \bar{\vec{v}}\, t$
re-written in ordinary notation, since only single vector on either side

$$\bar{v} = \frac{x}{t}$$

2) Since acceleration is CONSTANT.
Average & instantaneous accelerations are the same.
Definition of acceleration becomes:

$$a = \frac{v_f - v_0}{t} \Rightarrow \boxed{v_f = v_0 + at}$$

Final velocity = Initial velocity (v_0) + change in velocity (at)

3) For an object undergoing uniform acceleration:
velocity changes linearly from v_0 to v_f

Thus average velocity between A & B is:

$$\bar{v} = \frac{v_0 + v_f}{2}$$

Equations: $\bar{v} = x/t$ ①, $v_f = v_0 + at$ ② and

③ $\bar{v} = \dfrac{v_0 + v_f}{2}$ apply to uniformly accelerated motion.

Example 1
A car starts from rest and accelerates uniformly to 5.0 m/s in 10 s as it travels along the x-axis. Find the acceleration and distance travelled in this time.

Solution:

From rest implies $V_0 = 0$

To $5ms^{-1}$ " $V_f = 5.0ms^{-1}$ at $t = 10s$

motion in x-direction ∴ 1-D motion.

For acceleration we use the eqn. with V_0, V_f & t since the unknowns are a, & x

$$a = \frac{V_f - V_0}{t} = \frac{5.0ms^{-1} - 0ms^{-1}}{10s} = 0.50ms^{-2}$$

Distance travelled involves use of average velocity, so find it first.

$$\bar{V} = \frac{V_f + V_0}{2} = \frac{5.0ms^{-1} + 0ms^{-1}}{2} = 2.5ms^{-1}$$

average velocity

Given $t = 10s$

∴ distance covered is: $x = \bar{V}t$

$= (2.5m\cancel{s}^{-1})(10\cancel{s})$

$= \underline{25m}$

Example 2

A car travelling at $5.0ms^{-1}$ is brought to rest in a distance of 20m. Find its acceleration and time it takes to stop. Assuming motion in x-axis and constant acceleration.
deceleration

Solution

Given $V_0 = 5ms^{-1}$

Coming to rest implies $V_f = 0$

in 20m ⇒ change takes place over this distance

$$\bar{v} = \frac{v_0 + v_f}{2} = \frac{(5ms^{-1} + 0ms^{-1})}{2} = 2.5ms^{-1}$$

Then time it takes to stop is:

$$\boxed{x = \bar{v}.t} \Rightarrow t = \frac{x}{\bar{v}} = \frac{20m}{2.5ms^{-1}} = 8s$$

Since we know t, we can get acceleration from:

$$a = \frac{v_f - v_0}{t} = \frac{0ms^{-1} - 5.0ms^{-1}}{8.0s}$$

$$= -0.625ms^{-2}$$

-ve sign implies direction of a is opposite direction of $\vec{v}$ hence car slowing down.

ADDITIONAL EQUATIONS OF MOTION

displacement $\boxed{\vec{s} = \bar{v}t}$ (a)

average velocity $\bar{v} = \frac{v_0 + v_f}{2}$ (b) } true for constant accln.

if initial vel. $= v_0$ & final vel. $= v_0 + at$

$\boxed{v_f = v_0 + at}$

Let us substitute for v_f in (b)

$$\bar{v} = \frac{v_0 + (v_0 + at)}{2} = \boxed{v_0 + \tfrac{1}{2}at} \quad (c)$$

The displacement is accordingly:

subsitute for $\bar{v}$ with result from (C) into (a)

$$\vec{s} = \bar{v}t = (v_0 + \tfrac{1}{2}at)t = v_0t + \tfrac{1}{2}at^2$$

(d) $\boxed{\vec{s} = v_0t + \tfrac{1}{2}at^2}$ under constant acceleration.

Case of $v_0 = 0$

then eqn. reduces to $\boxed{\vec{s} = \frac{1}{2}at^2}$

We can also find relationships among s, v_0, v_f

which do not involve t

re-writing $a = \frac{v_f - v_0}{t}$ as "$\boxed{t = \frac{v_f - v_0}{a}}$"

and substituting for t in displacement eqn.

(d)

$$\vec{s} = v_0 t + \left(\tfrac{1}{2}a\right)t^2 = v_0\left(\frac{v_f - v_0}{a}\right) + \frac{1}{2}a\left[\frac{(v_f - v_0)^2}{a^2}\right]$$

$$= \frac{v_0 v_f}{a} - \frac{v_0^2}{a} + \frac{v_f^2}{2a} - \frac{v_0 v_f}{a} + \frac{v_0^2}{2a}$$

$$s = \frac{v_f^2 - v_0^2}{2a} \qquad \times \text{ b.s. by } 2a$$

$$\Rightarrow \quad 2a\vec{s} = v_f^2 - v_0^2$$

normally re-written as

$$\boxed{v_f^2 = v_0^2 + 2as}$$

Example

A car starts from rest and accelerates at $4.0\,ms^{-2}$ through a distance of $20.0\,m$.

a) How fast is it then going?

b) How long did it take to cover 20 m?

Solution:

We know $V_0 = 0$, $a = 4.0\,ms^{-2}$, $s = 20\,m$

need to find V_f at 20 m and elapsed time.

eqns with t as only unknown are:

i) $V_f = V_0 + at$ & ii) $S = V_0 t + \frac{1}{2}at^2$

i) easier to solve, linear eqn.
ii) difficult " " , quadratic.

a) Use $V_f^2 = V_0^2 + 2as$ to get V_f

$\therefore V_f^2 = 0 + (2)(4.0\,ms^{-2})(20\,m) = 160\,m^2s^{-2}$

$\Rightarrow V_f = \pm\sqrt{160\,m^2s^{-2}} = \pm 12.6\,ms^{-1}$

take +ve answer since it is accelerating.

b) use $V_f = V_0 + at$, re-write as:

$$t = \frac{V_f - V_0}{a} = \frac{12.6\,ms^{-1} - 0\,ms^{-1}}{4\,ms^{-2}}$$

$$= \underline{3.15\,s}$$

SUMMARY

5 eqns. used in solving problems involving uniformly accelerated motion <u>along a straight line</u>

i) $\vec{x} = \bar{\vec{v}}t$ (averagevel. x time)

ii) $\bar{\vec{v}} = \frac{V_0 + V_f}{2}$ average velocity.

iii) $V_f = V_0 + \vec{a}t$ final velocity

iv) $V_f^2 = V_0^2 + 2ax$ final velocity squared

v) $\vec{x} = V_0 t + \frac{1}{2}at^2$ – displacement (distance)

Example

A car starts from rest and accelerates at $4 m/s^2$ through a distance of 20 m.

a) How fast is it then going?
b) How long did it take to cover 20m?

Solution:
We know $V_0 = 0$, $a = 4 m/s^2$, $s = 20 m$,
we need to find V_f at 20m and elapsed time.
Look for equations with t as the only unknown.
$\Rightarrow$ i) $V_f = V_0 + at$ & ii) $s = V_0 t + \frac{1}{2}at^2$

a) Use $V_f^2 = V_0^2 + 2as$ to get V_f

$$V_f^2 = 0^2 + (2)(4 m/s^2)(20 m) = 160 m^2 s^{-2}$$

$$\Rightarrow V_f = \pm\sqrt{160 m^2 s^{-2}} = \pm 12.6 m s^{-1}$$

car is accelerating $\therefore$ we take +ve value

b) We choose eqn. (i) to get time

$V_f = V_0 + at$ re-write in terms of t

$$t = \frac{V_f - V_0}{a}$$

$$= \frac{12.6 m/s - 0 m/s}{4 m s^{-2}}$$

$$= 3.15 s$$

N.B. Choose appropriate eqns. in solving problems!

Example

A car has an acceleration of $8\ m/s^2$.

a) How much time is needed for it to reach a velocity of 24 m/s if it starts from rest?

b) How far does it go during this period?

Solution:

a) $V_f = 24\ m/s$, $a = 8\ m/s$

$$a = \frac{V_f - V_o}{t} \Rightarrow a = \frac{V_f}{t} \quad \text{since } V_o = 0$$

$$\therefore \quad t = \frac{V_f}{a} = \frac{24\ m/s}{8\ m/s^2} = \underline{3s.}$$

b) displacement $s = V_o t + \frac{1}{2}at^2$, $V_o = 0$

$$\therefore \quad s = \frac{1}{2}at^2 = \frac{1}{2}(8\ m/s^2)(3s)^2$$

$$= \underline{36\ m}$$

Example

The brakes of a car whose initial velocity is 30 m/s are applied, and the car receives an acceleration of $-2\ m/s^2$. How far will it have gone

a) when its velocity has decreased to 15 m/s and

b) when it has come to a stop?

Solution:

a) We use the equation - $V_f^2 = V_o^2 + 2as$

$$\therefore \quad s = \frac{V_f^2 - V_o^2}{2a} = \frac{15^2 - 30^2}{(2)(-2)} = 169\ m.$$

b) come to a stop $\equiv V_f = 0$

$$\therefore \quad s = \frac{0 - 30^2}{(2)(-2)} = 225\ m.$$