

2010

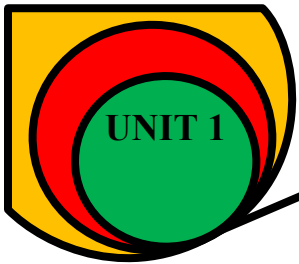


MODULE I: Introductory Mechanics.

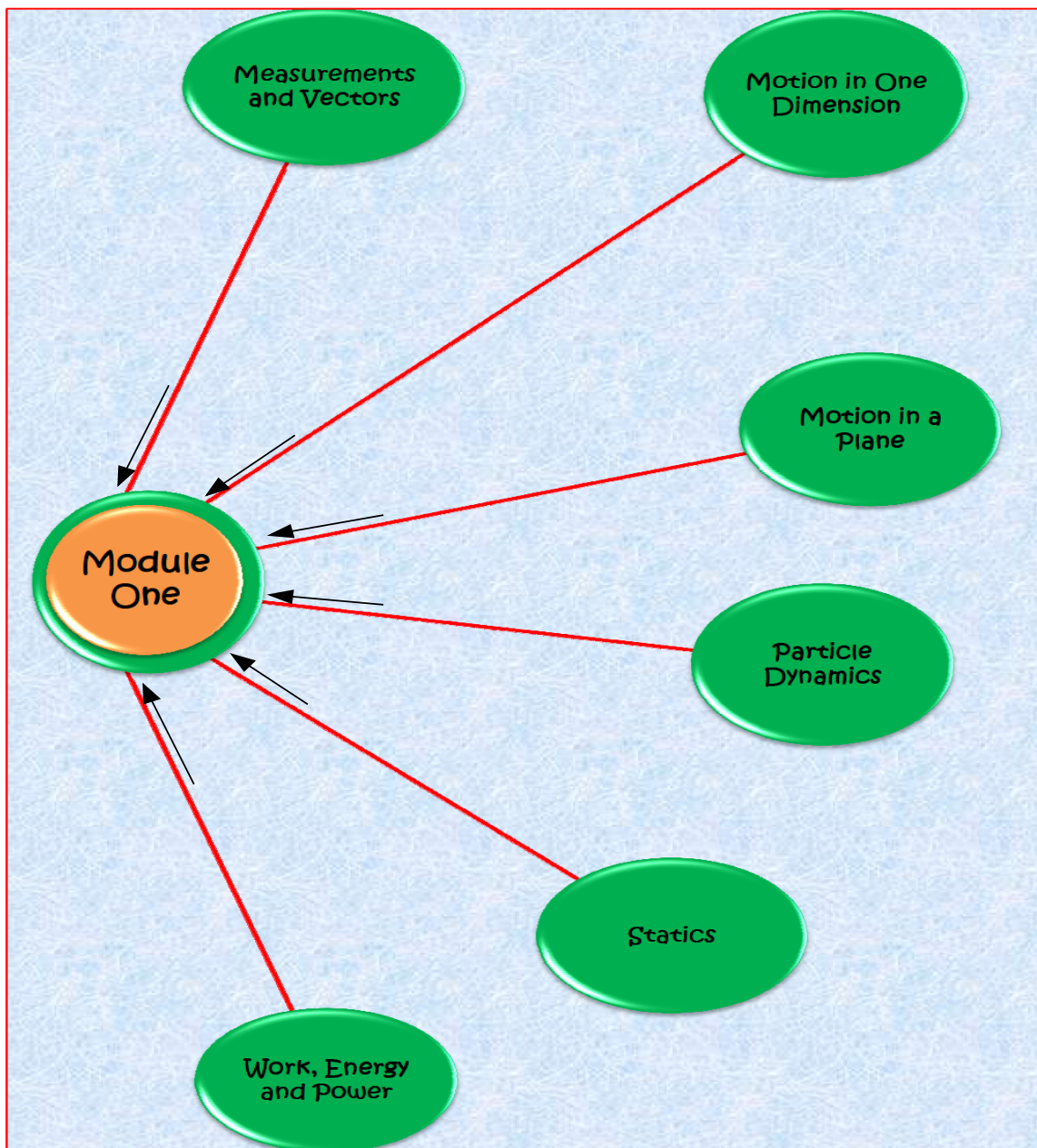


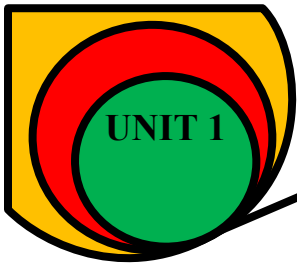
Steven Mudenda

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[University of Zambia]



Module I: Introductory Mechanics





Module I: Introductory Mechanics



Abstract

Classical Mechanics is essential to understanding physics and the world around us. A thorough understanding of it is essential to solving problems involving motion of objects as well as successful assimilation of modern physics. Welcome to Module 1!!

Time:

This Module can be completed in 87 hours. Here is the breakdown:

Unit 1 - 15 hrs

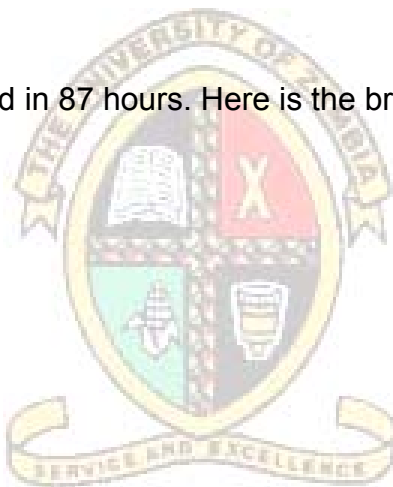
Unit 2 - 15 hrs

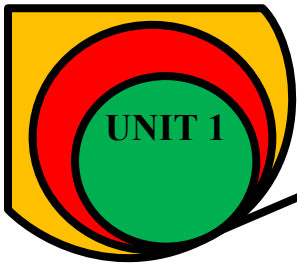
Unit 3 - 12 hrs

Unit 4 – 15 hrs Δ

Unit 5 – 15 hrs

Unit 6 - 15 hrs





Module I: Introductory Mechanics



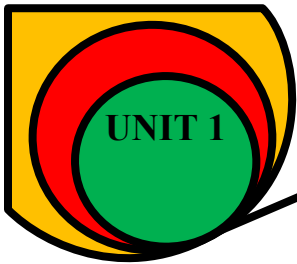
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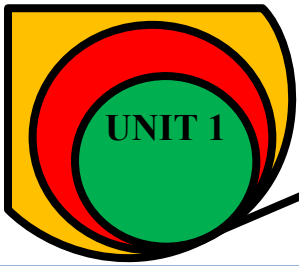


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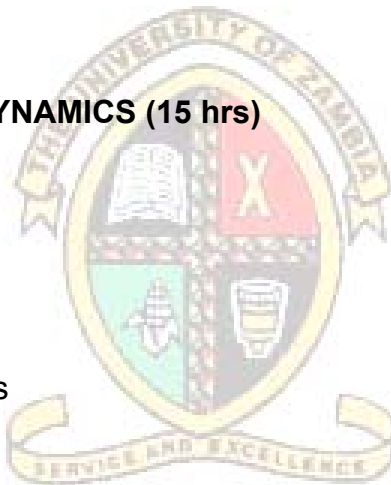
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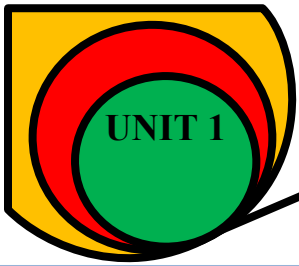


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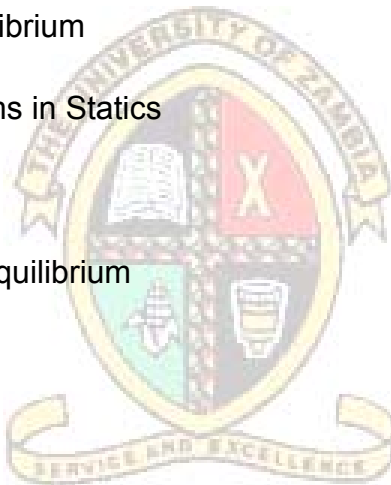
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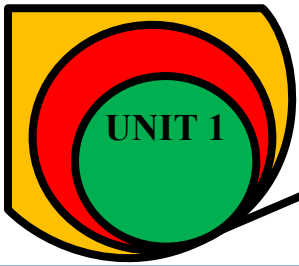
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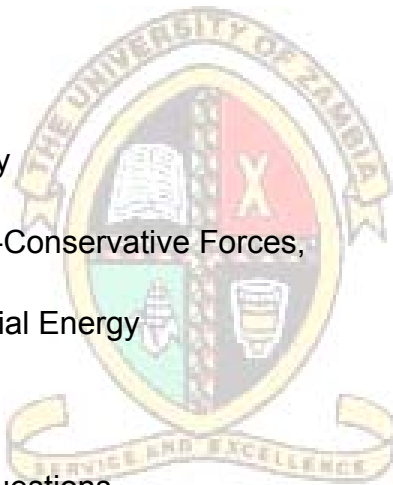


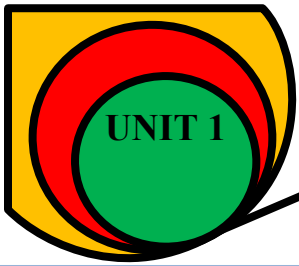


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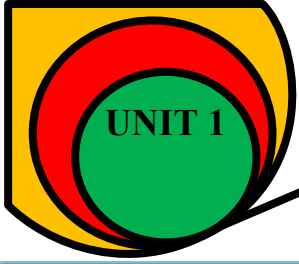


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Measurements and Vectors.



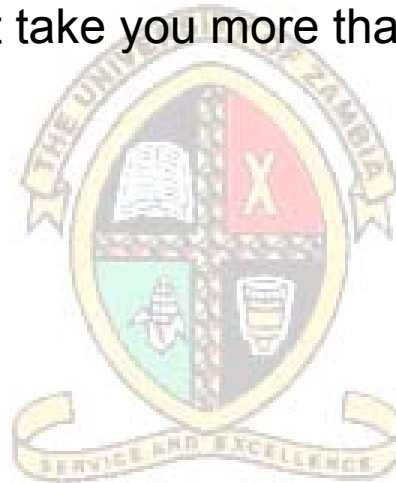


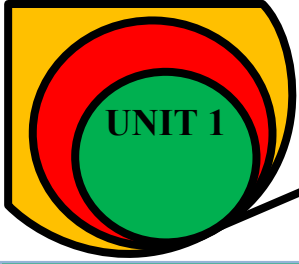
Measurements and Vectors



UNIT 1: Measurements and Vectors: (15 hours)

This unit should not take you more than 15 hours.



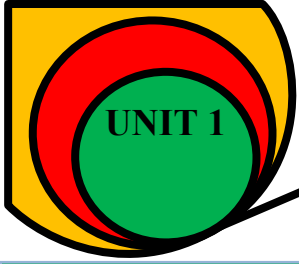


1.1 Introduction

Physics! Why physics? What is it that makes an understanding of the fundamentals of science so essential in today's world? Unlike our distant ancestors, we no longer rely on our innate senses but employ high technology to learn about the world around us. We no longer rely on the position of the sun in the sky to know the time. We no longer rely just on the strength of our own bodies to build or move but also on machines. We no longer gather food or building materials only from our local environment but also move raw materials around the globe with ease and on a daily basis. We are always striving to improve our lives by accessing information in ever greater amounts and at ever greater rates. Our society is heavily reliant on technology and all technology is rooted in the basic sciences. To gain an understanding of the workings of our society, to actively contribute to it as scientists or engineers, or to make use of it as consumers, we need to understand some of the basics of physics.

Now, **Welcome to Unit 1: *Measurements and Vectors*!!**

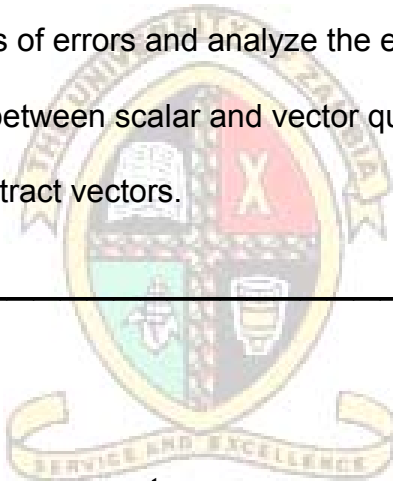
In this unit we will describe the *essentials of measurement* and the meaning of *accuracy* in measurement. We will explain the role of the *fundamental quantities* of our physical world, namely space, time, and mass. We will describe the system of units that allows us to communicate the results of our observations in a universal way; finally, we will discuss *vectors*, a significant descriptive tool for the real world.



1.2 Unit Objectives

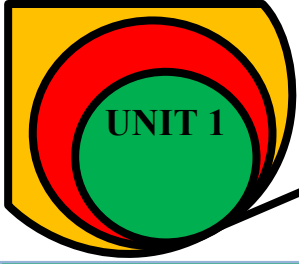
After completing this unit, you should be able to

- Identify the seven fundamental quantities in the SI system
- Measure quantities such as length, mass and some related derived units
- Identify types of errors and analyze the errors when measuring
- Distinguish between scalar and vector quantities
- Add and subtract vectors.



1.3 Measurements

The laws of physics are expressed as mathematical relationships among physical quantities that we will discuss throughout this module. Most of these quantities are *derived quantities*, in that they can be expressed as combinations of a small number of *basic quantities*.



1.3.1. SI Units

In mechanics, the three basic quantities are **length**, **mass**, and **time**. All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a *standard* must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 “glitches” if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit.

Likewise, if we are told that a person has a mass of 75 kilograms and our unit of mass is defined to be 1 kilogram, then that person is 75 times as massive as our basic unit.1 Whatever is chosen as a standard must be readily accessible and possess some property that can be measured reliably. Measurements taken by different people in different places must yield the same result.



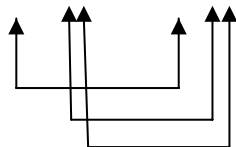
In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the **SI (Système International)**, and its units of length, mass, and time are the *meter*, *kilogram*, and *second*, respectively. Length is designated by a dimension $[L]$, time $[T]$ and mass $[M]$. Other SI standards established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*). In total there are seven standard units. All other units are derived from this seven.

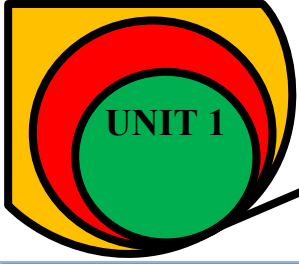
- An example is velocity, which is just m/s , and from this, it can be observed that it is derived from length $[L]$ and time $[T]$ that is,

$$\frac{[L]}{[T]} = \frac{m}{s} \text{ or simply } ms^{-1}.$$

- Similarly to derive the units for **force** we need to know its formula being **mass times acceleration** as well as the units of the individual parameters in the equation. Therefore we have;

$$F = ma = [M][LT^{-2}] = kg \ ms^{-2}$$





Measurements and Vectors



The arrows show the units for a particular dimension. Therefore the units are $kgms^{-2}$ which is also known as the Newton (**N**)

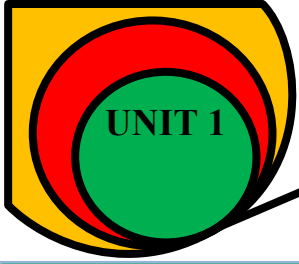


Self-help task 1.1

What are the SI units for the following (a) Pressure? (b) Work?
Check the answers at the end of the chapter! Did you get them right?
If No: **Rework**, If Yes then

PROCEED



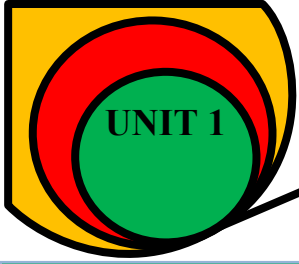


1.3.2 Uncertainty in measurements

Physics rests on experiment, and experiment requires measurement. But measurements are, at best, only approximate more or less so depending on the instrument doing the measurement. Although you might be happy to know the distance of the route you took between Lusaka and Ndola to within the tenth of a mile possible on your car's odometer, you would use a tape measure rather than the car's odometer to measure the length of your driveway. An uncertainty is an indication of the accuracy of a measurement. The uncertainty depends on the accuracy and calibration of the instrument that is making the measurement and on how well the instrument can be read.

When you measure the thickness of a book with an instrument and record your answer as 2.3 cm, what this means is that you are sure of the two "2" but uncertain of the three "3." For a mass such as 4.56 kg, you are sure of the 4.5 but not sure of the last digit in the decimal six "6." Hence we shall talk about uncertainties in measurements.

We can best illustrate the meaning of uncertainty with an example. If the width of a page of paper is measured with a ruler to be 21.6 cm with an



Measurements and Vectors



uncertainty of 1 mm (or 0.1 cm) which is about the best you could manage with a typical ruler, it would be correct to say that the width is $21.6 \text{ cm} \pm 0.1 \text{ cm}$, or $21.6 \pm 0.1 \text{ cm}$. (The \pm is read as "plus or minus.") Here, 21.6 cm is called the *central value* or *best estimate* and 0.1 cm the uncertainty around that central value. This implies that at least we are sure of the central value, 21 cm but we are not sure of the 0.6 cm. The width lies between $21.6 - 0.1 = 21.5 \text{ cm}$ and $21.6 + 0.1 = 21.7 \text{ cm}$.

The most common way to show the range of values that we believe includes the true value is:

$$\text{Measurement} = \text{best estimate} \pm \text{uncertainty (units)} \quad (1.1)$$

The general guideline and quick way of finding this limit of precision or uncertainty is stated as follows:

"The limit of precision of a measuring device is $\pm 1/2$ the smallest division of measurement a device is able to display."

Therefore,

- If the smallest division in a meter rule is 1 mm, then the limit of precision is $\pm 1/2 = \pm 0.5 \text{ mm}$
- For stop watch ruled in 0.5 seconds intervals, the precision is $\pm 0.25 \text{ sec}$
- For the scale in Figure 1.1, it reads to the nearest 0.1 kg. Therefore the precision is $\pm 0.05 \text{ kg}$. Suppose you want to find the weight of a heavyweight boxer, his weight would lie between 138.15 kg and 138.25 kg that is $138.2 \pm 0.05 \text{ kg}$.



When reporting the final value, it is important to observe the number of decimal figures, implying that the uncertainties have to be rounded off to conform to the number of decimal places in the best estimate. For the weight of the heavy boxer in the above example, the weight would be 138.2 ± 0.1 kg.

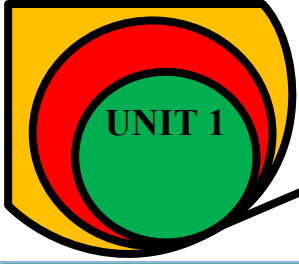


Figure 1.1: bathroom scale



Self-help task 1.2

Determine the limits of precision for: (a) an ammeter that reads 2.23 Amperes, b) a digital clock that reads 2.14 seconds.



Measurement errors may be classified as either **random** or **systematic**, depending on how the measurement was obtained.

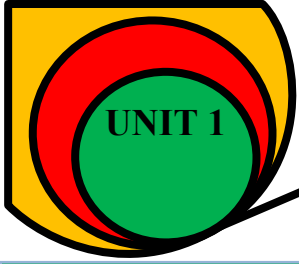
To learn about these errors, proceed to the next section.

1.3.1.1 Systematic errors

This error involves the possibility of incorrect design or calibration of an instrument. They cause the measurement to be reproducibly higher or lower than the true value. Such a measurement is said to be inaccurate. If a systematic error is identified when calibrating against a standard, applying a correction or correction factor to compensate for the effect can reduce the bias. Additionally great care in instrument design, calibration and reading can reduce this kind of error. However such errors cannot be detected or reduced by increasing the number of observations.

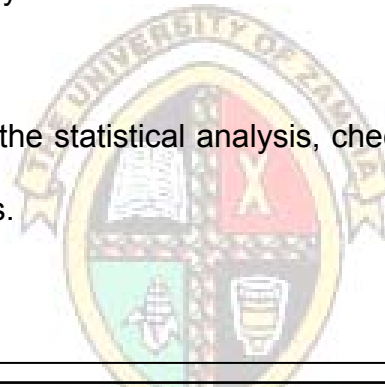
1.3.1.2 Random errors

If multiple measurements of the same quantity are taken using the same instrument, the results would often differ by more than the precision of the instrument. They are caused by fluctuations in the physical property being measured, such as changes in temperature, electrical voltage, pressure and so on.. These errors cannot be avoided nor eliminated but can be reduced by increasing the number of measurements/observations.



Their effect on the accuracy of the measured value can be calculated by using a statistical analysis as we shall see later. An example of a set of data that are subject to random error is for finding the period of a pendulum. You would record readings such as: $t = 4.23 \text{ s}$, 4.20 s , 4.24 s , 4.21 s , 4.22 s . You can see that these values are all close to the average value (4.22 s). To find the uncertainty we use statistics.

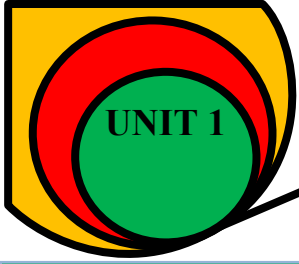
Before you go to the statistical analysis, check if you are able to categorize the types of errors.



Self-help task 1.3

Determine the type of error involved in the following

- | | |
|-------------------------|-----------------------------------|
| (a) Personal errors | (d) Wind blowing in the lab |
| (b) Instrument drift | (e) Failure to zero an instrument |
| (c) Physical Variations | (f) Environmental factors |



If you have finished identifying the kinds of errors in the self-help task and you are sure of them now, we can delve into calculating the associated error on these repeated measurements.

1.3.1.3 Estimating uncertainty in repeated measurements (Random errors).

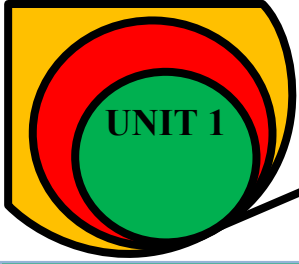
A good experimental practice in reducing errors is to have repeated measurements. If you want to find the period of a pendulum, it is not advisable to just do “one run” of measurement. A number of measurements guarantee you a best estimate for that particular measurement.

Now consider an experiment where “ N runs” or N repeated measurements are made. This implies that you have N values. If this value is x_i where

$$i = 1, 2, 3 \dots N,$$

then the best estimate of this value is the average or mean, i.e,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots x_N}{N} = \frac{\sum x_i}{N} \quad (1.2)$$



Measurements and Vectors



So that our final answer to be reported is ;

Measurement = best estimate \pm uncertainty (units)

Or $measurement = \bar{x} \pm \sigma$ (1.3)

Where σ (sigma) is the uncertainty or standard deviation of the mean. This sigma is the one to be calculated statistically.

One way to express the variation among the measurements is to use the **average deviation**. This statistic tells us on average (with 50% confidence) how much the individual measurements vary from the mean or best estimate. The average deviation is given by

(1.4)

However, the **standard deviation, s** is the most common way to characterize the spread of a data set. The *standard deviation* is always slightly greater than the *average deviation*.

To calculate the standard deviation, the following is the procedure:

1. Sum all the measurements and divide by N to get the **average**, or **mean**.
2. Now, subtract this **average** from each of the N measurements to obtain N “**deviations**”.
3. **Square** each of these N **deviations** and add them all up.



4. Divide this result by $(N-1)$ and take the square root.

We can write out the formula for the standard deviation as follows. Let the N measurements be called x_1, x_2, \dots, x_N . Let the average of the N values be called \bar{x} . Then each deviation is given by

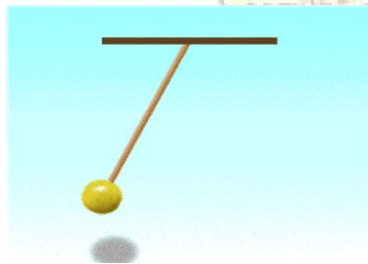
$$\delta x_i = x_i - \bar{x} \quad (1.5)$$

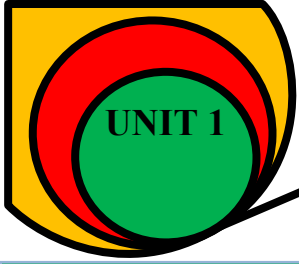
for $i = 1, 2, \dots, N$. The **standard deviation** is:

$$s = \sqrt{\frac{\delta x_1^2 + \delta x_2^2 + \dots + \delta x_N^2}{N-1}} = \sqrt{\frac{\sum \delta x_i^2}{N-1}} \quad (1.6)$$



EXAMPLE 1.1 : Suppose you measure the oscillation period of a pendulum with a stopwatch five times and record your findings in a table as shown:





When we report the average value of N measurements, the uncertainty we should associate with this average value is the *standard deviation of the mean*, often called the *standard error* (SE). This is the uncertainty we need.

Standard Deviation of the Mean, or Standard Error (SE),

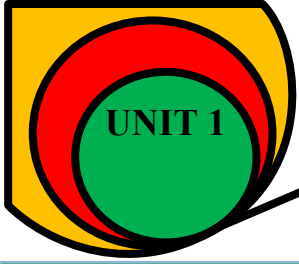
$$\sigma = \sqrt{\frac{\delta x_1^2 + \delta x_2^2 + \dots + \delta x_N^2}{N(N-1)}} = \sqrt{\frac{\sum \delta x_i^2}{N(N-1)}} = \frac{s}{\sqrt{N}} \tag{1.7}$$

The *standard error* is smaller than the *standard deviation* by a factor of $\frac{1}{\sqrt{N}}$. This reflects the fact that we expect the uncertainty of the average value to get smaller when we use a larger number of measurements, N .

Trial number, i	1	2	3	4	5
Measured Value, x_i (seconds)	3.9	3.5	3.7	3.4	3.5

Note that in this case $N=5$.

To best determine the standard deviation or uncertainty, we can use a table as shown:



Trial number, <i>i</i>	Measured value, x_i	mean, \bar{x}	deviation, $\delta x = x_i - \bar{x}$	δx^2
1	3.9	3.6	0.3	0.09
2	3.5	3.6	-0.1	0.01
3	3.7	3.6	0.1	0.01
4	3.4	3.6	-0.2	0.04
5	3.5	3.6	-0.1	0.01
TOTAL			0.0	0.16

The standard error σ therefore becomes,

$$\sigma = \sqrt{\frac{\delta x_1^2 + \delta x_2^2 + \dots + \delta x_N^2}{N(N-1)}} = \sqrt{\frac{\sum \delta x_i^2}{N(N-1)}} = \sqrt{\frac{0.16}{5(5-1)}} = \sqrt{\frac{0.16}{5(4)}} = 0.09 = 0.1$$

Therefore the measurement for period is:

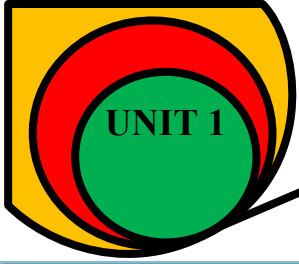
$$\text{measurement} = \bar{x} \pm \sigma = (3.6 \pm 0.1)s$$

You got it? Now it's your turn...



Self-help task 1.4

Consider the measurement of the width of a piece of paper using a meterstick. The width of the paper is measured at a number of points on the sheet, and the values obtained in mm were: 31.33, 31.15, 31.26, 31.02, and 31.20.



You got it right? If not Rework or otherwise

PROCEED

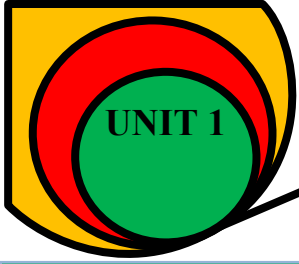


1.4 Scalars and Vectors

In our study of physics, we often need to work with physical quantities that have both numerical and directional properties. These are either *scalar* or *vector* quantities. A scalar quantity has *magnitude only*; no direction is associated with it.

Examples are pure numbers and physical quantities such as mass, time, volume, distance and speed. To completely specify a scalar quantity, one has to state the magnitude and the corresponding unit, if any. For example 10 books, 5 chairs, 3 tables *etc.*

A vector quantity has *both magnitude and direction*. Examples include displacement, velocity, acceleration, force, and electric field strength.



To make a complete statement about a vector quantity, one has to state both magnitude and direction. For example, if we say that a car is moving at 50 km per hour, we are talking only about its speed or its magnitude, which is a scalar quantity. This does not tell us which direction the car is moving. On the other hand if we say that the car is moving at 50 km per hour due east, we are talking about the velocity of the car which has a magnitude of 50 km per hour directed east. Hence this is a vector quantity.

1.4.1 Vector Addition and Subtraction

Before we add vectors, we should first describe *displacement*. The displacement is a position vector whose magnitude is the straight line distance from A to B and its direction is that of an arrow that points from A to B.

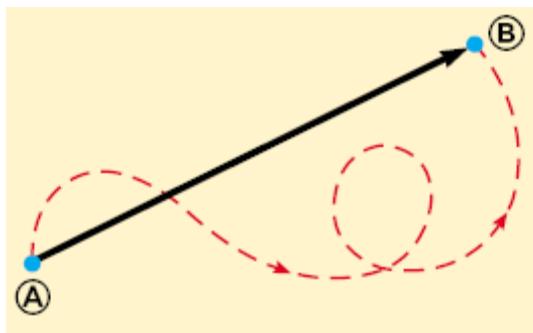
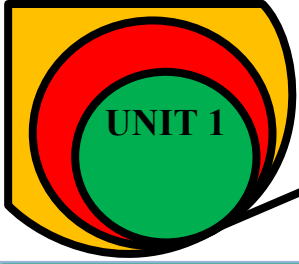


Figure 1.2



In this module, we use a boldface letter, such as **A**, to represent a vector quantity. Another notation is useful when boldface notation is difficult, such as when writing on paper or on a chalkboard—an arrow is written over the symbol for the vector: \vec{A} . The magnitude of the vector **A** is written as either A or $|A|$.

The magnitude of a vector has physical units, such as meters for displacement or meters per second for velocity. The magnitude of a vector is *always* a positive number.

1.4.2 Vector Addition and Subtraction by Graphical Methods

To add vector **B** to vector **A**, first draw vector **A** on graph paper, with its magnitude represented by a convenient length scale, and then draw vector **B** to the same scale with its tail starting from the tip of **A**, as shown in Figure 1.3. The **resultant** vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is the vector drawn from the tail of **A** to the tip of **B**.

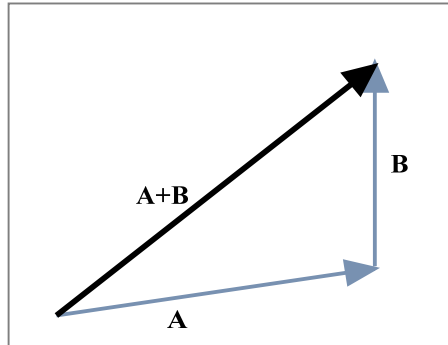


Figure 1.3

A geometric construction can also be used to add more than two vectors. This is shown in Figure 1.4 for the case of four vectors. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$ is the vector that completes the polygon.

In other words, \mathbf{R} is the vector drawn from the tail of the first vector to the tip of the last vector.

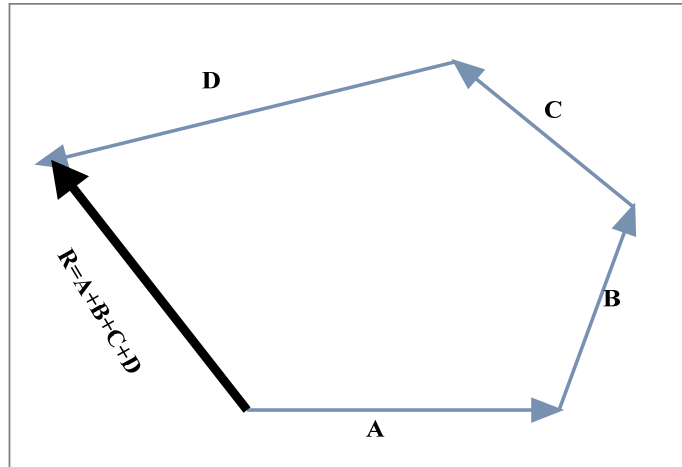


Figure 1.4.

It should be observed that the result of adding vectors does not depend on the order in which you add them.

1.4.3 Graphical Subtraction of Vector

Given a vector V , we define the negative of this vector ($-V$) to be a vector with the same magnitude as V but opposite in direction as shown in Figure 1.5.

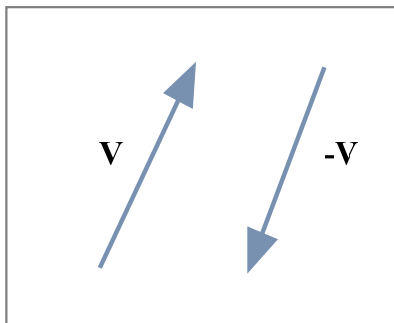
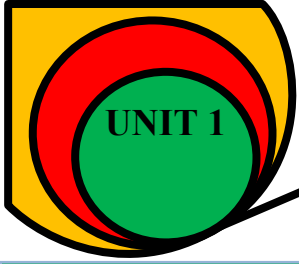


Figure 1.5



It is important to note that no vector is ever negative in the sense of its magnitude; the magnitude of every vector is positive. Rather, a minus sign tells us about its direction.

How about subtraction? What do you think?

Well, we can now define the subtraction of one vector from another: the difference between two vectors $V_2 - V_1$ is defined as

$$V_2 - V_1 = V_2 + (-V_1)$$

Therefore the difference between two vectors is equal to the sum of the first plus the negative of the second. Thus the rules for addition of vectors can be applied as shown in Figure 1.6 using the tail-to-tip method.

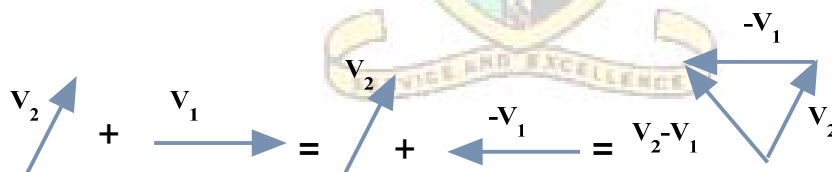


Figure 1.6



1.4.1 Vector Addition by Analytical Method

The graphical method of adding a vector is not recommended whenever high accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the projections of vectors along coordinate axes. These projections are called the components of the vector. Any vector can be completely described by its components.

We shall now briefly review some simple trigonometric functions of a right angles triangle.

Consider a vector \mathbf{A} lying in the xy plane and making an arbitrary angle θ with the positive x axis, as shown in Figure 1.7. This vector can be expressed as the sum of two other vectors \mathbf{A}_x and \mathbf{A}_y .

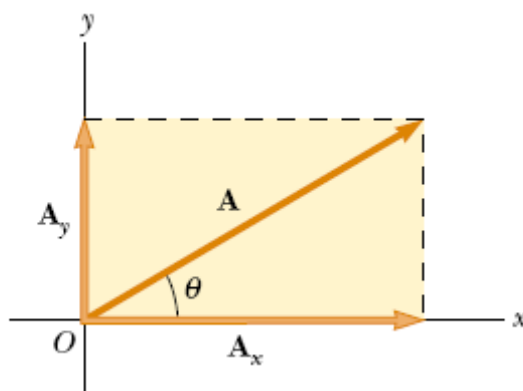


Figure 1.7.



We can also see that the three vectors form a right triangle and that $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$. We shall often refer to the “components of a vector \mathbf{A} ,” written A_x and A_y (without the boldface notation).

The component A_x represents the projection of \mathbf{A} along the x axis, and the component A_y represents the projection of \mathbf{A} along the y axis. These components can be positive or negative. The component A_x is positive if A_x points in the positive x direction and is negative if A_x points in the negative x direction. The same is true for the component A_y .

From Figure 1.7 and the definition of sine and cosine, we see that

$$\cos \theta = \frac{A_x}{A}$$

and

$$\sin \theta = \frac{A_y}{A}$$

So that

$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

For example, if $\theta = 120^\circ$, then A_x is negative and A_y is positive. If $\theta = 225^\circ$, then both A_x and A_y are negative. Figure 1.8 summarizes the signs of the components when \mathbf{A} lies in the various quadrants.



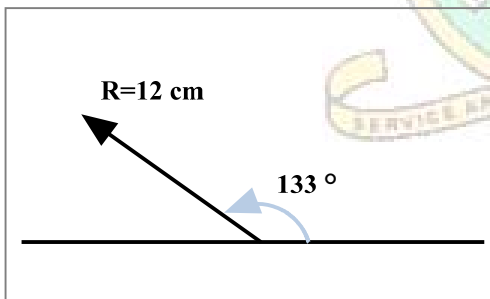
When solving problems, you can specify a vector **A** either with its components A_x and A_y or with its magnitude and direction A and θ .

	y	
A_x negative		A_x positive
A_y positive		A_y positive
		x
A_x negative		A_x positive
A_y negative		A_y negative

Figure 1.8



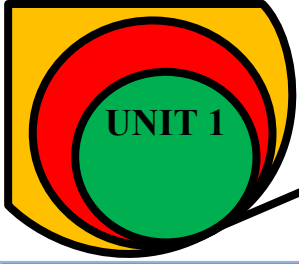
EXAMPLE 1.2: Calculate the x and y component of the vector **R**



$$R_x = R \cos \theta = 12 \cos 133^\circ =$$

$$R_y = R \sin \theta = 12 \sin 133^\circ =$$

Finish the working! What are the values? Use a calculator.



Self-help task 1.5

Before proceeding be sure you can find the x and y components of the vectors shown in Figure 1.9. Note also that the direction of each component is indicated by an algebraic

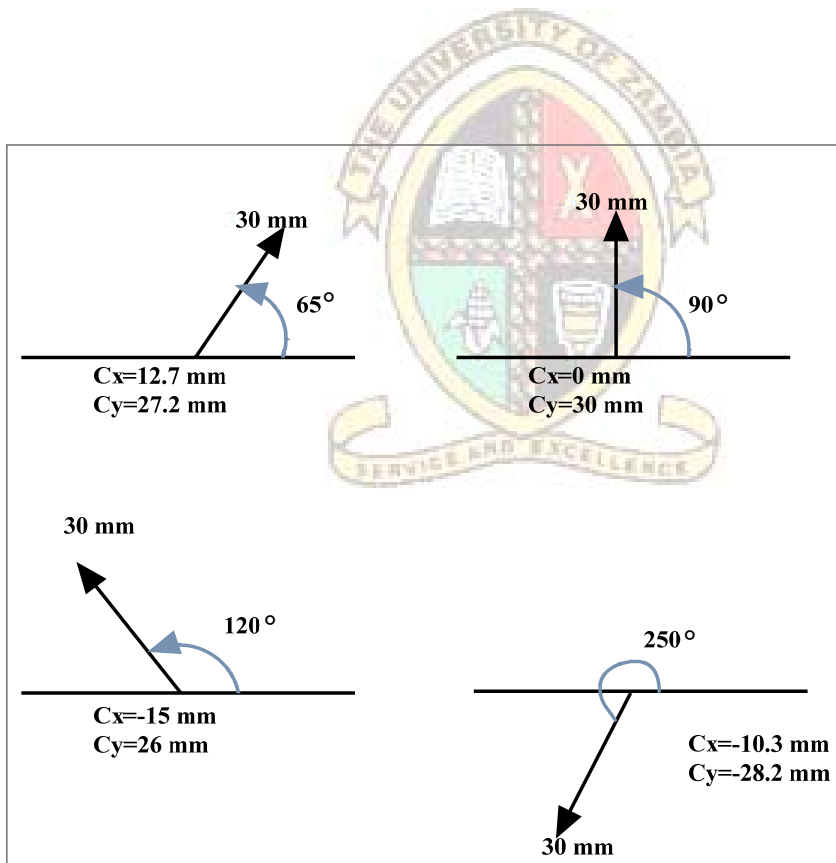
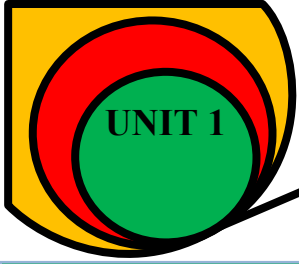


Figure 1.9



Did you get them right? If you didn't please rework otherwise

PROCEED



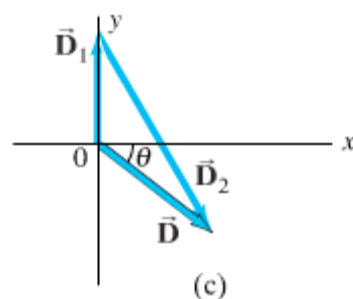
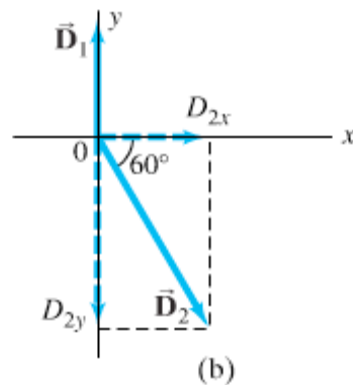
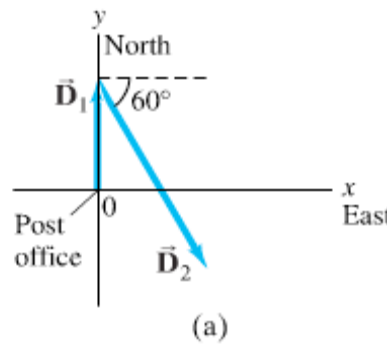
Finally here is a table showing tips on how to add/subtract vectors analytically.

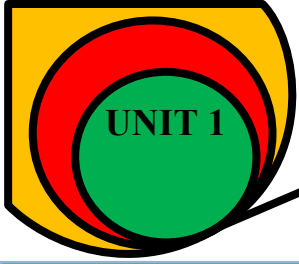
PROBLEM SOLVING: ADDING VECTORS	
<p>1. Draw a diagram, adding vectors graphically by either the parallelogram or tail-to-tip method</p>	<p>4. Calculate each component using sines and cosines. If θ is the angle that the vector \mathbf{V} makes with the positive axis, then</p> $V_x = V \cos \theta \quad \text{and} \quad V_y = V \sin \theta$ <p>Pay careful attention to signs: any component that points along the negative x or y axis gets a - sign.</p>
<p>2. Choose x and y axes. Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors so that the vector will have only one component.)</p>	<p>5. Add the x components together to get the x component of the resultant. Ditto for y:</p> $R_x = A_x + B_x + \text{any others}$ $R_y = A_y + B_y + \text{any others}$ <p>This is the answer: the components of the resultant vector.</p>
<p>3. Resolve each vector into its x and y components, showing each component along its appropriate (x or y) axis as a dashed arrow.</p>	<p>6. If you want to know the magnitude and direction of the resultant vector,</p> $R = \sqrt{R_x^2 + R_y^2} \quad \tan \theta = \frac{R_y}{R_x}$ <p>The vector diagram you already drew helps to obtain the correct position of the angle θ.</p>



EXAMPLE 1.3 : A rural carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction 60.0° south of east for 47.0 km.

What is her displacement from the post office?





To add the vectors, we add the x and y components separately, i.e,

$$R_x = D_{1x} + D_{2x} \quad \text{and} \quad R_y = D_{1y} + D_{2y}$$

This is best done by drawing tables as shown below:

Vector	x-component	y-component
D_1	0	20
D_2	$D_2 \cos 60$	$-D_2 \sin 60$
$R=D_1+D_2$	$0 + D_2 \cos 60$	$20 +(- D_2 \sin 60)$

After punching the calculator, the table looks like this :

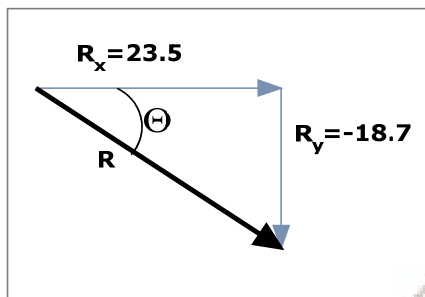
Vector	x-component	y-component
D_1	0	20
D_2	23.5	-40.7
$R=D_1+D_2$	23.5	-18.7



From the table,

$$R_x = 23.5 \text{ km and } R_y = -18.7 \text{ km}$$

So that we have



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(23.5)^2 + (-18.7)^2} = 30.0 \text{ km}$$

And the direction is

$$\tan \theta = \frac{23.5}{-18.7} = -0.796$$

A calculator with INV TAN, ARC TAN or TAN^{-1} key gives $\theta = \tan^{-1}(-0.796) = -38.5^\circ$.

The negative implies the angle is below the x-axis. Therefore the displacement is 30.0 km directed at 38.5° below the positive x-axis

or at $360^\circ - 38.5^\circ = 321.3^\circ$ as measured from the positive axis.



EXAMPLE 1.4 : A bug is crawling along a tabletop and undergoes the following displacements:

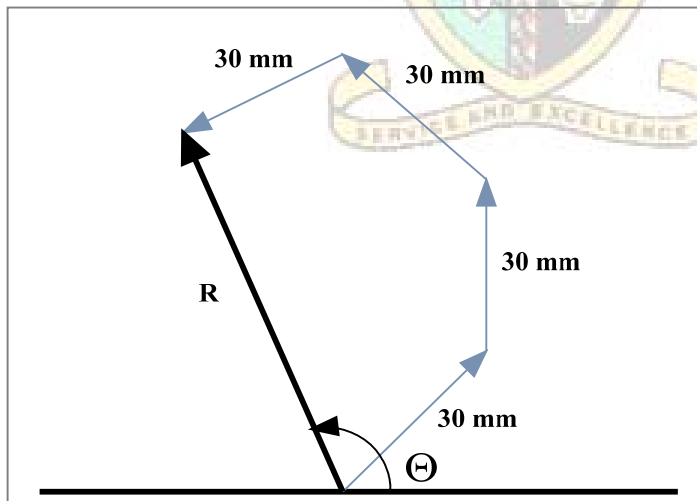
30.0 mm at 65.0° to the positive x -axis (east)

30.0 mm at 90.0°

30.0 mm at 120.0°

30.0 mm at 250.0° .

Graphically, the vectors can be solved as shown below:



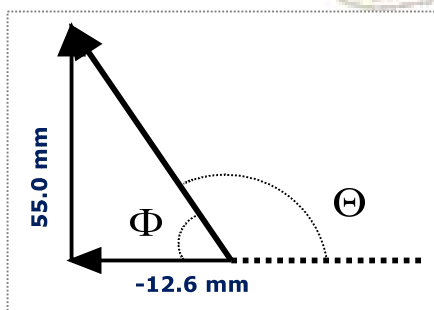
and then measuring the length of R and angle θ .



Analytically, we can solve it by first drawing a table to simply the problem.

Vector	x-component	y-component
A	$30.0 \cos 60.0^\circ$	$30.0 \sin 60.0^\circ$
B	0	30.0
C	$30.0 \cos 120.0^\circ$	$30.0 \sin 120.0^\circ$
D	$30.0 \cos 250.0^\circ$	$30.0 \sin 120.0^\circ$
R=A+B+C+D	-12.6 mm	55.0 mm

So that $R_x = -12.6 \text{ mm}$ and $R_y = 55.0 \text{ mm}$



$$R = \sqrt{((-12.6)^2 + (55.0)^2)} = \sqrt{3184} = 56.4 \text{ mm},$$



$$\tan \varphi = \frac{R_x}{R_y} = \frac{55.0}{12.6} = 4.37$$

$$\text{so that } \varphi = \tan^{-1}(4.37) = 77.0^\circ$$

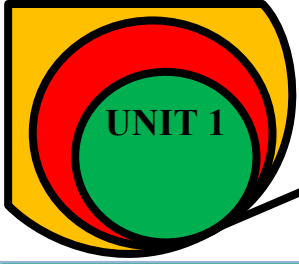
Therefore, the displacement is 56.4 mm at 77.0° above the negative x-axis or $180^\circ - 77^\circ = 103.0^\circ$ as measured from the positive x-axis.



EXAMPLE 1.5: Two displacements **A** and **B** lie in the xy plane. **A** is 49 cm at $\theta = 42^\circ$ and **B** is 32 cm at 115° . What are the displacements **A+B** and **A-B**?

Solution: The table will look like this:

Vector	x-component	y-component
A	$49 \cos 42.0^\circ$	$49 \sin 42.0^\circ$
B	$32 \cos 115.0^\circ$	$32 \sin 115.0^\circ$
A + B	$49 \cos 42.0^\circ + 32 \cos 115.0^\circ$	$49 \sin 42.0^\circ + 32 \sin 115.0^\circ$
A - B	$49 \cos 42.0^\circ - 32 \cos 115.0^\circ$	$49 \sin 42.0^\circ - 32 \sin 115.0^\circ$



So that we have

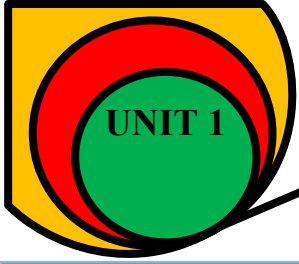
Vector	x-component	y-component
A	$49 \cos 42.0^\circ$	$49 \sin 42.0^\circ$
B	$32 \cos 115.0^\circ$	$32 \sin 115.0^\circ$
A + B	22.9	61.8
A - B	49.9	3.78

$$A + B = \sqrt{(22.9^2 + 61.8^2)} = 65.9 \text{ cm}$$

And

$$\theta = \tan^{-1}\left(\frac{61.8}{22.9}\right) = 69.7^\circ \text{ above the positive x-axis.}$$

Similarly for **A - B** prove that it is **50.0 cm** and that the angle is **4.33°** above the positive x-axis.



You need **not** make two tables to solve a particular problem; this was just done for mere clarification. A single table is enough as long as you have a calculator by your side.



1.5. Unit Summary

The three fundamental physical quantities of mechanics are length, mass, and time, which in the SI system have the units meters (m), kilograms (kg), and seconds (s), respectively. All the others are derived.

Numbers that represent physical quantities can be measured only to a certain accuracy. An explicit way to indicate this accuracy is to write a physical quantity x as a *best estimate* \pm *an uncertainty*. Calculations involving physical quantities are meaningful only to within the known accuracy of those quantities.

There are two types of errors: Random and Systematic. Random errors are statistical fluctuations (in either direction) in the measured data due to the precision limitations of the measurement device. Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations (see standard error).



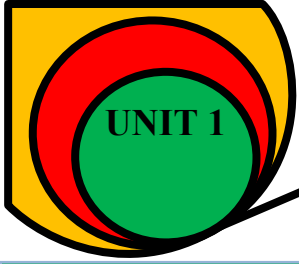
Systematic errors are reproducible inaccuracies that are consistently in the same direction. These errors are difficult to detect and cannot be analyzed statistically. If a systematic error is identified when calibrating against a standard, applying a correction or correction factor to compensate for the effect can reduce the bias. Unlike random errors, systematic errors cannot be detected or reduced by increasing the number of observations.

Physical quantities can either be scalars or vectors. A scalar is a quantity that has magnitude only while a vector has both magnitude and direction. Vectors can be added or subtracted by adding the x and y components separately and then using the Pythagoras theorem, the magnitude can be found i.e.:

$$A+B = \sqrt{R_x^2 + R_y^2}$$

Where $R_x = A_x+B_x$ and $R_y=A_y+B_y$

$$\text{Tan } \theta=R_y/R_x$$



1.6. Answers to self-help tasks

1.1. (a). $P = F/A = \text{N/m}^2$ OR $P = F/A = ma/A = \text{kgms}^{-2}/\text{m}^2 = \text{kgm}^{-1}\text{s}^{-2}$

(b). $W = F \cdot d = \text{Nm}$ OR $W = ma \cdot d = \text{kgms}^{-2} \cdot \text{m} = \text{kgm}^2\text{s}^{-2} = \text{joule} = \text{J}$

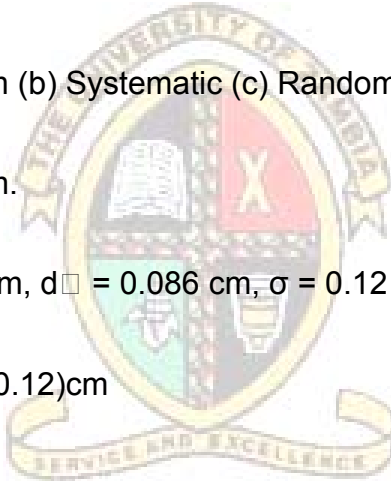
1.2 (a) For an ammeter that reads 2.23 A, the smallest division is 0.01. The limit of precision is $0.01/2 = 0.005$ A

1.3 (a) Random (b) Systematic (c) Random (d) Random (e) Systematic

(f) Random.

1.4 $x_{\square} = 31.19$ cm, $d_{\square} = 0.086$ cm, $\sigma = 0.12$ cm. The measurement is (

$(31.19 \pm 0.12)\text{cm}$





1.7. References

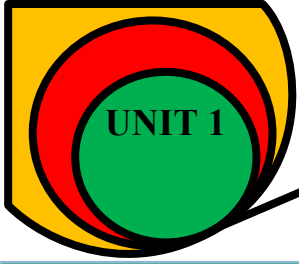
F.J Bueche, D.A Jerde, *Principles of Physics*.6th Edition, McGraw-Hill, 1995.

ISBN 0- 13.978-00700

D. Giancoli, *Physics:Principles with Applications*, 6th Edition.Pearson Prentice Hall, ISBN 0-13-060620-0

P.M Fishbane, S.G Gasiorowicz,S.T Thornton, *Physics for Scientists and Engineers with Modern Physics*,3rd Edition, Pearson Prentice Hall , ISBN 0-13-191182-1

Sears and Zemansky, *University Physics (with Modern Physics)*, 12th Edition, Hugh D Young and Roger, A Freedman,Pearson/Addison Wesley,2007, ISBN 0-8053-2187-X.



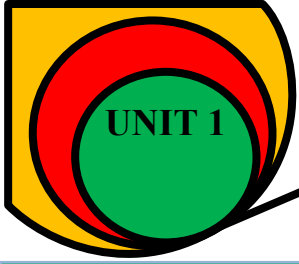
1.8. Exercise

1.1 Can the magnitude of a particles displacement be greater than the distance travelled? Explain

Answer. No, the magnitude of the displacement is always less than or equal to the distance traveled. If two displacements in the same direction are added, then the magnitude of their sum will be equal to the distance traveled. Two vectors in any other orientation will give a displacement less than the distance traveled. If you first walk 3 meters east, and then 4 meters south, you will have walked a total distance of 7 meters, but you will only be 5 meters from your starting point.

1.2 One car travels due east at 40 km/h, and a second car travels north at 40 km/h. Are there velocities equal? Explain.

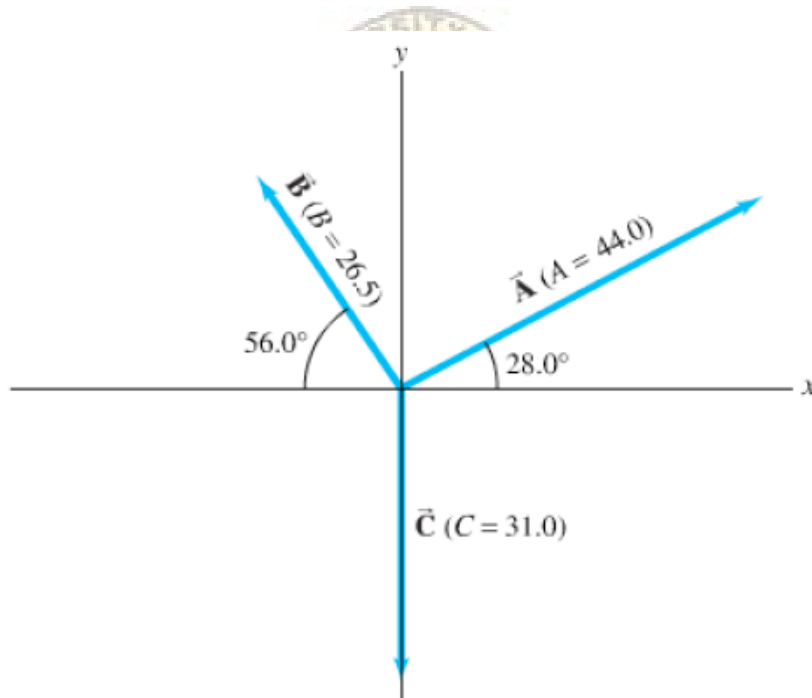
Answer. No. Velocity is a vector quantity with both direction and magnitude. The fact that the cars were going in different directions makes velocity different, though the magnitude is the same, 40 km/h.



1.3 A Zambeef delivery track travels 18 km north, 10 km east, and 16 km south. What is its final displacement from the origin?

Answer. 10.8 km, north east

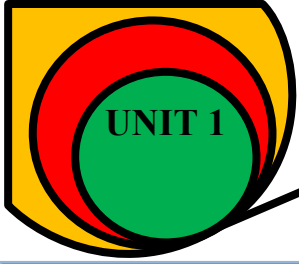
1.4 Three vectors are shown below. Their magnitudes are in arbitrary units.



Determine

i. $\mathbf{A + B + C}$

Answer: 26.7 at $\theta = 25.8^\circ$



Measurements and Vectors



ii. **A – C** **Answer:** 64.6 at $\theta = 53.1^\circ$

iii. **B – A** **Answer:** 53.7 at $\theta = 1.4^\circ$ above the negative x-axis OR $\theta = 180 - 1.4 = 178.6^\circ$

1.5 Vector A has a magnitude of 40 m at $\theta = 225^\circ$. If we want to add to **A** a vector **B** so as to produce a resultant along the positive x-axis that has a magnitude of 20 m, what must be the components of **B**?

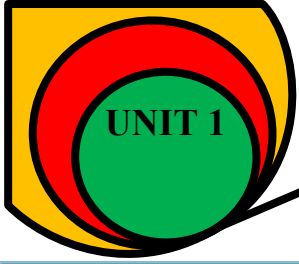
Answer:

Vector	x-component	y-component
A	-28.3	-28.3
B	B_x	B_y
R=A+B	20	0

$$R_x = A_x + B_x : 20 = -28.3 + B_x \text{ therefore } B_x = 48.3 \text{ m}$$

Similarly

$$R_y = A_y + B_y : 0 = -28.3 + B_y \text{ therefore } B_y = 28.3 \text{ m}$$



Measurements and Vectors



1.6 When displacement **B** is added to displacement **A** the result is a displacement **C** that has components $C_x = -3.70 \text{ cm}$ $C_y = +2.25 \text{ cm}$ and

$C_z = +4.60 \text{ cm}$. Displacements **A** and **B** are in the same direction, but the magnitude of **A** is only one-third that of **B**. Find the components of **A**.

Answer:

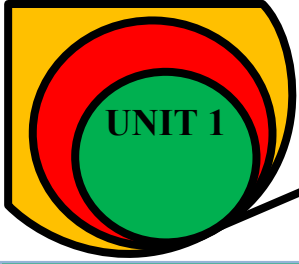
Vector	x-component	y-component	z-component
A	A_x	A_y	A_z
B	$3A_x$	$3A_y$	$3A_z$
C=A+B	-3.70	2.25	4.60

$$C_x = A_x + B_x : -3.70 = A_x + 3A_x \text{ therefore } A_x = -0.925 \text{ cm}$$

$$C_y = A_y + B_y : 2.25 = A_y + 3A_y \text{ therefore } A_y = 0.563 \text{ cm}$$

And

$$C_z = A_z + B_z : -3.70 = A_z + 3A_z \text{ therefore } A_z = -0.925 \text{ cm}$$



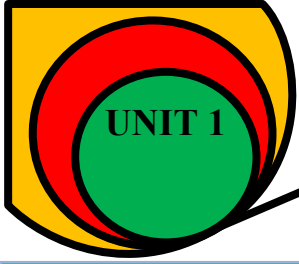
Measurements and Vectors



Got them right?

Good! Proceed to unit 2 then...



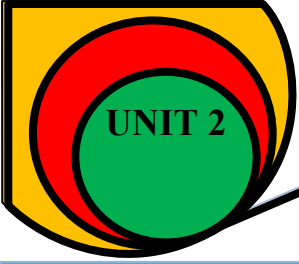


Measurements and Vectors



The logo of The University of Zambia is centered in the background. It features a circular emblem with a shield in the center, divided into four quadrants of different colors (green, red, blue, and yellow). Above the shield is an open book, and below it is a banner with the motto "SERVICE AND EXCELLENCE". The text "THE UNIVERSITY OF ZAMBIA" is written around the top of the circular border.

Motion in One Dimension



Motion in One Dimension



UNIT 2: Motion in One Dimension (15 hrs):

This unit should not take you more than 15 hours.

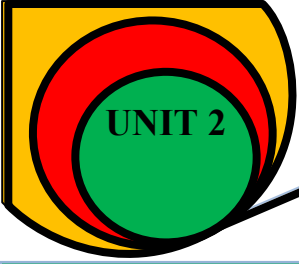




2.1 Introduction

As a first step in studying classical mechanics, we describe motion in terms of space and time while ignoring the agents that caused that motion. This portion of classical mechanics is called *kinematics*. In this chapter we consider only motion in one dimension, that is, motion along a straight line. We first define position, displacement, velocity, and acceleration. Then, using these concepts, we study the motion of objects traveling in one dimension with a constant acceleration. We shall also study the relationships of displacement, velocity and acceleration as a function of time through graphs. Finally we shall look at falling bodies and relative motion.

Welcome to Unit 2!



2.2 Unit Objectives

After completing this unit, you should be able to

- Derive and use equations of linear motion
- Interpret $s-t$, $v-t$ and $a-t$ graphs of linear motion
- Solve problems in the kinematics of 1-D motion including free fall
- Calculate the relative velocity of moving bodies

2.3 Particle Kinematics:

The motion of a particle is completely known if the particle's motion in space is known at all times. A particle's *position* is the location of the particle with respect to a chosen reference point that we can consider the origin of a coordinate system.

In everyday conversation, we use the terms *speed* and *velocity* interchangeably but in science it is not the case as we shall discover soon in the next section



2.3.1 Displacement

We now develop the concept of velocity. We first need to differentiate between *distance* travelled and the *displacement* of a particle which is the change in position of the body.

Displacement (s), thus, is how far the object is from its starting point. To see the distinction between total distance and displacement, imagine a person walk 70 m to the east and then turning around and then walking back (west) a distance 30 m (see Fig 2.1).

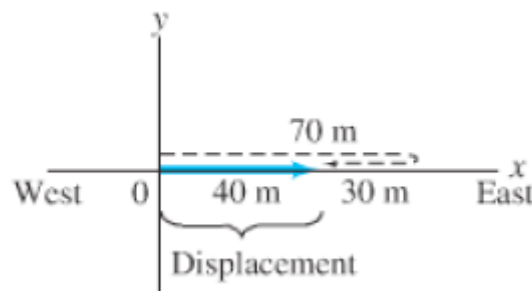


Figure 2.1

The total *distance* travelled is 100m, but the *displacement* is only 40 m since the person is now only 40 m from the starting point.



The difference between average speed and average velocity is similarly defined. Therefore displacement is the position vector that points from “start (position A) to finish (position B),” that is,

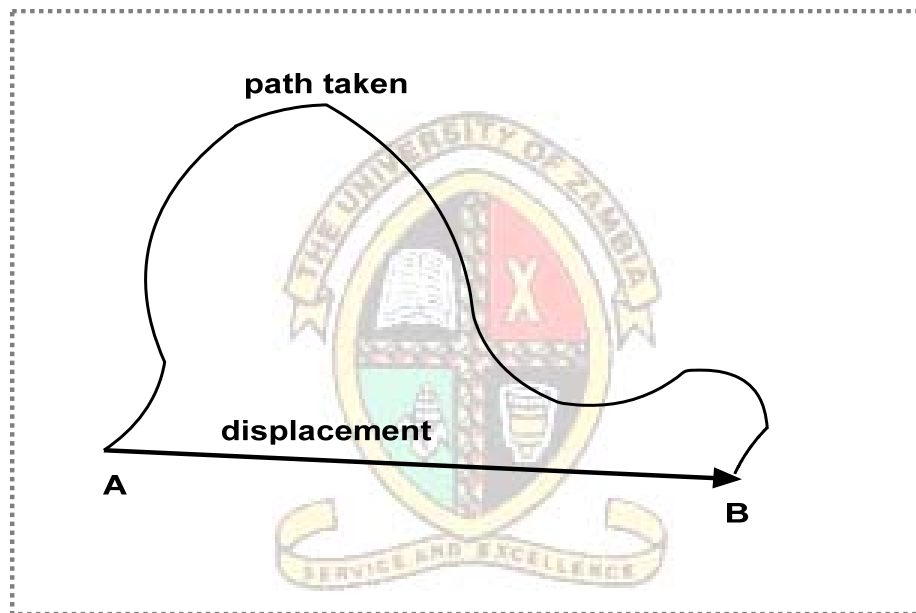


Figure 2.2

The displacement \mathbf{s} is

$$\mathbf{s} = \mathbf{x}_B - \mathbf{x}_A \quad (2.1)$$



2.3.2 Average speed and velocity,

Average speed is defined in terms of actual distance travelled and so depends on the path taken. *Average velocity* (\bar{v}), on the other hand, is a vector defined as the displacement from start to finish divided by the time taken.

Therefore,

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{d}{t} \quad (2.2)$$

While

$$\text{Average velocity} = \bar{v} = \frac{\text{displacement vector}}{\text{time taken}} = \frac{\Delta s}{\Delta t} = \frac{x_f - x_i}{t_f - t_0} \quad (2.3)$$

Therefore if the person takes 10 minutes to reach the end point, then the

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{100\text{m}}{10 \text{ min}} = 10\text{m} / \text{min}$$

While

$$\text{Average velocity} = \bar{v} = \frac{\text{displacement vector}}{\text{time taken}} = \frac{40\text{m}}{10 \text{ min}} = 4\text{m} / \text{min}$$

This shows that speed is different from velocity.



Instantaneous velocity

If you drive a car 100 km along a straight road in one direction for 2.0 h, the average velocity is 50 km/h. It is unlikely that you were moving at 50 km/h at every instant. Hence we talk about *instantaneous velocity*.

This is the velocity at an instant of time. Its magnitude is the number that is always indicated on a speedometer. At any instant of time the speedometer of a car will indicate the magnitude of the velocity of the car. That is the instantaneous velocity. Therefore the magnitude of the instantaneous velocity at any point is equal to the instantaneous speed at that point.

2.3.3 Average acceleration and instantaneous acceleration

Acceleration

Suppose that at a certain instant an object has a velocity (not speed) v_0 and its velocity at a later time t is v_f . (The subscripts o and f stand for “original” and “final”).



The average acceleration \bar{a} of the object during this time interval is defined to be

$$\bar{a} = \text{Change in velocity/ time taken} = \frac{v_f - v_0}{t} \quad (2.4)$$

where t is the time interval given by

$$t = t_f - t_0$$

In other words acceleration is the change of velocity per unit time.

When the value of acceleration is negative, then the object was slowing down and we call this *deceleration*.

Instantaneous acceleration

We can now define instantaneous acceleration. As an example, suppose a race car driver is driving along a straightway as shown in Figure 2.3. To define instantaneous acceleration at P_1 , we take the second point P_2 to be closer and closer to P_1 so that the average acceleration is computed over shorter and shorter time intervals.



Figure 2.3



The instantaneous acceleration is the limit of the average acceleration as the time interval approaches zero. In the language of calculus, instantaneous acceleration equals the instantaneous rate of change of velocity with time.

Wait a minute !!

If you have not done calculus in your life before, don't worry, you'll do it in the mathematics modules but it won't do you harm if you read a bit to have an idea of how to differentiate and integrate functions.

Thus the instantaneous acceleration in the x-direction (straight-line motion) is

$$a_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad \text{(instantaneous x-acceleration, straight-line motion)} \quad (2.5)$$

Note that a_x in equation (2.5) is really the x-component of the acceleration vector or the instantaneous x-acceleration in a straight line motion and all other components are zero. From now on, the term acceleration will always mean instantaneous acceleration and not average acceleration.



EXAMPLE 2.1: Suppose the x -velocity of the car in Fig 2.3 at any time is given by the equation

$$v_x = (60 + 0.5t^2) \text{ m/s}$$

(a). Find the average x -acceleration in the time interval between $t_1 = 1.0 \text{ s}$ and $t_2 = 3.0 \text{ s}$

(b) Find the instantaneous x -acceleration at time $t_1 = 1.0 \text{ s}$ by taking $\Delta t = 0.1 \text{ s}$

(c) Derive an expression for the instantaneous acceleration at any time and use it to find the acceleration at $t_1 = 1.0 \text{ s}$

SOLUTION

(a) $\bar{a} = \text{Change in velocity} / \text{time taken} = \frac{v_f - v_0}{t}$

$$v_{1x} = 60 + 0.5(1.0)^2 = 60.5 \text{ m/s}$$

$$v_{2x} = 60 + 0.5(3.0)^2 = 64.5 \text{ m/s}$$

the time interval is

$$\Delta t = t_f - t_0 = 3.0 - 1.0 = 2.0 \text{ s} = t$$



Therefore the average acceleration is

$$\bar{a} = \frac{v_f - v_0}{t} = \frac{64.5 - 60.5}{2.0} = 2.0 \text{ m/s}^2$$

(b) When $\Delta t = 0.1 \text{ s}$, $t_2 = 1.1 \text{ s}$. Therefore

$$v_{1x} = 60 + 0.5(1.0)^2 = 60.5 \text{ m/s}$$

$$v_{2x} = 60 + 0.5(1.1)^2 = 60.605 \text{ m/s}$$

the average acceleration therefore becomes

$$\bar{a} = \frac{v_f - v_0}{t} = \frac{60.605 - 60.5}{0.1} = 1.05 \text{ m/s}^2$$

(c) The instantaneous acceleration is

$$a = \frac{dv_x}{dt} = \frac{d}{dt}(60 + 0.5t^2) = 0.5(2t) = 1.0t$$

At $t_1 = 1.0 \text{ s}$,

$$a = 1.0t = 1.0(1.0) = 1.0 \text{ m/s}^2$$

Note that the value for instantaneous acceleration in (c) is not equal to the average acceleration in (b). That's because the car's instantaneous acceleration varies with time.



2.3.4 Uniformly accelerated motion

We shall restrict ourselves to situations where acceleration is constant. The initial conditions will be described with subscript “0” that is, x_0 , v_0 , and $t_0 = 0$ (from this point onward (with a few exceptions), we will set the initial time to zero). The final conditions will be denoted by a subscript “f” or without any subscript at all.

For constant acceleration, average acceleration = instantaneous acceleration

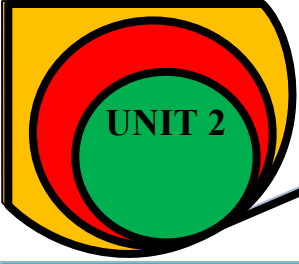
If an object has acceleration a and passes a point A with initial velocity v_0 , and after time t , its velocity is v_f then,

The displacement is

$$x = x_f - x_0$$

So that the average velocity \bar{v} for the trip is

$$\bar{v} = \frac{\text{displacement}}{\text{time}} = \frac{x}{t} \tag{2.6}$$



Motion in One Dimension



This is a restatement of equation (2.3). Hence

$$x = \bar{v}t \quad (2.7)$$

Because the acceleration is constant,

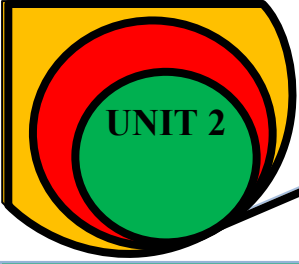
$$a = \frac{v_f - v_0}{t_f - t_0} = \frac{v_f - v_0}{t - 0} = \frac{v_f - v_0}{t}$$

So that

$$v_f = v_0 + at \quad (2.8)$$

Because the velocity is changing from v_0 to v_f , the average velocity of these two values is

$$\bar{v} = \frac{v_f + v_0}{2} \quad (2.9)$$



Motion in One Dimension



Substituting equation (2.9) into the expression for displacement in equation (2.7) yields

$$x = \frac{1}{2}(v_0 + v_f)t \quad (2.10)$$

Which when substituted into equation (2.8) gives

$$x = v_0 t + \frac{1}{2}at^2 \quad (2.11)$$

not forgetting of course that x is the displacement.

The same equation (2.10) when substituted into equation (2.8) and eliminating t gives

$$v_f^2 = v_0^2 + 2ax \quad (2.12)$$

Note: the acceleration is constant.



Motion in One Dimension



Here is a brief guide to help you translate problems involving motion:

Question or Statement	Statement
When	What value of t ?
Where	What value of position (x, y or z)?
Starts from rest	$v_0 = 0$
How fast	What value of v or v_f ?
How long does it take	What value of Δt ? : $\Delta t = t_f - t_0$
How far	What value of $x_f - x_0$?
Comes to rest	$v_f = 0$

Finally tips to solving one dimensional problems...

Problem-Solving Strategy for Accelerated Motion in 1-D:

- a) Make sure all of the units of the parameters in the problem are consistent.
- b) Choose a coordinate system (and draw a picture).
- c) Make a list of all of the quantities given in the problem and a separate list of those to be determined.
- d) Select those equations that will allow you to determine the unknown parameters (i.e., variables).



e) Make sure your answer is consistent with the diagram drawn in part (b).



EXAMPLE 2.2. A car starts from rest at a stop sign and accelerates (at a constant rate) to the posted speed limit of 50 km/hr. The car reaches the speed limit after travelling 107 m. What is the average velocity and acceleration of the car? How long does it take you to reach the speed limit?

SOLUTION

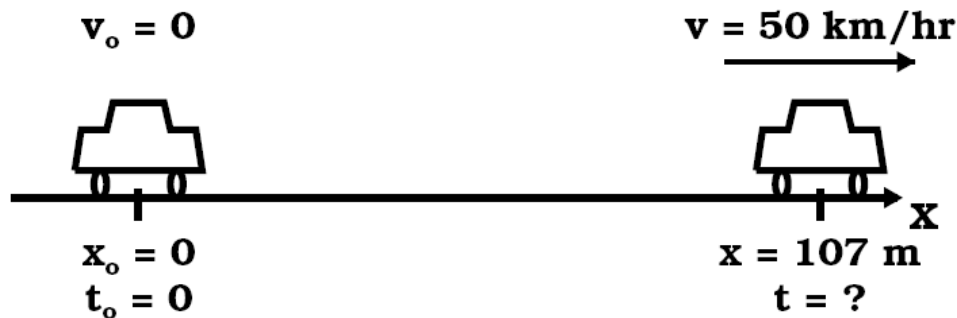
a) Units are not consistent; we have to use the SI units, (m/s for velocity say) $x = 107$ m & $v = 50$ km/hr. You want SI units, so change km into m:

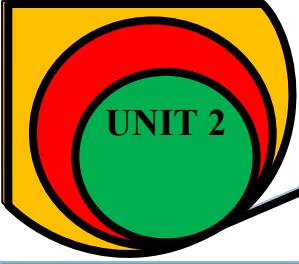
$$v = 50 \frac{\text{km}}{\text{hr}} \cdot 10^3 \frac{\text{m}}{\text{km}} = 5.0 \times 10^4 \text{ m/hr.}$$

Change hours to seconds:

$$v = 5.0 \times 10^4 \frac{\text{m}}{\text{hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} = 13.89 \text{ m/s} = 14. \text{ m/s.}$$

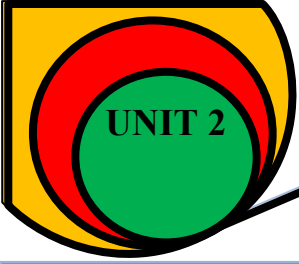
b) Choose a coordinate system (and draw picture):





Motion in One Dimension





Motion in One Dimension



Any question?? A task for you as well...



Self-help task 2.1

Suppose a car travelling at 5.00 m/s is brought to rest in a distance of 20.0 m. Find its acceleration and the time it takes to stop. Assume the motion is along the x-axis and that acceleration is constant.

Check the answers at the end of the unit.....Got them?

Another example...

EXAMPLE 2.3: A motorist travelling with constant speed of 15 m/s passes a school-crossing corner, where the speed limit is 10 m/s. Just as the motorist passes, a police officer on a motorcycle parked at the corner starts off in pursuit with constant acceleration of 3.0 m/s^2 . (a) How much time elapses before the officer's catches up with the motorist? (b) What is the officer's speed at that point? (c) What is the total distance each vehicle has travelled at that point?





SOLUTION

- The police officer and the motorist both move with constant acceleration (equal to zero for the motorist).
- We take the origin at the corner, so that $x_0=0$ for both and we take the positive direction to the right.
- Let x_p (for the police) be the officer's position and x_m (for motorist) be the motorist's position at any time t .

- (a) To find the value of the time t when the motorist and the police officer are at the same position, we apply equation (2.11): $x = v_0 t + \frac{1}{2} a t^2$ to each vehicle:

$$x_m = v_{om} t + \frac{1}{2} a t^2 = v_{om} t + \frac{1}{2} (0) t^2 = v_{om} t$$

For the motorist and for the police officer it is

$$x_p = v_{op} t + \frac{1}{2} a t^2 = (0) t + \frac{1}{2} a_p t^2 = \frac{1}{2} a_p t^2$$

Since $x_p = x_m$ at time t , we set these two expressions equal to each other and solve for t :

$$v_{om} t = \frac{1}{2} a_p t^2$$

$$t = 0 \text{ or } t = \frac{2v_{om}}{a_p} = \frac{2(15)}{3} = 10s$$



There are *two* times when both the vehicles have the same x -coordinate. The first, $t=0$, is the time when the motorist passes the parked motorcycle at the corner. The second, $t=10$ s, is the time when the officer catches up with the motorist.

(b) We want the officer's velocity v_p at the time t found in part (a). This is

$$v_p = v_0 + at = v_{op} + a_p t = 0 + (3)(10) = 30 \text{ s.}$$

So after 10 s, the officer overtakes at a speed twice as fast as the motorist.

(c) In 10 s, the distance travelled by the motorist is

$$x_m = v_{om} t = 15(10) = 150 \text{ m}$$

And the distance travelled by the officer is

$$x_p = v_{op} t + \frac{1}{2} at^2 = (0)t + \frac{1}{2} a_p t^2 = \frac{1}{2} a_p t^2 = \frac{1}{2} (3)10^2 = 150 \text{ m}$$

This verifies that at a time the officer catches the motorist, they have gone equal distances.

PROCEED





2.4 Graphical Analysis of Rectilinear motion

Modern mathematical notation is a highly compact way to encode ideas. Equations can easily contain the information equivalent of several sentences. Galileo's description of an object moving with constant speed (perhaps the first application of mathematics to motion) required one definition, four axioms, and six theorems (google them...) All of these relationships can now be written in a single equation. When it comes to depth, nothing beats an equation.

Well, almost nothing. Think back to the previous section on the equations of motion. You should recall that the three (or four) equations presented in that section were only valid for motion with constant acceleration along a straight line. Since, as I rightly pointed out, "no object has ever traveled in a straight line with constant acceleration anywhere in the universe at any time" these equations are only approximately true, only once in a while.

Equations are great for describing idealized situations, but they don't always cut it. Sometimes you need a picture to show what's going on — a mathematical picture called a graph. Graphs are often the best way to convey descriptions of real world events in a compact form. Graphs of motion come in several types depending on which of the kinematic quantities (time, displacement, velocity, acceleration) are assigned to which axis.



2.4.1 Distance – time ($s-t$) graph

This is where the position x of an object is tracked with time. Let us first consider an example of motion at a constant velocity as shown in the figure 2.4.

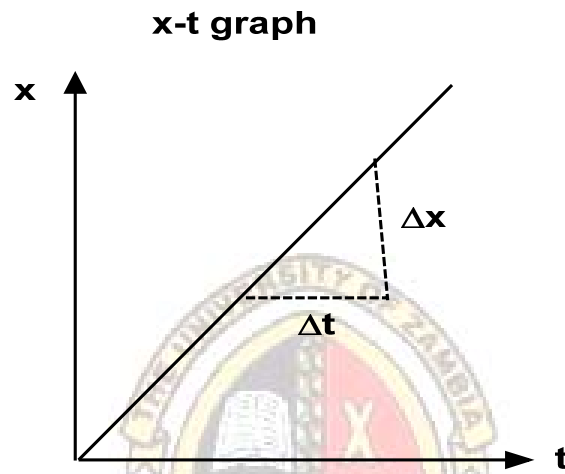


Figure 2.4

Compare the displacement-time equation for constant velocity with the classic slope-intercept equation taught in introductory algebra.

$$x = x_0 + v\Delta t$$

$$y = c + mx$$



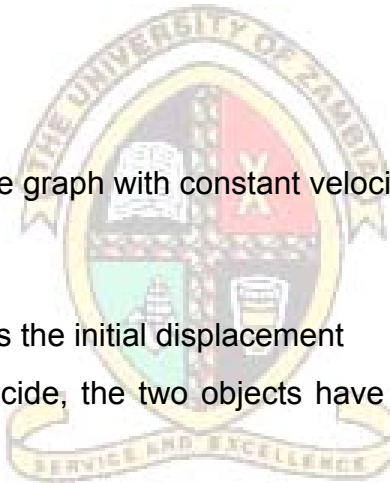
Thus velocity corresponds to slope and initial displacement to the intercept on the vertical axis (commonly thought of as the "y" axis). Since each of these graphs has its intercept at the origin, each of these objects had the same initial displacement. This graph could represent a race of some sort where the contestants were all lined up at the starting line. If it were a race, then the contestants were already moving when the race began, since the curve has a non-zero slope at the start. Note that the initial position being zero does not necessarily imply that the initial velocity is also zero. The height of a curve tells you nothing about its slope.

The slope therefore is the velocity i.e.

$$\text{slope} = v = \frac{\Delta x}{\Delta t}$$

Therefore on a distance – time graph with constant velocity,

- Slope equals velocity
- The "y" intercept equals the initial displacement
- When two curves coincide, the two objects have the same displacement at that time.





EXAMPLE 2.4: We adapt the question in example 2.3. What is the total distance travelled by the officer on the motorcycle when he/she catches up with the motorist? Use graphs.

SOLUTION

We write the displacements as obtained in example 2.3:

For the motorist we have

$$x_m = v_{om}t = 15t$$

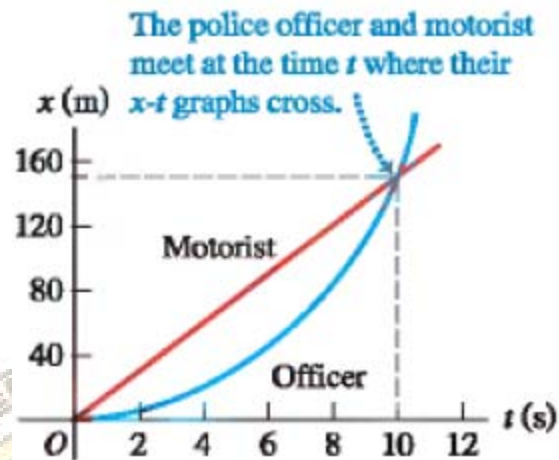
Which gives a straight line since it is moving at constant velocity while for the police officer:

$$x_p = v_{op}t + \frac{1}{2}at^2 = (0)t + \frac{1}{2}a_p t^2 = \frac{1}{2}(3)t^2 = 1.5t^2$$

That is

$$x_p = 1.5t^2$$

Plotting these two displacement equations the two curves coincide/meet at 150 m. This means they travel the same distance at that time $t = 10$ s.



In contrast to the previous graph in figure 2.4, the displacement of an object with a constant, non-zero acceleration starting from rest at the origin has a graph shown in figure 2.5.

The primary difference between this curve and those on the previous graph is that this curve actually curves. The relation between displacement and time is quadratic when the acceleration is constant and therefore this curve is a parabola.

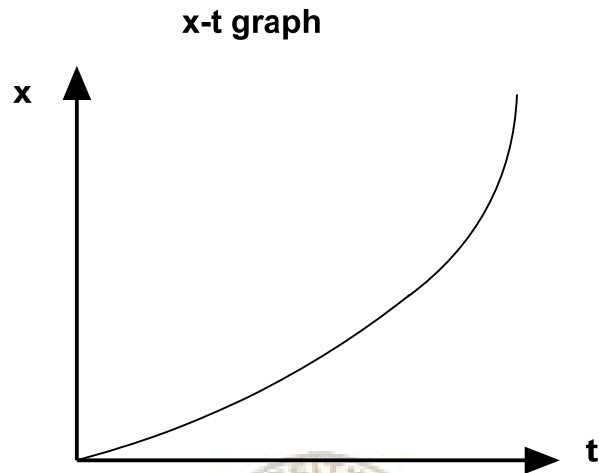


Figure 2.5

This is a quadratic relation with y intercept which is x_0 in our case being zero as initial displacement. The relation is the usual equation

$$x = x_0 + v_0 \Delta t + \frac{1}{2} a t^2$$

analogous to the famous quadratic equation

$$y = a + bx + cx^2$$

When a displacement-time graph is curved, it is not possible to calculate the velocity from its slope.



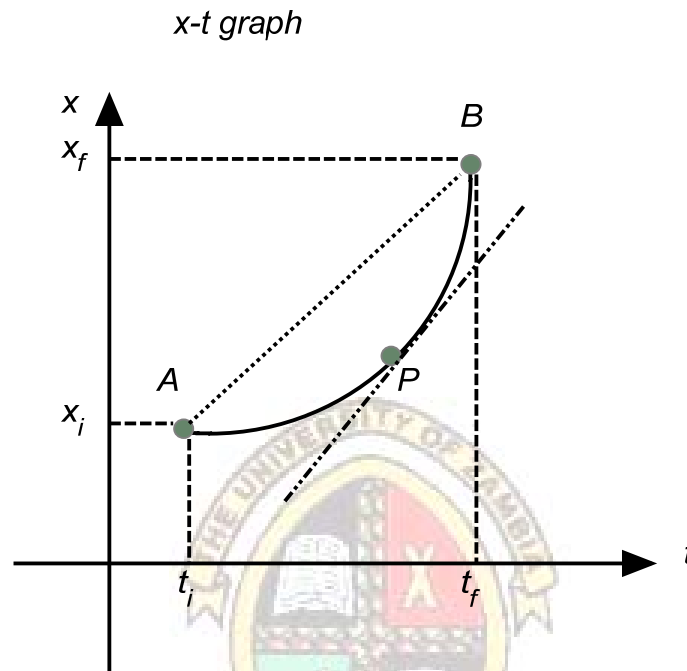
Slope is a property of straight lines only. Such an object doesn't have a velocity because it doesn't have a slope. The words "the" and "a" are underlined here to stress the idea that there is no single velocity under these circumstances. The velocity of such an object must be changing. It's accelerating.

Therefore on a displacement-time graph

- Straight lines imply constant velocity
- Curved lines imply acceleration and
- An object undergoing constant acceleration traces a portion of a parabola

Although our hypothetical object has no single velocity, it still does have an average velocity and a continuous collection of instantaneous velocities. The *average velocity* of any object can be found by dividing the total displacement by the total time.

This is the same as calculating the slope of the straight line connecting the first (A) and last (B) points on the curve as shown in figure 2.6.

**Figure 2.6**

As we found out from equation (2.3) the average velocity is

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Instantaneous velocity is the limit of average velocity as the time interval shrinks to zero.

As the endpoints of the line of average velocity get closer together, they become a better indicator of the actual velocity. When the two points coincide, the line is tangent to the curve and this is the tangent at point P in figure 2.6.

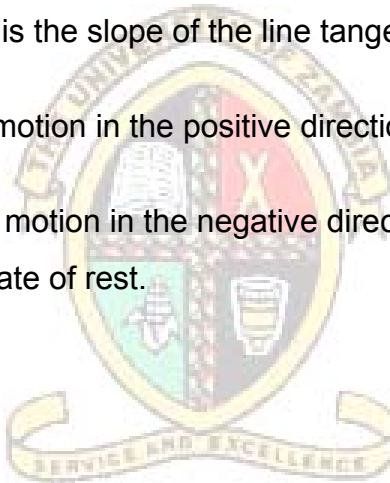


When a slope is calculated for a certain trajectory, negative slope does not automatically mean driving backward, or walking left, or falling down. The choice of signs is always arbitrary. About all we can say in general, is that when the slope is negative, the object is traveling in the negative direction.

Finally on displacement time graph,

- Average velocity is the slope of the straight line connecting the endpoints of a curve.
- Instantaneous velocity is the slope of the line tangent to a curve at any point.
- Positive slope implies motion in the positive direction.
- Negative slope implies motion in the negative direction
- Zero slope implies a state of rest.

A task for you as well...



Self-help task 2.2

A car is parked 10 m from home for 10 minutes. Draw a displacement-time and velocity-time for the motion. Label all the axes.



PROCEED



2.4.1 Velocity – time ($v-t$) graph

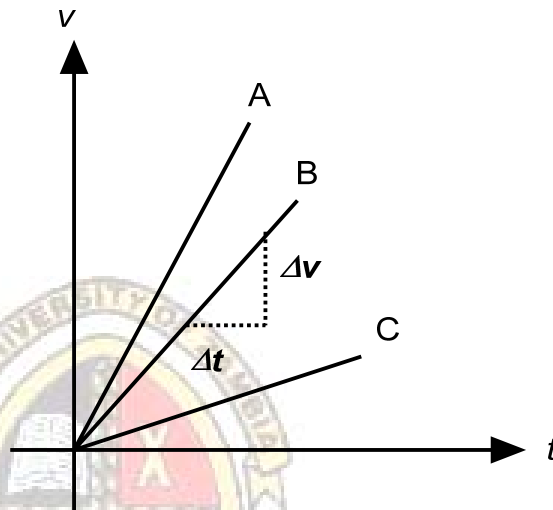
The most important thing to remember about velocity-time graphs is that they are velocity-time graphs, not displacement-time graphs. There is something about a line graph that makes people think they're looking at the path of an object.

Don't look at these graphs and think of them as a picture of a moving object. Instead, think of them as the record of an object's velocity. In these graphs, higher means faster not farther.

A $v-t$ graph that has a horizontal curve indicates that the initial velocity of the object is the same as the final velocity and that the velocity is the same in between.

Conversely, when the curve on a velocity-time graph is straight but not horizontal, the velocity is changing. The three curves in figure 2.7 each have a different slope.

The graph with the steepest slope experiences the fastest change in velocity. That object has the greatest acceleration.

*v-t graph***Figure 2.7**

Compare the velocity-time equation for constant acceleration with the classic slope-intercept equation taught in introductory algebra.

$$v = v_0 + a\Delta t$$

$$y = a + bx$$

You should see that acceleration corresponds to slope and initial velocity to the intercept on the vertical axis. Since each of these graphs has its intercept at the origin, each of these objects was initially at rest. The initial velocity being zero does not mean that the initial position must also be zero, however. This graph tells us nothing about the initial position of these objects. For all we know they could be on different planets.



Therefore on a velocity-time graph,

- slope equals acceleration
- the "y" intercept equals the initial velocity.
- when two curves coincide, the two objects have the same velocity at that time.

The curves on the previous graph were all straight lines. A straight line is a curve with constant slope. Since slope is acceleration on a velocity-time graph, each of the objects represented on this graph is moving with a constant acceleration. Were the graphs curved, the acceleration would not have been constant.

- On a velocity-time graph ...
 - straight lines imply uniform acceleration.
 - curved lines imply non-uniform acceleration.
 - an object undergoing constant acceleration traces a straight line.

Since a curved line has no single slope we must decide what we mean when asked for the acceleration of an object. If the average acceleration is desired, draw a line connecting the endpoints of the curve and calculate its slope. If the instantaneous acceleration is desired, take the limit of this slope as the time interval shrinks to zero, that is, take the slope of a tangent. These descriptions follow directly from the definitions of average and instantaneous acceleration.



Therefore on velocity-time graphs (which are not straight lines),

- positive slope implies an increase in velocity in the positive direction,
- negative slope implies an increase in velocity in the negative direction and
- zero slope implies motion with constant velocity

Anyone who has taken a calculus course should have known this before they read it here (or at least when they read it they should have said, "Oh yeah, I remember that"). The first derivative of displacement with respect to time is velocity. The derivative of a function is the slope of a line tangent to its curve at a given point. The inverse operation of the derivative is called the integral. The integral of a function is the cumulative area between the curve and the horizontal axis over some interval. This inverse relation between the actions of derivative (slope) and integral (area) is so important that it's called the fundamental theorem of calculus. This means that it's an important relationship. Learn it! It's "fundamental". You haven't seen the last of it.

Therefore on a velocity time graph,

- the area under the curve equals the change in displacement.



2.4.3 Acceleration – time ($a-t$) graph

The acceleration-time graph of any object traveling with a constant velocity is the same. This is true regardless of the velocity of the object. An airplane flying at a constant 600 mph (270 m/s), a sloth walking with a constant speed 1 mph (0.4 m/s), and a couch potato lying motionless in front of the TV for hours will all have the same acceleration time graphs — a horizontal line collinear with the horizontal axis.

That's because the velocity of each of these objects is constant. They're not accelerating. Their accelerations are zero. As with velocity-time graphs, the important thing to remember is that the height above the horizontal axis doesn't correspond to position or velocity, it corresponds to acceleration.

Acceleration and velocity are different quantities. Going fast does not imply accelerating quickly. The two quantities are independent of one another. A large acceleration corresponds to a rapid change in velocity, but it tells you nothing about the values of the velocity itself.

When acceleration is constant, the acceleration-time curve is a horizontal line. The rate of change of acceleration with time is a meaningless quantity so the slope of the curve on this graph is also meaningless. Acceleration need not be constant, but the time rate of change of this number has no name. On the surface, the only information one can glean from an acceleration-time graph is the acceleration at any given time.

Therefore on an acceleration-time graph,

- slope is meaningless.
- the "y" intercept equals the initial acceleration.



- when two curves coincide, the two objects have the same acceleration at that time.
- an object undergoing constant acceleration traces a horizontal line.
- zero slope implies motion with constant acceleration.

Acceleration is the rate of change of velocity with time. We can obtain an acceleration-time graph from a velocity-time graph by calculating the slope of a line tangent to the curve at any point. (In calculus, this is called finding the derivative.) The reverse process entails calculating the cumulative area under the curve. (In calculus, this is called finding the integral.) This number is then the change of value on a velocity-time graph.

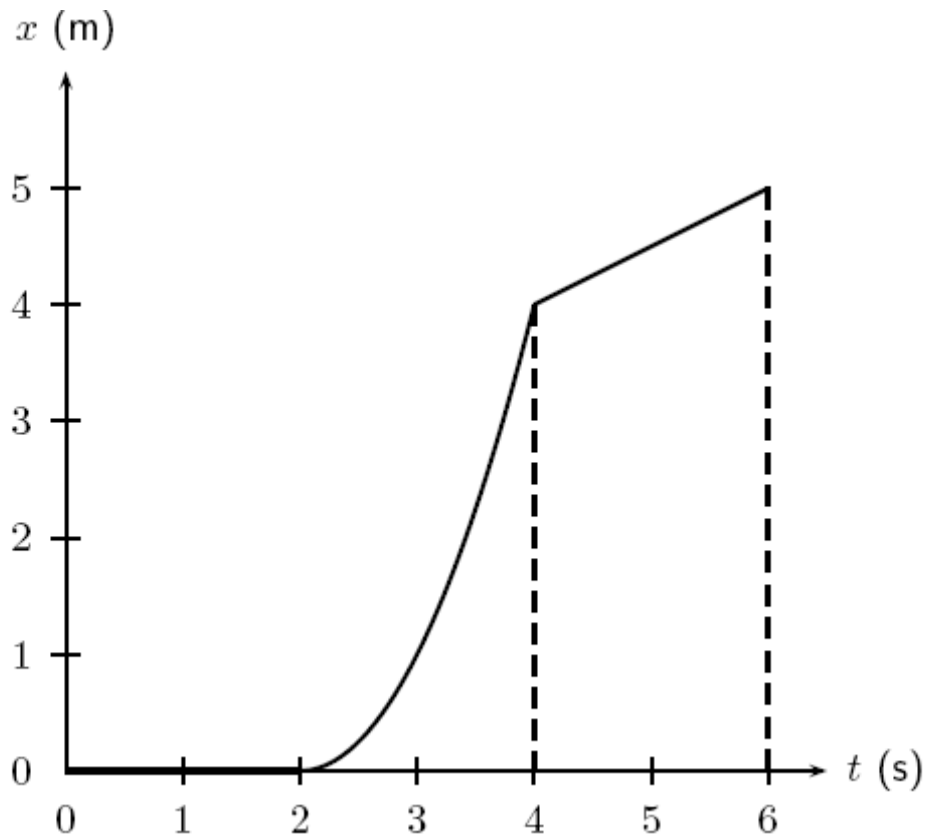
Therefore on an acceleration-time graph,

- the area under the curve equals the change in velocity.

There are more things one can say about acceleration-time graphs, but they are trivial for the most part.



EXAMPLE 2.6: Description of motion based on a position-time graph
Question: The position vs. time graph for the motion of a car is given below. Draw the corresponding velocity vs. time and acceleration vs. time graphs, and then describe the motion of the car.



SOLUTION

Step 1 : Identify what information is given and what is asked for. The question gives a position vs. time graph and the following three things are required:

1. Draw a v vs. t graph.
2. Draw an a vs. t graph.
3. Describe the motion of the car.



To answer these questions, break the motion up into three sections: 0 - 2 seconds, 2 - 4 seconds and 4 - 6 seconds.

Step 2 : Velocity vs. time graph for 0-2 seconds

For the first 2 seconds we can see that the displacement remains constant - so the object is not moving, thus it has zero velocity during this time. We can reach this conclusion by another path too: remember that the gradient of a displacement vs. time graph is the velocity. For the first 2 seconds we can see that the displacement vs. time graph is a horizontal line, ie. it has a gradient of zero. Thus the velocity during this time is zero and the object is stationary.

Step 3 : Velocity vs. time graph for 2-4 seconds

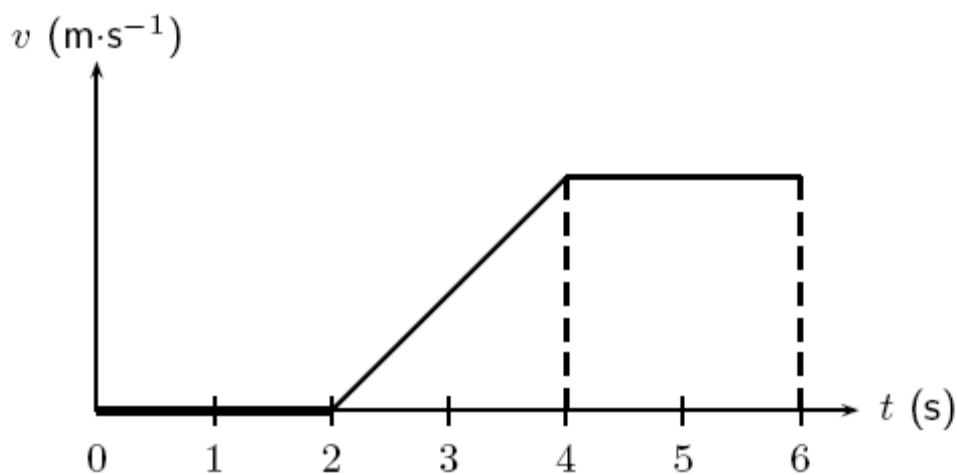
For the next 2 seconds, displacement is increasing with time so the object is moving. Looking at the gradient of the displacement graph we can see that it is not constant. In fact, the slope is getting steeper (the gradient is increasing) as time goes on. Thus, remembering that the gradient of a displacement vs. time graph is the velocity, the velocity must be increasing with time during this phase.

Step 4 : Velocity vs. time graph for 4-6 seconds

For the final 2 seconds we see that displacement is still increasing with time, but this time the gradient is constant, so we know that the object is now travelling at a constant velocity, thus the velocity vs. time graph will be a horizontal line during this stage. We can now draw the graphs:



So our velocity vs. time graph looks like this one below. Because we haven't been given any values on the vertical axis of the displacement vs. time graph, we cannot figure out what the exact gradients are and therefore what the values of the velocities are. In this type of question it is just important to show whether velocities are positive or negative, increasing, decreasing or constant.



Once we have the velocity vs. time graph its much easier to get the acceleration vs. time graph as we know that the gradient of a velocity vs. time graph is the just the acceleration.

Step 5 : Acceleration vs. time graph for 0-2 seconds

For the first 2 seconds the velocity vs. time graph is horizontal and has a value of zero, thus it has a gradient of zero and there is no acceleration during this time.



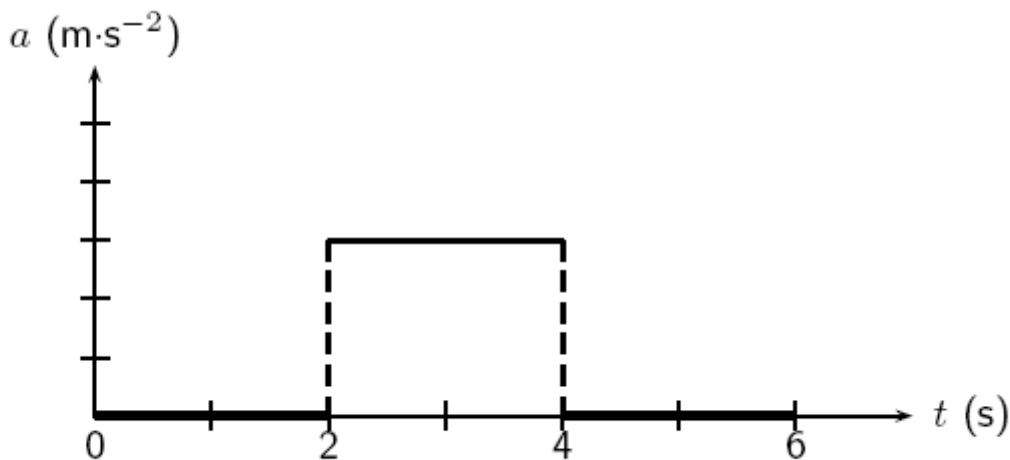
(This makes sense because we know from the displacement time graph that the object is stationary during this time, so it can't be accelerating).

Step 6 : Acceleration vs. time graph for 2-4 seconds

For the next 2 seconds the velocity vs. time graph has a positive gradient. This gradient is not changing (i.e. its constant) throughout these 2 seconds so there must be a constant positive acceleration.

Step 7 : Acceleration vs. time graph for 4-6 seconds

For the final 2 seconds the object is traveling with a constant velocity. During this time the gradient of the velocity vs. time graph is once again zero, and thus the object is not accelerating. The acceleration vs. time graph looks like this:



Step 8 : A description of the object's motion



A brief description of the motion of the object could read something like this: At $t = 0$ s and object is stationary at some position and remains stationary until $t = 2$ s when it begins accelerating. It accelerates in a positive direction for 2 seconds until $t = 4$ s and then travels at a constant velocity for a further 2 seconds.

2.5 Freely Falling Bodies

This is the case where objects fall under the influence of gravity. The Italian scientist **Galileo Galilei** the father of experimental physics carried out a variety of experiments in mechanics in the study of motion.

He found out that

- Near the surface of the Earth, bodies fall at the same rate of acceleration independent of the body's mass.
- Air resistance can affect the rate at which a body falls.

A freely falling body is an object moving under the influence of gravity, regardless of its initial velocity. The following points relates to free fall:

- Objects thrown upward or downward and those released from rest are all falling freely once they are released (even if they are initially going up).



- Once a body is in free fall, all objects have an acceleration directed downward. This free fall acceleration is called *acceleration due to gravity* g .
- A gravitating body's gravity depends upon the total mass and the size of the gravitating body.
- The Earth's gravity at sea level is $g = 9.80 \text{ m/s}^2$
- Bodies dropped in the earth's atmosphere will experience a frictional force from air resistance in addition to the gravitational force. In this module, we will ignore air resistance (but it is important in reality).

Whether the body is falling or rising, g is always 9.8 m/s^2 and directed downward. When the body is rising, the velocity decreases at a rate of 9.8 m/s per second until at the top where v eventually becomes zero. Similarly when it is falling it gains velocity at a rate of 9.8 m/s per second.



Self-help task 2.3

An object is falling under free fall. What is its velocity after 2, 3 and 5 seconds?



When solving problems relating to falling and rising objects, it is important to choose a convention or coordinate system with the launch point being the origin or zero. In this module, we will chose **up (positive)** and **down (negative)** as shown in figure 2.8 but you are free to chose which convention to use.

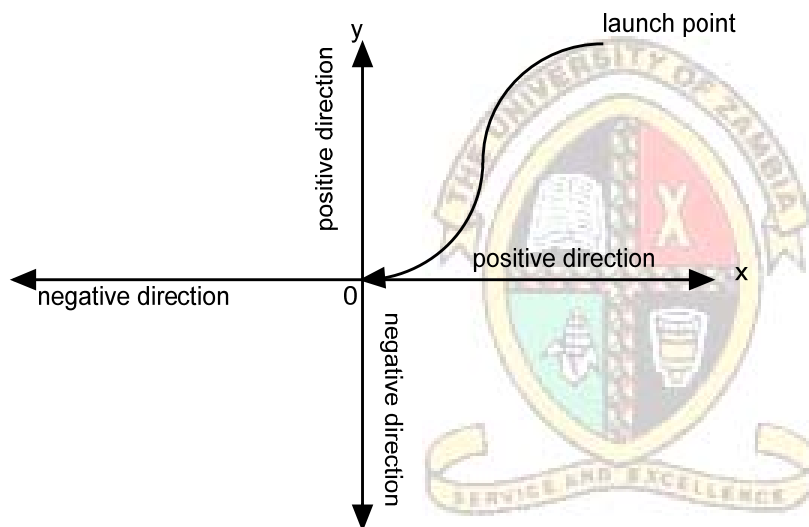


Figure 2.8

This means, in the case of the former convention, for example, a body falling downwards will be assigned a negative (-) sign and for a rising body a positive (+) sign. Acceleration due to gravity g is always directed downwards so it is assigned a negative.



In the equations of motion, a is replaced by $-g$ so that the equations become:

$$v = v_0 - gt$$

$$v^2 = v_0^2 - 2gy$$

(2.13)

$$y = v_0t - \frac{1}{2}gt^2$$

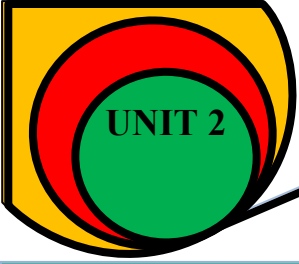
If an object falls freely, then $v_0 = 0$ so that

$$v = v_0 - gt = -gt$$

and the distance of fall is

$$y = v_0t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2.$$

The velocity will be negative implying that the object was going downwards. The displacement is also negative implying that it is below the launch point, below the origin.



Self-help task 2.4

A bullet is fired vertically with an initial velocity of 20 m/s.

What is its velocity after 4 seconds?

The sign of the velocity or displacement tells us a great deal of information about the trajectory. The following table shows how:



Sign	Velocity (v)	Displacement (y)
Positive (+)	The body is still going up	<i>The body travels y distance above the launch point</i>
Negative(-)	The body is going down	<i>The body travels y distance below the launch point</i>



EXAMPLE 2.7: A ball is thrown vertically upwards with an initial velocity of 20 m/s. Is the ball still going up or going down after (a) 1 second? (b) 1.5 seconds?

(c) 3 seconds?

(d) What is its displacement after 2 seconds?

SOLUTION

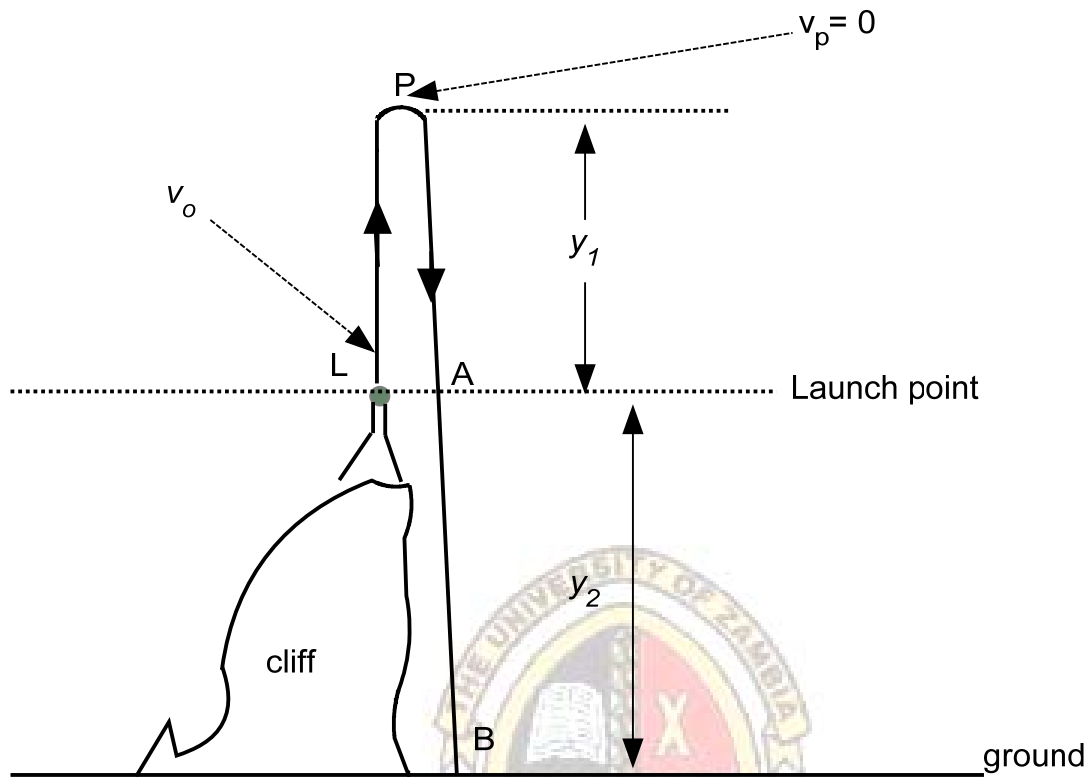
(a). $v = v_0 - gt = 20 - 9.8(1) = +10.2 \text{ m/s}$ Still going up since it is positive

(b). $v = v_0 - gt = 20 - 9.8(1.5) = +5.3 \text{ m/s}$ Still going up

(c). $v = v_0 - gt = 20 - 9.8(3) = -9.4 \text{ m/s}$ Going downward

(d). $y = v_0 t - \frac{1}{2}gt^2 = 20(2) - 4.9(2^2) = 20.4 \text{ m}$

Consider an object fired vertically by a canon on a cliff at a launch point A of height y_2 as shown in figure 2.9. If the object rises to a maximum distance of y_1 and then hits the ground at B, the following is the analysis of the trajectory.

**Figure 2.9**

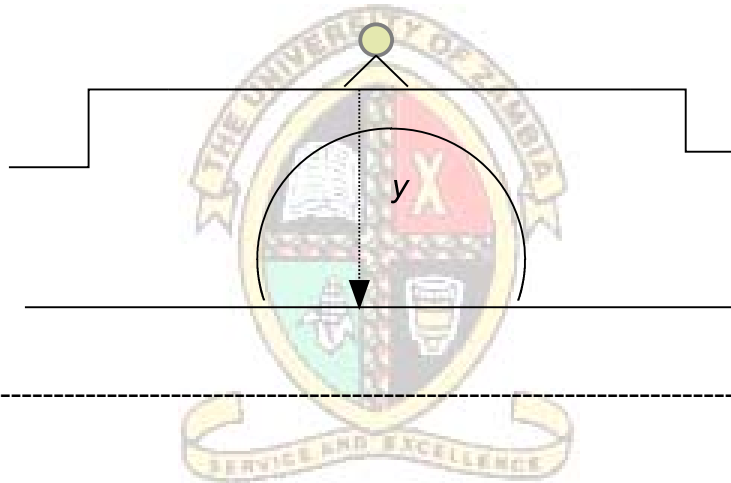
- The velocity at the tip of the trajectory is $v_p = 0$. This also gives the maximum height reached y_1 .
- When the body moves from L to A, the displacement is $y = 0$. This is because the initial position is equal to the final position.
- When the body moves from L to B, the displacement is $y = -y_2$. This is because it is below the launch point.



So... what have we been talking about? Let us see some examples...



EXAMPLE 2.8: You drop a stone from a bridge. If it takes 3.0 s for the stone to hit the water below, how high above the water was your hand when you let go of the stone?



SOLUTION

This is free fall. So that:

$$v_0 = 0, \quad t = 3.0 \text{ s}, \quad y = ?$$

$$\begin{aligned} y &= v_0 t - \frac{1}{2} g t^2 \\ &= 0(3.0) - \frac{1}{2} 9.8(3.0^2) \\ &= -44 \text{ m} \end{aligned}$$

So it is 44 m. The negative just implies it is below the launch point.



EXAMPLE 2.9: A ball is thrown upward with an initial speed of 15 m/s and then falls and is caught by the person who threw it.

- (a) How high does the ball go?
- (b) How fast is it going the instant it is caught?
- (c) How long is it in the air?

SOLUTION

- (a) As usual we use our convention as **up positive and down negative**. The data that we have is that, we have the initial velocity and the final velocity. At the peak, the ball momentarily comes to rest so that the final velocity is zero.

Therefore

$$v_0 = 15 \text{ m/s}$$

$$v_f = 0$$

so that

$$v_f^2 = v_0^2 - 2gy$$

$$0 = (15)^2 - 2gy$$

So that

$$y = (15)^2 / 2(9.8) = +11.5 \text{ m.}$$



The positive (+) sign means the distance is above the launch point.

- (b) When it is caught, the initial position and the final position are the same so that displacement is zero, i.e. $y = 0$.

We therefore use the equation : $v_f^2 = v_0^2 - 2gy$

So that

$$v_f^2 = 15^2 \rightarrow v_f = \pm 15.0 \text{ m/s}$$

Using our convention and knowing that the ball is going down, then

$$v_f = -15.0 \text{ m/s.}$$

- (c) One way to find the time of flight is to use the equation below

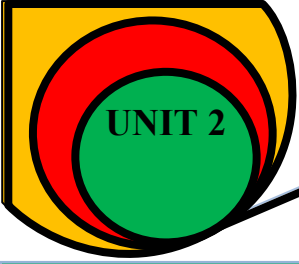
$$v_f = v_0 - gt$$

$$-15.0 = 15.0 - 9.8t$$

$$30.0 = 9.8t$$

$$\therefore t = 3.1 \text{ s}$$

One can also use the displacement equation keeping in mind that $y=0$.



Self-help task 2.5

A helicopter is ascending vertically with a speed of 5.20 m/s. At a height of 125 above the earth, a package is dropped from a window. How much time does it take for the package to reach the ground?

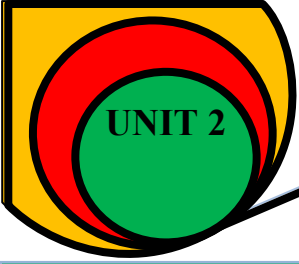
2.6. Summary

After a particle moves along the x axis from some initial position x_i to some final position x_f , its displacement is

The average velocity of a particle during some time interval is the displacement Δx divided by the time interval Δt during which that displacement occurs:

The average speed of a particle is equal to the ratio of the total distance it travels to the total time interval during which it travels that distance:

The instantaneous velocity of a particle is defined as the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero.



Motion in One Dimension

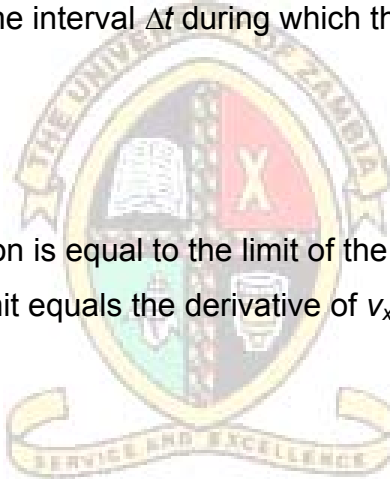


By definition, this limit equals the derivative of x with respect to t , or the time rate of change of the position:

The instantaneous speed of a particle is equal to the magnitude of its instantaneous velocity.

The average acceleration of a particle is defined as the ratio of the change in its velocity Δv_x divided by the time interval Δt during which that change occurs:

The instantaneous acceleration is equal to the limit of the ratio $\Delta v_x / \Delta t$ as Δt approaches Zero (0). By definition, this limit equals the derivative of v_x with respect to t , or the time rate of change of the velocity:



When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.

Graphical interpretation of one dimensional motion: On a graph of x versus t , the slope at any instant of time is equal to the velocity at that instant. On a graph of velocity versus time, the slope of the graph at any instant represents the instantaneous acceleration at that instant.



The equations of kinematics for a particle moving along the x axis with uniform acceleration a_x (constant in magnitude and direction).

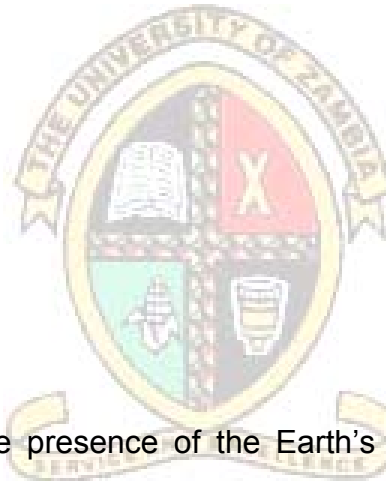
$$a = \frac{v_f - v_0}{t_f - t_0} = \frac{v_f - v_0}{t - 0} = \frac{v_f - v_0}{t}$$

$$v_f = v_0 + at$$

$$\bar{v} = \frac{v_f + v_0}{2}$$

$$x = v_0 t + \frac{1}{2} at^2$$

$$v_f^2 = v_0^2 + 2ax$$



An object falling freely in the presence of the Earth's gravity experiences a free-fall acceleration directed toward the center of the Earth. If air resistance is neglected, if the motion occurs near the surface of the Earth, and if the range of the motion is small compared with the Earth's radius, then the free-fall acceleration g is constant over the range of motion, where g is equal to 9.80 m/s^2 .

For free fall, the motion has constant acceleration. The magnitude of acceleration due to gravity is a positive quantity, g . The acceleration of a body in free fall is always downward. Hence using the convention of “**up positive and down negative**” acceleration in the equations of motion is replaced by $-g$ since it is pointing downward.



2.7. References

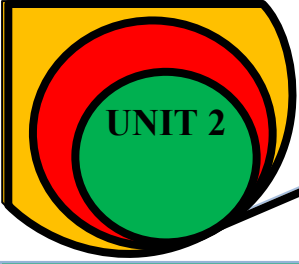
F.J Bueche, D.A Jerde, *Principles of Physics*.6th Edition, McGraw-Hill, 1995.

ISBN 0- 13.978-00700

D. Giancoli, *Physics:Principles with Applications*, 6th Edition.Pearson Prentice Hall, ISBN 0-13-060620-0

P.M Fishbane, S.G Gasiorowicz,S.T Thornton, *Physics for Scientists and Engineers with Modern Physics*,3rd Edition, Pearson Prentice Hall , ISBN 0-13-191182-1

Sears and Zemansky, *University Physics (with Modern Physics)*, 12th Edition, Hugh D Young and Roger, A Freedman,Pearson/Addison Wesley,2007, ISBN 0-8053-2187-X.



2.8. Answers to Self-help Questions

2.1 To find acceleration use

$$v^2 = v_o^2 + 2ax$$

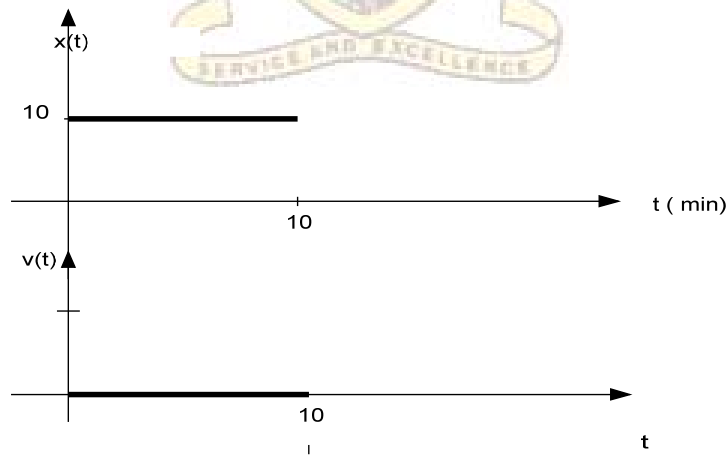
$$\therefore a = -0.625 \text{ m/s}^2$$

To find time, we use

$$v = v_o + at$$

$$\therefore t = 8 \text{ s}$$

2.2 Since it is stationary, the velocity is zero but time is ticking...





$$v = v_o - gt$$

$$= -gt \quad \text{since the initial velocity is zero}$$

$$\therefore v_2 = -9.8(2) = -19.6 \text{ m/s}$$

$$v_3 = -9.8(3) = -29.4 \text{ m/s}$$

$$v_2 = -9.8(5) = -49.0 \text{ m/s}$$

The negative implies that object was falling. Remember our convention...?

2.4 We use the same equations of motion to find $v_y = 19.2 \text{ m/s}$

2.5 Not forgetting our convention and that the helicopter is going up, then the initial velocity of the parcel is $+5.2 \text{ m/s}$. The displacement is -125 m . It is a negative because it is below the launch point (level). Using

$$y = v_{oy}t - \frac{1}{2}gt^2$$

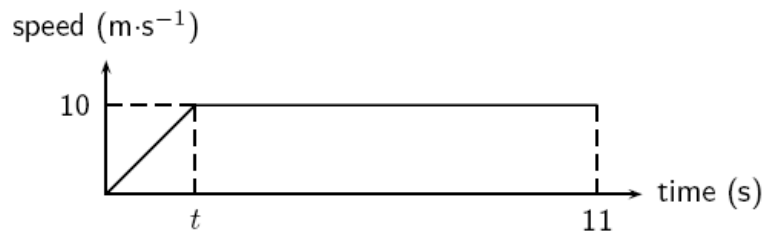
$$-125 = 5.2t - 4.9t^2 \quad \text{and solving for the quadratic equation in } t \text{ yields}$$

$$t = 5.73 \text{ s.}$$



2.9 EXERCISES

2.1 The graph shows how the speed of an athlete varies with time as he/she sprints over 100 m.



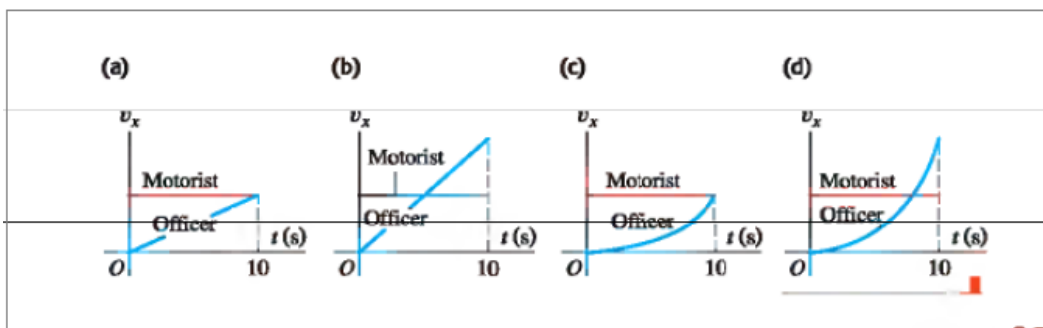
Which of the following equations can be used to correctly determine the time t for which he accelerates?

- (a) $100 = (10)(11) - \frac{1}{2}(10)t$
- (b) $100 = (10)(11) + \frac{1}{2}(10)t$
- (c) $100 = 10t + \frac{1}{2}(10)t^2$
- (d) $100 = \frac{1}{2}(0)t + \frac{1}{2}(10)t^2$

[(a): Use the area of the trapezium]



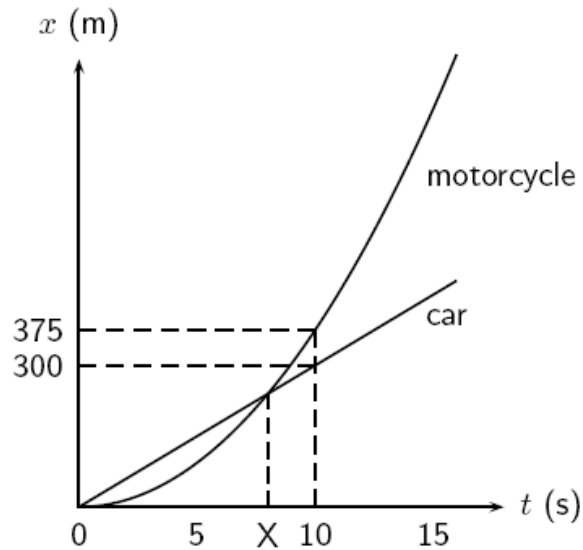
- 2.2 For possible v - t graphs are shown for the two vehicles in example 2.3. which graph is correct?



[(d): The motorist is moving with constant velocity, hence the graph is a horizontal straight line. For the police officer, it is accelerating since the velocity is $v_{po} = a_x t$. Therefore the options are (a) and (b), but after 10 seconds,

$$v_{mo} = 15 \text{ m/s} \quad \text{and} \quad v_{po} = 30 \text{ m/s} \quad \text{hence we settle for (b)}$$

- 2.3 A car, travelling at constant velocity, passes a stationary motor cycle at a traffic light. As the car overtakes the motorcycle, the motorcycle accelerates uniformly from rest for 10 s. The following displacement-time graph represents the motions of both vehicles from the traffic light onwards.



- (a) Use the graph to find the magnitude of the constant velocity of the car. [30 m/s]
- (b) Use the information from the graph to show by means of calculation that the magnitude of the acceleration of the motorcycle, for the first 10 s of its motion is 7.5 m/s^2
- (c) Calculate how long (in seconds) it will take the motorcycle to catch up with the car (point X on the time axis). [8 s]
- (d) How far behind the motorcycle will the car be after 15 seconds? [394 m]

2.4 A bullet travelling 220 m/s strikes a tree and penetrates 4.33 cm before stopping. Find the average acceleration and the time it takes to stop. [$a = -5.54 \times 10^5 \text{ m/s}^2$, $t = 3.94 \times 10^{-4} \text{ s}$]



2.5 A locomotive is pulling a train 580 m long, including the locomotive. The locomotive accelerates uniformly from rest and reaches a road crossing 1.35 km from its starting point in 9.66 min. (a) How long after the locomotive reaches the crossing does the caboose reach the crossing, assuming the train maintains its uniform acceleration? (b) How fast is the train moving when the caboose reaches the crossing? [(a) 113 min (b) 5.6 m/s]

2.6 Just as a car starts to accelerate from rest at a constant 2.44 m/s^2 , a bus moving at a constant speed of 19.6 m/s passes the car in a parallel lane. (a) How long before the car overtakes the bus? (b) How fast is the car going then? (c) How far has the car gone at that point? [(a) 16.1 s (b) 39.2 m/s (c) 316 m].

2.7 Two cars both travelling at 30.5 m/s are headed toward each other and headed for collision. When they are 250 m apart they see each other and begin to decelerate at the same rate. What must this deceleration be if the cars are to barely avoid collision? [-3.72 m/s^2]

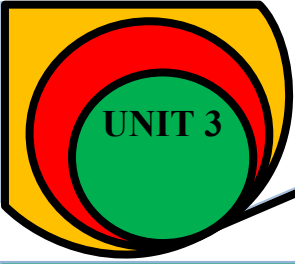
2.8 An elevator in which a woman is standing is moving upward at a constant speed of 3.35 m/s. The woman drops a coin from a height of 1.25 m above the elevator floor. How long does it take the coin to strike the elevator floor? [0.505 s]

2.9 A ball is thrown straight upward with a speed v_o from a point h m above the ground. Show that the time the ball takes to reach the ground is given by

$$\frac{v_o}{g} \left(1 + \sqrt{1 + \frac{2hg}{v_o^2}} \right)$$

The logo of The University of Zambia is centered in the background. It features a circular emblem with a shield in the center, divided into four quadrants of different colors (green, red, blue, and yellow). Above the shield is an open book. The text "THE UNIVERSITY OF ZAMBIA" is written in a semi-circle above the emblem. Below the emblem is a ribbon with the motto "SERVICE AND EXCELLENCE".

Motion in a Plane



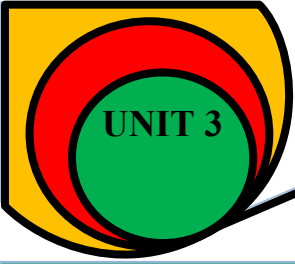
Motion in a Plane



UNIT 3: Motion in a Plane: (12 Hrs)

This unit should not take you more than 12 hours.





Motion in a Plane



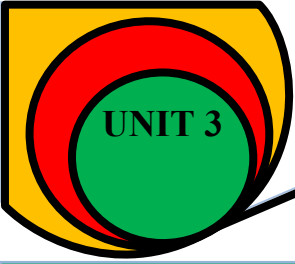
3.1 Introduction

We have been dealing with motion in one direction or straight line in unit 2. We also dealt with a purely vertical motion of falling bodies undergoing acceleration due to gravity. In the real world motion in a plane or two dimensions is very possible. Examples of such motions are circular motion, projectile motion, motion of a bug on a wall, a cue-ball on a pool table, motion of a football referee on a pitch, kicked footballs and speeding bullets to mention but a few.

With a two dimensional motion, we mean an object moves in the x and y directions simultaneously.

In this unit, we shall learn about two dimensional motions such as a projectile motion and circular motions.

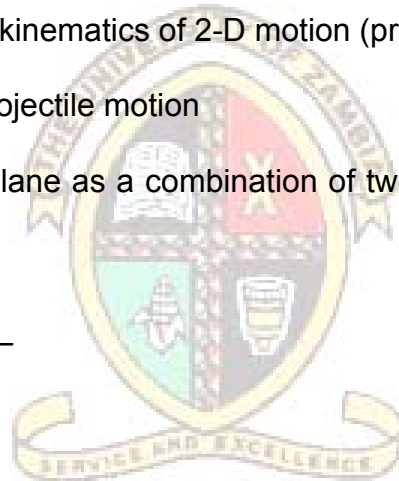
Welcome to Unit 3!



3.2 Unit Objectives

After completing this unit, you should be able to

- Extend derivation and use of equations of linear motion to 2-D
- Solve problems in the kinematics of 2-D motion (projectile, uniform circular...)
- Derive equations of projectile motion
- Interpret motion in a plane as a combination of two linear motions along x and y axes



3.3 Displacement, Velocity and Acceleration:

For motion in a plane, think of a skater on a lake whose skates leave marks that specify the trajectory of the motion. Figure 3-1 depicts a particle, for example the skater moving in a two-dimensional plane. We label the plane as the xy -plane and introduce a Cartesian coordinate system that contains an origin and x - and y axes.

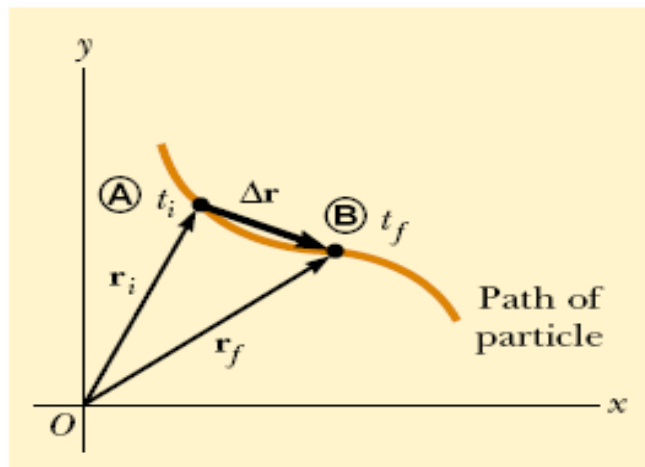


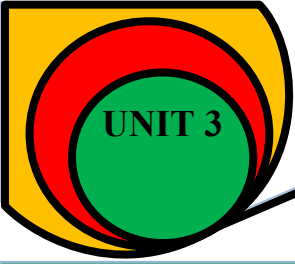
Fig. 3.1



In moving from point A to Point B, the skater traverses a displacement vector Δr so that

$$(3.1)$$

The direction is indicated in Figure 3.1. Its magnitude is less than the distance travelled. From these, we obtain the following relations:



Motion in a Plane



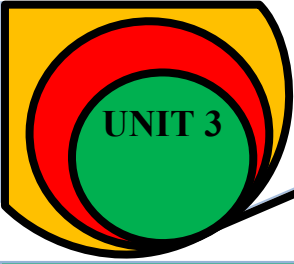
1. Displacement



Since the motion is in two dimensions, it has x and y components. Therefore for velocity, it is v_x and v_y and similarly for displacement we have Δr_x and Δr_y . The same is true for acceleration.

The magnitude as we found out in unit one on vectors is found by

$$v = \sqrt{v_x^2 + v_y^2}$$



Motion in a Plane



We can write the equations of motion in component form as shown

$$v_f = v_0 + at \Rightarrow \{v_{fx} = v_{0x} + a_x t \quad \& \quad v_{fy} = v_{0y} + a_y t \quad (3.2)$$

$$r_f = v_0 t + \frac{1}{2} at^2 \Rightarrow \{r_{fx} = v_{0x} t + \frac{1}{2} a_x t^2 \quad \& \quad r_{fy} = v_{0y} t + \frac{1}{2} a_y t^2 \quad (3.3)$$

These components are illustrated in Figure 3.2. The component form of the equations for v_f and r_f show us that two-dimensional motion at constant acceleration is equivalent to two *independent* motions—one in the x direction and one in the y direction having constant accelerations a_x and a_y .

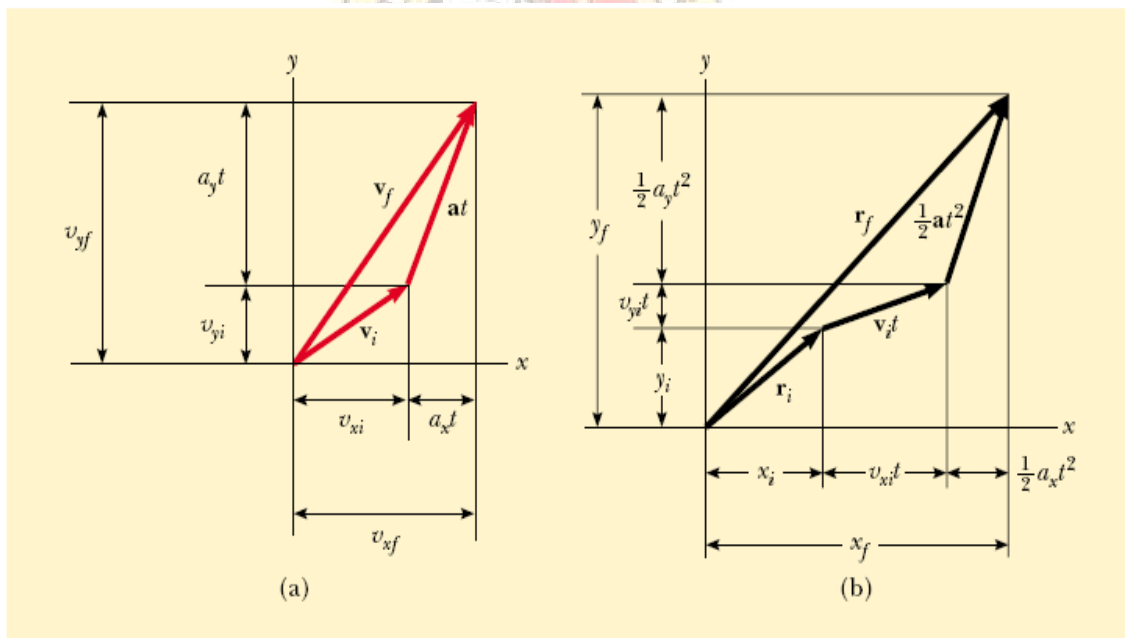


Figure 3.2



3.4 Projectile Motion

Any object launched in an arbitrary direction in space with an initial velocity, which then follows a path determined by the acceleration due to gravity is called **projectile** and its motion is called **projectile motion**.

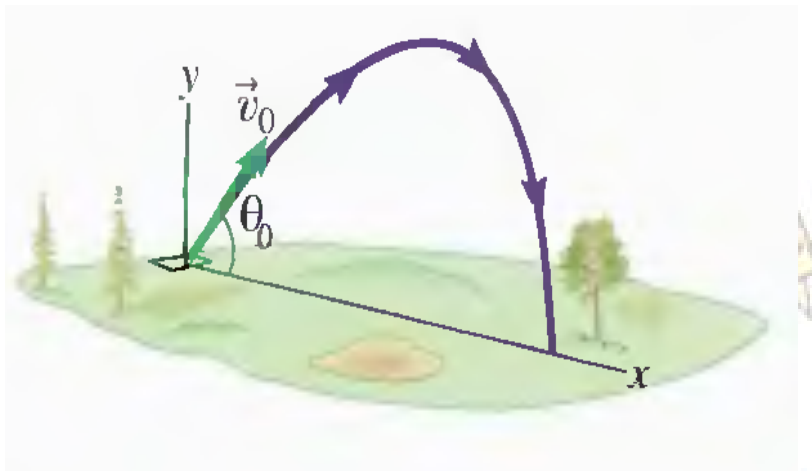
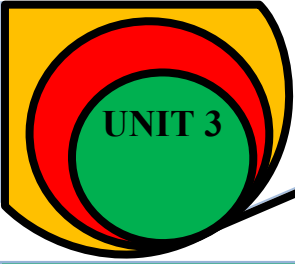


Figure 3.3: A golf ball leaves the tee with initial velocity of magnitude v_0 .

A soccer ball in motion in the air kicked by a goalkeeper is an example of a projectile that moves under the effect of gravity. Another example is a golf ball teed in the air as shown in Figure 3.3. In the absence of air resistance, what is the trajectory of a projectile? The motion is that of constant acceleration due to gravity, and this constant acceleration g has only a vertical component. The path of a projectile is called its trajectory. This trajectory is always a parabola in shape.

Therefore for a projectile the following assumptions will be made and followed throughout the unit:



Motion in a Plane



- The free-fall acceleration, $\sim g$, has a magnitude of 9.80 m/s^2 , is constant over the range of motion, and is directed downward.
- The effect of air resistance is negligible (hence the size of the object is relatively small and its surface is smooth).
- The rotation of the Earth does not affect the motion (hence the distance traveled is small with respect to the Earth's radius)
- The motion can be divided into two components, (1) motion in the x direction and (2) motion in the y direction.
- There is constant velocity in the x direction, hence acceleration is zero while in the y direction the object falls under gravity so that acceleration $a = -9.8 \text{ m/s}^2$ (keeping in mind the convention: **up positive and down negative**)

3.4.1 Projectile Fired Horizontally

Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately.

For convenience, we assume that the motion begins at $t = 0$ at the origin of an xy coordinate system.

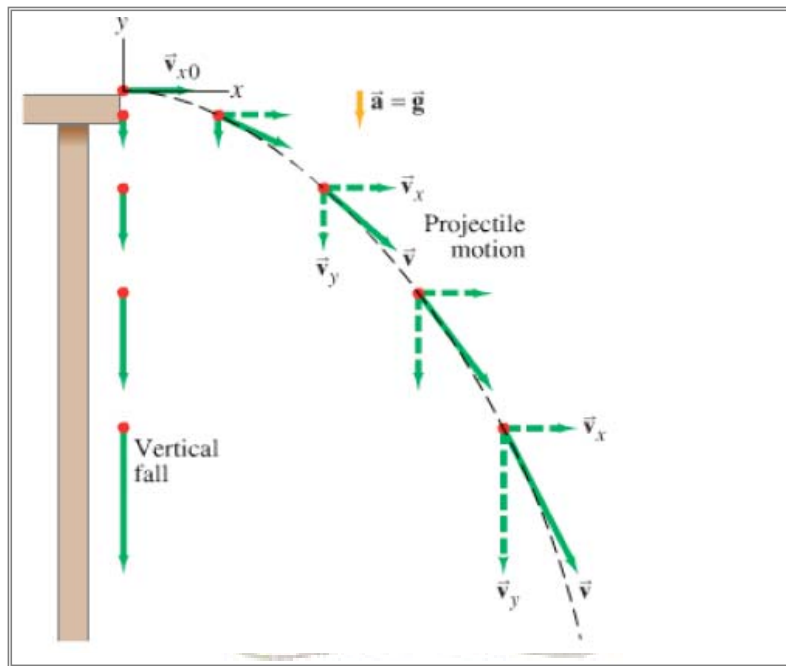
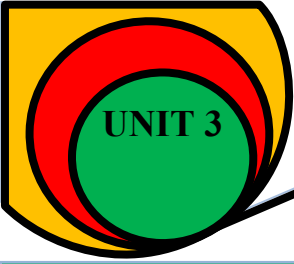


Figure 3.4: The projectile motion of a small object projected horizontally.

Let us look at a tiny ball rolling off the end of a horizontal table with an initial velocity in the horizontal (x) direction v_{x0} . See Figure 3.3 where an object falling vertically is also shown for comparison. The velocity \mathbf{v} at each instant points in the direction of the ball's motion at that instant and is always tangent to the path. Following Galileo's ideas, we treat the horizontal and vertical components of the velocity, v_x and v_y , separately and we apply the kinematic equations to the separate components.



Motion in a Plane



First we examine the vertical (y) component of the motion. At the instant the ball leaves the table, it has only the x component of the velocity and experiences a vertically downward acceleration g , the acceleration due to gravity. Thus v_y is initially zero ($v_{oy}=0$) but increases continually in the downward direction until the ball hits the ground.

Using our now familiar convention, $a_y = -g$, and from equation 2.11 from unit 2, we have

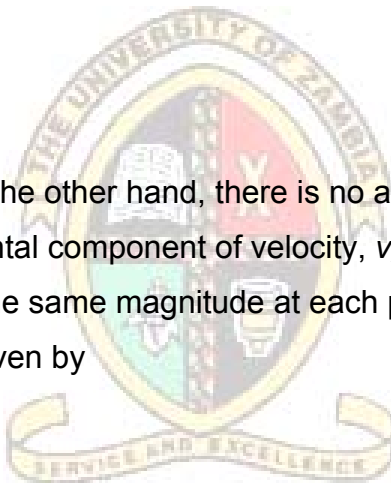
$$v_y = v_{oy} - gt = -gt.$$

The vertical displacement is given by

$$y = v_{oy}t - \frac{1}{2}gt^2 = -\frac{1}{2}gt^2.$$

In the horizontal direction on the other hand, there is no acceleration (we are ignoring air resistance). So the horizontal component of velocity, v_x , remains constant, equal to its initial value, v_{ox} . Thus it has the same magnitude at each point on the path. The horizontal displacement is given by

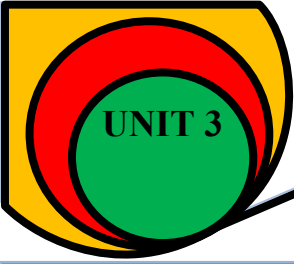
$$x = v_{ox}t - \frac{1}{2}gt^2 = v_{ox}t$$



The vector components \mathbf{v}_x and \mathbf{v}_y can be added vectorially at any instant to obtain the velocity \mathbf{v} at any point as shown in Figure 3.3 that is,

$$v = \sqrt{v_x^2 + v_y^2}$$

The result of this analysis, which Galileo himself predicted, is that *an object projected horizontally will reach the ground in the same time as an object dropped vertically*. This is because the vertical motions are the same in both cases.



3.4.2 Projectile Motion Fired at an Angle

If an object is projected at an upward angle as shown in Figure 3.4, the analysis is similar to the previous one in section 2.4.1 except that now there is an initial vertical component, v_{0y} . Because of the downward acceleration of gravity, v_y gradually decreases with time until the object reaches the highest point on its path, at which point, $v_y = 0$. Subsequently the object moves downwards and v_y increases in the downward direction. As before the horizontal component v_x remains constant (that is $v_{0x} = v_x$).

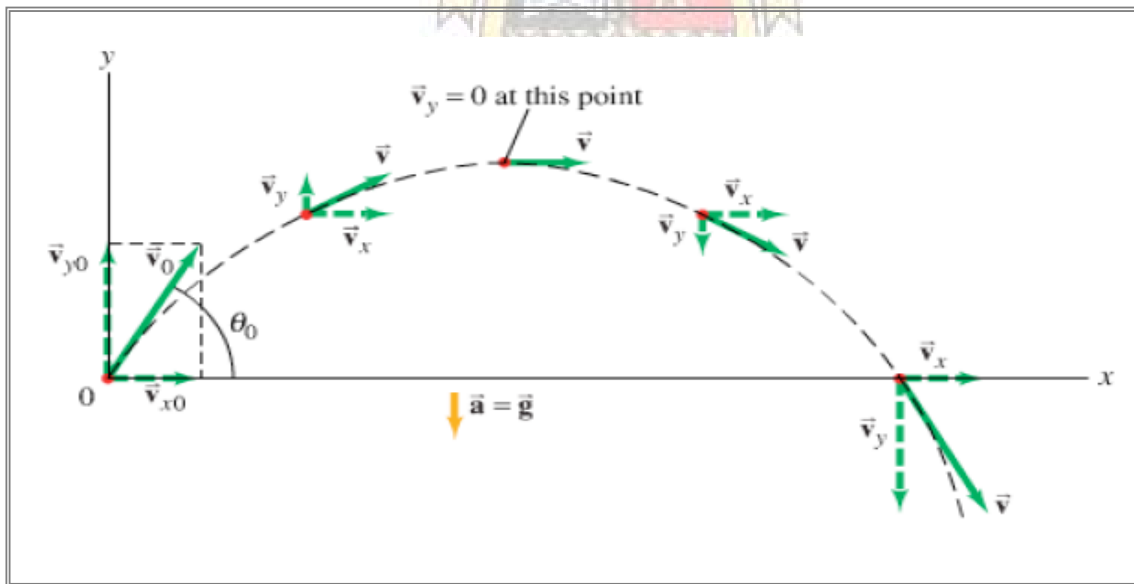
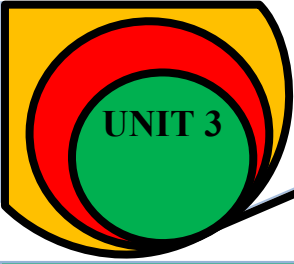


Figure 3.4: Projectile fired at an angle θ



Motion in a Plane



Thus, when analyzing projectile motion, consider it to be the superposition of two motions: (1) **constant-velocity motion in the horizontal direction** and (2) **free-fall motion in the vertical direction**. The horizontal and vertical components of a projectile's motion are completely independent of each other and can be handled separately, with time t as the common variable for both components.

To find the initial velocity in both directions, we resolve the velocity into components.

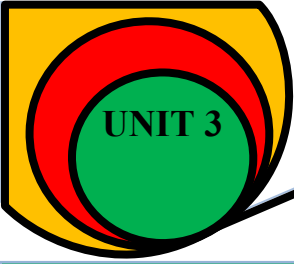
Do you still remember how to do it, from unit 1? (Use the famous **SOHCAHTOA**). This is how...!!

$$v_{ox} = v_0 \cos \theta_0 \tag{3.4}$$

$$v_y = v_0 \sin \theta_0 \tag{3.5}$$



To solve problems involving projectile motion, it is helpful to draw a table to aid in separating the two different motions. This is shown in Table 3.1.



Motion in a Plane



Table 3.1: Two component motions of a projectile motion.

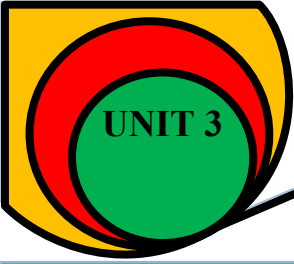
Horizontal Motion	Vertical Motion
$v_{0x} = v_0 \cos \theta = v_x$ (eqn :3.6)	$v_{0y} = v_0 \sin \theta$ (eqn :3.7)
$x = v_{0x}t$	$v_{fy} = v_{oy} - gt$ $= v_0 \sin \theta - gt$ (eqn :3.9)
$x = (v_0 \cos \theta)t$ (eqn :3.8)	$y = v_{oy}t - \frac{1}{2}gt^2$ $= (v_0 \sin \theta)t - \frac{1}{2}gt^2$ (eqn 3.10)
	$v_{fy}^2 = v_{oy}^2 - 2gy$ $= (v_0 \sin \theta)^2 - 2gy$ (eqn :3.11)

The velocity at which the projectile hits the ground is given by

$$v = \sqrt{v_{xf}^2 + v_{yf}^2} \tag{3.12}$$

At an angle of

$$\tan \theta = \frac{v_{fy}}{v_{fx}} = \frac{v_{fy}}{v_{ox}} \tag{3.13}$$



Motion in a Plane



We can also derive the relationship between the vertical displacement (y) versus the horizontal distance (x). Using the equations in Table 3.1 we can eliminate the time variable and derive such relationship, called the **trajectory equation**.

From equation 3.8, the equation for x we can make t as a function of x that is,

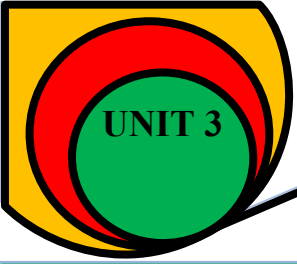
$$t = \frac{x}{v_0 \cos \theta} \tag{3.12}$$

Substituting this into equation 3.10

$$y = (v_0 \sin \theta) \frac{x}{(v_0 \cos \theta)} - \frac{1}{2} g \left(\frac{x}{v_0 \cos \theta} \right)^2 \tag{3.13}$$
$$= (\tan \theta)x - \left(\frac{g}{2v_0^2 \cos^2 \theta} \right) x^2$$

We have made use of the fact that $\sin \theta / \cos \theta = \tan \theta$. This is a quadratic relationship of the form $y = ax^2 + bx$ with $a = \tan \theta$, and $b = \frac{g}{2v_0^2 \cos^2 \theta}$.

This is the equation of a parabola passing through the origin with its axis parallel to the y -axis. The trajectory of all objects moving with constant acceleration is parabolic. This is shown in Figure 3.5.



Motion in a Plane



It is worth noting that when the object returns to its launch point, the displacement is zero i.e $y = 0$. At the peak of the trajectory, $v_y = 0$.

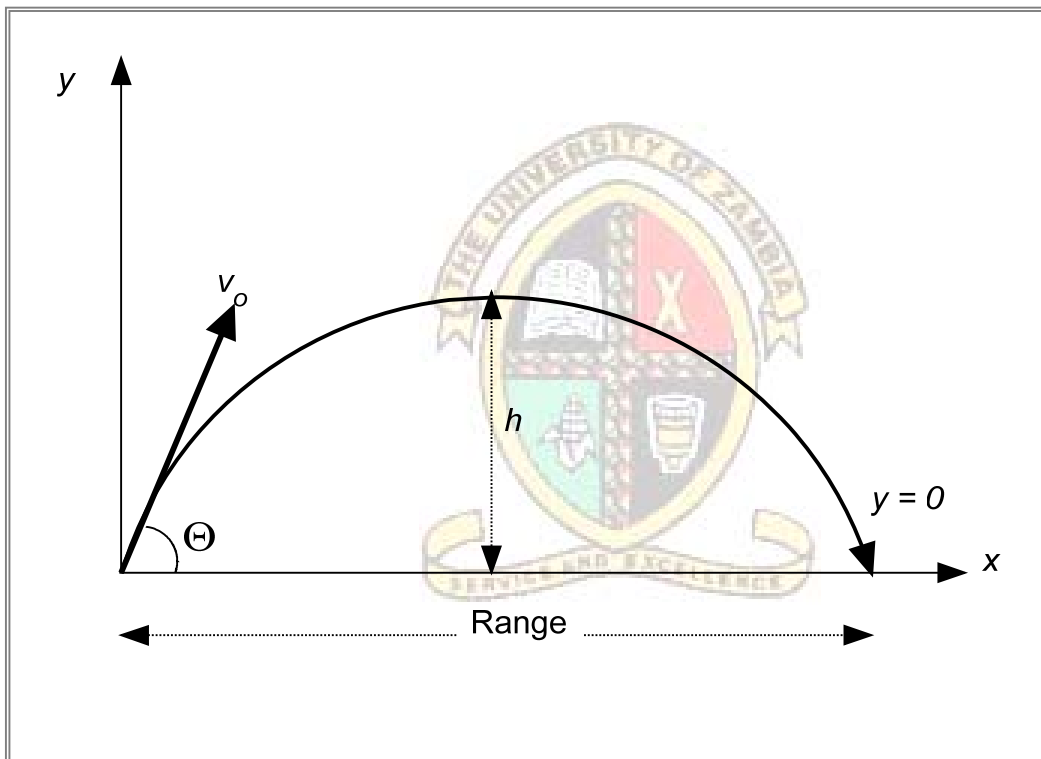
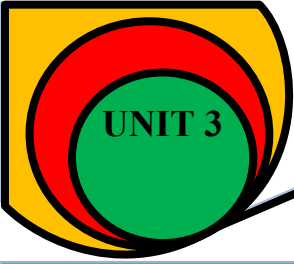


Figure 3.4: Trajectory of the projectile



Motion in a Plane



Range: We define the range R of a projectile shown in Figure 3.5 launched from the ground ($y = 0$) to be the horizontal distance that the projectile travels over level ground; that is, it lands at the same height from which it started. The quantity R is the value of x when the projectile has returned to the ground, that is, when y again equals zero. If we insert $y = 0$ into equation 3.10, we have

$$t = \frac{2v_0 \sin \theta}{g} \tag{3.14}$$

Substituting this value of t of equation 3.14 into equation 3.8 and setting $x = R$, we get

$$R = v_{ox}t = v_o \cos \theta \frac{2v_0 \sin \theta}{g} = \frac{2v_0^2 \sin \theta \cos \theta}{g} \tag{3.15}$$

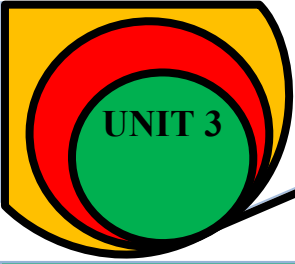
$$\therefore R = \frac{v_0^2 \sin 2\theta}{g} \tag{3.16}$$

We have used the trigonometric identity

$$2 \sin \theta \cos \theta = \sin 2\theta$$

The maximum height y_{max} (= H) is reached when the vertical component of velocity v_y becomes zero. According to equation 3.11, v_y is given by

$$v_y^2 = v_{oy}^2 - 2gy = (v_o \sin \theta)^2 - 2g \text{ substituting } y = y_{max} = H \text{ and } v_y = 0 \text{ yields}$$



$$v_0^2 \sin^2 \theta = 2gH$$

OR

$$H = \frac{v_0^2 \sin^2 \theta}{2g}$$

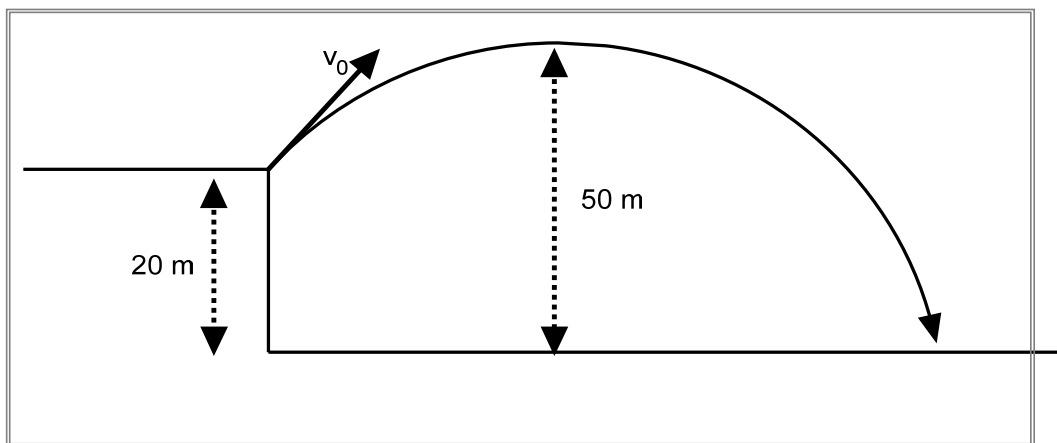
(3.17)

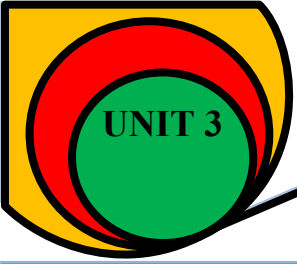
It should be noted that Equation 3.16 is useful for calculating R only for a **symmetric path**, as shown in Figure 3.4. If the path is not symmetric, **do not use this equation**. The general expressions given by Equations 3.8 and 3.10 are the *more important* results, because they give the position and velocity components of *any* particle moving in two dimensions at *any* time t .



Self-help task 3.1

What is the vertical displacement in Figure 3.5 below?





Motion in a Plane



Figure 3.5: Projectile fired on a cliff

Finally the problem-solving strategy for the projectile motion:

- Select a coordinate system (typically Cartesian).
- Resolve the initial velocity vector into x and y components.
- Treat the horizontal motion and the vertical motion independently.
- Horizontal motion has constant velocity.
- Vertical motion has constant acceleration.

Follow this procedure religiously and you will be okay!

Let us do some examples now to put in perspective what we have discussed so far.



Self-help task 3.2

As a projectile thrown upward moves in its parabolic path (such as in Figure 3.4), at what point along its path are the velocity and acceleration vectors for the projectile perpendicular to each other? (a) nowhere, (b) the highest point, (c) the launch point.



EXAMPLE 3.1: A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s .

- (a) How far does he jump in the horizontal direction? (Assume his motion is equivalent to that of a particle.)
- (b) What is the maximum height reached?
- (c) At what speed will he land on the ground?

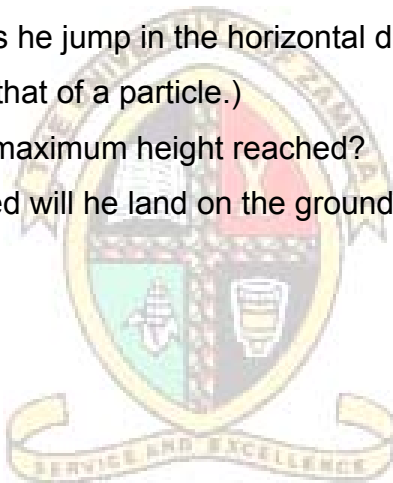
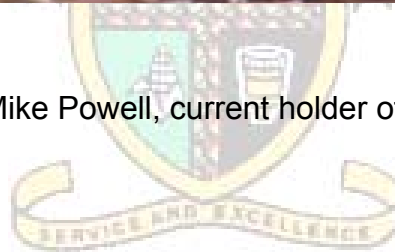




Figure 3.6 : (Example 3.1) Mike Powell, current holder of the world long jump record of 8.95 m.

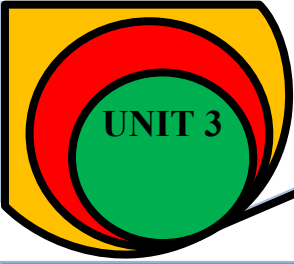


SOLUTION

We separate the motion as usual into two components, the vertical motion and horizontal motion.

Horizontal motion

Vertical Motion



Motion in a Plane



$$v_{ox} = v_0 \cos \theta$$

$$= 11 \cos 20^\circ = 10.34 \text{ m/s}$$

$$v_{oy} = v_0 \sin \theta$$

$$= 11 \sin 20^\circ = 3.76 \text{ m/s}$$

$$\text{Range} = R = v_{ox}t$$

$$y = v_{oy}t - 1/2gt^2$$

(a) When the athlete lands on the ground, his vertical displacement is zero (**why?**) that is

$$y = 0$$

Therefore

$$0 = v_{oy}t - 1/2gt^2$$

So that

$t = 0.76$ seconds, which is the time of flight.



Since time is the only parameter that connects the two motions, we substitute into the range equation to give,

$$\text{Range} = R = v_{ox}t = 10.34(0.76) = 7.73 \text{ m}$$

(b) The maximum height is reached when $v_y = 0$. Therefore using equation 3.11, we have

$$v_y^2 = v_{oy}^2 - 2gy = v_y^2 = v_{oy}^2 - 2gy_{max}$$



$$y_{\max} = \frac{v_{0y}^2}{2g} = 0.72 \text{ m}$$

(c) The velocity at which he will land...

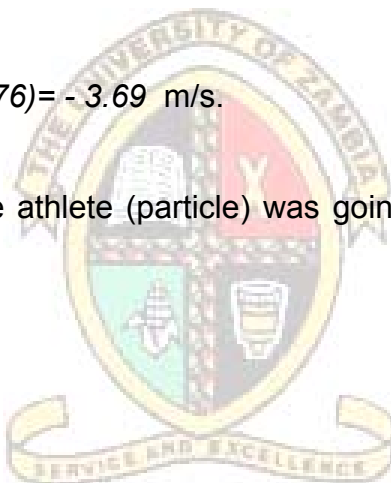
We know that in the horizontal direction the velocity is constant so

$$v_{0x} = v_0 \cos \theta = v_{fx} = 10.34 \text{ m/s.}$$

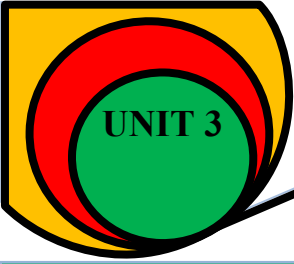
In the vertical direction,

$$v_y = v_{0y} - gt = 3.76 - (9.8)(0.76) = -3.69 \text{ m/s.}$$

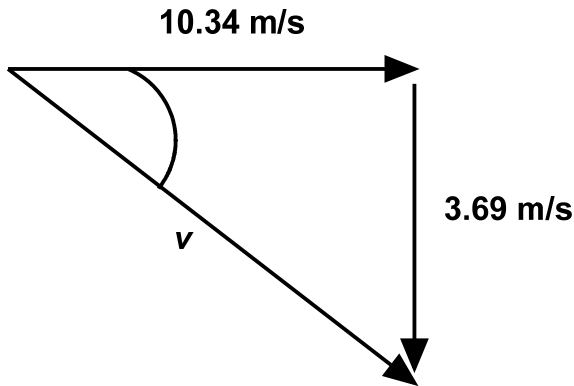
The negative implies that the athlete (particle) was going downwards (**remember our convention?**)



Therefore we have



Motion in a Plane



$$v = \sqrt{(v_{fx}^2 + v_{fy}^2)} = \sqrt{(10.34^2 + (-3.69)^2)} = 10.98 \text{ m/s}$$

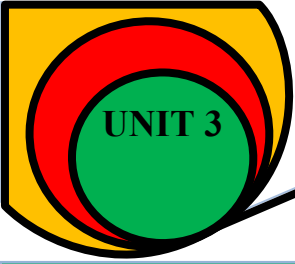
$$\tan \theta = \frac{3.69}{10.34} \text{ so that } \theta = \tan^{-1}\left(\frac{3.69}{10.34}\right) = 19.6^\circ \text{ below the positive x-axis.}$$

One could have used the direct equations for finding the range and maximum height reached using equations 3.16 and 3.17 respectively.

The sign of the vertical velocity is important in knowing if the object is still going up or descending. If it is positive then the object is still going up but when it is negative then it is going downwards.

Let us do another example.





Motion in a Plane



EXAMPLE 3.2: A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal with an initial speed of 20.0 m/s , as shown in Figure 4.14. If the height of the building is 45.0 m ,

- (a). how long does it take the stone to reach the ground?
- (b). what is the speed of the stone just before it strikes the ground?

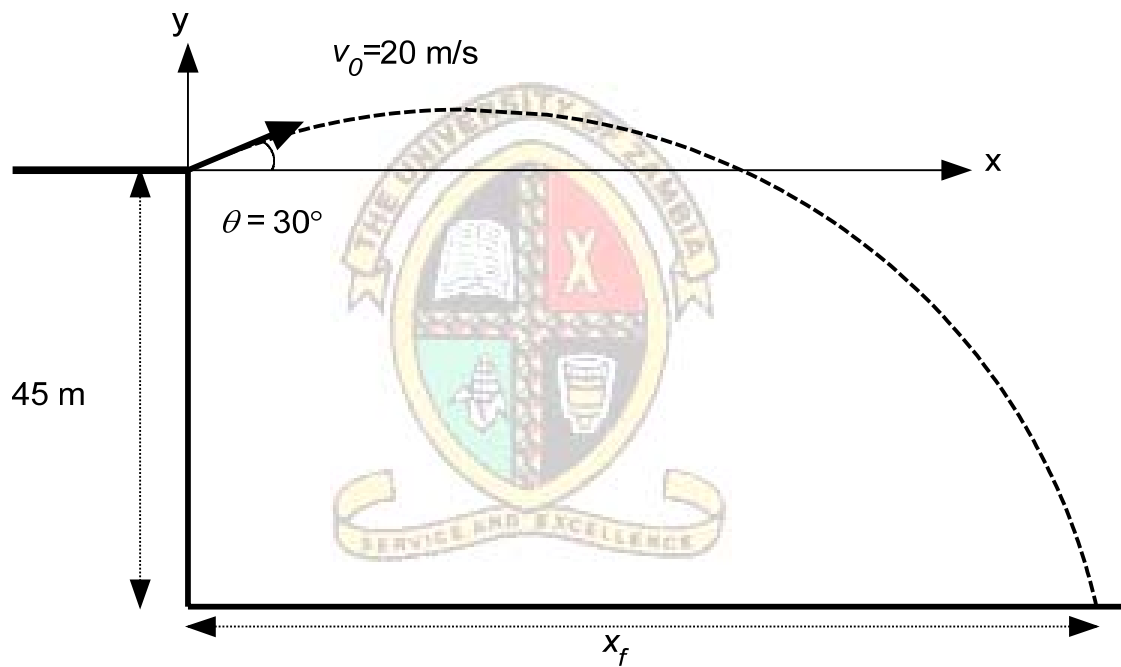
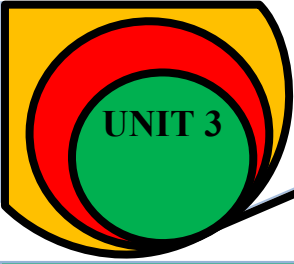


Figure 3.7: Stone thrown from the top of the building. Example 3.2



As usual we split the motions into horizontal and vertical motions.

Horizontal motion

$$v_{ox} = v_0 \cos \theta$$

$$= 20.0 \cos 30.0^\circ = 17.3 \text{ m/s}$$

$$x_f = v_{ox}t$$

Vertical Motion

$$v_{oy} = v_0 \sin \theta$$

$$= 20.0 \sin 30.0^\circ = 10.0 \text{ m/s}$$

$$y = v_{oy}t - 1/2gt^2$$

(a). Since the stone falls a distance 45 m below the launch point, the displacement is $y = -45.0 \text{ m}$.

Substituting into equation 3.10, the only unknown is t which is the time of flight.

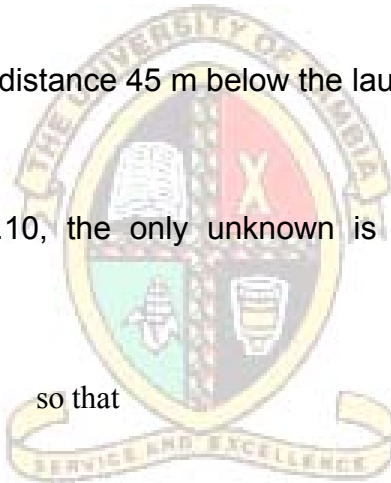
Therefore we have

$$y = v_{oy}t - \frac{1}{2}gt^2$$

so that

$$-45.0 = 10.0t - 4.9t^2$$

Solving this quadratic equation for t yields, for the positive root, $t = 4.22 \text{ s}$. To finalize this part, think: Does the negative root have any physical meaning?





(b) The horizontal component of velocity is constant. Therefore

$$v_{ox} = v_{xf} = 17.3 \text{ m/s}$$

In the vertical direction the final velocity is

$$v_y = v_{oy} - gt$$

$$= 10.0 - 9.8(4.22) = -31.4 \text{ m/s}$$

We therefore have the horizontal and vertical components of the final velocity.

Therefore

$$v = \sqrt{v_{fx}^2 + v_y^2} = \sqrt{(17.3)^2 + (-31.4)^2} = 35.9 \text{ m/s}$$

$$\tan \theta = \frac{-31.4}{17.3} \text{ so that } \theta = \tan^{-1}\left(\frac{-31.4}{17.3}\right) = -61.1^\circ \text{ below the positive x-axis.}$$

One more example to go...



EXAMPLE 3.3: An arrow is shot with a velocity of 30.0 m/s at an angle of 37.0° above the horizontal. The arrow is initially 2.0 m above the ground and 15.0 m from a wall.

- At what height above the ground does it hit the wall?
- Is it still going up just before it hits or it is already on its way down?

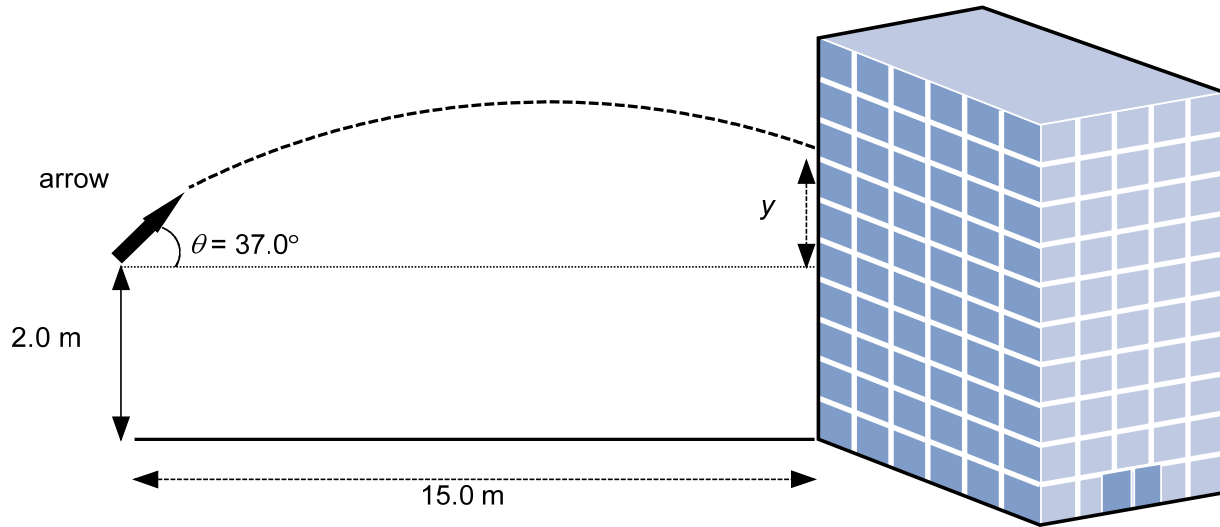


Figure 3.8: Arrow shot at angle to a wall. Example 3.3.

SOLUTION

Horizontal motion

$$v_{ox} = v_0 \cos \theta$$

$$= 30.0 \cos 37.0^\circ = 24.0 \text{ m/s}$$

$$x_f = v_{ox} t$$

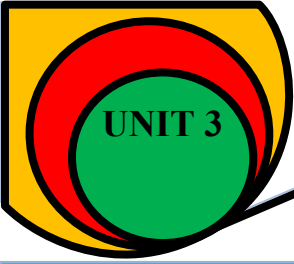
Vertical Motion

$$v_{oy} = v_0 \sin \theta$$

$$= 30.0 \sin 37.0^\circ = 18.1 \text{ m/s}$$

$$y = v_{oy} t - 1/2 g t^2$$

(a) From the expression of the horizontal displacement, we can obtain the time of flight.



That is

$$x_f = v_{ox}t \quad \text{so that we get}$$

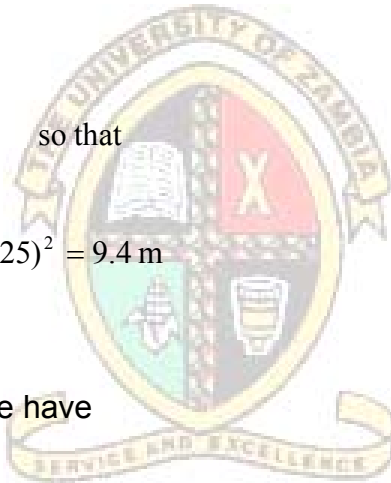
$$15 = (24.0)t$$

$$\therefore t = 0.625 \text{ s}$$

From equation 3.10, we can obtain the vertical displacement with respect to the launch level.

$$y = v_{oy}t - \frac{1}{2}gt^2 \quad \text{so that}$$

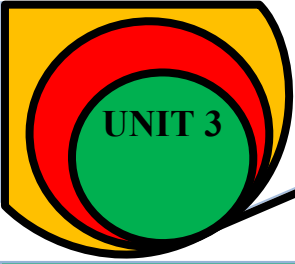
$$y = 18.1(0.625) - 4.9(0.625)^2 = 9.4 \text{ m}$$



With respect to the ground, we have

$$y = 2 + 9.4 = 11.4 \text{ m}$$

(b) Without forgetting our convention and depending on the sign of the final vertical component of velocity, we are able to determine if the arrow was still going up or was on its way down.



$$v_y = v_{oy} - gt$$

$$= 18.1 - 9.8(0.625) = +12.0 \text{ m/s}$$

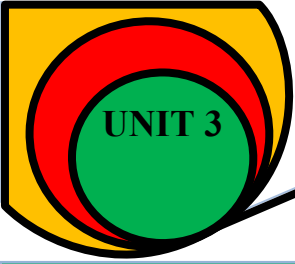
Since it is positive the arrow was still going up as opposed to the way it is shown in the graphic of Figure 3.8.



Self-help task 3.3

How can you throw a projectile so that it has zero speed at the top of its trajectory? So that it has nonzero speed at the top of its trajectory?

Your turn to solve as well!

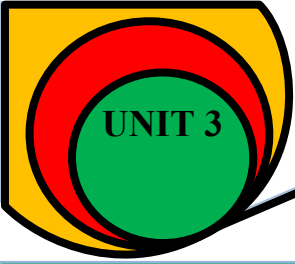


Self-help task 3.4

Suppose you fire a projectile 35° above horizontal with an initial velocity of 200 m/s. It lands in a valley 300 m below the launch point. What is the range (the horizontal distance from the point the projectile was fired to the point it struck the ground) of the projectile and what is the projectile's time of flight?

Did you get it? Then go to the next section and learn about circular motion...





3.5 Uniform Circular Motion

We are quite familiar with objects that move in circles at constant speeds, for example a carnival merry-go-round, the platter of a microwave oven, or a phonograph turntable. We are in fact being carried in a circular motion with Earth as it rotates each day.

An object undergoes uniform circular motion when it travels at a constant speed along a circular path (see Fig. 3-9), Circular motion is a special case of motion on a plane. We can best describe uniform circular motion by specifying a radius r and an angle θ , with the origin of the coordinate system at the center of the circle.

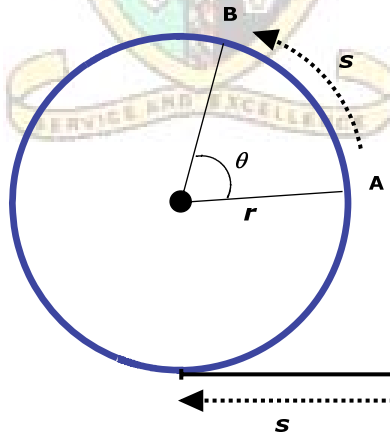
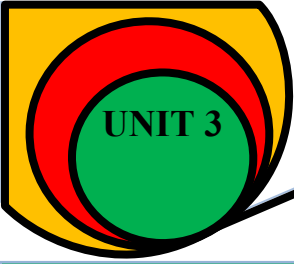


Figure 3.9: Wheel turning from A to B



3.5.1 Angular displacement θ

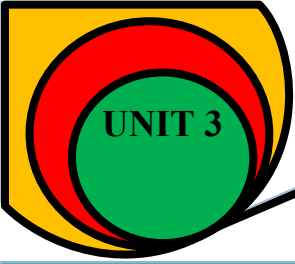
To describe the motion of an object along a line, we need a coordinate along the line, which we often take to be the x -coordinate. To describe the motion of an object on a circular path or the rotation of a wheel on an axle (axis), we need a coordinate to measure angles, the rotational representative of linear displacement. Therefore in the concept of circular motion we talk about angular displacement just as we talk of linear displacement for linear motion.

Now consider the two positions of the wheel shown in Figure 3.9. When the wheel has turned from point A to B, it has turned through an angle θ .

There are three ways in which θ is measured.

- We can measure it in terms of *degrees* (deg):
1 full circle $\equiv 360^\circ$
- It can also be measured in terms of *revolutions* (rev):
1 full circle $\equiv 1$ revolution
- Finally it can be measured in terms of *radians* (rad):

To explain the meaning of radians, we consider Figure 3.9. When the wheel turns, the distance covered s is given by the arc s .



Motion in a Plane



The angle in radians is found from the well known high school expression for finding the arc given the angle and radius. In fact the angle is in radians if you didn't know!

Therefore

$$\theta(\text{rad}) = \frac{s}{r} \quad (3.18)$$

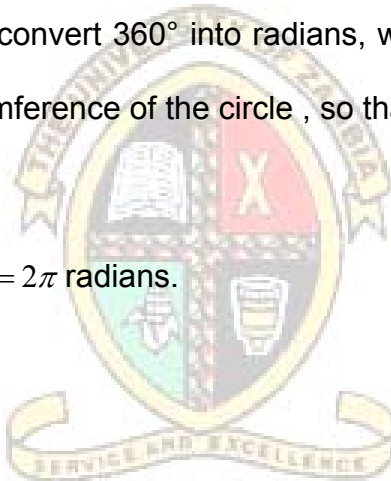
So that if you want to convert 360° into radians, we appreciate that this angle is equivalent to the circumference of the circle, so that $s = 2\pi r$

Therefore

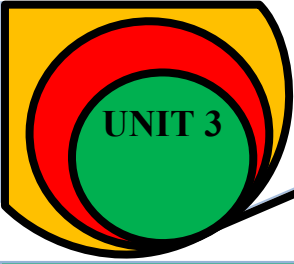
$$\theta(\text{rad}) = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ radians.}$$

Therefore

$$\mathbf{1 \text{ rev} = 360^\circ = 2\pi \text{ rad}}$$



Notice that degrees, revs, and rads are all dimensionless quantities. That is, they do not involve any of the basic dimensions of physical measurement. Thus when they are involved in a calculation, they do not change the units of the terms in the equation. However, it is important to indicate which way an angle is measured in order that the result of the calculation can be interpreted correctly.



Motion in a Plane



EXAMPLE 3.4 : Convert a 70.0° angle into radians and revolutions.

SOLUTION:

Using the conversion factors $2\pi \text{ rad} / 360^\circ$ and $1 \text{ rev} / 360^\circ$, we have

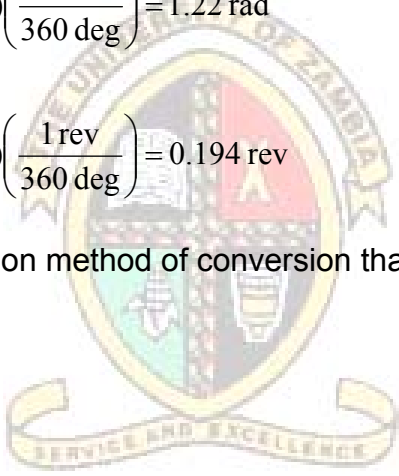
$$70.0^\circ = (70.0 \text{ deg}) \left(\frac{2\pi \text{ rad}}{360 \text{ deg}} \right) = 1.22 \text{ rad}$$

$$70.0^\circ = (70.0 \text{ deg}) \left(\frac{1 \text{ rev}}{360 \text{ deg}} \right) = 0.194 \text{ rev}$$

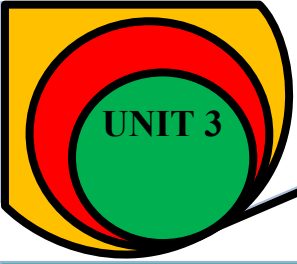
The most common method of conversion that you are familiar with is

$$2\pi \text{ rad} = 360^\circ$$

$$x(\text{rad}) = 70^\circ$$



then cross multiply to find x (*rad*). It is up to you to choose which method you are familiar or comfortable with.



Self-help task 3.5

Make the following conversions:

- (a) 0.210 rad to degrees and revolutions
- (b) 1.0 rad to degrees and revolutions

3.5.2 Angular Displacement ω

This describes how fast something is rotating. Analogously to linear motion, where average velocity is defined to be displacement divided by time, we define

$$\text{Average angular velocity} = \frac{\text{angular displacement}}{\text{time taken}}$$

Or

$$\bar{\omega} = \frac{\theta}{t} \tag{3.19}$$



Where ω (Greek omega) is the angular velocity. Typical units for omega as one would guess are radians per second (rad/s), degrees per second (deg/s) and revolutions per minute (rev/min) or famously **rpm**.



EXAMPLE 3.5: A wheel turns through 1800 rev in 10 min. Find its average angular velocity in rad/s.

$$\bar{\omega} = \frac{\theta}{t} = \frac{1800 \text{ rev}}{60 \text{ s}} = 30 \text{ rev/s}$$

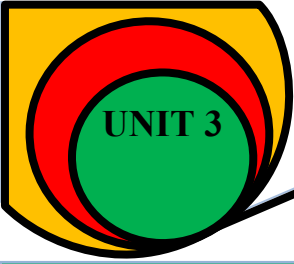
Then

$$30 \text{ rev/s} = \left(30 \frac{\text{rev}}{\text{s}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) = 60\pi \text{ rad/s} = 190 \text{ rad/s}$$

3.5.3 Angular Acceleration α

For cases of rotating bodies, we are often interested in how they speed up or slow down. Hence we are concerned with angular acceleration, that is, the rate of change of angular velocity.

There is a lot of analogy between linear and circular motion as is shown below:



Linearly, $a = \frac{v_f - v_i}{t}$

Angularly, $\alpha = \frac{\omega_f - \omega_i}{t}$ (rad/s²) (3.20)

Similarly the average angular velocity is

$\bar{\omega} = \frac{\omega_f + \omega_i}{2}$ (3.21)

Here are the kinematic equations showing analogies between linear and angular motions.

Linear motion	Angular motion
$s = \bar{v}t$	$\theta = \bar{\omega}t$
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$\bar{v} = \frac{v_i + v_f}{2}$	$\bar{\omega} = \frac{\omega_i + \omega_f}{2}$
$v_f^2 = v_i^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$
$s = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$



You have seen how analogous these equations are, right? An example...



EXAMPLE 3.6: A wheel turning at 3.00 rev/s coasts to rest uniformly in 18.0 s. What is the deceleration? How many revolutions does it turn through while coming to rest?

SOLUTION

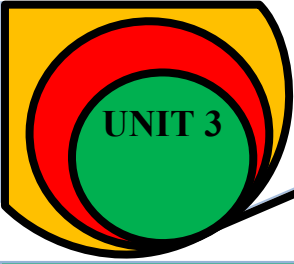
$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 3.00}{18.00} = -0.167 \text{ rad/s}$$

$$\begin{aligned} \theta &= \omega_i t + \frac{1}{2} \alpha t^2 = 3(18.0) + \frac{1}{2} (-0.167)(18.0)^2 \\ &= 27 \text{ rev} \end{aligned}$$

You can also use the expression for average angular velocity:

$$\bar{\omega} = \frac{\omega_f + \omega_i}{2} = \frac{0 + 3.00}{2} = 1.5 \text{ rad/s}$$

$$\therefore \theta = \bar{\omega} t = 1.5(18.0) = 27 \text{ rev}$$



3.5.4 Tangential Quantities

When you are in a moving car, the wheels can be rotating but you are being transported from one point to another. You are not rotating along with the wheels. What does that mean? It means there is a close connection between rotational motion and translational motion. This is how.

When the wheel in Figure 3.10 (a) is rotating with an angular speed ω , the centre is translated with a translational speed v_T . Now we establish the connection as illustrated in Figure 3.10 (b).

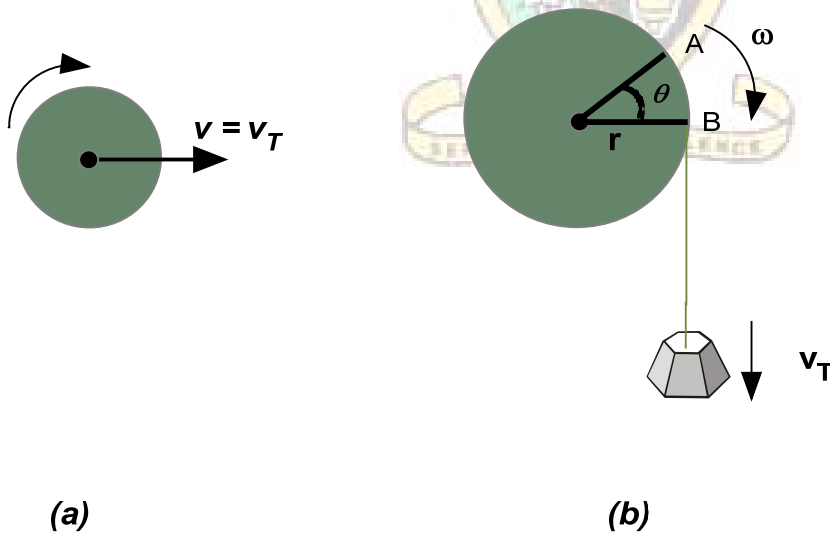
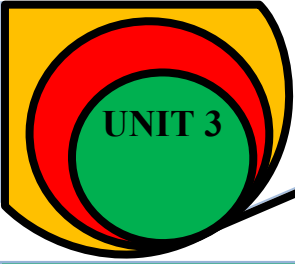


Figure 3.10: connection between rotational and translational motion

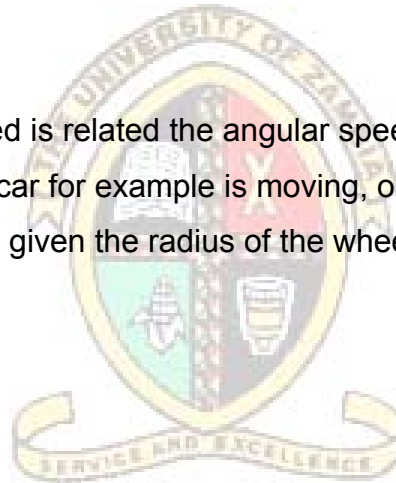


We know that

$$\omega = \frac{\theta}{t} = \frac{\frac{s}{r}}{t} = \frac{s}{t} * \frac{1}{r} = \frac{v}{r}$$

$$\therefore v_T = \omega r$$

Therefore the tangential speed is related to the angular speed through the radius. Given the linear velocity at which a car for example is moving, one can know the speed at which the wheels are rotating given the radius of the wheels.



In short

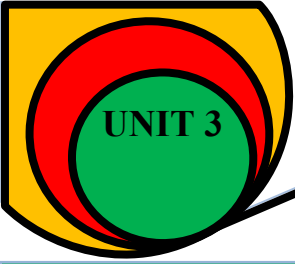
$$\omega = \frac{v}{r}$$

Or

$$v = \omega r$$

(3.21)

The tangential speed is just the translational speed.



This connection can also be extended to angular acceleration. This is how:

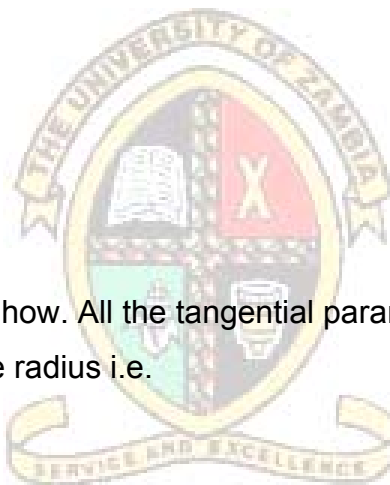
$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{\frac{v_f}{r} - \frac{v_i}{r}}{t} = \frac{v_f - v_i}{rt} = \frac{a}{r}$$

Therefore we have the relationship

$$\therefore \alpha = \frac{a}{r}$$

Or

$$a = \alpha r$$



(3.22)

Now all is shaping up. This is how. All the tangential parameters s , v and a are related to angular counterparts by the radius i.e.

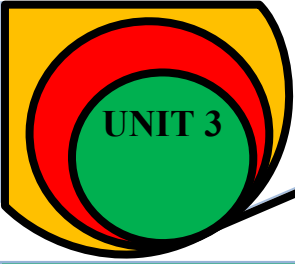
$$s = \frac{\theta}{r}$$

$$v = \frac{\omega}{r}$$

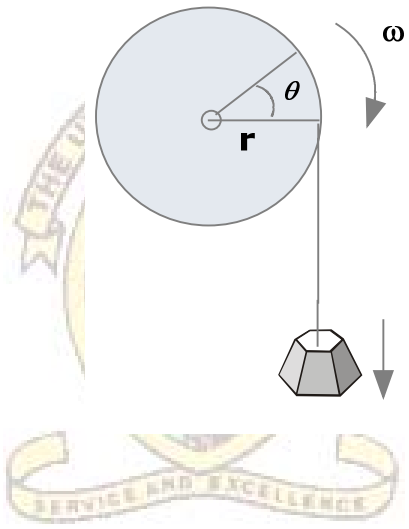
$$a = \frac{\alpha}{r}$$

Clear ?!!

Let us check an example...



EXAMPLE 3.7: Suppose that the mass starts from rest and accelerates downward at 8.6 m/s^2 . If the radius of the spool is 20 cm, what is the rotation rate after 3.0 s?

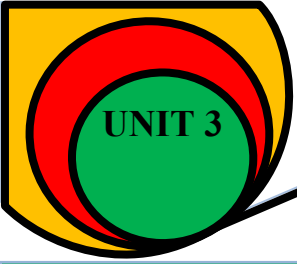


SOLUTION

We convert the given acceleration in angular parameters.

$$\alpha = \frac{a}{r} = \frac{8.6}{0.2} = 43 \text{ rad/s}^2$$

$$\begin{aligned} \omega_f &= \omega_i + \alpha t \\ &= 0 + 43(3.0) \\ &= 130 \text{ rad/s} \end{aligned}$$



3.5.5 Centripetal Acceleration

Whenever we hear about acceleration by now we definitely know that's it is a time rate of change of velocity. For a motorcycle travelling around a curve in Figure 3.11 with constant speed, one would be tempted to say acceleration is zero. This is not the case. The motorcycle is accelerating. Since acceleration is the rate of change of velocity and that velocity is a vector quantity, the change can either be

- Change in magnitude or
- Change in direction which is our case in Figure 3.11.

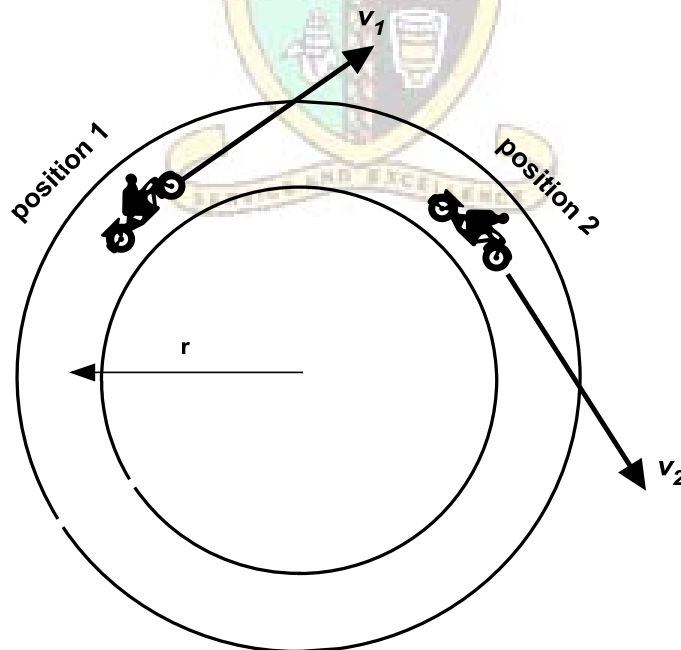
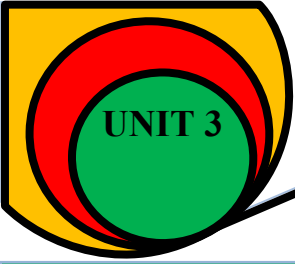


Figure 3.11: Motorcycle moving in a circular track with constant speed.



Now because the direction of velocity at position 1 is not the same as that at position 2, the velocity changes as the motorcycle moves along the track.

Therefore the average acceleration of the car between 1 and 2 is

$$\bar{a} = \frac{\text{change in velocity}}{\text{time taken}}$$

Now in order to calculate the acceleration of the motorcycle in Figure 3.11, we redraw it in Figure 3.12. The y component of the car's velocity is v_y at position 1 and $-v_y$ at position 2.

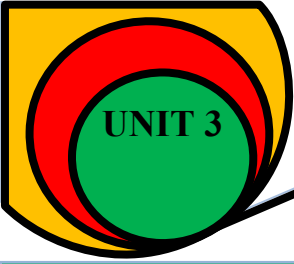
Therefore in going from position 1 to position 2, we have

$$(\text{change in velocity})_y = v_{y2} - v_{y1} = -v_y - v_y = -2v_y.$$

For the x – component

$$(\text{change in velocity})_x = v_{x2} - v_{x1} = v_x - v_x = 0$$

The time the motorcycle takes to go from 1 to 2 is $t = s/v$, where v is the constant tangential velocity of the car along the track and s is the arc length from 1 to 2.



Motion in a Plane

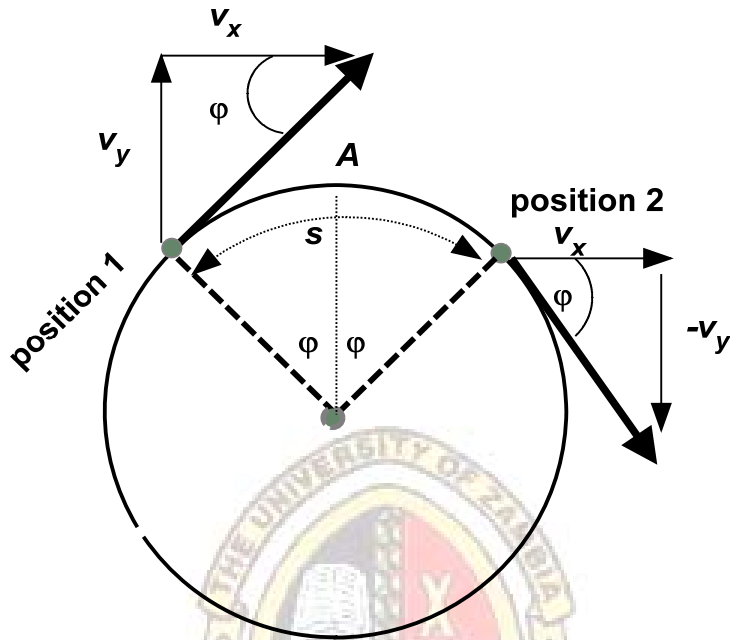


Figure 3.12: In going from position 1 to position 2 the change in velocity is $-2v_y$

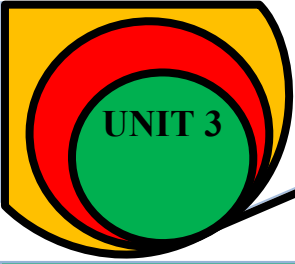
From the definition of radian measure ,

$$\theta = s/r$$

So that we have

$$2\phi = \frac{s}{r} \quad \text{or} \quad s = 2r\phi$$

Since s subtends an angle 2ϕ in this case.



Thus we find that

$$t = \frac{s}{v} = \frac{2r\phi}{v}$$

Knowing the change in velocity, $-2v_y$ and the time taken, $2r\phi/v$. Hence

$$\bar{a} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{-2v_y}{2r\phi/v} = -\frac{vv_y}{r\phi}$$

From Figure 3.12, $v_y = v \sin \phi$

So that

$$\bar{a} = -\frac{v^2 \sin \phi}{r\phi}$$

*

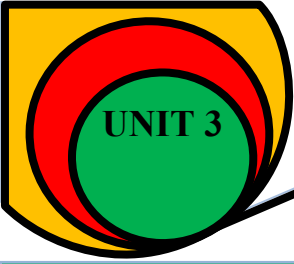
We are interested in the instantaneous acceleration at a point such as A. To obtain it we simply shrink the angle ϕ to a very small value.

Before we proceed lets check out this mathematical fact:



Self-help task 3.6

- Change a small angle say $\theta = 9^\circ$ to radians θ (rad)
- Change the mode on your calculator to rad
- Find the sine of the radian you found in bullet 1: $\sin(\theta)$
- What did you find? Compare it with the angle in radians in bullet 1



Motion in a Plane



If you did the task well, the following is the finding

$$\sin \varphi \approx \varphi \quad \text{in radians when } \varphi \text{ is very small.}$$

Substituting into *

We obtain the acceleration at A as

$$a = -\frac{v^2 \sin \varphi}{r\varphi} = -\frac{v^2 \varphi}{r\varphi} = -\frac{v^2}{r}$$

This is the acceleration of the car as it goes by point A. Since the speed is constant, all points on the circle are equivalent, and so the magnitude of acceleration is $a = v^2 / r$ no matter where we choose point A.

To find the direction of acceleration, we recall that by definition, Δv and acceleration a point in the same direction. Taking point A as our point of reference, and that $\Delta v = -2v_y$, the negative sign indicates a vector from A to the negative y direction (pointing towards the centre). Hence that is the direction of acceleration since they point in the same direction as Δv .

Point A was chosen arbitrary. We could always choose our y axis to pass through any other selected point. Thus the conclusion we draw for this point which also applies to other points on the circle is that:



- ❖ An object moving with constant speed v along a circular path of radius r is undergoing an acceleration directed toward the centre of the circle.
- ❖ This is called **centripetal** (centre seeking) **acceleration** a_c . The magnitude of this acceleration is

$$a_c = \frac{v^2}{r} = \omega^2 r \quad (3.23)$$

where we have used $v = \omega r$

This acceleration represents the rate of turning because it describes the rate of change of the direction of motion.

In many situations it is convenient to describe the motion of a particle moving with constant speed in a circle of radius r in terms of the period T , which is defined as the time required for one complete revolution. In the time interval T the particle moves a distance of $2\pi r$, which is equal to the circumference of the particle's circular path.

Therefore, because its speed is equal to the circumference of the circular path divided by the period,

$$v = \frac{2\pi r}{T} \quad (3.24)$$



EXAMPLE 3.8: A 150 g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions in a second. What is its centripetal acceleration?

SOLUTION

The centripetal acceleration is $a_c = \frac{v^2}{r}$. We are given r but not the speed. We can find the speed from the given radius and period ($T = \frac{1}{f} = \frac{1}{2} = 0.5 \text{ s}$) where f is the frequency.

$$\therefore v = \frac{2\pi r}{T} = \frac{2(3.14)(0.600 \text{ m})}{(0.500 \text{ s})} = 7.54 \text{ m/s}$$

The centripetal acceleration is

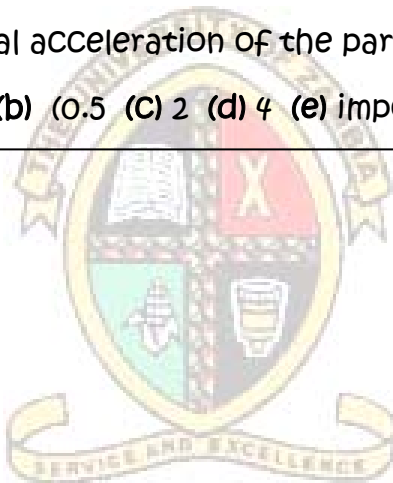
$$a_c = \frac{v^2}{r} = \frac{(7.54)^2}{0.600} = 94.7 \text{ m/s}^2$$

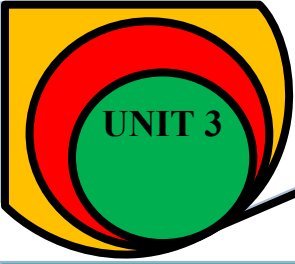
A task for you...



Self-help task 3.7

A particle moves in a circular path of radius r with speed v . It then increases its speed to $2v$ while traveling along the same circular path. The centripetal acceleration of the particle has changed by a factor of (a) 0.25 (b) 0.5 (c) 2 (d) 4 (e) impossible to determine





3.6 Summary

- In projectile motion with negligible or zero air resistance,
 - ✓ $a_x = 0$, $a_y = -g$
 - ✓ The velocity is constant in the x-direction that is :

$$v_{ox} = v_o \cos \theta = v_{fx}$$

$$v_{oy} = v_o \sin \theta$$

$$✓ \quad v_y = v_{oy} - gt$$

$$x = v_{ox}t = (v_o \cos \theta)t$$

$$y = v_{oy}t - \frac{1}{2}gt^2$$



- ✓ The velocity at which it hits the target is

$$v = \sqrt{v_x^2 + v_y^2}$$



- Circular motion is also a motion in a plane. The following was found:
 - ✓ The rotational motion is related to the linear motion by the following

$$\theta = \frac{s}{r}$$

$$\omega = \frac{v}{r}$$

$$\alpha = \frac{a}{r}$$

- ✓ When an object is moving with a constant speed around a circle of radius r , there is an acceleration directed towards the centre of the circle called the centripetal acceleration.

$$a_c = \frac{v^2}{r}$$



3.7. References

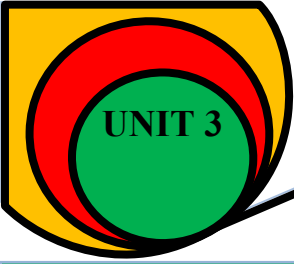
F.J Bueche, D.A Jerde, *Principles of Physics*.6th Edition, McGraw-Hill, 1995.

ISBN 0- 13.978-00700

D. Giancoli, *Physics:Principles with Applications*, 6th Edition.Pearson Prentice Hall, ISBN 0-13-060620-0

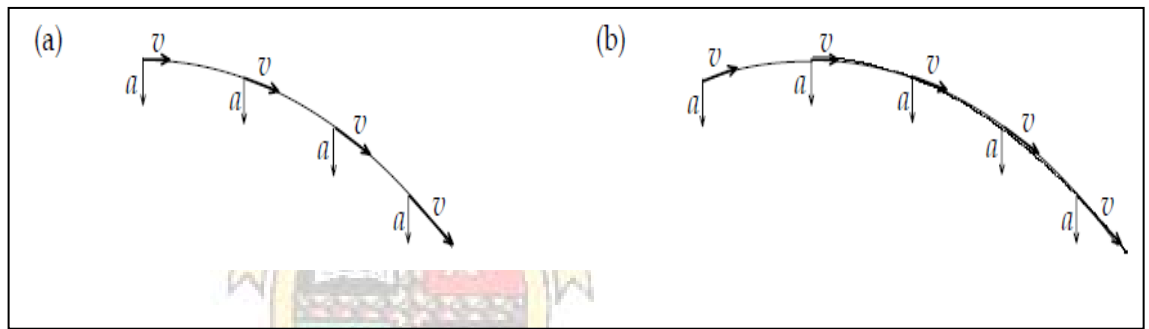
P.M Fishbane, S.G Gasiorowicz,S.T Thornton, *Physics for Scientists and Engineers with Modern Physics*,3rd Edition, Pearson Prentice Hall , ISBN 0-13-191182-1

Sears and Zemansky, *University Physics (with Modern Physics)*, 12th Edition, Hugh D Young and Roger, A Freedman,Pearson/Addison Wesley,2007, ISBN 0-8053-2187-X.



3.8. Answers to Self-help Questions

- 3.1 When the projectile goes up and then back to the launch point, the displacement is zero, then it proceeds 20 m down. So that $y = -20$ m.
- 3.2 It happens at the peak of the trajectory, i.e.



- 3.3 Straight up. Throwing the ball any other direction than straight up will give a nonzero speed at the top of the trajectory
- 3.4 Hint:

The vertical displacement is $y = -300$ m

$$v_{ox} = v_0 \cos \theta$$

$$= 200.0 \cos 35.0^\circ = \underline{\hspace{2cm}}$$

$$v_{oy} = v_0 \sin \theta$$

$$= 200.0 \sin 35.0^\circ = \underline{\hspace{2cm}}$$

- We then find then time of flight t using equation 3.10, with $y = -300$ m
That gives a quadratic equation. Solve for positive t .
- Use this t to obtain the horintal distance (range) from equation

$$\text{Range} = v_{ox}t$$



3.5 (a) 12.0°

((b) 0.0334 rev

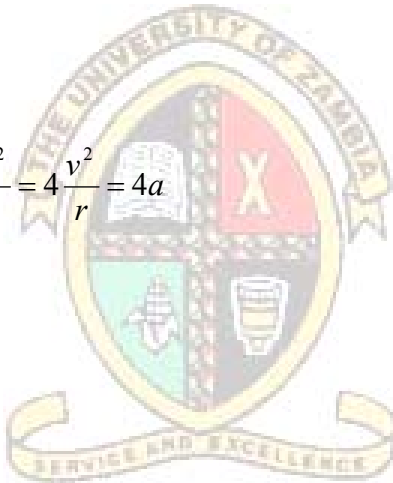
3.6 $\sin(\theta(\text{rad})) \approx \theta(\text{rad})$

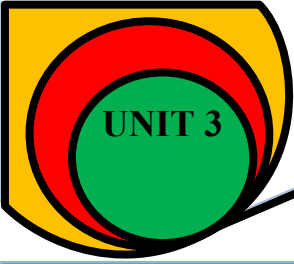
3.7 (d)

$$a = \frac{v^2}{r}$$

$$v_1 = 2v$$

$$\therefore a_1 = \frac{v_1^2}{r} = \frac{(2v)^2}{r} = 4 \frac{v^2}{r} = 4a$$





3.9 EXERCISES

3.1 A punter kicks a football during a critical football game; the ball leaves his foot from ground level with a speed of 28 m/s at an angle of 50° to the horizontal. At the very top of its flight the ball hits a wandering seagull. The ball and the seagull each stop dead and fall vertically from the point of collision below. Assumption: ignore air resistance. (a) With what speed is the ball moving when it strikes the seagull? (b) How high was the unfortunate seagull when it met the ball? (c) What is the speed of the seagull when it hits the ground? See Fig. 3.9.1

[(a) 18.00 m/s (b) 23.47 m (c) 21.45 m/s]

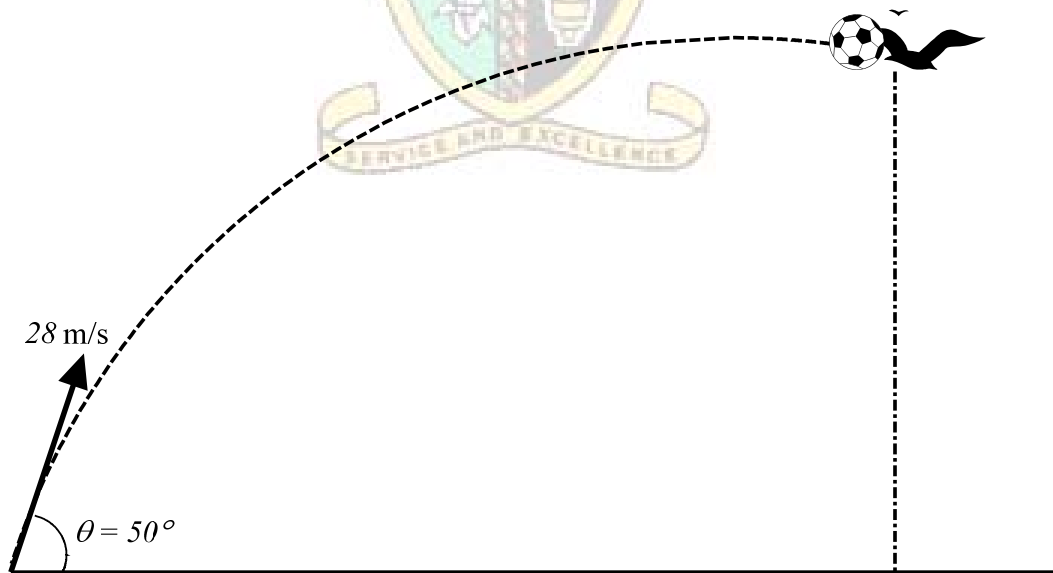
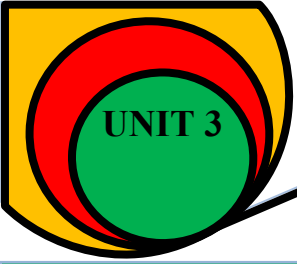


Figure 3.9.1 : Problem 3.1



- 3.2 A projectile is shot at an angle of 25° to the horizontal over level ground. Assuming air resistance plays no role, what angle does the projectile make with the horizontal when it lands? [25° below horizontal]
- 3.3 Find the initial angle if the range of a projectile is twice its maximum height.
[63.4°]
- 3.4 The stunt driver in Figure 3.9.2 wishes to shoot off the ramp and land on the platform. How fast must the motorcycle be going as it leaves the ramp if the stunt is to succeed? [13.9 m/s]

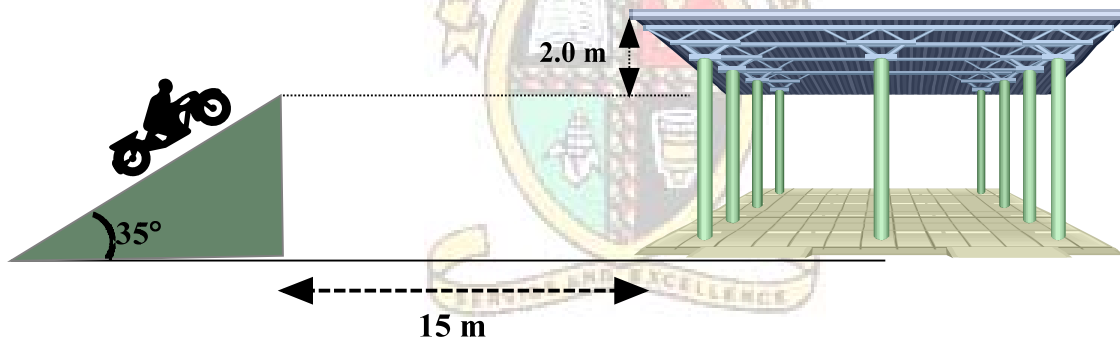


Figure 3.9.2: Problem 3.4

- 3.5 A basketball player who is 2.00 m tall is standing on the floor 10.0 m from the basket, as in Figure P3.9.3. If he shoots the ball at a 40.0° angle with the horizontal, at what initial speed must he throw so that it goes through the hoop without striking the backboard? The basket height is 3.05 m. [10.67 m/s]

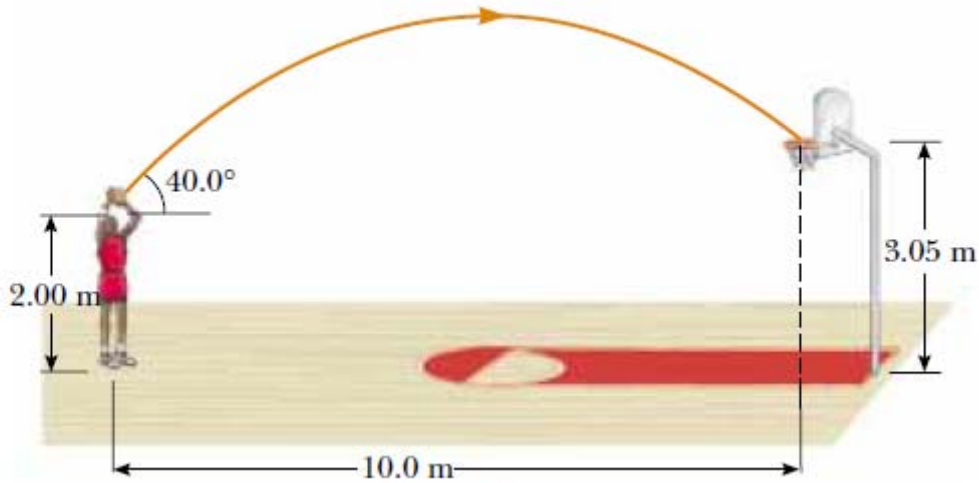
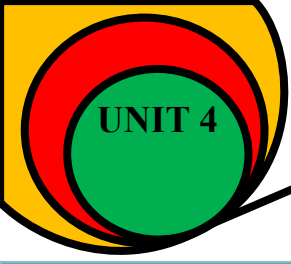


Figure 3.93: Problem 3.5

- 3.6 A merry-go-round takes 22 s to accelerate from rest to its operating speed of 3.75 rev/min. Find
- (a) Its acceleration in revolutions per second squared. [0.00284 rev/s²]
 - (b) The number of revolutions turned in this time. [0.69 rev]
- 3.6 The space shuttle is in a circular orbit 220 km above Earth's surface and completes an Earth revolution every 89 min.
- (a) What is the shuttle's speed? [7.8 x 10³ m/s]
 - (b) Its acceleration? [9.1 m/s² towards the earth's centre]
- 3.7 A car accelerates uniformly from rest to 17.5 m/s in 23.6 s. Find the angular acceleration of one of its wheels and the number of revolutions turned by a wheel in the process. The radius of the car wheel is 0.40 m. [1.85 rad/s², 81.5 rev]

The logo of the University of Zambia is centered in the background. It features a shield with a red and green design, topped by a crest. The shield is encircled by the text 'THE UNIVERSITY OF ZAMBIA'. Below the shield is a ribbon with the motto 'SERVICE AND EXCELLENCE'.

Particle Dynamics

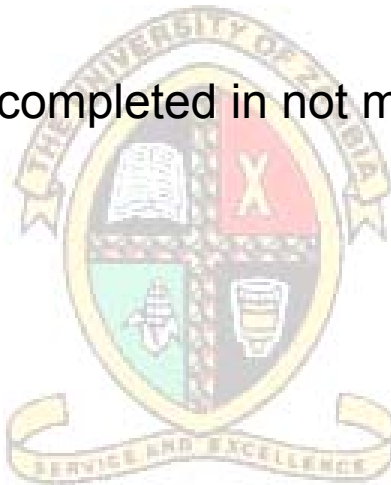


Particle Dynamics



UNIT 4: Particle Dynamics: (15 Hrs)

This unit should be completed in not more than 15 hours.





4.1 Introduction

In unit 3, we defined and discussed velocity and acceleration without considering what causes an object's motion. We now investigate how accelerations are caused by the forces. In so doing, we shall describe Newton's three laws of motion. These laws are of primary importance in physics and our physical world.

We shall also look at circular motion and at motion on an incline. It is a common experience that above a certain speed, one cannot negotiate a curve successfully, especially during the rain season when the surface is very slippery. Now we ask: what is the minimum speed that you must travel at in order to negotiate a curve? All this will be tackled.

Welcome to Unit 4!

4.2 Unit Objectives

After completing this unit, you should be able to

- Integrate Newton's 3 laws of motion into different physical conditions
- Solve problems using the laws of motion
- Derive and use the equations of uniform accelerated motion
- Solve problems involving centripetal force



4.3 Concept of Force:

Everyone has a basic understanding of the concept of force from everyday experience. When you push your empty dinner plate away, you exert a force on it. Similarly, you exert a force on a ball when you throw or kick it. In these examples, the word **force** is associated with muscular activity and some change in the velocity of an object.

Forces do not always cause motion, however. For example, as you sit reading this unit, a gravitational force acts on your body and yet you remain stationary. As a second example, you can push (in other words, exert a force) on a wall and not be able to move it.

What force (if any) causes the Moon to orbit the Earth? Newton answered this and related questions by stating that forces are what cause any change in the velocity of an object. The Moon's velocity is not constant because it moves in a nearly circular orbit around the Earth. We now know that this change in velocity is caused by the gravitational force exerted by the Earth on the Moon. Because only a force can cause a change in velocity, we can think of force as *that which causes an object to accelerate*.

In this unit, we are concerned with the relationship between the force exerted on an object and the acceleration of that object.

What happens when several forces act simultaneously on an object? In this case, the object accelerates only if the net force acting on it is not equal to zero. The net force acting on an object is defined as the vector sum of all forces acting on the object.



(We sometimes refer to the net force as the *total force*, the *resultant force*, or the *unbalanced force*.) If the net force exerted on an object is zero, the acceleration of the object is zero and its velocity remains constant. That is, if the net force acting on the object is zero, the object either remains at rest or continues to move with constant velocity.

Forces can be categorized into two. **Contact** forces and **Field** (non-contact) forces.

Now pause!!



Self-help task 4.1

Can you list the forces which are non-contact.

Figure 4.1 shows categories of forces which are contact and non-contact forces.

- When a coiled spring is pulled, as in Figure 4.1 a, the spring stretches.
- When a stationary cart is pulled sufficiently hard that friction is overcome, as in Figure 4.1b, the cart moves.
- When a football is kicked, as in Figure 4.1c, it is both deformed and set in motion.

These situations are all examples of a class of forces called *contact forces*. That is, they involve physical contact between two objects.

Other examples of contact forces are the force exerted by gas molecules on the walls of a container and the force exerted by your feet on the floor.

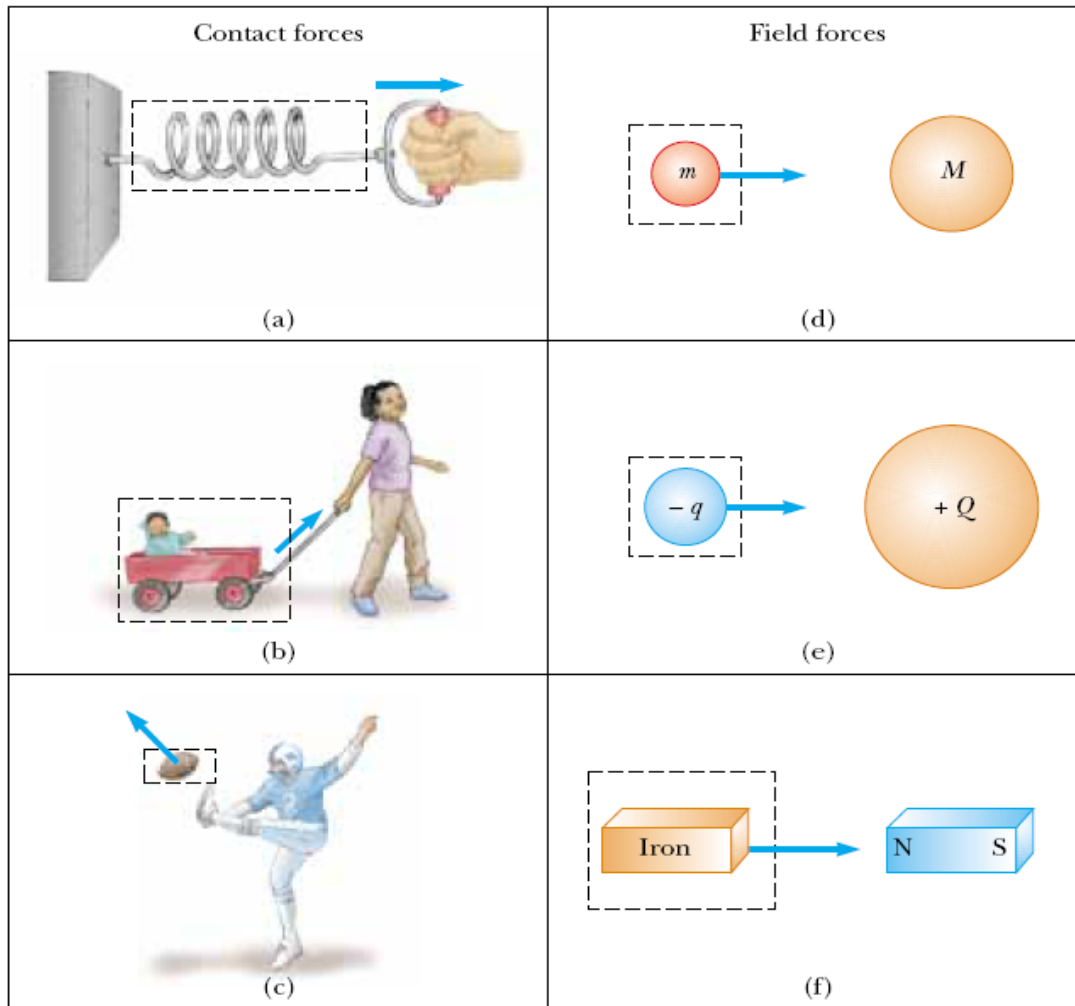


Figure 4.1: Categories of forces



Another class of forces, known as *field forces*, do not involve physical contact between two objects but instead act through empty space.

- The gravitational force of attraction between two objects, illustrated in Figure 4.1d, is an example of this class of force. This gravitational force keeps objects bound to the Earth and the planets in orbit around the Sun.
- Another common example of a field force is the electric force that one electric charge exerts on another (Fig. 4.1e). These charges might be those of the electron and proton that form a hydrogen atom.
- A third example of a field force is the force a bar magnet exerts on a piece of iron (Fig. 4.1f).

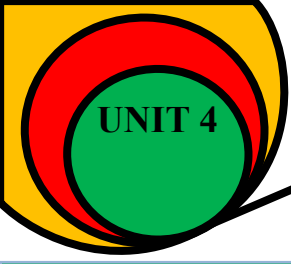
4.4 Newton's First Law:

Newton's first law of motion pertains to situations in which there is zero resultant force acting on a body. This means that, although there may be a number of individual forces acting, the vector sum of those forces is zero. We often say the **net force** is zero.

If an object is at rest, Newton's first law states,

“An object at rest remains at rest if there is zero resultant force acting on it”

We are familiar with many such situations. One such is a book lying on a table. It experiences the downward pull of gravity, balanced by the upward support supplied by the rigid table



What about if the object is moving, what happens to the law? The first law can be generalized as follows:

“A moving object continues to move with constant velocity if the vector sum of all external forces acting on the object is zero”

In other words an object continues to move in a straight line unless a net external force (unbalanced) is acted upon it. This also applies when $v = 0$.

4.4.1 Inertia and Mass

This is closely linked to the first law. The common definition is :

“Inertia is the tendency of an object at rest to remain at rest and of an object in motion to remain in motion with its original velocity.”

A common experience of this is that it is more difficult to stop a cement truck than a child's wagon because it has more inertia. When you are in a car that is in motion, the moment it comes to a stop suddenly, you jerk forward. This is because of inertia. It tends to keep you moving forward.



To quantify inertia we shall define the concept of mass. A certain metal object carefully preserved in France is defined to have a mass of exactly one kilogram (1 kg). By definition, any object that has the same inertia as this standard mass has a mass of 1 kg.

4.5 Newton's Second Law

Before we discuss the second law, let us describe what the word NET means. "This what remains after all deductions." It can be a negative or positive number. On your pay slip the net pay is the money that goes into your bank.

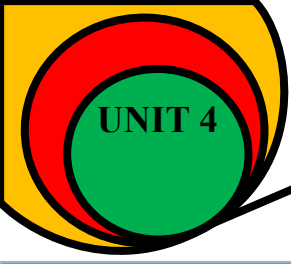
In physics we can also talk about "net force." This is the unbalanced force. Depending on the net force acting on an object, one can determine the direction of motion of the object.

It is also important to have a coordinate system. Forces pointing to the left are assigned negative and those pointing to the right are assigned positive. The same applies to motion and forces in the vertical direction.

In Figure 4.2 (a), the net force is $-12\text{ N} + (-4\text{ N}) + 18\text{ N} = +2\text{ N}$ pointing to the right. Hence the object accelerates to the right.

By knowing the direction motion of the object, one can determine the net force. The net force in Figure 4.2 (c) is found as follows;

Since the object is going from left to right $F_2 \cos \theta$ is greater than F_1 so that the net force is $F_{net} = F_2 \cos \theta - F_1$.



Self-help task 4.2

1. Determine the net force and the direction of movement of block in Fig 4.2 (b)
2. Determine the net force in Fig 4.2 (d)

Second law

When the net force acting on an object is zero, the object maintains constant velocity. What happens to this object when a (nonzero) net force acts on it?

Newton's second law answers this question both qualitatively and quantitatively. We can start by saying: An object acted upon by a net force accelerates. Further, the object will accelerate or change its velocity, in the same direction as the net force.

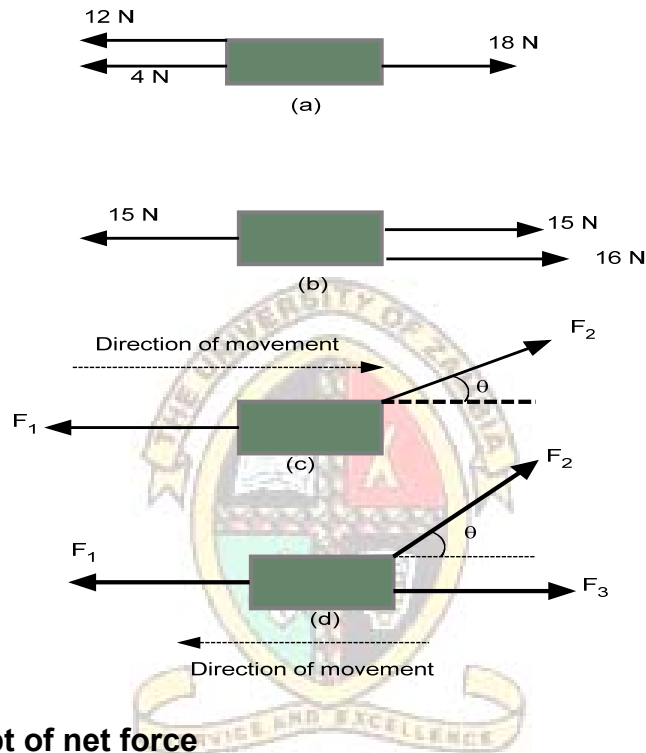


Figure 4.2: Concept of net force

Newton's second law states that

“The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to its mass. The direction of the acceleration is in the direction of the net force acting on the object.”



Thus, we can relate mass, acceleration, and force through the following mathematical statement of Newton's second law:

$$\sum \vec{F} = \vec{F}_{net} = m\vec{a} \quad (4.1)$$

In both the textual and mathematical statements of Newton's second law above, we have indicated that the acceleration is due to the *net force* $\sum F$ acting on an object.

The net force on an object is the vector sum of all forces acting on the object. In solving a problem using Newton's second law, it is imperative to determine the correct net force on an object. There may be many forces acting on an object, but there is only one acceleration.

Note that Equation (4.1) is a vector expression and hence is equivalent to three component equations:

$$\begin{aligned} (F_{net})_x &= \sum F_x = ma_x \\ (F_{net})_y &= \sum F_y = ma_y \\ (F_{net})_z &= \sum F_z = ma_z \end{aligned} \quad (4.2)$$



The SI unit of force is the **Newton**. A net force of one Newton is that force that causes an acceleration of 1 m/s^2 on a 1 kg mass. Therefore

$$1 \text{ N} = (1 \text{ kg}) (1 \text{ m/s}^2) = 1 \text{ kgms}^{-2}.$$



EXAMPLE 4.1: A 900 kg car is accelerated from rest to 12.0 m/s in 8.00 s along a straight road. How large the force?

$$\begin{aligned} F &= ma \\ &= m \left(\frac{v_f - v_o}{t} \right) \\ &= 900 \left(\frac{12.0 - 0}{8.00} \right) = 1350 \text{ N} \end{aligned}$$

4.5.1 Mass and Weight

We are well aware that all objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the gravitational force F_g . This force is directed toward the center of the Earth, and its magnitude is called the weight of the object W .



We saw in Unit 2 that a freely falling object experiences an acceleration g acting toward the center of the Earth. Applying Newton's second law $\sum F = ma$ to a freely falling object of mass m , with $a = g$ and $\sum F = F_g$, we obtain

$$F_g = mg \quad (4.3)$$

Thus, the weight of an object, defined as the magnitude of F_g , is equal to mg . Because it depends on g , weight varies with geographic location. Because g decreases with increasing distance from the center of the Earth, objects weigh less at higher altitudes than at lower ones.

For example, suppose a student has a mass of 70.0 kg. The student's weight in a location where $g = 9.80 \text{ m/s}^2$ is $F_g = mg = 686 \text{ N}$.

At the top of a mountain, however, where $g = 9.77 \text{ m/s}^2$, the student's weight is only 684 N.

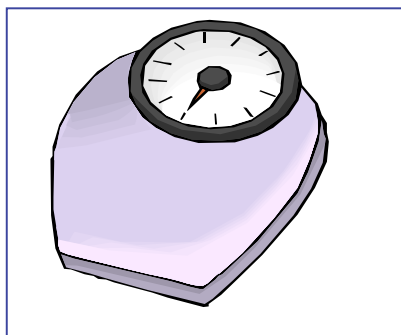


Figure 4.3: Bathroom scale



Therefore there is a difference between mass and weight. The scales that are used in clinics, hospitals and “*kaponyas*” as shown in Figure 4.3 measure mass not weight. When they tell you to measure your weight, tell them that you want to measure your mass!

4.5.2 Free Body Diagrams (FBD)

If you exert some force onto a wall, what do you feel? Of course some hardness on the wall, right? It is all because the wall is exerting a force on you. We shall discuss this later but, if you are to draw the forces, you’d definitely draw two forces acting in opposite directions.

That brings us to “*Free body diagrams*”

A free body diagram is a picture that isolates and identifies only those forces acting on the object of interest. Even when the magnitudes of some forces acting on the body are unknown, we can usually indicate them on the FBD by a symbol and identify their directions. Such a diagram simplifies writing the sums of forces.

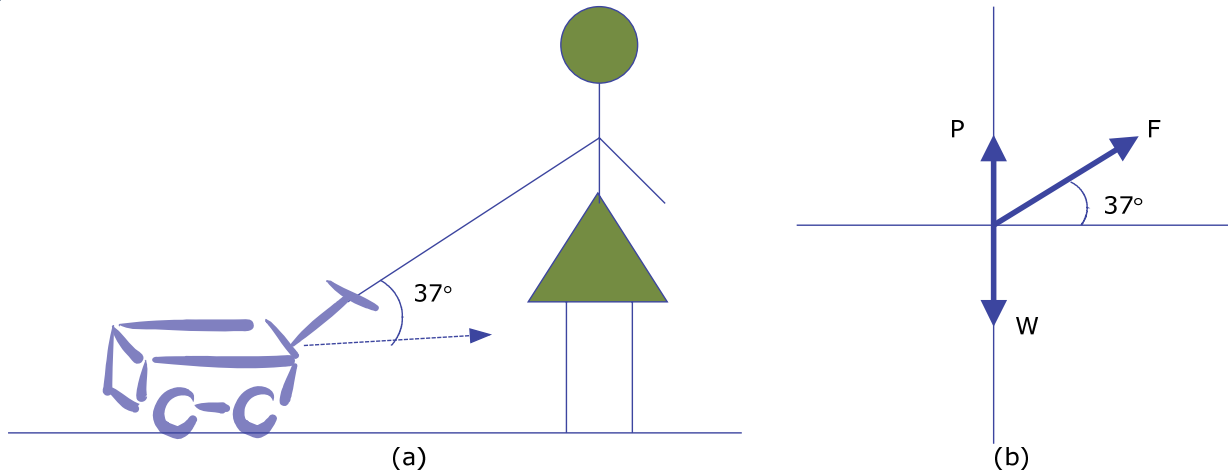


Figure 4.4: Forces exerted on a wagon in (a) are shown in an FBD in (b)

When you draw a free body diagram the following should be taken into consideration:

- Only forces acting on the body matter. But the sum $\sum F$ includes all the forces that act on the body in question. Once you have chosen the body in question identify all the forces on the body and NOT the forces exerted by that body on another body.
- When you have a complete the FBD, you must be able to answer for each force the questions
 - ✓ What other body is applying this force?
 - ✓ If you cannot answer the question, you may be dealing with a non-existent force. Non-existent forces are forces such as the force of acceleration or the $m\vec{a}$ force
- When a problem involves more than one body you have to take the problem apart and draw a separate FBD for each body.



Figure 4.5 (a) shows accelerating bodies on a pulley. The bodies in question have been separated. Each body has its own FBD as shown in Figure 4.5 (b).

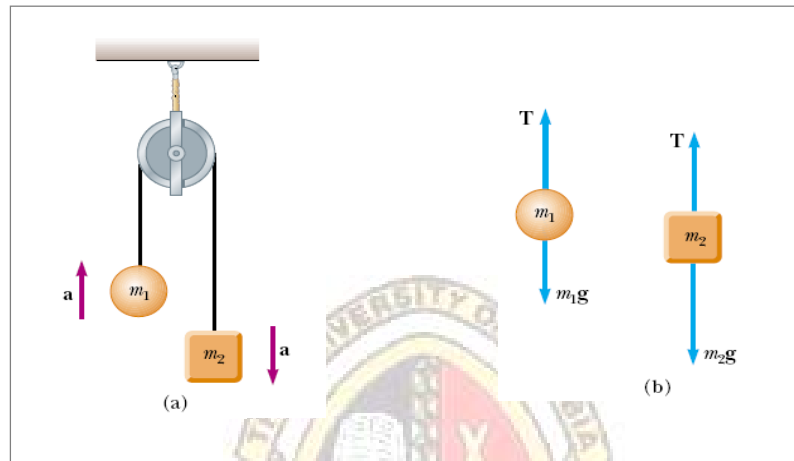


Figure 4.5: A pulley shown the FBD

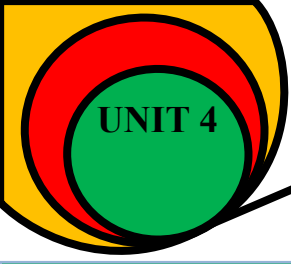
4.6 Newton's Third Law: Action and Reaction

As mentioned earlier, if you push on a wall, the wall will push back on you. This is an example of Newton's third law of motion which states:

“ If object A exerts a force on \mathbf{F} on object B, then B exerts a force $-\mathbf{F}$ on object A, equal in magnitude but opposite in direction.”

One of these forces is the **action force** and the other is the **reaction force**. For the push on the wall example, the action force is the push ONTO the wall and the reaction is the push BY the wall. Mathematically,

$$\mathbf{F} = -\mathbf{F} \quad (4.4)$$



Self-help task 4.3

Can you list situations involving action-reaction forces.

4.7 Applications of Newton's Second Law

We now delve into the application part of Newton's laws. In Unit 3 we learnt about circular motion and now we shall learn what causes it. We shall also study motion on an incline. Before we do that, we study **friction**.

4.7.1 Frictional Forces

Until now we have ignored friction, but it must be taken into account in most practical situations. Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Figure 4.6.

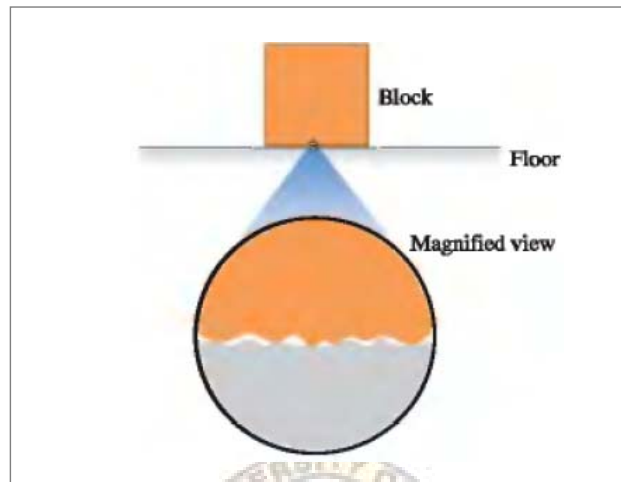


Figure 4.7: A block moving to the right on a table or floor. The two surfaces in contact are rough, at least on a microscopic scale.

Friction is a familiar concept. It is a contact force that impedes sliding, and we experience it in all aspects of our lives. Sometimes friction is useful to us: It is friction that holds nails and screws in place; if there were no friction between our feet and the ground, we would not be able to walk, and if there were no friction between the wheels of a car and the road, the engine would cause the wheels to spin but there would be no forward or backward motion. The force of friction is always opposite to the direction of motion. See Figure 4.8.

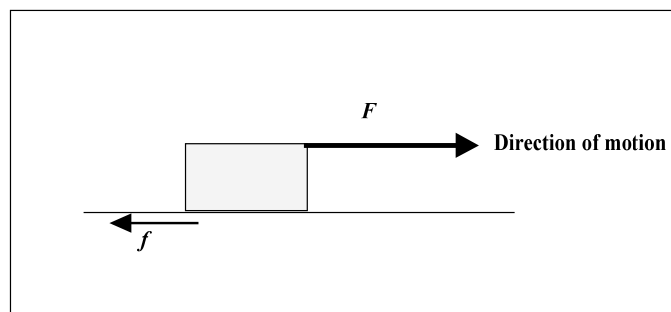


Figure 4.8: Frictional force is always opposite the direction of motion.



Friction is highly dependent on the force pressing the two surfaces together such that the jagged points on one surface penetrate the valleys of the other. This force is called the **normal force** F_N . The normal force is perpendicular to the surface.

When the jagged points on each surface lock each other, it is difficult to slide the object with ease. The frictional force that resists this is called **static friction** , f_c .

As the object at rest is slowly pushed or pulled, a critical force f_c is reached at which the object will start moving. Then to keep it moving at constant velocity, a smaller force of magnitude f_k is sufficient, where “k” stands for kinetic or moving. This is the friction force encountered when the object is in motion. It is called **Kinetic friction** f_k .

Static friction prevents the start of sliding motion while kinetic friction is the resistance felt when the object is in motion.

The magnitude of the friction force is dependent on the normal force as mentioned earlier on.

Thus we have

$$f_c = \mu_s F_N$$

and

$$f_k = \mu_k F_N$$

(4.5)



where μ_s and μ_k are the coefficients of static and kinetic friction respectively. The value of this coefficient is between zero and one, that is ,

$$0 \leq \mu \leq 1$$

When it is one, then the surface is very rough and when $\mu = 0$ then the surface is very smooth, it is frictionless.



Self-help task 4.4

Why is static friction greater than kinetic friction?

Check the answer at the end of this unit to compare with what you wrote...

The determination of the normal force is very crucial in finding the force of friction. For example Figure 4.9 (a), the weight \mathbf{W} is acting on the surface and by Newton's third law the normal force is perpendicular to the surface but in the opposite direction.

Therefore $F_N = W$

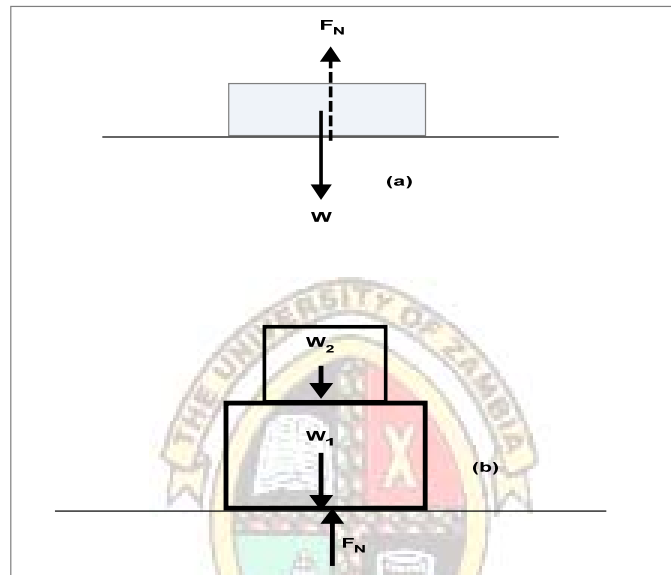


Figure 4.9 : Determination of the normal force

For Figure 4.9 (b), the weight pushing down on the surface is the sum of two blocks. So the supporting force by Newton's third law is $F_N = W_1 + W_2$. Knowing the normal force, one can then find the friction force.

Since the friction force is highly dependent on the normal force, figuring out the normal force is of utmost importance.



EXAMPLE 4.2: A man pushes on a box at an angle of θ with a force F as shown in Figure 4.10 (a). The FBD is shown in (b). Determine the normal force F_N .

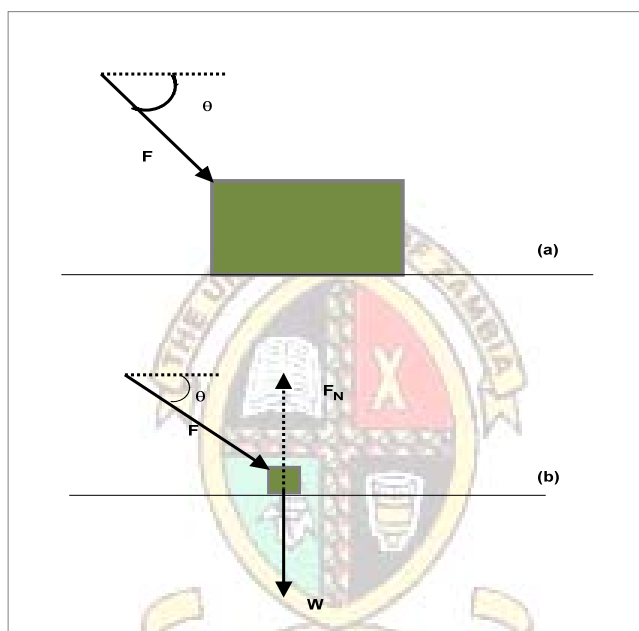


Figure 4.10: Example 4.2

SOLUTION

When we resolve the force F into x and y components we have

$$F_x = F \cos \theta$$

and

$$F_y = F \sin \theta$$



We are currently interested in the y component. It points in the negative y direction.

Since there is no motion in the y direction, acceleration is zero, therefore from Newton's second law we have

$$\sum F_y = ma = 0$$

$$F_N - W - F \sin \theta = 0$$

$$\therefore F_N = W + F \sin \theta$$

We now have the necessary background for applying Newton's second law. Before we start solving let us point out the general procedure that we should follow.

1. Sketch a picture of the problem
2. Isolate the object for which you wish to write $F = ma$.
3. Draw a free body diagram for the isolated object, showing all the forces acting on it. Do not include forces that do not act on it directly.
4. Choose a convenient coordinate system for the FBD and find the components of the forces.
Important: Always choose the positive direction of motion to be the direction in which the body moves.
5. Write $F = ma$ in component form for forces in the FBD. When you do so, F should be in newtons, mass in kilograms and a in m/s^2 .
6. Solve the components equations for the unknowns.
7. Check the reasonableness of the result.

And now examples...



EXAMPLE 4.3: Two blocks of masses m_1 and m_2 , are placed in contact with each other on a frictionless, horizontal surface, as in Figure 4.13 (a). A constant horizontal force, $F = 9 \text{ N}$ is applied to m_1 as shown. $m_1 = 1.0 \text{ kg}$, $m_2 = 2.0 \text{ kg}$.

- (a) Find the magnitude of the acceleration of the system.
- (b) Determine the contact force between the two blocks

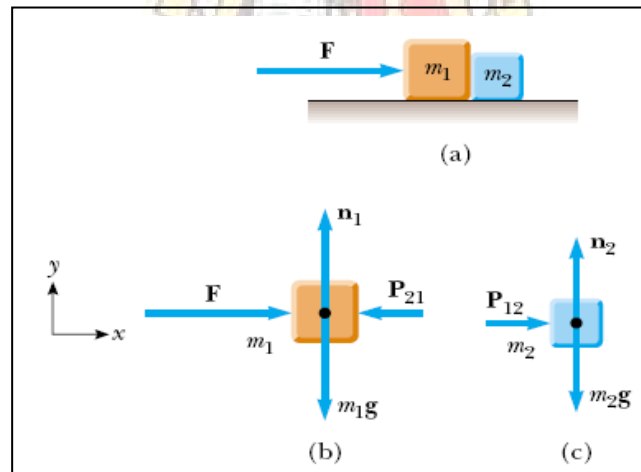


Figure 4.13: Example 4.3

SOLUTION

-
- (a) Since the force moves the blocks with a common acceleration in the x - direction we have the component acceleration given as



$$\sum F_x(\text{system}) = F = (m_1 + m_2)$$

$$a_x = \frac{F}{(m_1 + m_2)} = \frac{9.0}{(1.0 + 2.0)} = 3.0 \text{ m/s}^2$$

- (b) To find the force of compression, we draw a free body diagram for each block as shown. Figure 4.13 (b) shows the FBD for block 1. P_{21} is the compression force on block 1 due to block 2.

Writing Newton's second law for block 1 we have

$$\begin{aligned} \sum F_x(\text{block 1}) &= F - P_{21} = m_1 a_x \\ \therefore P_{21} &= F - m_1 a_x \\ &= 9.0 - (1.0)(3.0) \\ &= 6.0 \text{ N} \end{aligned}$$

By Newton's third law, the force exerted by block 1 on block 2 P_{12} is also equal to 6.0 N, which is the compression force.

To prove this, let us solve for block 2

$$\begin{aligned} \sum F_x(\text{block 2}) &= P_{12} = m_2 a_x \\ &= (2.0)(3.0) \\ &= 6.0 \text{ N} \end{aligned}$$



EXAMPLE 4.4: The two masses in Figure 4.14 are tied to opposite ends of a massless rope hung over a massless and frictionless pulley. Find the acceleration of the masses. (This device is called an Atwood machine).

$$m_1 = 5.0 \text{ kg}, m_2 = 10.0 \text{ kg}.$$

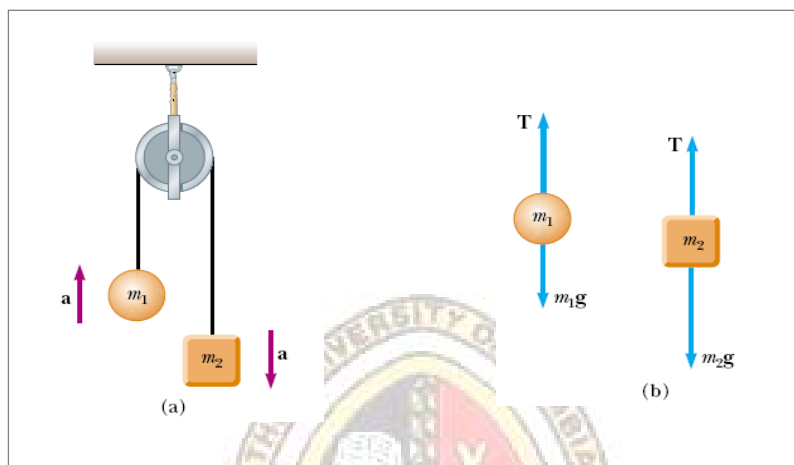


Figure 4.14: Atwood's machine

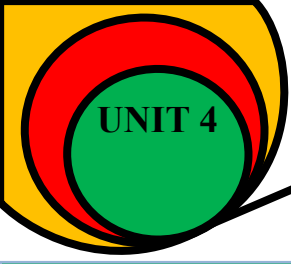
SOLUTION

We shall treat the bodies separately. Remember when applying the second law, we choose the positive direction of motion to be the direction in which the body moves.

Therefore for the 5 kg block we have

$$\sum F_y(\text{mass 1}) = T - m_1g = m_1a \quad (1)$$

For mass 2 we have



Particle Dynamics



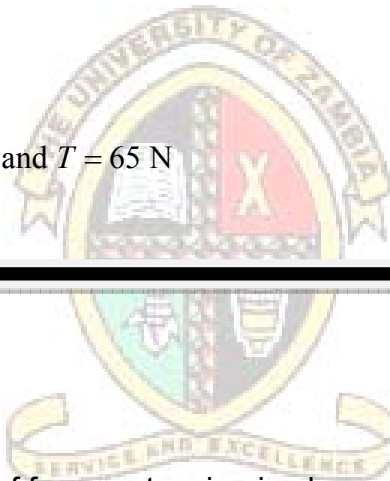
$$\sum F_y(\text{mass } 2) = m_2g - T = m_2a \quad (2)$$

Solving these simultaneous equations (1) and (2) we obtain

$$T - 49 = 5a$$

$$98 - T = 10a$$

Therefore $a = 3.3 \text{ m/s}^2$ and $T = 65 \text{ N}$



IMPORTANT: The direction of force or tension is always away from the body.



Self-help task 4.5

You can hold a box against a rough wall (Figure 4.15) and prevent it from slipping down by pressing hard horizontally.

How does the application of a horizontal force keep an object from moving vertically?

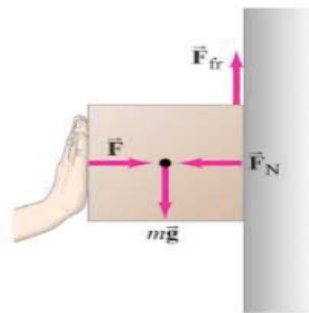


Figure 4.15: A box on a wall

You got it right?

PROCEED





EXAMPLE 4.5: A box weighing 500 N is being pushed with a force F directed at 30° below horizontal. See Figure 4.16.

(a) What must F be in order for the box to slide?

((b) If the same force is maintained once the box starts sliding, what will be the acceleration? $\mu_s = 0.7$, $\mu_k = 0.4$.

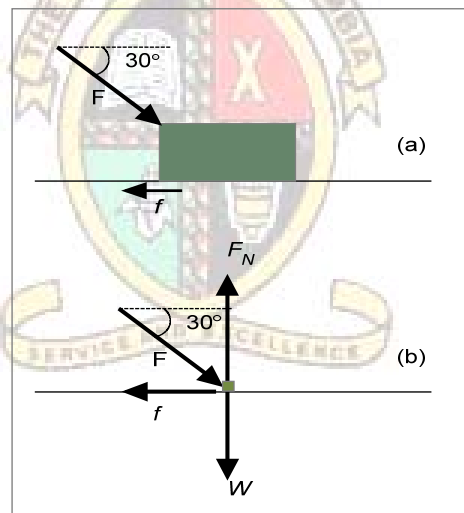


Figure 4.16: A box being pushed.

SOLUTION

(a) The FBD is shown in (b). From Newton's second law we have



$$\sum F_y = F_N - W - F \sin 30^\circ = 0 \quad \text{because there is no motion in the y-direction}$$

$$\therefore F_N = W + F \sin 30^\circ$$

$$\sum F_x = F \cos 30^\circ - f = ma$$

Just before it begins to slide, $a = 0$, so that we use $\mu_s = 0.7$.

$$F \cos 30^\circ - f = ma$$

$$f \cos 30^\circ - \mu_s F_N = 0$$

Or

$$F \cos 30^\circ - \mu_s (W + F \sin 30^\circ)$$

$$\Rightarrow F = \frac{\mu_s W}{(\cos 30^\circ - \mu_s \sin 30^\circ)} = \frac{0.7 \times 500}{(\cos 30^\circ - 0.7 \times \sin 30^\circ)} = 678 \text{ N}$$

(b) When it begins to slide and F is maintained, we have $F = 678 \text{ N}$, $\mu_k = 0.4$ so that

$$\sum F_x = F \cos 30^\circ - f = ma$$

Or

$$a = \frac{F \cos 30^\circ - f}{m} = \frac{F \cos 30^\circ - \mu_k F_N}{W/g}$$

$$= 4.9 \text{ m/s}^2$$

One more example...



EXAMPLE 4.6: A block of mass m_1 ($= 3$ kg) on a rough, horizontal surface is connected to a ball of mass m_2 ($= 1$ kg) by a lightweight cord over a lightweight, frictionless pulley, as shown in Figure 4.17 (a). A force of magnitude $F = 30$ N at an angle 37° with the horizontal is applied to the block as shown in Figure 4.17. The coefficient of kinetic friction between the block and surface is 0.4. Determine the magnitude of the acceleration of the two objects.

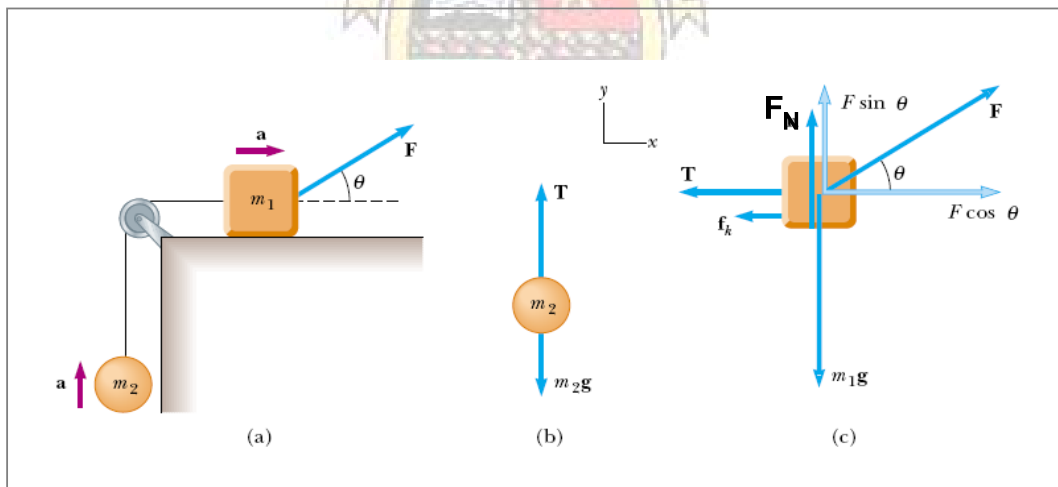
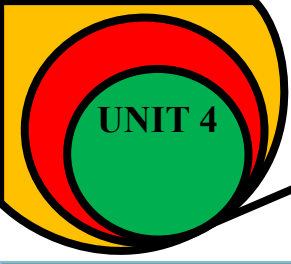


Figure 4.17: Example 4.6

SOLUTION

As usual we draw the FDBs as shown in (b) and (c). For mass 2, we have



Particle Dynamics



$$\sum F_y = T - m_2g = m_2a$$

Or

$$T - 9.8 = a \tag{1}$$

$$\sum F_x(\text{mass 1}) = F \cos \theta - T - f_k = m_1a \tag{2}$$

But $f_k = \mu_k F_N$

To find F_N on mass 1 we have

$$\sum F_y = F_N + F \sin \theta - m_1g = 0$$

$$\therefore F_N = m_1g - F \sin \theta$$

so that

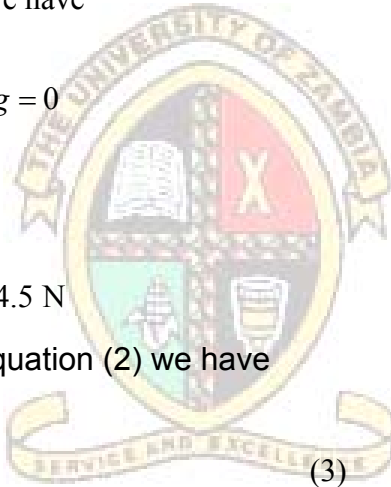
$$f_k = \mu_k(m_1g - F \sin \theta) = 4.5 \text{ N}$$

Substituting this into equation (2) we have

$$19.4 - T = 3a \tag{3}$$

Solving equations (3) and (1) simultaneously we obtain

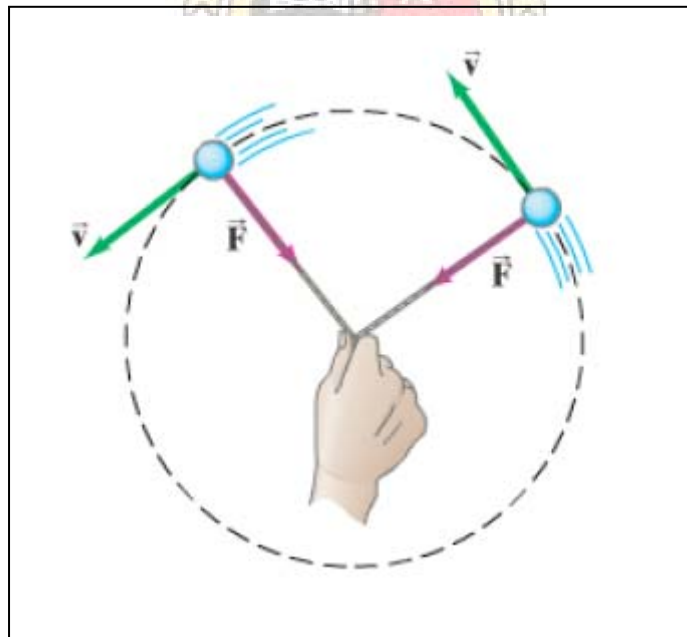
$$a = 2.4 \text{ m/s}^2 \text{ and } T = 12.2 \text{ N.}$$





4.7.2 Circular Motion Dynamics

Newton's first law states that a net force must be applied to deflect an object moving in a straight line. Similarly an object being twirled in a circular motion is kept in this motion because of this net force otherwise the object would fly off at a tangent in a straight line. This force is called the **centripetal force**. The fact that we have centripetal acceleration means we have the centripetal force which causes this acceleration or hold the object in a circular path. This force is directed towards the centre of the circle just as acceleration is directed towards the centre. See Figure 4.18.



.Figure 4.18: Centripetal force is always directed towards the centre of the circle.



From Newton's second law, we have

$$\sum F_c = F_{net} = ma_c$$

Or

$$F_c = m \frac{v^2}{r} \quad (4.6)$$

Since $a = \frac{v^2}{r}$



EXAMPLE 4.7: A 1420-kg car moving at 21.2 m/s is rounding a curve of radius 37.5 m. How large a horizontal force is needed to hold the car in its path?

SOLUTION

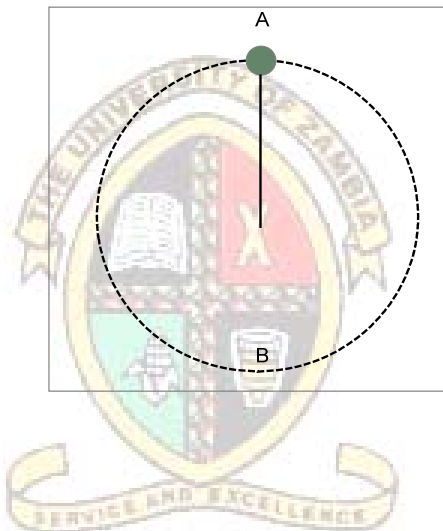
The horizontal force needed to hold the car in its path is the centripetal force.

Therefore

$$F_c = \frac{mv^2}{r} = \frac{(1420)(21.2)^2}{37.5} = 17040 \text{ N}$$

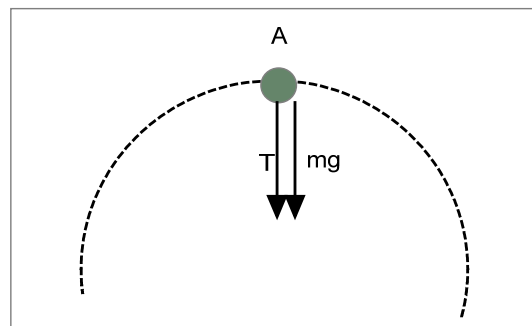


EXAMPLE 4.8: A ball is tied to the end of string in a vertical circle of radius r as shown. What is the tension in the string when the ball is at point A and point B if the ball's speed is v at that point? Do not neglect the force of gravity.

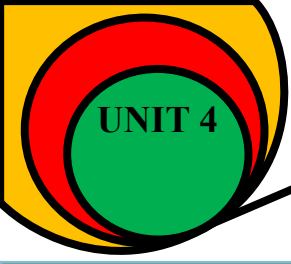


SOLUTION

Tension at point A: The FBD is shown below:



These are both directed towards the centre of the circle so that the net force is just the centripetal force.



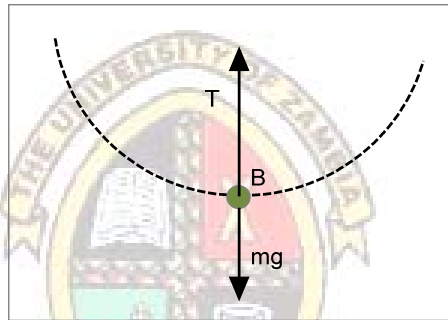
Particle Dynamics



$$\sum F_y = F_{net} = T + mg = \frac{mv^2}{r}$$

$$\therefore T = \frac{mv^2}{r} - mg$$

Tension at B: The FBD is as well shown below:



The ball is being pulled towards the centre of the circle so that the net force is

$$\sum F_y = F_{net} = T - mg = ma_c = \frac{mv^2}{r}$$

$$\therefore T = \frac{mv^2}{r} + mg$$

Now we discuss the conical pendulum through an example. Do you have an idea what a conical pendulum is? Think about a cone...



EXAMPLE 4.9: A small object of mass m is suspended from a string of length L . The object revolves with constant speed v in a horizontal circle of radius r , as shown in Figure 4.19. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find an expression for v .

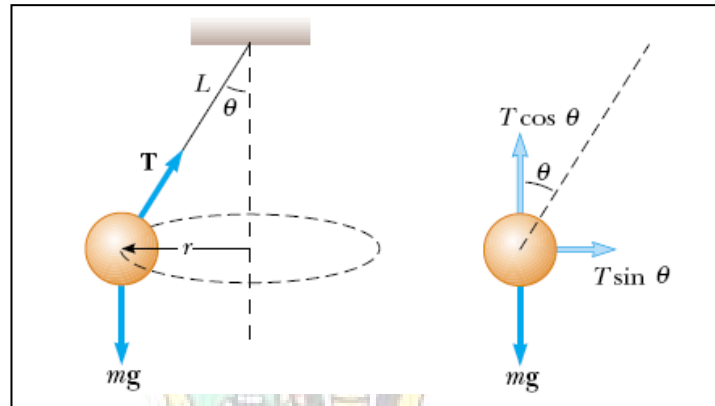


Figure 4.19: Conical pendulum

The free body diagram is shown on the right. By applying Newton's second law we have

$$\sum F_y = T \cos \theta - mg = ma = 0$$

Or

$$T = \frac{mg}{\cos \theta} \quad (1)$$

In the x-direction we have



$$\sum F_x = ma_c$$

$$T \sin \theta = \frac{mv^2}{r}$$

(2)

Substituting equation 1 into 2, we obtain

$$v = \sqrt{rg \tan \theta}$$

but $r = L \sin \theta$

$$\therefore v = \sqrt{Lg \sin \theta \tan \theta}$$

Highway Curves, Banked and Unbanked

Another example of circular dynamics occurs when a vehicle rounds a curve say to the left. In such a situation, you may feel that you are pushed outward toward the right door. There is no mysterious force pulling on you. What is happening is that you tend to move in a straight line when the car has begun to follow a circular path. To make you go in a curved path, the seat (friction) or the door of the car (direct in contact) exerts a force on you. See Figure 4.20. The car also must have a force exerted on it toward the centre of the curve if it is to move in that curve. This is provided by the frictional force between the tires and the road.

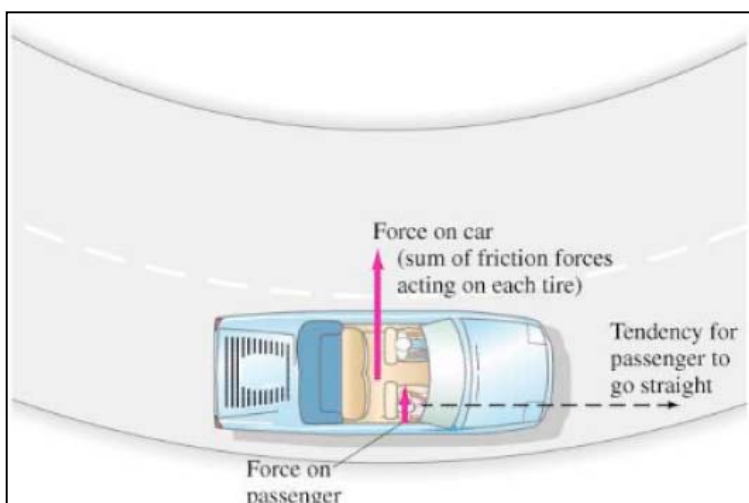


Figure 4.20: Car rounding a curve

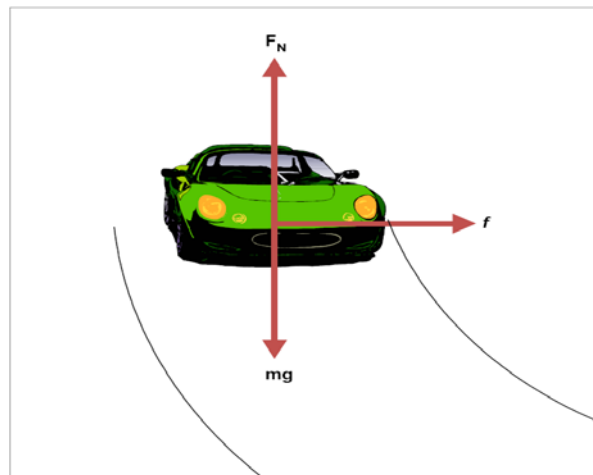
If the wheels and tires of the car are rolling normally without slipping or sliding, the bottom of the tire is at rest against the road at each instant; so the friction force the road exerts on the tires is static friction. But if the static friction force is not great enough, as under icy conditions, sufficient friction force cannot be applied and the car will skid out of a circular path into a more nearly straight path. Once the car skids or slides, the friction force becomes kinetic friction, which is less than static friction.



EXAMPLE 4.10: A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 50 km/h (14 m/s). Will the car follow the curve, or will it skid? Assume: (a) the road is dry and the coefficient of static friction is $\mu_s = 0.60$; (b) the road is icy and $\mu_s = 0.25$.

SOLUTION

The diagram and FBD is shown in the figure below:



(a) Since there is no motion in the y-direction, then,

$$\sum F_y = F_N - mg = 0$$

$\therefore F_N = mg$ Therefore friction force becomes

$$f = \mu_s F_N = \mu_s mg = 5900 \text{ N}$$

The force needed to provide the centripetal force is



$$\sum F_r = ma_c = \frac{mv^2}{r} = \frac{1000(14)^2}{50} = 3900 \text{ N}$$

Therefore since the only horizontal force is the friction force which is 5900 N, and is more than the 3900N which is the force required to make the curve, then the car will not skid.

(b) When the road is icy, the friction is

$$f = \mu_s F_N = 0.25(1000)(9.8) = 2500 \text{ N}$$

Since 3900 N is the required amount of force needed to make the curve, the available 2500 is not enough, and therefore the car will skid.

Banked Highways

The other way of limiting skidding is through banking. On stretches that are hilly, where the road curves a lot, say, from Kafue to Chirundu, the roads are designed in such a way as to tilt inside. This is called **banking**. See Figure 4.21.

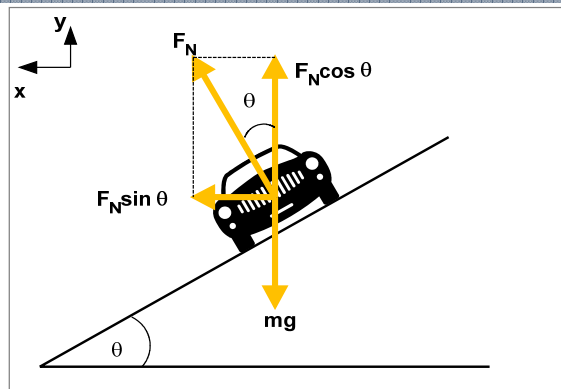


Figure 4.21: A car on a bank angled at θ .

The banking of curves reduces the chance of skidding. The normal force exerted by a banked road, acting perpendicular to the road, will have a component toward the centre of the circle as shown in the figure, thus **reducing the reliance on friction**.

For any given banking angle θ , there will be a speed below which no friction at all is needed. This will be the case when the horizontal component of the normal force toward the curve, $F_N \sin \theta$ is just equal to the force required to give the car its centripetal acceleration:

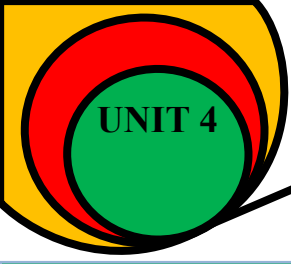
$$\sum F_x = F_N \sin \theta = \frac{mv^2}{r} \quad (1)$$

but since there is no motion y-direction

$$\sum F_y = F_N \cos \theta - mg = 0$$

so that

$$F_N = \frac{mg}{\cos \theta} \quad (2)$$



substituting (2) into (1) we obtain

$$g \tan \theta = \frac{v^2}{r}$$

Or

$$v = \sqrt{gr \tan \theta} \tag{4.7}$$

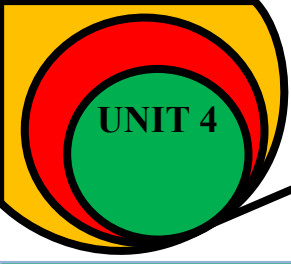
This is the maximum velocity required to make the curve without the need for friction.



Self-help task 4.6

Now extend the previous case to the case where there is friction. Draw the free body diagram and write an expression for the maximum speed a car can have without skidding if the coefficient of static friction is μ_s .

Finally here is the problem solving strategy for circular motion...



PROBLEM SOLVING:	Uniform Circular Motion
<p>1. Draw a free body diagram, showing all the forces acting on the body under consideration</p>	<p>4. Apply Newton's second law to the radial component:</p> $\left(\sum F\right)_R = ma_R = m\frac{v^2}{r}$
<p>2. Determine which of the forces, or which of their components, act to provide the centripetal acceleration- that is, all the forces or components that act radially, toward or away from the center of the circular path. The sum of these forces provides the centripetal acceleration.</p>	
<p>3. Choose a convenient coordinate system, preferably with one axis along the acceleration direction</p>	

4.7.3 Motion on an Incline

Now we consider what happens when an object slides down an incline, such as a hill or a ramp. Such problems are interesting because gravity is the accelerating force, yet the acceleration is not vertical. Solving problems is usually easier if we choose the xy coordinate system so that the x axis points along the incline and the y axis is perpendicular to the incline as shown in Figure 4.20. Note also that the normal force is not vertical, but it is perpendicular to the sloping surface on the plane.

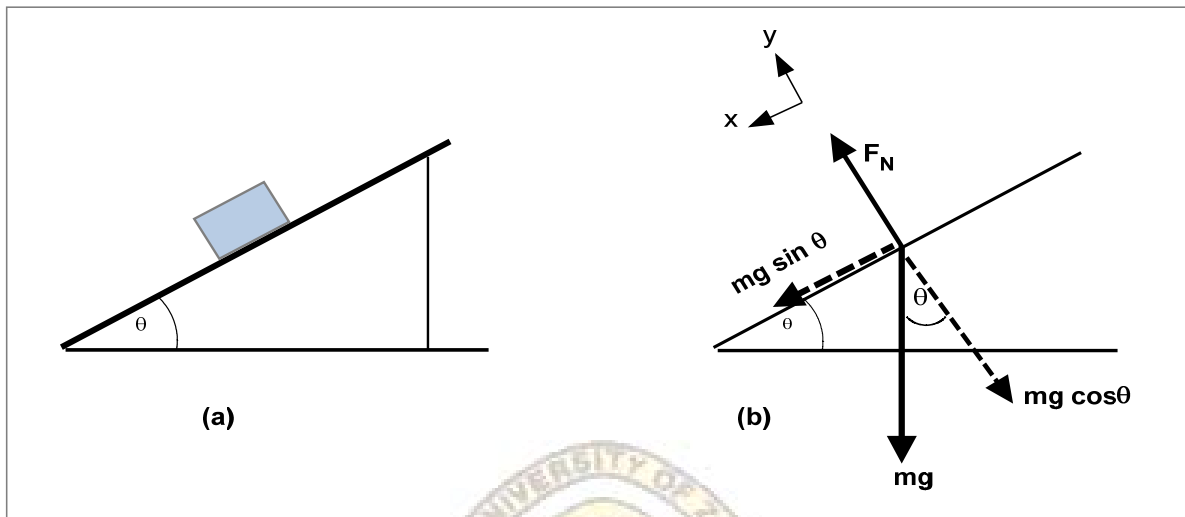


Figure 4.20: Motion on an incline

Figure 4.20 (b) shows the free body diagram. The weight is resolved into $mg \cos \theta$ in the y-direction and $mg \sin \theta$ in the x-direction.

- Since no motion is allowed perpendicular to the ramp (y-direction), we have

$$\sum F_y = F_N - mg \cos \theta = 0$$

Or

$$F_N = mg \cos \theta$$

- The motion is entirely along the x-direction and is governed by Newton's second law:



$$\sum F_x = mg \sin \theta = ma$$

or

$$a = \frac{mg \sin \theta}{m} = g \sin \theta \quad (4.7)$$

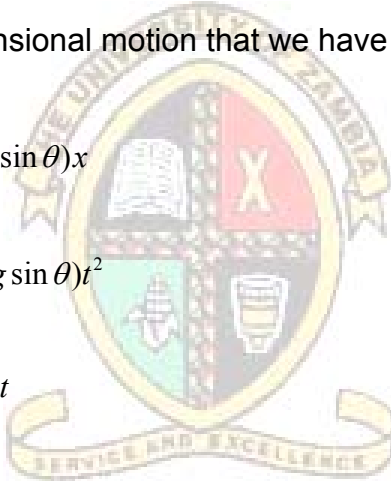
This is the acceleration down the incline.

Therefore if the surface is frictionless, g is replaced by $g \sin \theta$ and the same equations of one dimensional motion that we have used before apply, that is:

$$v^2 = v_o^2 + 2ax = v_o^2 + 2(g \sin \theta)x$$

$$x = v_o t + \frac{1}{2}gt^2 = v_o t + \frac{1}{2}(g \sin \theta)t^2$$

$$v = v_o + at = v_o + (g \sin \theta)t$$



where the positive direction is conveniently chosen down the incline in the direction of motion.

For motion on an incline with friction, everything is the same except that the frictional force is added and opposing the motion. See Figure 4.21.

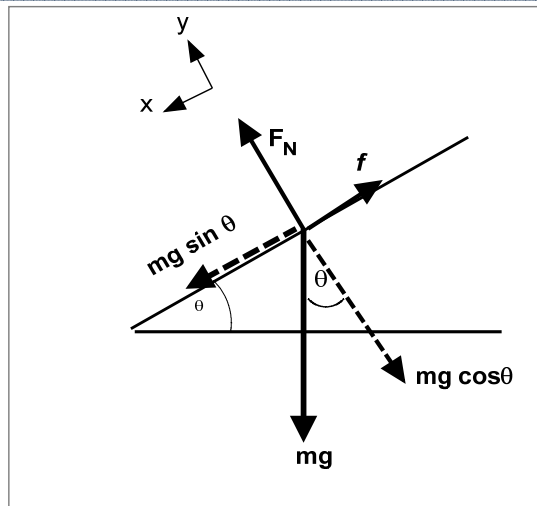


Figure 4.21: Motion on an incline with friction.

$$f = \mu F_N \quad \text{where} \quad F_N = mg \cos \theta$$

so that

$$f = \mu mg \cos \theta$$

From Newton's second law, we have

$$mg \sin \theta - f = ma$$

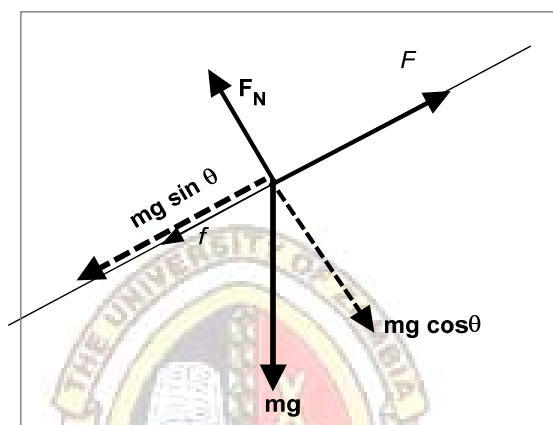


EXAMPLE 4.10: A man wants to push a package of shingles of total mass 15 kg up a roof being built at an angle of 27° . The coefficient of kinetic friction between the package and the roofing paper already in place is $\mu_k = 0.55$, (a) How much force does the man have to exert on the package directly along the slope of the roof to cause the package to accelerate at 0.15 m/s^2 ? (b) If the coefficient of static friction is 0.58, will the package remain on the roof?



SOLUTION

The free body diagram is shown below. Friction is opposing the motion.



(a) From Newton's second law we have

$$\sum F_x = F - mg \sin \theta - f = ma$$

$$\therefore F = ma + mg \sin \theta + f$$

$$= ma + mg \sin \theta + \mu_k mg \cos \theta$$

$$= 15(0.15) + 15(9.8)(\sin 27^\circ) + (0.55)(15)(9.8)(\cos 27^\circ)$$

$$= 141.0 \text{ N}$$



(b) We assume the man is no longer pushing. Therefore the forces acting along the incline are $mg \sin \theta$ directed down the incline and the friction force f opposing this sliding. We now find their values.

$$mg \sin \theta = 66.7 \text{ N}$$

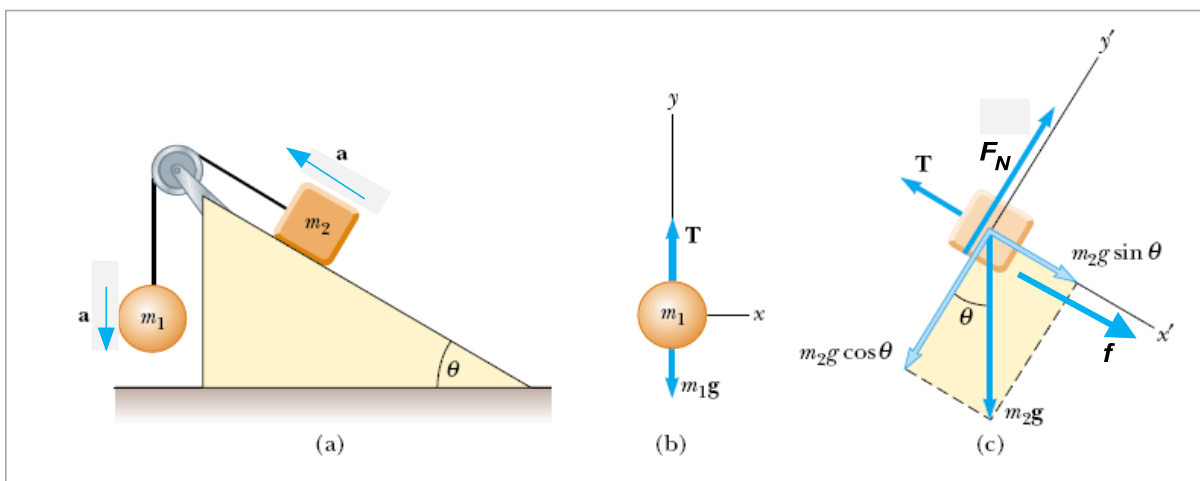
$$f = \mu_s mg \cos \theta = 72.0 \text{ N}$$

Since the friction is greater than the sliding force, the package will not move.

A more complex problem...



EXAMPLE 4.11: A system of blocks is placed on a 37° incline as shown below. Assume that the coefficient of friction between the incline and the m_2 ($=5 \text{ kg}$) block are $\mu_s = 0.70$ and $\mu_k = 0.50$. If $m_1 = 7 \text{ kg}$, (a) show that the system will start to slide once it is let go, (b) what acceleration will the blocks have?



SOLUTION

The free body diagrams are shown in (a) and (b). For mass 1, we have

$$\sum F_y = m_1 g - T = 0$$

$$T = 69 \text{ N}$$

(1)

For mass 2, we have

$$\sum F_{x'} = T - m_2 g \sin \theta - f = 0$$

and substituting into this equation yields

$$f = 69 - 29 = 40 \text{ N}$$

This is the friction that would prevent sliding.



Now let us calculate the actual value of the force of friction

$$f_c = \mu_s F_N = 0.7(5)(9.8)(\cos 37^\circ) = 27 \text{ N}$$

Therefore we can conclude that sticking is not possible in this situation and the system will slide.

(b) Using Newton's second law, we have

$$\sum F_y (\text{mass1}) = m_1 g - T = m_1 a$$

$$69 - T = 7a \quad (2)$$

$$\sum F_x = T - m_2 g \sin \theta - f = m_2 a$$

or

$$T - m_2 g \sin \theta - \mu_k m_2 g \cos \theta = m_2 a$$

or

$$T - 49 = 5a \quad (3)$$

Solving for a gives, $a = 1.7 \text{ m/s}^2$.

4.8 Unit Summary

The following has been learnt so far in this unit:

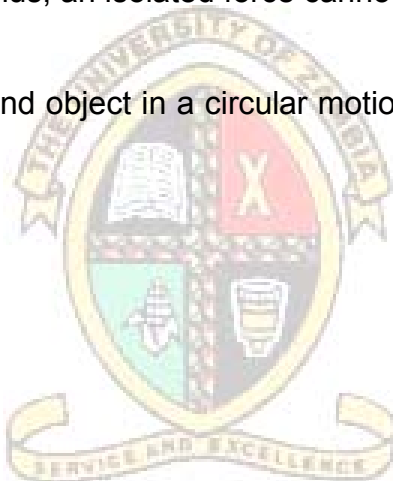
- Newton's first law states that an object at rest remains at rest and an object in uniform motion in a straight line maintains that motion.
- Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.



The net force acting on an object equals the product of its mass and its acceleration: $\sum F = F_{net} = ma$.

- The gravitational force exerted on an object is equal to the product of its mass (a scalar quantity) and the free-fall acceleration: $W = mg$. The weight of an object is the magnitude of the gravitational force acting on the object.
- Newton's third law states that if two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1. Thus, an isolated force cannot exist in nature.
- The force that keeps an object in a circular motion is the centripetal force given

by $F_c = \frac{mv^2}{r}$



4.9 References

F.J Bueche, D.A Jerde, *Principles of Physics*.6th Edition, McGraw-Hill, 1995. ISBN 0-13.978-00700

D. Giancoli, *Physics:Principles with Applications*, 6th Edition.Pearson Prentice Hall, ISBN 0-13-060620-0



P.M Fishbane, S.G Gasiorowicz, S.T Thornton, *Physics for Scientists and Engineers with Modern Physics*, 3rd Edition, Pearson Prentice Hall, ISBN 0-13-191182-1

Sears and Zemansky, *University Physics (with Modern Physics)*, 12th Edition, Hugh D Young and Roger, A Freedman, Pearson/Addison Wesley, 2007, ISBN 0-8053-2187-X.

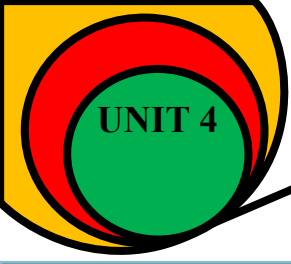


4.10 Answers to Self-help Questions

4.1 Check Figure 4.1

4.2 (i) $\sum F_x = 15 + 16 - 15 = +16$. It accelerates to the right.

(ii) Remember the positive of the coordinate system direction is the direction of motion of the body. Therefore



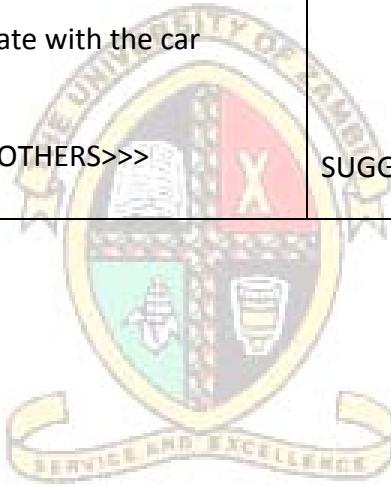
Particle Dynamics



$$\sum F_x = F_1 - F_3 - F_2 \cos \theta$$

4.3

ACTION	REACTION
<ul style="list-style-type: none"> • Your weight pressing down on a chair • The forward force of the car seat on you, which causes you to accelerate with the car <p>SUGGEST OTHERS>>></p>	<ul style="list-style-type: none"> • The rigid chair pushing up on you • The backward force you exert on the car seat, which causes you to “sink” into it <p>SUGGEST OTHERS>>></p>



4.4 When the object is at rest and the jagged points of one surface are embedded in the valleys of the other surface there is so much resistance to sliding but once the sliding has begun, the surfaces do not have time to settle down onto each other completely. As a result less force is required to keep them moving than to start the motion.



4.5 The force that you push provides the normal force by Newton's third law.

$$F = F_N$$

$$\therefore f = \mu F_N = \mu F$$

For the box to remain stationary there must be a force to balance the weight mg and therefore this is friction.

$$f = mg$$

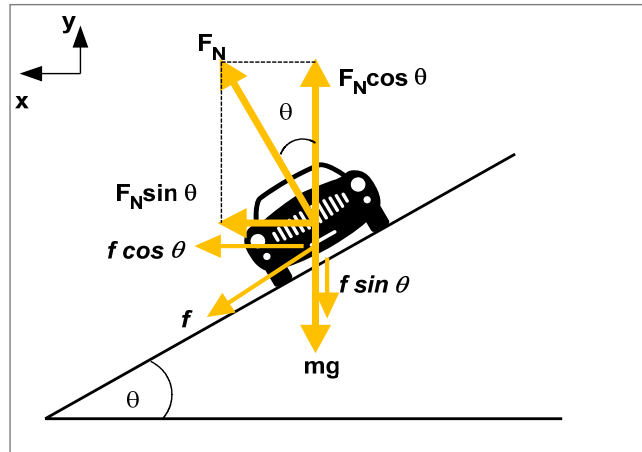
or

$$\mu F = mg$$

The force that you would push therefore is

$$F = \frac{mg}{\mu}$$

4.6. The friction is directed down the incline and when it is resolved, it adds to the $F_N \cos \theta$ which is directed towards the centre of the curve hence making it more possible to make the curve without skidding even at a high speed which we shall show. The free body diagram is shown next:



The forces that provide the centripetal force are those pointing directed towards the centre of the curve, therefore

$$F_N \sin \theta + f \cos \theta = \frac{mv^2}{r}$$

or

$$F_N \sin \theta + \mu F_N \cos \theta = \frac{mv^2}{r}$$

Since there is no motion in the y-direction, we have

$$\sum F_y = F_N \cos \theta - f \sin \theta - mg = 0$$

$$\therefore F_N = \frac{mg}{\cos \theta - \mu \sin \theta}$$

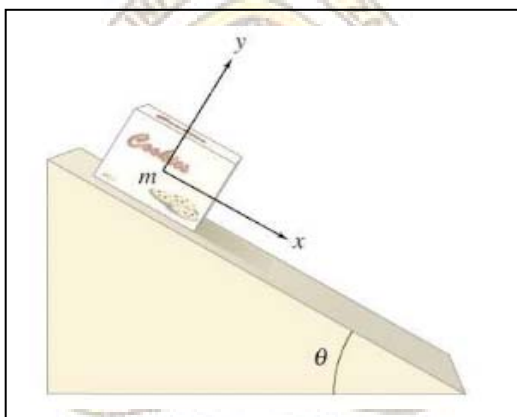
substituting in the centripetal force equation we obtain

$$v = \frac{gr(\sin \theta + \mu \cos \theta)}{\cos \theta - \mu \sin \theta} = v_{\max}$$

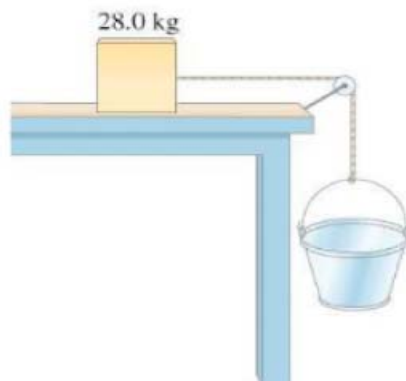




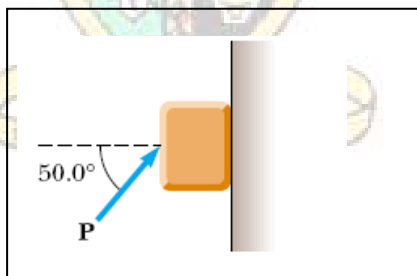
- 4.1 A rubber ball is dropped onto the floor. What force causes the ball to bounce?
- 4.2 What is wrong with the statement “Because the car is at rest, there are no forces acting on it”? How would you correct this sentence?
- 4.3 A carton is given an initial speed of 3.0 m/s up the 22.0° incline as shown below. (a) How far up the plane will it go? (b) How much time elapses before it returns to its starting point? Ignore friction. [(a) 1.2 m up the incline, (b) 1.6 s]



- 4.4 A 28-kg block is connected to an empty 1.35-kg bucket by a cord running over a frictionless pulley. The $\mu_s = 0.45$ and $\mu_k = 0.325$. If sand is gradually added to the bucket until it just begins to move. (a) Calculate the mass of the sand added. (b) What is the acceleration of the system? [(a) 11.3 kg of sand added, (b) 0.88 m/s^2]

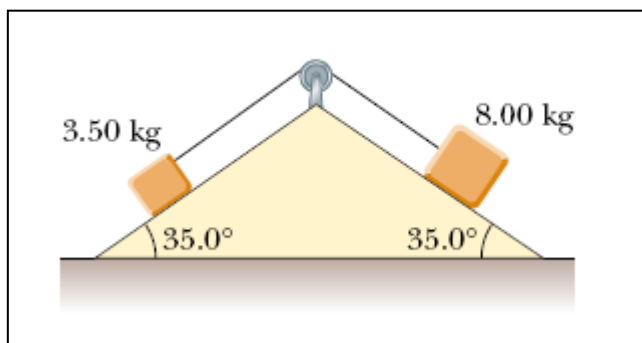
**Problem 4.4**

- 4.5 A block of mass 3.00 kg is pushed up against a wall by a force \mathbf{P} that makes a 50.0° angle with the horizontal as shown below. The coefficient of static friction between the block and the wall is 0.250. Determine the possible values for the magnitude of \mathbf{P} that allow the block to remain stationary. [$P_{\min} = 31.7 \text{ N}$, $P_{\max} = 48.6 \text{ N}$]

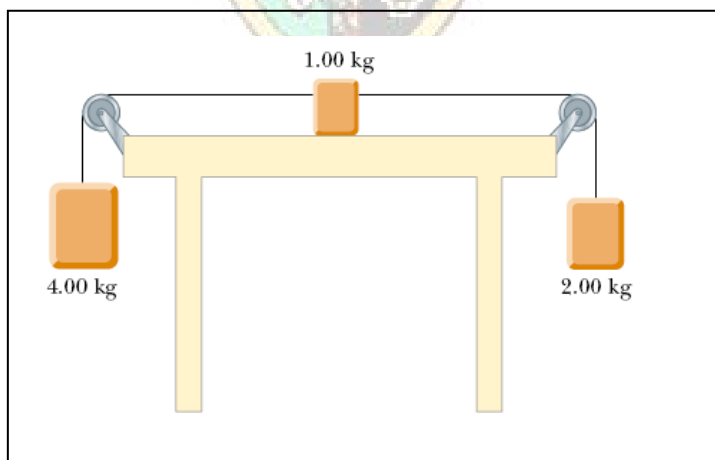




- 4.6 Two blocks of mass 3.50 kg and 8.00 kg are connected by a massless string that passes over a frictionless pulley as shown. The inclines are frictionless. Find (a) the magnitude of the acceleration of each block and (b) the tension in the string. [(a) $T=27.4$ N, (b) 2.20 m/s²]

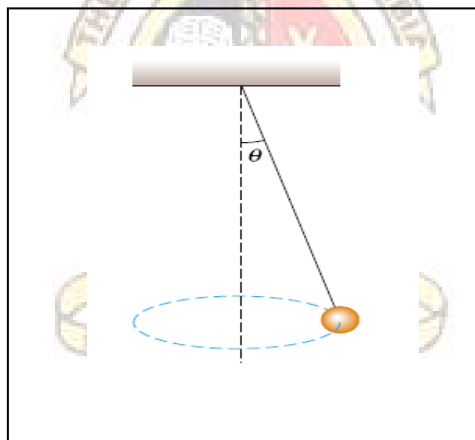


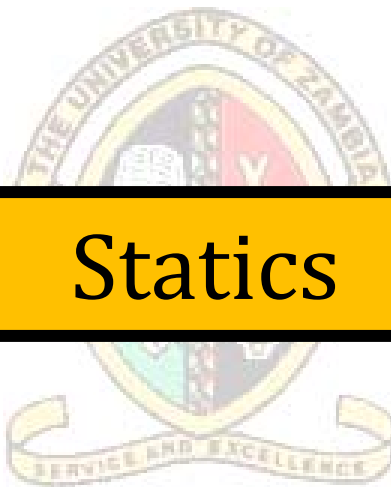
- 4.7 Three objects are connected on the table as shown. The table is rough and has a coefficient of kinetic friction of 0.350. The objects have masses of 4.00 kg, 1.00 kg, and 2.00 kg, as shown, and the pulleys are frictionless. Draw free-body diagrams of each of the objects. (a) Determine the acceleration of each object and their directions. (b) Determine the tensions in the two cords. [$a = 2.31$ m/s²]



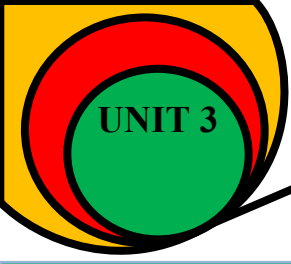


- 4.8 A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.600, how fast can the truck be moving without the crate sliding? [$v \leq 14.3$ m/s]
- 4.9 Consider a conical pendulum with an 80.0-kg bob on a 10.0-m wire making an angle of 5.00° with the vertical as shown. Determine (a) the horizontal and vertical components of the force exerted by the wire on the pendulum and (b) the radial acceleration of the bob. [(a) $T_x = 68.6$ N, $T_y = 784$ N, (b) $a = 0.857$ m/s²]





Statics



UNIT 5: Statics: (15 Hrs)



An Industrial transport bridge



A perspective of a rail bridge



5.1 Introduction

An important part of physics has to do with objects and systems at rest. This branch of physics is called **statics**.

Can you imagine how skyscrapers and tall buildings such as FINDECO house are able to hold their ground and not collapse even when there is a lot of wind or they are packed with people to full capacity?

The designs of such structures employ this branch of physics called statics. It is of great importance to those who design and build bridges, buildings and any other structures whose stability we depend on. All this employs Newton's laws of physics.

This unit treats objects that are at equilibrium. By equilibrium, we mean

- Objects at rest (static) and whose forces balance. Hence we call this “static equilibrium”
- Objects moving at constant velocity with forces balancing. We call this “dynamic equilibrium.”



In this unit we shall deal with static equilibrium. In doing this, two laws of equilibrium will be considered.

Welcome to Unit 5!

5.2 Unit Objectives

After completing this unit, you should be able to

- Add and subtract forces using analytical methods
- State the conditions for static equilibrium
- Solve problems related to the equilibrium of rigid bodies



5.3 First Condition of Equilibrium

When an object is at rest, we say it is in **static equilibrium**. By applying Newton's 1st law we know that the acceleration is zero and hence the net force is zero. This implies that the forces balance each other in all directions. This leads us to the first condition of equilibrium:

“For an object to be in equilibrium, the vector sum of the forces acting on it must be zero.”

Since it is at rest, $a = 0$, so that $\sum F = ma = m(0) = 0$. This leads us to the mathematical definition of the first condition:

$$\sum F_x = 0$$

$$\sum F_y = 0 \tag{5.1}$$

$$\sum F_z = 0$$

Do not forget that these are components of a vector.



Consider now Figure 5.1 below which is in equilibrium. Applying the first condition we have

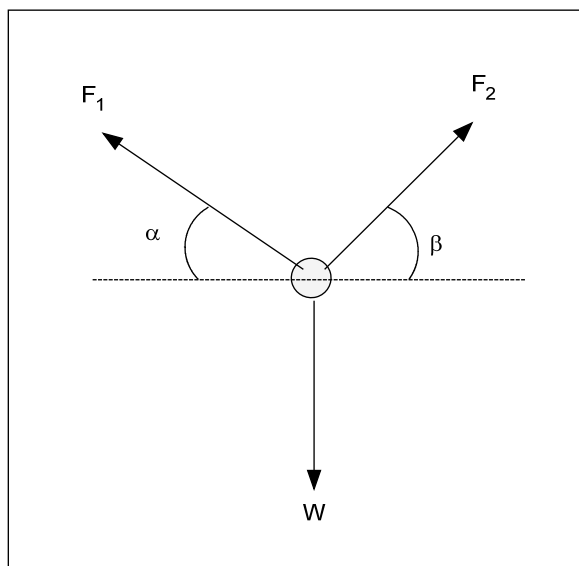


Figure 5.1: A ring held in equilibrium by three forces

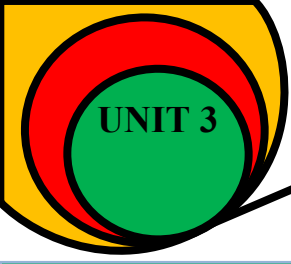
We resolve all the forces into components and apply the 1st condition: To do so we choose a convenient coordinate system. We choose UP and RIGHT as positive directions.

$$\sum F_x = F_2 \cos \beta - F_1 \cos \alpha = 0$$

$$\therefore F_2 \cos \beta = F_1 \cos \alpha$$

$$\sum F_y = F_1 \sin \alpha + F_2 \sin \beta - W = 0$$

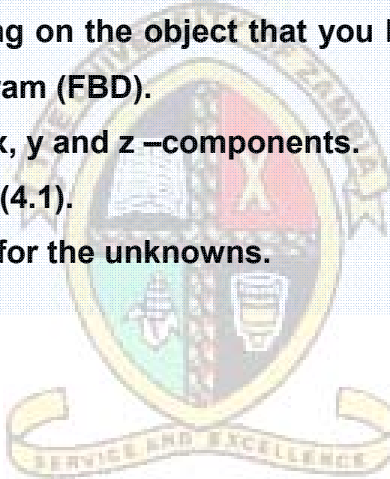
$$\therefore W = F_1 \sin \alpha + F_2 \sin \beta.$$



5.3.1 Solving Problems in Statics

Here are the few simple rules to help you solve problems in Statics. As long as you follow the rules, you'll be able to solve such problems with little effort.

- ✓ Isolate the object you are going to talk about. The forces acting on this object are the **ONLY** ones you need in writing equation (5.1).
- ✓ Draw the forces acting on the object that you have isolated and label them in the free body diagram (FBD).
- ✓ Split each force into x, y and z –components.
- ✓ Write down equation (4.1).
- ✓ Solve the equations for the unknowns.



Now see how it is done...



nt



EXAMPLE 5.1: A bag of cement of weight 325 N hangs from three wires as shown in Figure 5.2. Two of the wires make angles $\theta_1 = 60.0^\circ$ and $\theta_2 = 25.0^\circ$ with the horizontal. If the system is in equilibrium, find the tensions T_1 , T_2 , and T_3 in the wires.

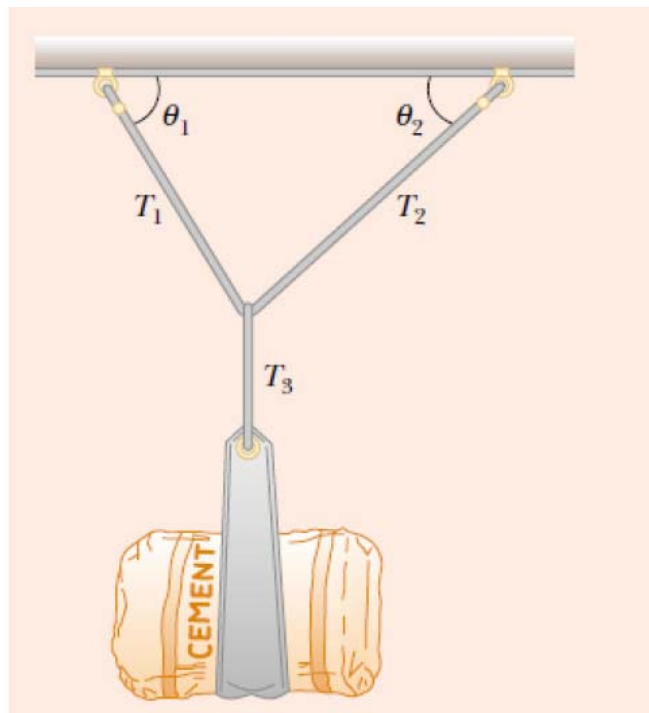
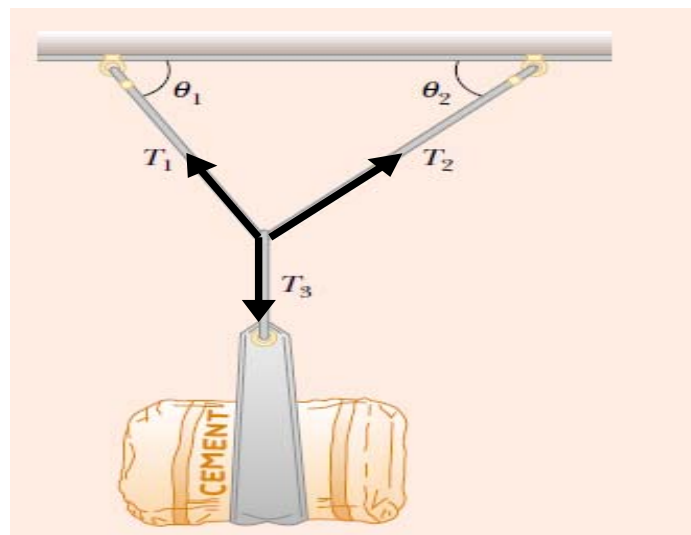


Figure 5.2: A bag of cement being held by two wires



SOLUTION

The free body diagram looks like this



Applying the laws, we have

$$\sum F_x = T_2 \cos \theta_2 - T_1 \cos \theta_1 = 0$$

$$\therefore T_2 = T_1 \frac{\cos \theta_1}{\cos \theta_2} \quad (1)$$



In the y-direction we have,

$$\sum F_y = T_2 \sin \theta_2 + T_1 \sin \theta_1 - T_3 = 0$$

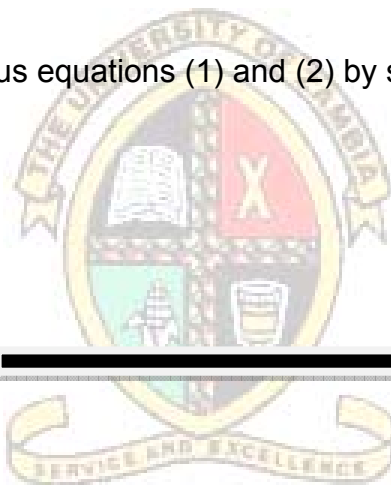
But $T_3 = W = mg$

$$T_2 \sin \theta_2 + T_1 \sin \theta_1 = W \quad (2)$$

Solving these two simultaneous equations (1) and (2) by substituting (1) into (1) gives

$$T_2 = 163 \text{ N}$$

$$T_1 = 296 \text{ N}$$



5.4 Torque

Before we go to the second condition of equilibrium, we shall review the concept of **torque**, also commonly known as the “moment of force.”

This measures the turning effect of an object hinged at a pivot.



Mathematically torque is given by

$$\text{Torque} = \tau = rF \quad (5.2)$$

where r = is the lever arm and

F = is the force.

Important: The lever arm is the perpendicular distance from the axis to the line of action of the force. In Figure 5.3, r_1 and r_2 are the lever arms for F_1 and F_2 respectively.

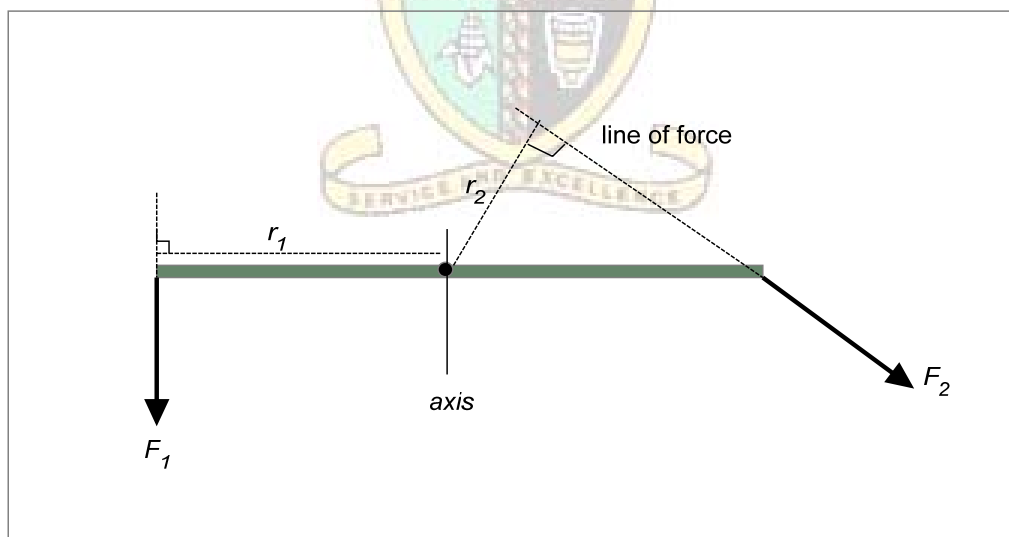


Figure 5.3: Illustration of the lever arm



The respective torques are:

$$\tau_1 = r_1 F_1$$

$$\tau_2 = r_2 F_2.$$

We can also observe that F_1 tends to turn the object in the counter-clockwise direction and F_2 in the clockwise direction. Therefore τ_1 is a counter-clockwise torque while τ_2 is a clockwise torque.

If the line of action of a force passes through the axis, then the lever arm is zero and so is the torque.

Torques that tend to produce a counter-clockwise (ccw) rotation about an axis will be assigned a **positive** value while those in the clockwise (cw) direction will be given a **negative** value.

Therefore for the torques in Figure 5.3,

$$\tau_1 = +r_1 F_1$$

$$\tau_2 = -r_2 F_2$$



If the system is in equilibrium or balanced, meaning it is not rotating, then the torques balance, that is, $\tau_1 = \tau_2$ or $r_1 F_1 = r_2 F_2$

Self-help task 5.1

Identify the lever arm and write down the torque for each of following forces with respect to rotation about O. See Figure 5.4.

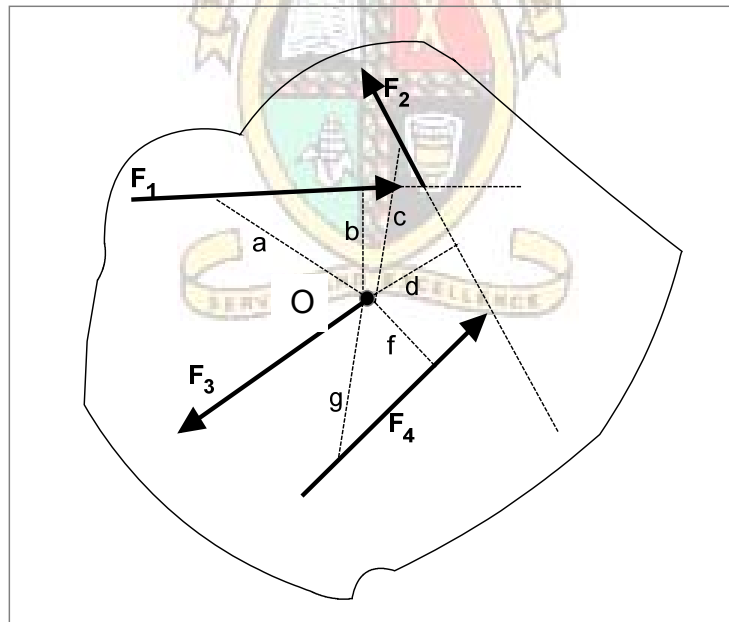
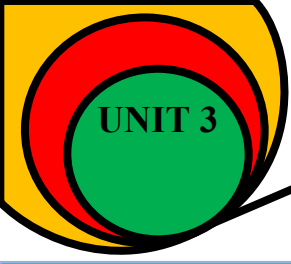


Figure 5.4: Finding the torques.



5.5 Second Condition of Equilibrium

Now that we can express the turning effect of a force in terms of torque, we are better equipped to talk about the second condition of equilibrium. Careful experiments show that, for an object to remain motionless, the clockwise torques acting on it must be balanced by the counter-clockwise torques. Using the sign convention for torques from the previous section, we can state that:

“For an object to be in static rotational equilibrium, the algebraic sum of clockwise and counterclockwise torques acting on it must be zero.”

This is the second condition of equilibrium. Mathematically we have

$$\sum \tau = 0 \tag{5.3}$$

We now have all the requirements for a body to be in equilibrium. To summarize, in two dimensions, we have:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum \tau = 0 \tag{5.4}$$

When using the second condition for equilibrium, it is necessary to show the correct placement of forces on an object's FBD.



EXAMPLE 5.2: The system in Figure 5.2 is in equilibrium. Find the force at the hinge and the tension in the wire. The mass of the beam is assumed negligible.

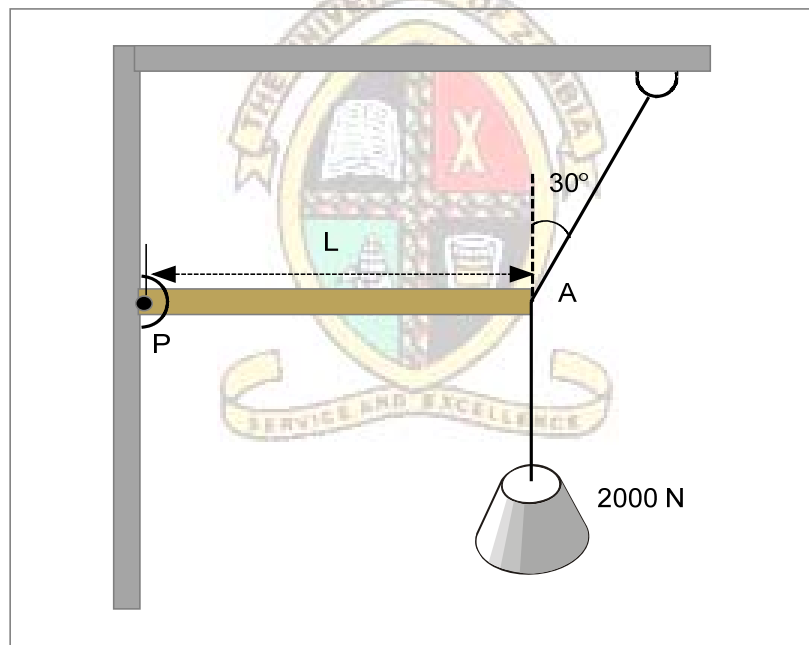
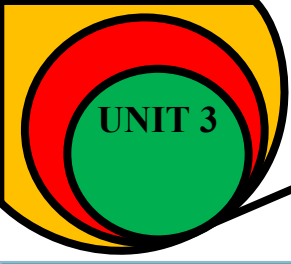
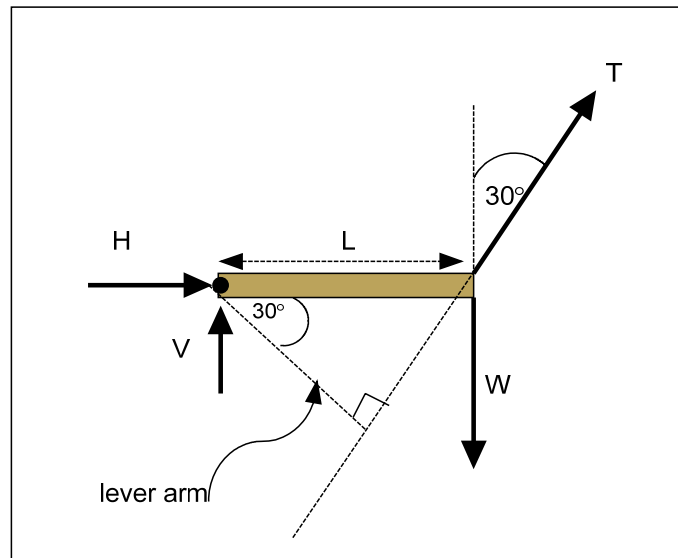


Figure 5.5



SOLUTION

The free body diagram looks like this



The direction of force acting at the hinge cannot be known now. All we can do is indicate that it has a horizontal (H) and vertical (V) component. If after solving, a value we have a negative sign, it means we didn't choose the correct direction. We should have given it the opposite direction.



The choice of the pivot is arbitrary. Let us choose the hinge as the pivot.

We now apply the two conditions

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum \tau = 0.$$

The first condition states

$$\sum F_x = 0: \quad H + T \sin 30^\circ = 0$$

$$\therefore H = -0.5T \quad (1)$$

$$\sum F_y = 0: \quad V + T \cos 30^\circ = W$$

$$\therefore V = 2000 - 0.866T \quad (2)$$

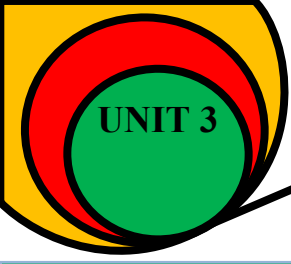
There are three unknowns but two equations. Hence we need a third equation. This is from the second condition.

$$\sum \tau = 0: \quad (WL) - (TL \sin 60^\circ) = 0$$

The L cancels out

$$\therefore T = \frac{2000}{\sin 60^\circ} = 2310 \text{ N}$$

Now that we have found T , we substitute into (1) and (2) to find H and V .



Statics



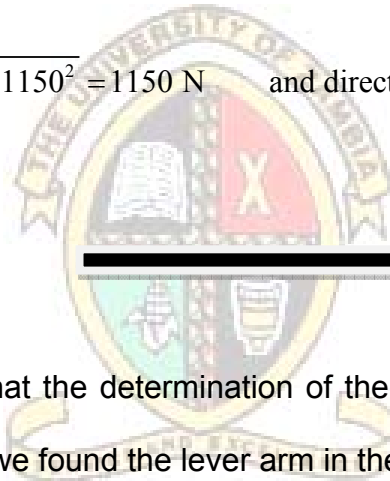
We therefore obtain

$$V = 0, \quad \text{and} \quad H = -1150 \text{ N}$$

We therefore know that the horizontal force is directed in the negative direction and not where it was assigned to point.

The force at the hinge therefore is

$$F_{\text{hinge}} = \sqrt{V^2 + H^2} = \sqrt{0 + 1150^2} = 1150 \text{ N} \quad \text{and directed to the negative x-direction.}$$



You should observe that the determination of the lever arm is very crucial. You should figure out how we found the lever arm in the previous example.

Is it clear? Go through it again if it is not , otherwise

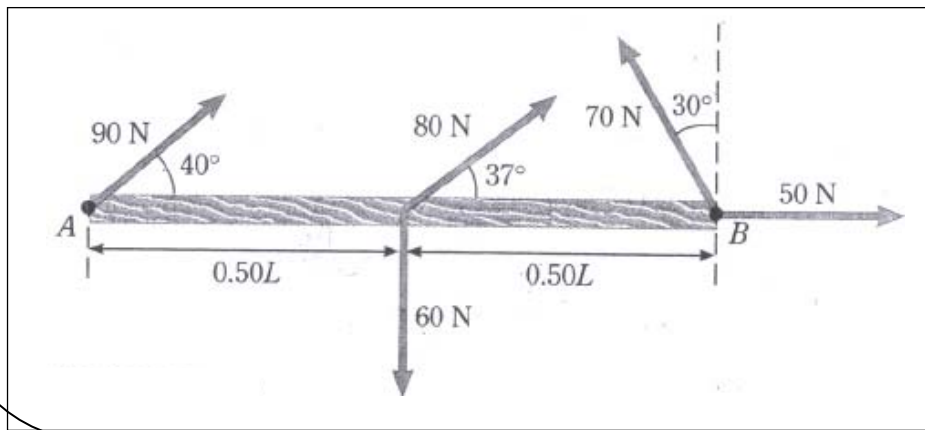
PROCEED





Self-help task 5.2

Find the torques about the axis A for the forces shown below if the length of the bar is $L=5.0$ m



EXAMPLE 5.3: A uniform beam weighs 50 N. The beam is secured to the wall by a hinge. See Figure 5.6. If the beam is in static equilibrium, how large is the tension in the upper cable and what is the force exerted by the hinge on the beam?

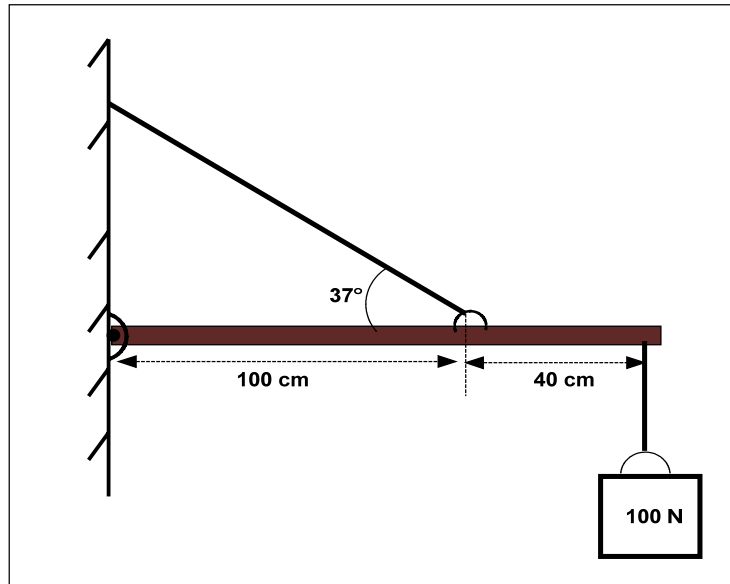
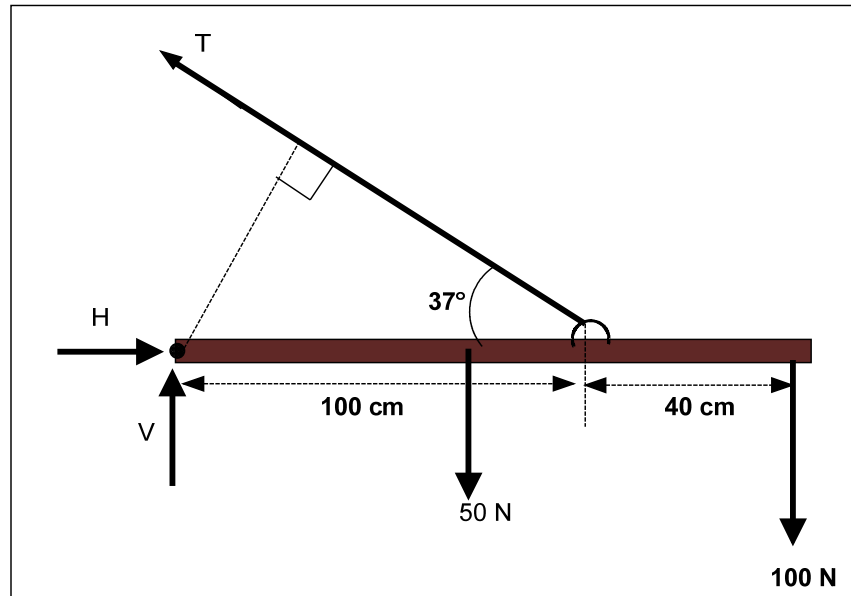


Figure 5.6: Beam held by the wall

SOLUTION

Since the beam is uniform, the centre of gravity is exactly at the centre. This is where the weight of the beam is acting. It is very important to show the correct directions of the forces. As usual the tension is always away from the body. The free body diagram now looks like this:



We shall now apply the two conditions of equilibrium starting with the second. We take the torque about the hinge,

$$\sum \tau = 0: \quad T \times (1.0 \sin 37) - 50 \times 0.7 - 100 \times 1.4 = 0$$

$$\therefore T = 290 \text{ N.}$$



We now apply the first condition

$$\sum F_x = 0: \quad H - T \cos 37^\circ = 0$$

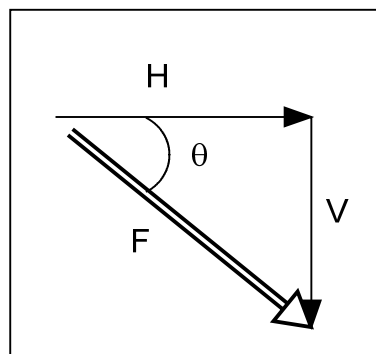
$$\therefore H = T \cos 37^\circ = 290 \cos 37^\circ = 230 \text{ N.}$$

$$\sum F_y = 0: \quad V + T \sin 37^\circ - 50 - 100 = 0$$

$$\therefore V = 150 - 290 \sin 37^\circ = -24 \text{ N.}$$

Again the negative sign indicates the wrong choice of direction of the vertical vector at the hinge.

Therefore the hinge exerts a force





$$F_{\text{hinge}} = \sqrt{V^2 + H^2} = \sqrt{(-24)^2 + (230)^2} = 231 \text{ N} \quad \text{and directed at } 6.0^\circ \text{ below the +ve x axis.}$$

5.6 Back Injury from Lifting

You may have been warned that there is a right and wrong way of lifting heavy objects. It can be at the gym or at school or at home, but you were warned. We shall use what we have learnt to see why this is the case.

In doing so we shall consider a boy lifting a 60 N bowling ball as shown in Figure 5.7.

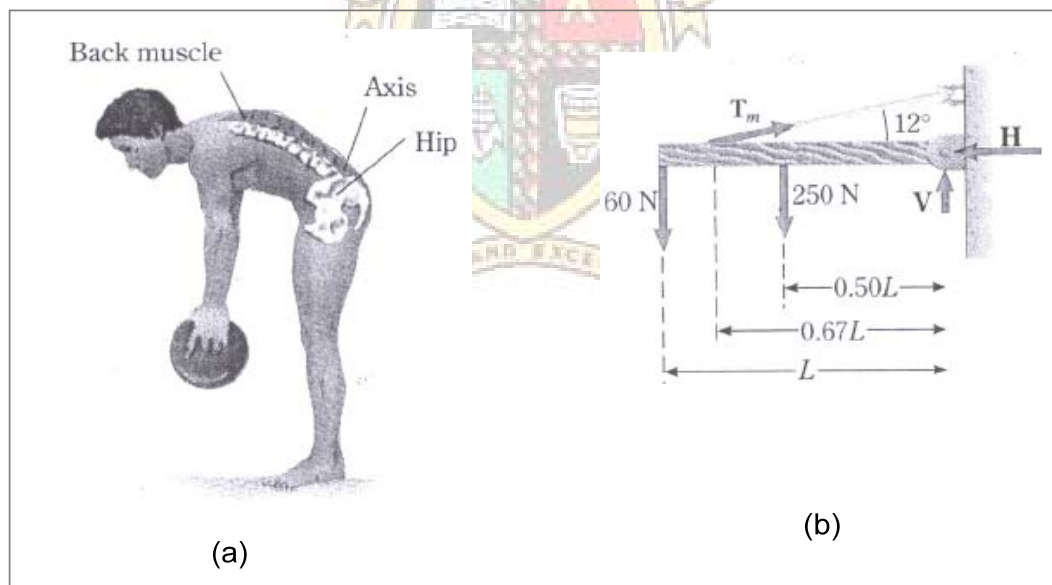


Figure 5.7: Back strain



Back strain can occur if the tension in the back muscle becomes so large or if the compression of the spine on the hip is so excessive.

To calculate the forces that are involved, we idealize the situation into a model (or free body diagram) as shown in 5.7 (b). In this model, the spine is represented as the horizontal beam pivoted at the hip, the back muscle as the tension T_m . We let the components of the force H and V be acting at the hinge (hip). Now if the weight of the boy is 250 N and he holds the ball at equilibrium then we have

$$\sum \tau = 0: \quad (60L) + (2500.5L) - (T_m 0.67L \sin 12^\circ) = 0$$

$$\therefore T_m = \frac{60 + 125}{0.67 \sin 12^\circ} = 1330 \text{ N.}$$

Remember the L cancels out?!

For the first condition we have

$$\sum F_x = 0: \quad T_m \cos 12^\circ - H = 0$$

$$\therefore H = T_m \cos 12^\circ = 1330 \cos 12^\circ = 1300 \text{ N}$$

$$\sum F_y = 0: \quad V + T_m \sin 12^\circ - 60 - 250 = 0$$

$$\therefore V = 310 - T_m \sin 12^\circ = 310 - 1330 \sin 12^\circ = 32 \text{ N.}$$



Notice how large these forces are. Although the ball only weighs 60 N, the back muscle is under a tension of 1330 N!! The force on the spine is also very large and is $\sqrt{1300^2 + 32^2}$. Therefore when you bend over you place a tremendous strain on the back.

Advice: When you intend to lift a heavy object, you should always **squat** and **hold your back erect** to reduce the forces.

That's it!





5.7 Summary

- In this unit, we learnt that an object which is at rest and which remains at rest indefinitely is said to be in static equilibrium.
- In solving problems involving statics, two conditions are satisfied.
 - ✓ First condition: The vector sum of all the forces which are in equilibrium is zero.

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$$

- ✓ Second condition: The algebraic sum of all the cw and ccw torques acting about any axis on an object in equilibrium must be zero.
$$\sum \tau = 0.$$
- In applying the first condition, the position of the forces does not matter. Only the direction of the forces is important.
- For the second condition, the correct position is important so that the torques are correctly calculated.
- The choice of the axis is arbitrary and even can be outside the object.
- Since the torque is zero when the line of action of the force passes through the axis, it is very helpful to choose an axis through which as many forces as possible pass.



5.8 References

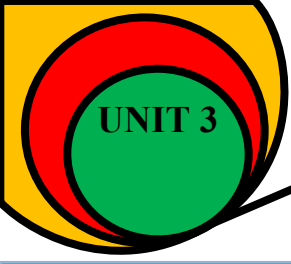
F.J Bueche, D.A Jerde, *Principles of Physics*.6th Edition, McGraw-Hill, 1995.

ISBN 0- 13.978-00700

D. Giancoli, *Physics:Principles with Applications*, 6th Edition.Pearson Prentice Hall, ISBN 0-13-060620-0

P.M Fishbane, S.G Gasiorowicz,S.T Thornton, *Physics for Scientists and Engineers with Modern Physics*,3rd Edition, Pearson Prentice Hall , ISBN 0-13-191182-1

Sears and Zemansky, *University Physics (with Modern Physics)*, 12th Edition, Hugh D Young and Roger, A Freedman,Pearson/Addison Wesley,2007, ISBN 0-8053-2187-X.



5.9 Answers to Self-help Questions

5.1 Remember our convention? Clockwise moments are assigned a negative value and a counterclockwise moment a positive value.

$$F_1: \tau_1 = -aF_1$$

$$F_2: \tau_2 = +dF_2$$

$$F_3: \tau_3 = 0 \quad \text{The line of force passes through the pivot.}$$

$$F_4: \tau_4 = +fF_4$$

5.2 First resolve the 80 N and 70 N forces along the x and y axes. Lines of action for the 50 and 90 N forces and those of x components of 80 N and 70 N forces pass through the pivot point A, so they contribute zero to the torque. Therefore the torques are:

$$\tau_{90} = \tau_{50} = 0$$

$$\tau_{80} = 80 \sin 37^\circ (2.5) = +120 \text{ Nm}$$

$$\tau_{70} = 70 \sin 60^\circ (5.0) = +303 \text{ Nm}$$

$$\tau_{60} = -60(2.5) = -150 \text{ Nm}$$



5.10 EXERCISES

- 5.1 A uniform beam in Figure 5.8 weighs 280 N. Find (a) the tension in the upper rope and (b) the horizontal and vertical components, H and V , of the force exerted by the hinge if $W = 840$ N. [$T = 1279$ N, $H = 822$ N, $V = 140$].

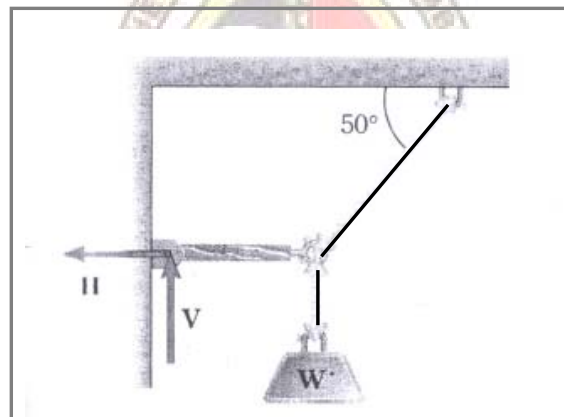


Figure 5.8 : Problem 5.1

- 5.2 The pulleys in Figure 5.9 are frictionless and have a negligible weights. At equilibrium $W_1 = 600$ N. Find the weight W_2 and the tensions T_1 , T_2 , T_3 and T_4 .

$$[T_1 = T_2 = T_4 = 300\text{N}, T_3 = 600\text{ N}].$$

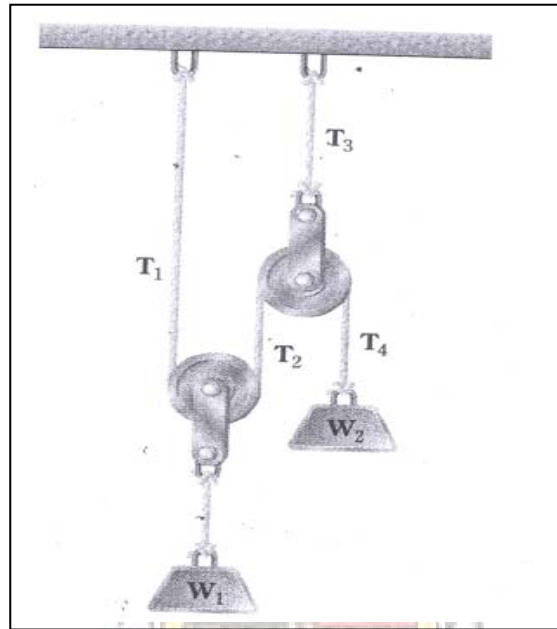


Figure 5.9:

- 5.3 In Figure 5.10 the vertical ropes with tensions T_1 supports two weights W_1 and W_2 at the ends of a weightless plank. If $T_1 = 240$ N and $W_2 = 280$ find the values of W_2 and T_2 . [$T_2 = 293$ N, $W_1 = 253$ N].

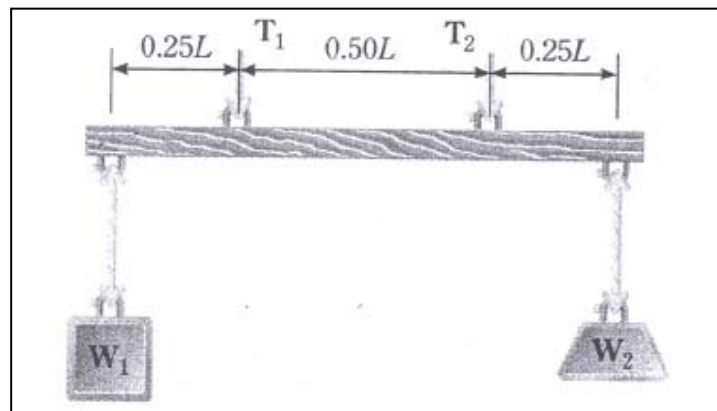


Figure 5.10: problem 5.3



- 5.4 A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in Figure 5.11. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. Will the traffic light remain hanging in this situation, or will one of the cables break? [$T_1 = 73.4$ N, $T_2 = 97.4$ N. Both of these forces are less than 100 N, so the cables will not break.]

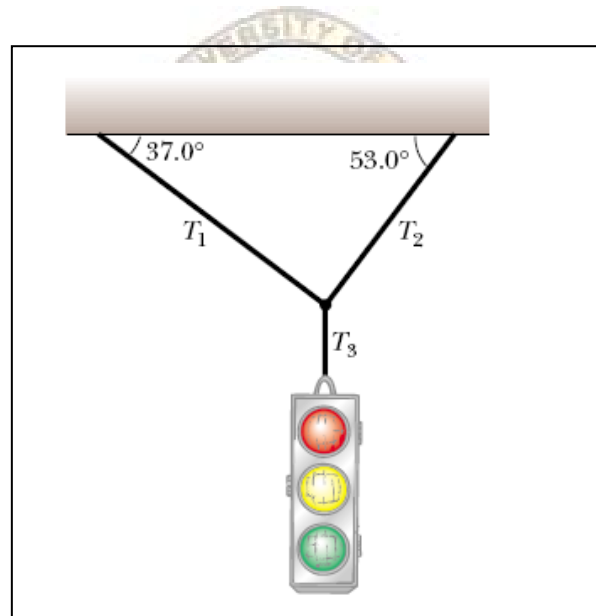


Figure 5.11: Problem 5.4



- 5.5 The uniform 450-N beam in Figure 5.12 supports the load as shown. (a) How large can the load be if the horizontal rope is able to hold 2800 N? (b) What are the horizontal and vertical components of the force at the base of the beam? [(a) $W = 11200$ N, (b) $H = 2800$ N, $V = 11652$ N].

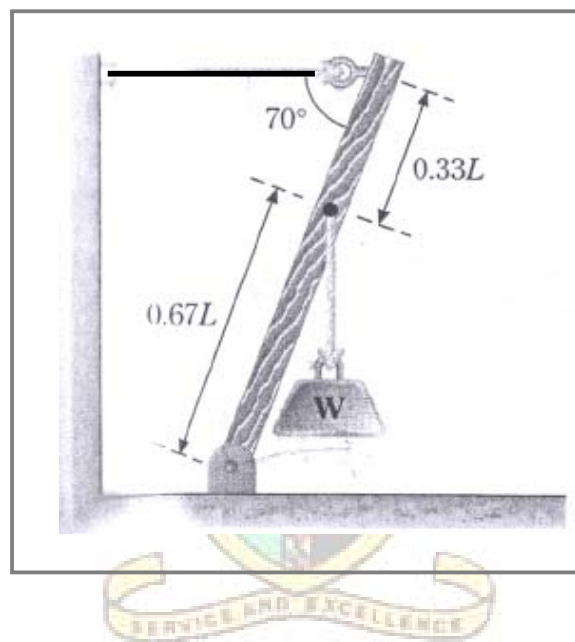
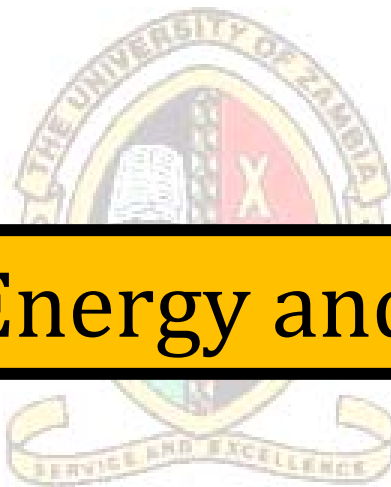
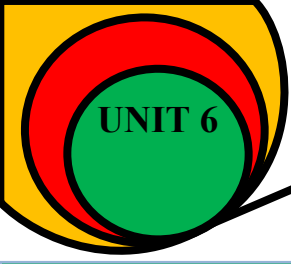


Figure 5.12: problem 5.5



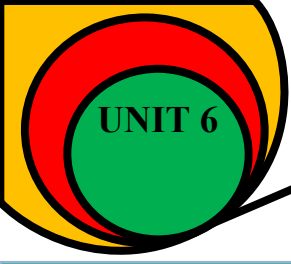
Work, Energy and Power



UNIT 6: Work, Energy and Power (15 hrs)



Wind energy being converted to electricity.



6.1 Introduction

Until now we have been studying the translational motion of an object in terms of Newton's three laws of motion. In this analysis, force has played a central role as the quantity determining the motion. However, in this unit we shall discuss an alternative analysis of translational motion of objects in terms of energy. The significance of energy is that it is conserved. This gives us a deeper insight into our physical world and an alternative approach to solving practical problems.

The concept of energy is one of the most important topics in science and engineering. In everyday life, we think of energy in terms of fuel for transportation and heating, electricity for lights and appliances, and foods for consumption. However, these ideas do not really define energy. They merely tell us that fuels are needed to do a job and that those fuels provide us with something we call energy.

In this unit we shall discuss the concept of work and potential energy.

Welcome to Unit 6!



6.2 Unit Objectives

After completing this unit, you should be able to

- Define work in the general case
- Derive the work-energy theorem and solve related problems
- Characterise work done by conservative and non-conservative forces
- Relate work, energy and power

6.3 Work

The word work has a variety of meanings in everyday language. But in physics, work is given a very specific meaning to describe what is accomplished when a force acting on an object moves through a distance.

When you sit at your desk studying these notes, you are not doing work! Imagine that? This does not mean that you are lazy or that learning physics is an effortless process. It is simply stating a fact that arises from the definition of work that scientists use.



6.3.1 Work done by a constant Force.

Consider an object being acted upon by a constant force F at an angle θ . Work done on an object by a constant force (constant in both magnitude and direction) is defined to be the product of the magnitude of the displacement s and the component of the force parallel to the displacement.

Here are the few simple rules to help you solve problems in Statics. As long as you follow the rules, you'll be able to solve such problems with little effort.

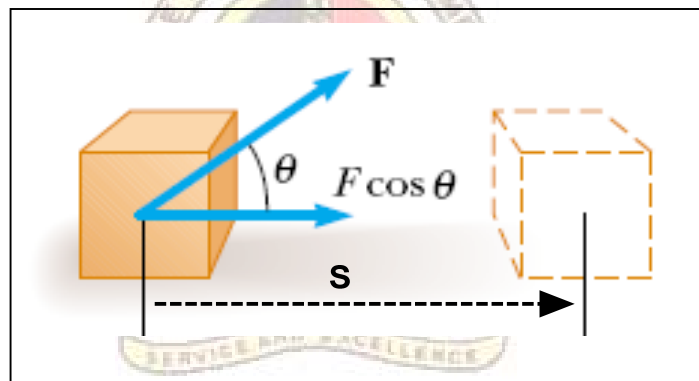


Figure 6.1: Work done by a force over a displacement s

Mathematically, the work done is

$$W = F_s s = (F \cos \theta) s = F s \cos \theta \quad (6.1)$$

where F_s is the force parallel to the displacement vector s .



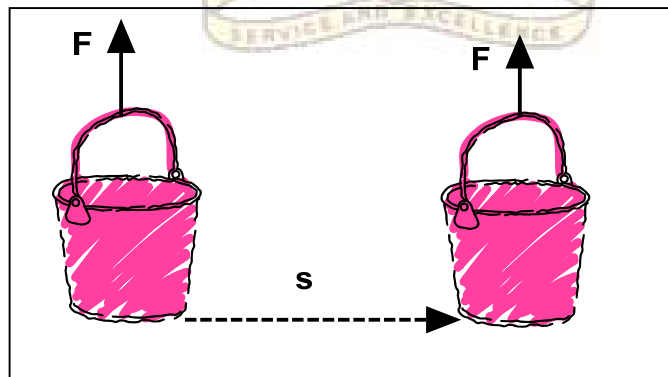
The presence of θ implies that work can either be negative or positive. It is positive for $0 < \theta < 90^\circ$ (implying that F has a component in the direction of displacement) and negative for $90^\circ < \theta < 180^\circ$ (implying that F has a component in the opposite direction of the displacement vector).

From the expression of work in (6.1), one is able to derive the SI unit for work. This is **N.m**. It is given a special name, the joule (J).

Therefore, a **joule** is the work done by a force of one Newton as it acts through a distance of one meter along the line of force. Therefore $1 \text{ J} = 1 \text{ Nm}$.



EXAMPLE 6.1: A person carries a pail containing water over a horizontal distance of 8.0 m at constant speed. How much work does F do?



SOLUTION



The force F is just the weight of the pail plus water. The angle between the F and the displacement vector is 90° (since he was carrying it over a horizontal distance) Therefore work is

$$W = Fs \cos 90^\circ = 0$$

Therefore no work is done by the vertical force because it has no component in the direction of motion.



Self-help task 6.1

Following the example 6.1, why do you get tired carrying the bucket even if you do no work?



EXAMPLE 6.2: How much work do you do on an object of weight mg as

- (a) you lift it a distance h straight up at constant speed?,
 (b) you lower it through this same distance again at constant speed?

SOLUTION

(a) In lifting to a height h , the displacement vector points up with the force $F = mg$ also directed up. Therefore the angle is zero, since F_h and h are parallel.

Therefore

$$W = Fs \cos \theta = mgh \cos 0 = mgh$$

This is the work you do in lifting the object a distance h .

(b) In lowering the object, the force vector F_h still points up but the displacement vector is directed downwards. Therefore the angle is 180° so that the work done is

$$W = Fs \cos \theta = mgh \cos 180^\circ = -mgh$$

From this we can say that you do negative work or the object did positive work on you. In other words, gravity did positive work on the object.



Here is a task for you...



Self-help task 6.2

The box is being pulled along the floor at a constant speed by a force F . The friction opposing the motion is 20 N and $m = 30\text{ kg}$. Find the magnitude of F and the work done by F as the box is being moved a distance 5.0 m . See Figure 6.2.

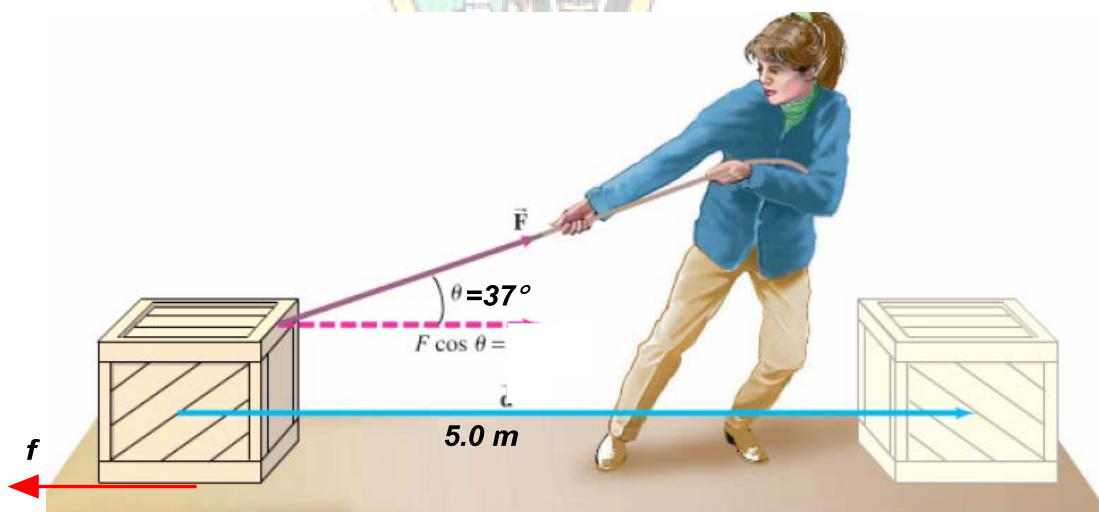
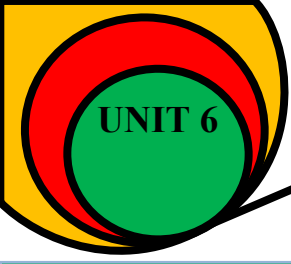


Figure 6.2: self-help task 6.2



6.3.2 Work done by a variable force

If the force acting on an object is constant, the work done by that force can be calculated using equation 6.1. But in many cases, the force varies in magnitude or direction during a process.



Self-help task 6.3

List some examples or situations where you can have a varying force.

You listed them? Confirm your list at the end of the unit.

Now we can proceed and find the work done by them...

The work done by a varying force is found by dividing the area under the force-displacement curve into small segments. When the object undergoes a very small displacement Δx , the x-component of the force is approximately constant. See Figure 6.3. The work done for this small displacement is the area of the small segment,

$$W \approx F_x \Delta x$$

If we add all the segments we obtain the area under the graph. Hence we obtain the work done by a varying force in moving an object from position x_i to x_f , that is

$$W = \sum_{x_i}^{x_f} F_x \Delta x \tag{6.2}$$

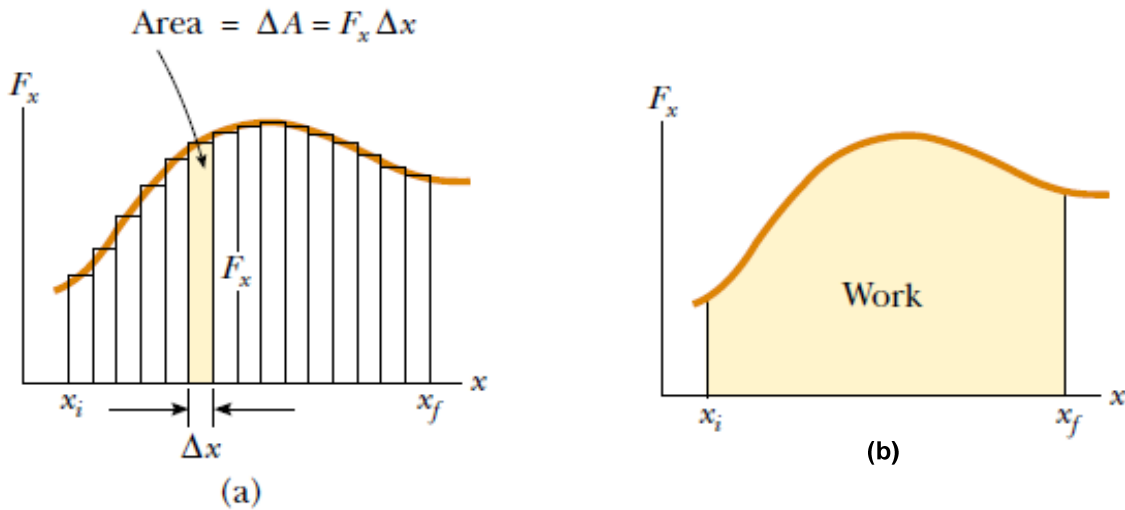
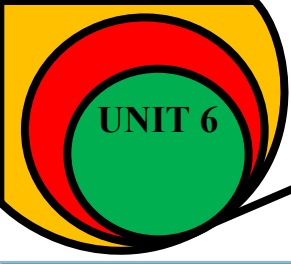


Figure 6.3: Work done by a varying force

If the displacement is allowed to approach zero, then

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$



Thus we have

$$W = \int_{x_i}^{x_f} F_x dx \tag{6.3}$$

From calculus you will recall that this is the formula for finding the area under a graph. If you have not done calculus already, don't worry you will do it but then basic calculus is easy to study. It won't kill you to browse through the topic.



Self-help task 6.4

How much work do you do in stretching a spring from the equilibrium position say $x_i = 0$ to a length of $x_f = x$? The restoring force is kx from Hooke's law.

6.4 Kinetic Energy

If an object can do work, we say it possesses energy. Therefore energy is the ability to do work. There are many kinds of energy but we shall for begin our study with kinetic energy.



What is **kinetic energy**?

When you hear the word *kinetic*, what comes into your mind? No doubt it is something to do with *moving*. Indeed **kinetic energy, KE, of an object is the energy possessed due to its motion**. Mathematically

$$KE = \frac{1}{2}mv^2 \quad (6.4)$$

where m is the mass in kilograms and v is the velocity in m/s. The unit of KE is the joule.

6.5 Work - Energy Theorem

For an object that is acted upon by a net force, the speed changes. Therefore when you move an object initially at rest, you produce a net force. In so doing you will move the object. There is therefore a connection between the net force and the kinetic energy.

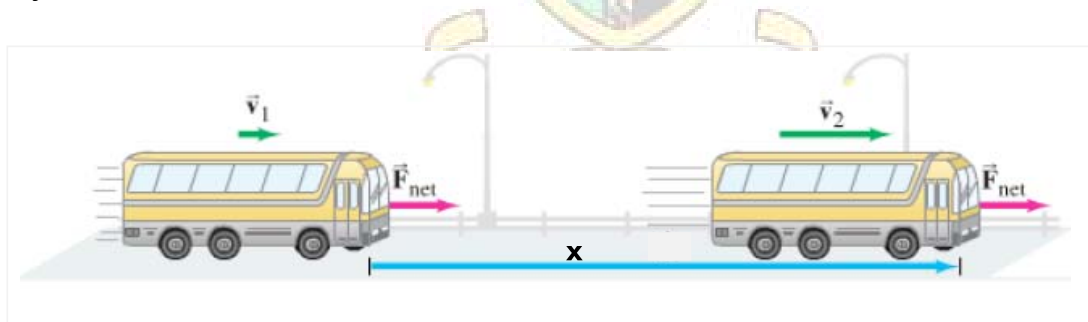


Figure 6.4: work-energy theorem



Now consider a bus whose engine drives it with a net force so as to accelerate the bus from a speed of v_1 to a speed of v_2 .

The net force is given by

$$F_{net} = ma \quad (1)$$

But from,

$$v_2^2 = v_1^2 + 2ax$$

we have

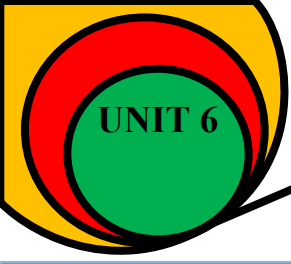
$$a = \frac{v_2^2 - v_1^2}{2x}$$

Substituting this into equation (1) we obtain

$$F_{net} = m \left(\frac{v_2^2 - v_1^2}{2x} \right)$$

Taking x to the other side we obtain

$$F_{net}x = m \left(\frac{v_2^2 - v_1^2}{2} \right)$$



From this equation one is able to identify the left hand side as the work done by the net force while the right hand side is the change in KE. Thus

$$F_{net}x = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \tag{6.5}$$

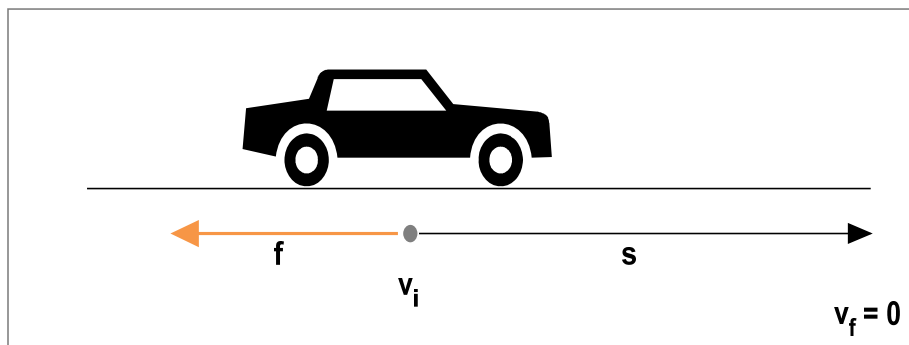
This is the work done by a net force. Equation 6.5 is called the **work-energy theorem**.



EXAMPLE 6.3: A 1000 kg car travelling at 20 m/s coasts to rest on level ground in a distance of 100 m. How large is the average frictional force acting on the car?

SOLUTION

There is no other force acting on the car apart from friction. Here is the FBD along with the displacement vector.



From the work-energy theorem, we have

Work done by friction (net force) is

$$W = fs \cos 180^\circ = -100f$$

Using the work energy theorem,

$$\text{Work by } F_{net} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$-100f = 0 - \frac{1}{2}(2000)(20)^2$$

$$\therefore f = 4000 \text{ N}$$

Let us see if you've grasped the principle. A task for you...



Self-help task 6.5

A 2 kg box is pushed from rest with a horizontal force of 100 N along a horizontal track of 5.0 m. What is its final velocity at the end of the track, if $\mu = 0.7$?

5.6 Power

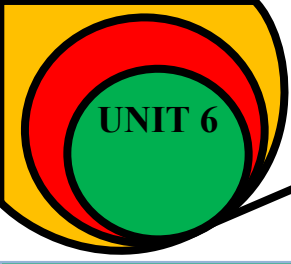
Power is the rate at which work is done. We tend to synonymously use power for energy. When one is said to be powerful, we mean he is very energetic. In science, that is not the case; the most powerful person is the one who does work in the shortest time possible.

We have

$$P = \frac{\text{Work done}}{\text{Time taken to do work}}$$

$$\therefore P = \frac{W}{t} \quad (6.6)$$

The SI unit is J/s or a **Watt** or N.m/s



For motors and engines, power is often measured in horsepower (hp) where

$$1 \text{ hp} = 746 \text{ W}$$

Power can also conveniently be expressed as

$$\therefore P = \frac{W}{t} = \frac{F_s s}{t} = F_s \frac{s}{t} = F_s v \tag{6.7}$$

or

$$P = (F \cos \theta) v \tag{6.8}$$

5.7 Potential Energy

This is the energy possessed by an object by virtue of its position. If you lift a mass m from point A to point B, the object at B will acquire a gravitational potential energy (GPE).

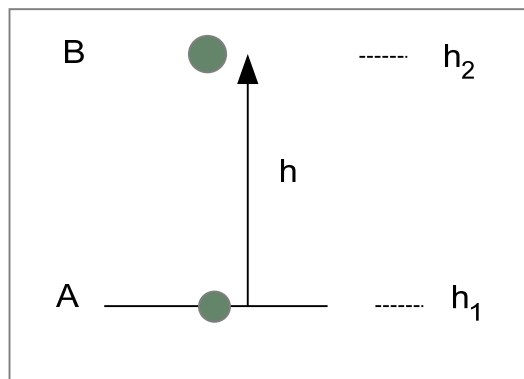


Figure 6.5: Gravitational potential energy



The GPE at B is the work that was done in lifting it to position B. Check example 6.2.

$$GPE = mgh \quad (6.9)$$

The SI unit is a joule.

The choice of reference is arbitrary. You can choose A as your zero or B as your zero.

- The change in GPE from A to B is

$$\Delta GPE_{AB} = mg(h_B - h_A) = mg(h_2 - h_1) = mgh$$

- The change in GPE from B to A is

$$\Delta GPE_{BA} = mg(h_A - h_B) = mg(h_1 - h_2) = -mgh$$

Now consider an object of mass m as shown in Figure 6.6.

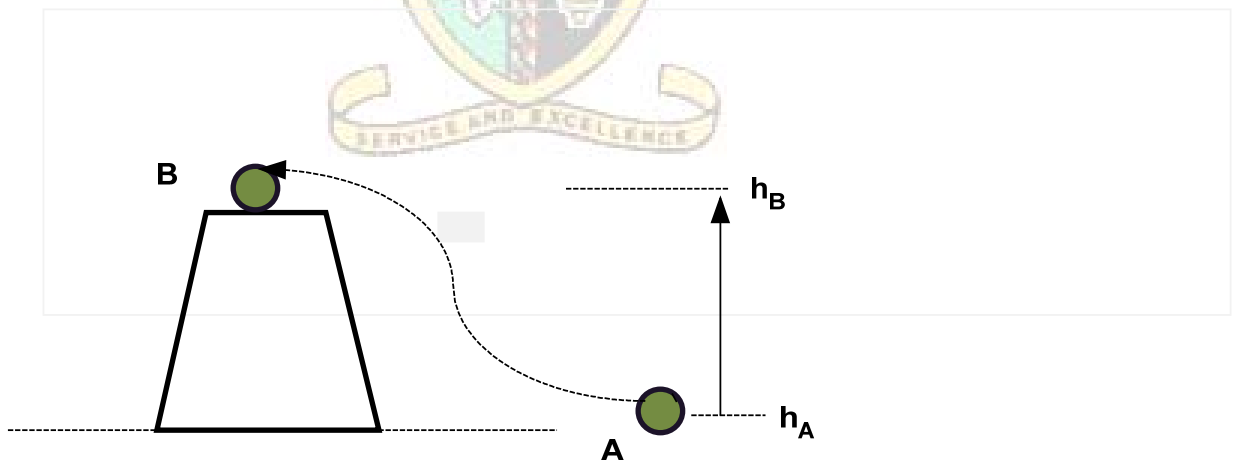
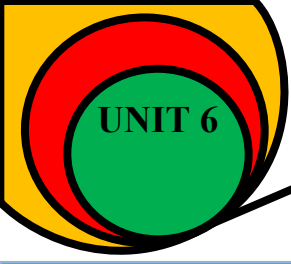


Figure 6.6: A conservative force



To lift the mass m from point A to point B, the work done against gravity is mgh .

Similarly in lowering the mass, the work done against gravity is $-mgh$. The gravitational force is an example of what we call a **conservative force**.

A force is said to be **conservative** if the work done in moving an object from A to B against the force is not dependent on the path taken for the movement.

An example of a **non-conservative** force is friction. The work done by the force depends on the path taken.



Self-help task 6.6

List some forces which are conservative.

5.7.1 Inter-conversion of kinetic and potential energy

Each time you throw a ball in the air or drop it there is an interchange of kinetic energy and potential energy.

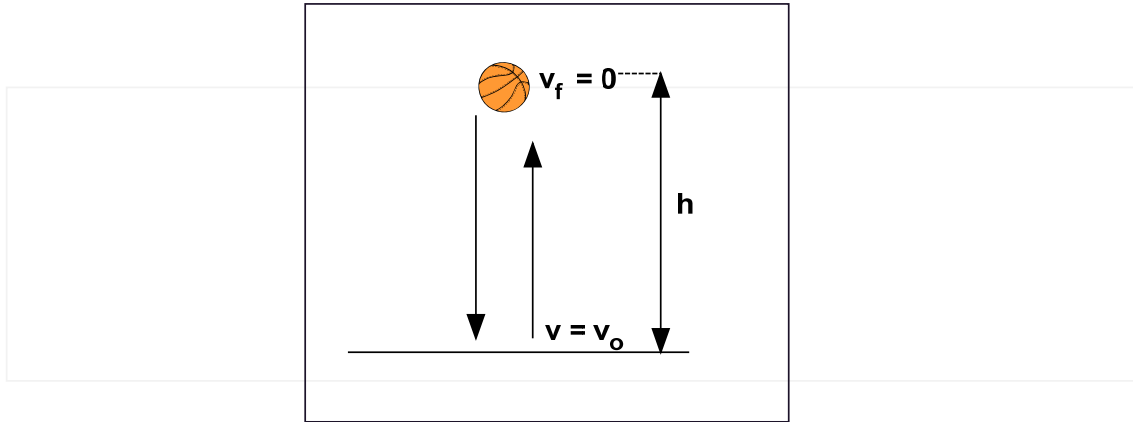


Figure 6.7: inter-conversion of energy

When the ball is at the bottom and is thrown with velocity v_o , then $KE = \frac{1}{2}mv_o^2$ and $PE = 0$.

When it is at the top, $v_f = 0$, so that $KE = 0$ but $PE = mgh$. Therefore

$$KE_{bottom} = PE_{top}$$

$$\frac{1}{2}mv_o^2 = mgh$$

Therefore when gravity alone acts on an object

$$\Delta KE + \Delta PE = 0 \tag{6.10}$$

For the previous case

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2 + mg(h_2 - h_1) = 0$$

$$0 - \frac{1}{2}mv_o^2 + mg(h - 0) = 0$$

$$\therefore mgh = \frac{1}{2}mv_o^2$$



5.7 Law of Conservation of Energy

When you keep in mind that energy is related to the ability to do work, it becomes clear that there are many other forms of energy. Coal, oil gasoline and other fuels possess energy which we call chemical energy. These can undergo chemical combustion and can do work.

The water at a hydroelectric power station has potential energy which then is converted to mechanical energy thereby turning the turbines and producing electrical energy. This is just one of the many other energy conversions that we have in nature.

This leads us to the concept of *energy conservation*. Energy conservation is a fundamental law of physics and is an enormously useful principle for the solution of problems.

Therefore for a closed system, when one form of energy is converted to another form, the sum total of energy remains the same. This is expressed mathematically in equation 6.10.

Another example of energy conservation is when you slide a book across a table. The kinetic energy you gave to the book disappears as the book comes to rest. Yet the book has not gained GPE since the floor is level. What happened to the KE, the original energy? To answer this, you should investigate the temperature of the book. You will discover that the temperature has increased. This is because of friction. The original KE has been entirely converted into doing work against friction and the result is the thermal energy TE that appears between the floor and the book. In this case, the energy conservation is



$$KE \Rightarrow TE_{\text{work against friction}}$$

$$\therefore KE = TE$$

or

$$\frac{1}{2}mv_o^2 = fs$$

In any physical process, there are always transformations of some forms of energy into other forms of energy. The law of conservation of energy states

“Energy can neither be created nor destroyed. When a loss in one form of energy occurs, an equal increase occurs in other forms.”

Mathematically we can write

$$\sum E_1 = \sum E_2$$

or

$$KE_1 + PE_1 + TE_1 = KE_2 + PE_2 + TE_2 \quad (6.11)$$

or

$$(KE_2 - KE_1) + (PE_2 - PE_1) + (TE_2 - TE_1) = 0$$

or

$$\Delta KE + \Delta PE + \Delta TE = 0 \quad (6.12)$$



Equation 6.11 is for energy conservation involving KE, PE and work against friction, the heat energy TE. TE_1 in this case is zero because, the object has not yet moved so the displacement is zero so that $TE_1 = fs = f \times 0 = 0$.



EXAMPLE 6.4: A 900 kg car is moving on a horizontal road at 20 m/s when its brakes are applied and the car skids to a stop in 30 m. Use the concept of work and energy to find the frictional force between the car's tyres and the road. Assume a horizontal road.

SOLUTION

We write the energy conservation law

$$PE_1 + KE_1 + TE_1 = PE_2 + KE_2 + TE_2$$

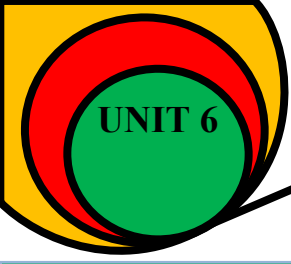
$$0 + \frac{1}{2}mv_o^2 + 0 = 0 + 0 + fs$$

$$\frac{1}{2}mv_o^2 = fs$$

$$\therefore f = \frac{\frac{1}{2}mv_o^2}{s} = 6000 \text{ N}$$

ΔGPE is zero because the track is horizontal. TE_1 is zero because at the time the brakes are applied the car has not yet moved any distance.

The heat generated in the tyres is $= fs = 6000 (30) = 180 \text{ kJ}$



Have you grasped the energy conservation concept? Here is a task for you...



Self-help task 6.7

A 50 kg crate falls off the roof of a building. By the time it hits the street 40 m below, it is moving at a speed of 20 m/s. Using the energy concepts find the average force of air drag during the fall.

Got it? Here is another example...



EXAMPLE 6.4: A particle of mass $m = 5.00$ kg is released from point **A** and slides on the frictionless track shown in Figure 6.8. Determine (a) the particle's speed at points **B** and **C** and (b) the net work done by the gravitational force in moving the particle from **A** to **C**.

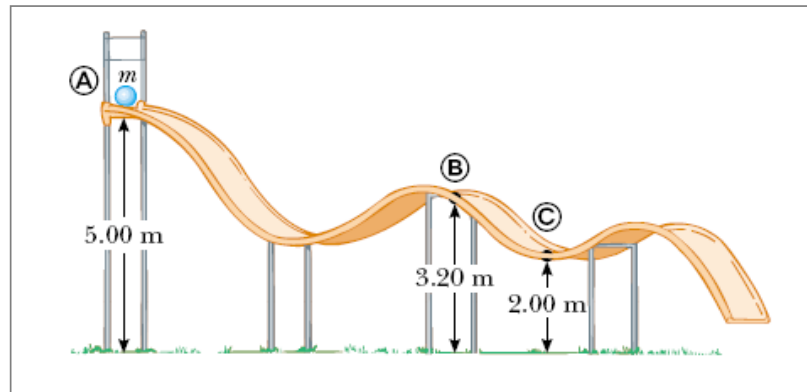


Figure 6.8: Example 6.4

(a): Velocity at A

$$PE_1 + KE_1 + TE_1 = PE_2 + KE_2 + TE_2$$

$$mgh_A + 0 + 0 = mgh_B + \frac{1}{2}mv_B^2 + 0$$

$$\therefore v_B = \sqrt{2g(h_A - h_B)} = \sqrt{2(9.8)(5.00 - 3.20)} = 5.44 \text{ m/s}$$

At B

$$PE_1 + KE_1 + TE_1 = PE_2 + KE_2 + TE_2$$

$$mgh_A + 0 + 0 = mgh_C + \frac{1}{2}mv_C^2 + 0$$

$$\therefore v_C = 7.67 \text{ m/s}$$

- (b) The net work is found from the work-energy theorem from equation 6.5, we have

$$W_{net} = \frac{1}{2}mv_C^2 - \frac{1}{2}mv_A^2$$

$$W = \frac{1}{2}mv_C^2 - 0 = \frac{1}{2}(5.00)(7.67)^2 = 147.07 \text{ J}$$

One more example in this section.



EXAMPLE 6.5: The coefficient of friction between the 3.00-kg block and the surface in Figure 6.9 is 0.400. The system starts from rest. What is the speed of the 5.00-kg ball when it has fallen 1.50 m?

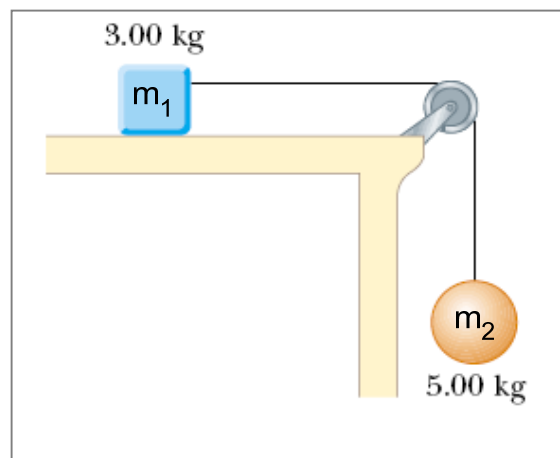


Figure 6.9: Example 6.5



SOLUTION

$$PE_1 + KE_1 + TE_1 = PE_2 + KE_2 + TE_2$$

The PE for m_1 is zero since m_1 is on a horizontal table. Therefore the potential energy change is just for m_2

Therefore

$$\therefore m_2gh_1 + 0 + 0 = mgh_2 + \frac{1}{2}(m_1 + m_2)v^2 + fs$$

When m_2 has fallen 1.5 m, the m_1 block moves 1.5 m so that $s = 1.5 = h$.

$$\therefore m_2g(h_1 - h_2) - fh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v = \frac{2[m_2g(1.5 - 0) - (\mu m_1g)h]}{m_1 + m_2} = 3.74 \text{ m/s}$$

This same problem can be solved using Newton's laws of motion. You therefore have two methods of solving such problems.

6.8 Springs: Elastic Potential Energy

There are many situations in which we encounter potential energy that is not gravitational in nature. One example is a catapult or a rubber slingshot. Work is done in stretching it. In this stretched position, it has potential energy or potential energy has been stored such that when released the PE is converted to KE and the stone flies off. This PE is called the **elastic potential energy (EPE)**.

Another example is a spring. See Figure 6.10. A force F_p is required to stretch the spring from the equilibrium position $x = 0$, to a new position x ,

$$F_p = kx \tag{6.13}$$

while the spring exerts a restoring force, F_s in the direction of opposite to F_p so that

$$F_s = -kx \tag{6.14}$$

The same is true for compression. K is the force constant in N/m .

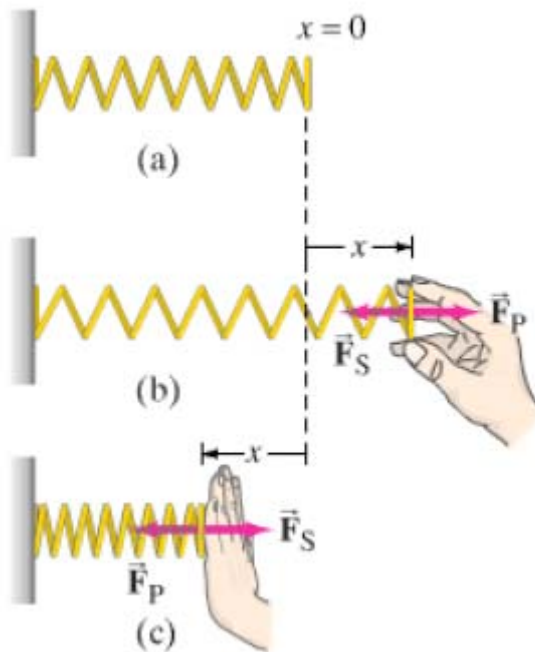


Figure 6.10: Stretching or compressing a spring



This force as you can see is a variable force of the kind discussed in Section 6.3.2. From equation 6.3, the work done on the spring to stretch it is

$$W = \int_0^x F(x)dx$$
$$= \int_0^x kx dx = \frac{1}{2} kx^2$$

This is the work required to stretch the spring or compress it to a position x from the equilibrium position. Therefore from the energy conversion, in the stretched position, it is not moving so that $KE = 0$ and all the energy is converted to the EPE.

$$\therefore EPE = \frac{1}{2} kx^2 \quad (6.15)$$

Using the law of conservation, we have

$$\Delta KE + \Delta PE + \Delta TE + \Delta EPE = 0 \quad (6.16)$$



EXAMPLE 6.6: Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 6.11. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k . The system is released from rest when the spring is un-stretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

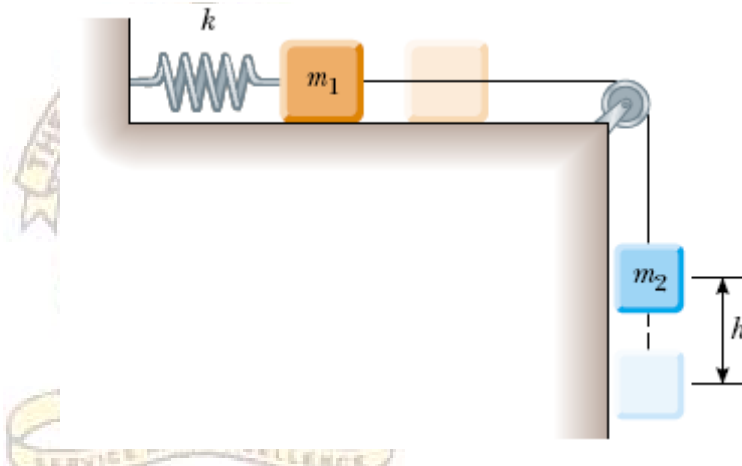


Figure 6.11: potential energy in a spring

SOLUTION

Using the conservation of energy, we see that the gravitational potential energy possessed by m_2 is converted to work done against friction and elastic potential energy in stretching the spring.



Therefore

$$E(\text{total})_1 = E(\text{total})_2$$

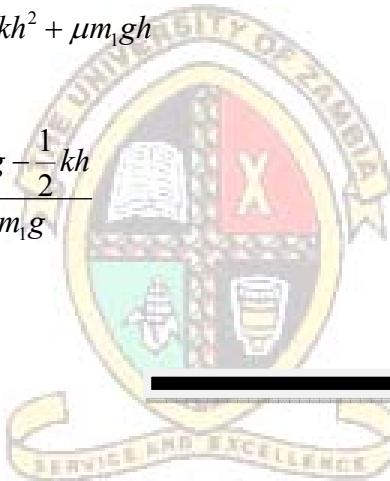
$$KE_1 + PE_1 + EPE_1 + TE_1 = KE_2 + PE_2 + EPE_2 + TE_2$$

$$0 + m_2gh + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2 + fs$$

The spring is stretched by h ,

$$m_2gh = \frac{1}{2}kh^2 + \mu m_1gh$$

$$\therefore \mu = \frac{m_2g - \frac{1}{2}kh}{m_1g}$$





6.9 Summary

- Work is done on an object by a force when the object moves through a distance s . If the direction of the constant force makes an angle θ with the direction of motion, then the work done is

$$W = Fs \cos \theta$$

- For a varying force, the work done in moving an object from x_i to x_f is given by the area under the $F(x)$ vs x graph. Mathematically

$$W = \int_{x_i}^{x_f} F(x) dx$$

- KE is the energy of motion, $KE = \frac{1}{2}mv^2$
- Potential energy is associated with forces that depend on position of the objects. It can be gravitational PE, given by

$$GPE = mgh$$

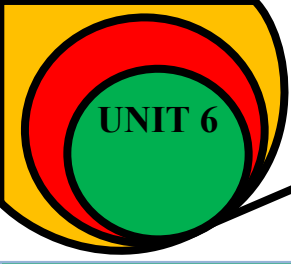
Or elastic PE, given

$$EPE = \frac{1}{2}kx^2$$

- The work-energy theorem states that the net work done on an object (by a net force) equals the change in KE.

$$W_{\text{net}} = F_{\text{net}}x = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

The law of conservation of energy states that: **“Energy can neither be created nor destroyed. When a loss in one form of energy occurs, an equal increase occurs in other forms.”**



Work, Energy and Power



That's it!

Do all the exercises and prepare for the exam!



6.10 References

F.J Bueche, D.A Jerde, *Principles of Physics*.6th Edition, McGraw-Hill, 1995.

ISBN 0- 13.978-00700

D. Giancoli, *Physics:Principles with Applications*, 6th Edition.Pearson Prentice Hall, ISBN 0-13-060620-0

P.M Fishbane, S.G Gasiorowicz,S.T Thornton, *Physics for Scientists and Engineers with Modern Physics*,3rd Edition, Pearson Prentice Hall , ISBN 0-13-191182-1

Sears and Zemansky, *University Physics (with Modern Physics)*, 12th Edition, Hugh D Young and Roger, A Freedman,Pearson/Addison Wesley,2007, ISBN 0-8053-2187-X.



6.11 Answers to Self-help Questions

6.1 Indeed you do no work carrying something horizontally. The reason we get tired is because of the lactic acid which is produced in the muscles during contractions.

6.2 We first use the Newton's second law to find the force.

$$F \cos 37 - 20 = ma = 0$$

$$\therefore F = \frac{20}{\cos 37} = 25 \text{ N}$$

work

$$W = Fs \cos \theta = F \cos 37 \times 5.0 = 100 \text{ N}$$

6.3 The variable force and situations are: The restoring force on a spring (force is proportional to the distance from its equilibrium), The force of gravity experienced by a rocket as it leaves the earth's gravity, the higher it goes the less the pull of gravity, electrostatic force experienced by a charge in the vicinity of another charge as well as magnetic forces.



6.4 Since this is a varying force,

$$W = \int_{x_i}^{x_f} F(x) dx$$

$$= \int_0^x kx dx = \frac{1}{2} kx^2$$

6.5 The work-energy theorem states that

$$F_{net} x = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$$

$$(100 - f)x = \frac{1}{2} mv_2^2 - 0$$

$$(100 - \mu mg)5 = \frac{1}{2} mv_2^2$$

$$\therefore v_2 = 20.8 \text{ m/s}$$

6.6 -Magnetic forces

-Electrostatic forces

6.7 We use the energy conservation law

$$PE_1 + KE_1 + TE_1 = PE_2 + KE_2 + TE_2$$

$$mgh_1 + 0 + 0 = 0 + \frac{1}{2} mv_2^2 + f_{drag} h$$

$$\therefore f_{drag} = \frac{(mgh_1 - \frac{1}{2} mv_2^2)}{h} = 240 \text{ N}$$



6.12 EXERCISES

- 6.1 When a particle rotates in a circle, a force acts on it directed toward the center of rotation. Why is it that this force does no work on the particle? [Because the angle between the direction of motion and the force is at 90°]
- 6.2 Can kinetic energy be negative? Explain. [Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.]
- 6.3 A block of mass 2.50 kg is pushed 2.20 m along a frictionless horizontal table by a constant 16.0-N force directed 25.0° below the horizontal. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, and (c) the gravitational force. (d) Determine the total work done on the block. [(a) 31.9 J, (b) 0, (c) 0, (d) 31.9 J]
- 6.4 A motor boat requires 100 hp to move at a constant speed of 16 m/s. What is the resistive force due to water at that speed? [4660 N]



- 6.5 An object of mass m starts from rest and slides a distance d down a frictionless incline of angle θ . While sliding, it contacts an unstressed spring of negligible mass as shown in Figure 6.12. The object slides an additional distance x as it is brought momentarily to rest by compression of the spring (of force constant k).

Find the initial separation d between object and spring. $[d = \frac{kx^2}{2mg \sin \theta} - x]$

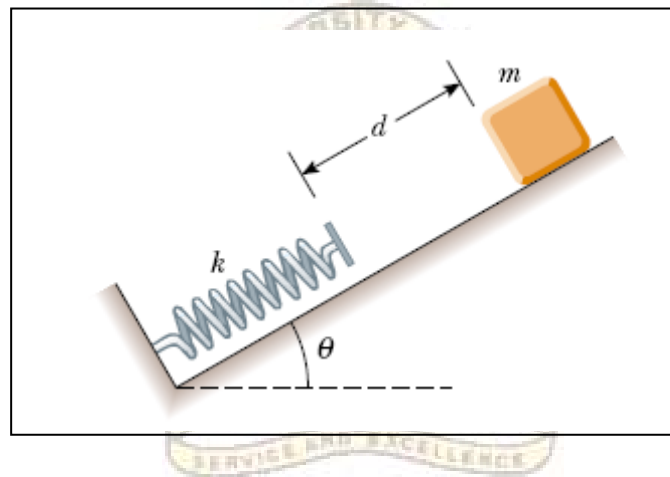


Figure 6.12: Problem 6.5

- 6.6 A 625 g block starts to slide up a 30° incline with an initial speed of 2.2 m/s. It stops after sliding. It stops after sliding 40 cm and slides back down. Assuming friction force impeding the motion to be constant, (a) how large is the friction force and (b) what is the block's speed as it reaches the bottom. [(a) 0.719 N (b) 1.73 m/s]



- 6.7 The roller-coaster car shown in Figure 6.13 is dragged up to point 1 where it is released from rest. Assuming no friction, calculate the speed at points 2, 3 and 4. [$v_2 = 26 \text{ m/s}$, $v_3 = 12 \text{ m/s}$, $v_4 = 20 \text{ m/s}$]

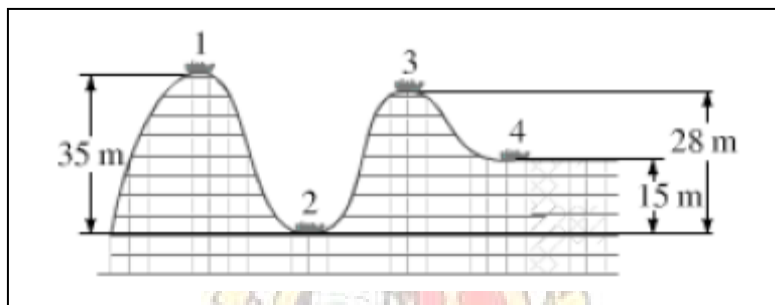


Figure 6.13: roller-coaster

- 6.8 A 280-g wood block is firmly attached to very light horizontal spring, Figure 6.14. The block can slide along a table where the coefficient of friction is 0.30. A force of 22 N compresses the spring 18 cm. If the spring is released from this position, how far beyond its equilibrium position will it stretch at its first maximum extension? [0.17 m]

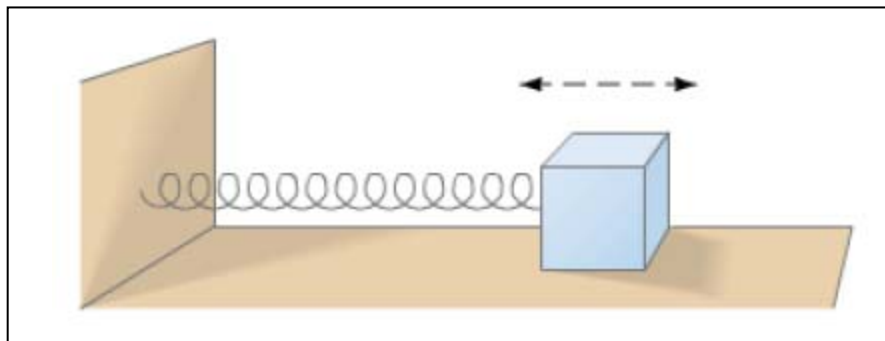


Figure 6.14: mass-spring system