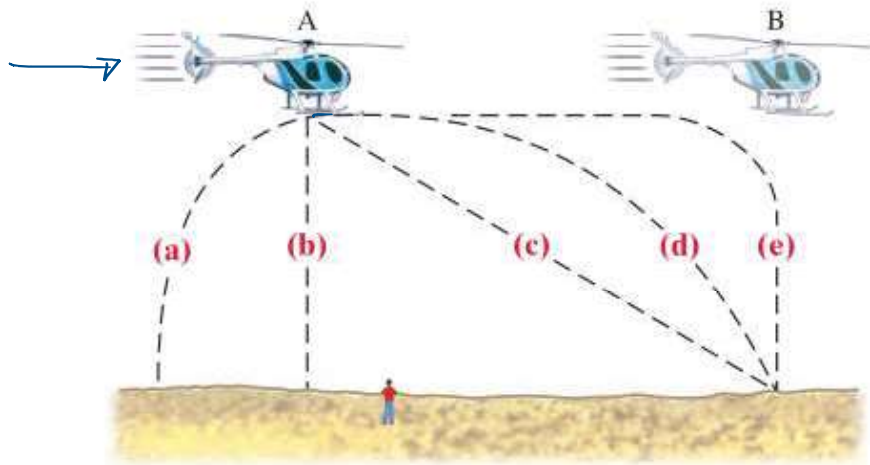


Projectile Motion



85

- An object that is thrown out (projected) is called a projectile.
- The path is called the trajectory.
- The trajectory of a projectile is a parabola.

86

Vertically they all fall with the same speed.

gedanken experiment
- thought exp

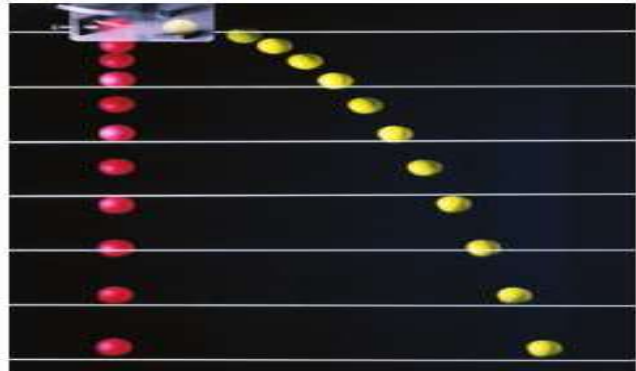
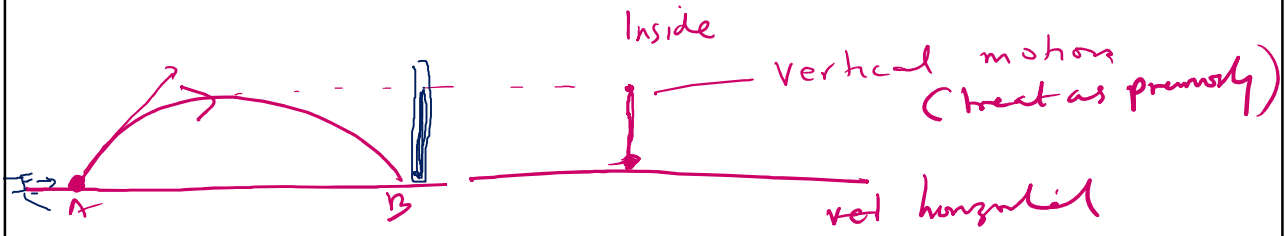


FIGURE 3-19 Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other ball was projected horizontally outward. The vertical position of each ball is seen to be the same at each instant.

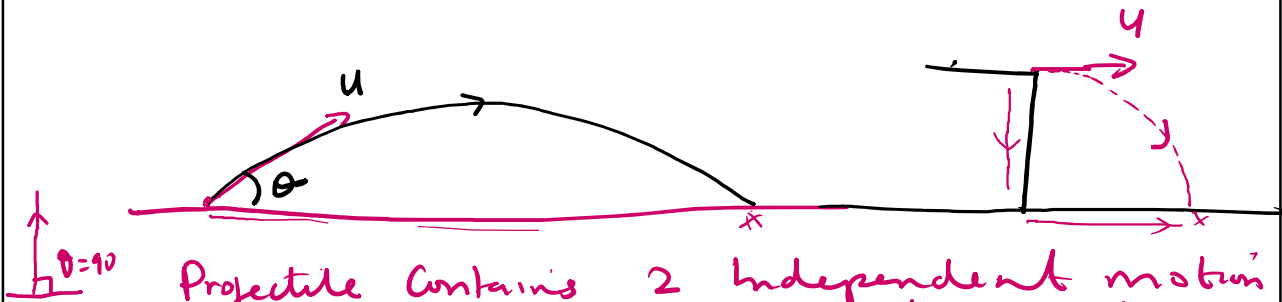
Gedanken experiment

This is a thought exp.



Projectile Motion

- This is the motion of an object projected at an angle and under g .

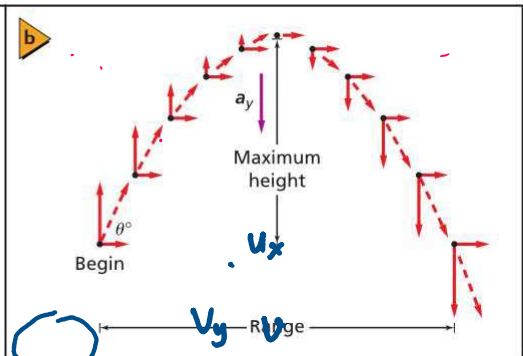
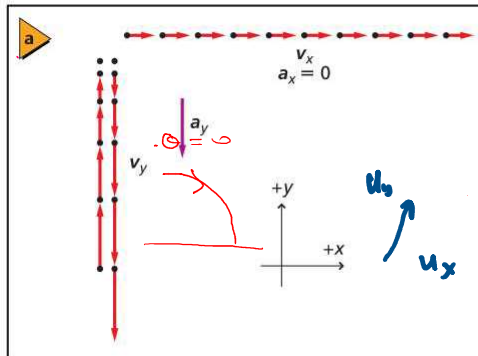
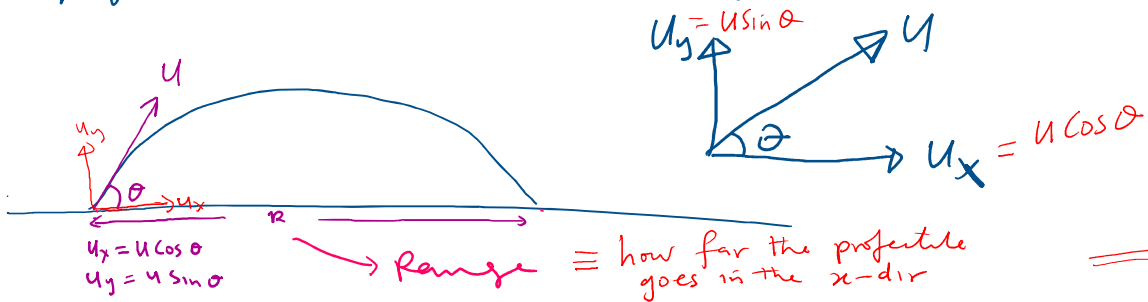


Projectile contains 2 independent motion motions but happening simultaneously.

1. Horizontal Motion.
2. Vertical motion.

89

Projectile Fired at an angle.



90

- Sm Procedure

- ▣ Since the two motions (horizontal & vertical) are independent but occur simultaneously, they are dealt with separately. The only parameter connecting the two is time since they happen at the same time

Horizontal	Vertical
$a = 0$	$a = -g$

91

- Sm Procedure

- ▣ Since the two motions (horizontal & vertical) are independent but occur simultaneously, they are dealt with separately. The only parameter connecting the two is time since they happen at the same time

Horizontal	Vertical
$a = 0$ $u_x = u \cos \theta$ $u_x = v_y = \text{constant}$	$a = -g$ $u_y = u \sin \theta$

92

- Sm Procedure

- Since the two motions (horizontal & vertical) are independent but occur simultaneously, they are dealt with separately. The only parameter connecting the two is time since they happen at the same time

Horizontal	Vertical
$a = 0$ $u_x = u \cos \theta$ $S_x = u_x t + \frac{1}{2} a t^2$ $= \text{Range}$ $R = u_x t$	$a = -g$ $u_y = u \sin \theta$ $S_y = u_y t - \frac{1}{2} g t^2$ $v_y = u_y + a t$ $v_y^2 = u_y^2 + 2 a S_y$
$t \leftrightarrow t$	

93

$$\text{Range} = R = u_x t$$

$$= u \cos \theta t$$

Range is max when
 $\theta = 45^\circ$

94

time of flight

$T = \frac{2 \times 27 \times \sin 30}{9.8}$

$0 = (u \sin \theta)t - \frac{1}{2}gt^2$

$$t = \frac{2u \sin \theta}{g}$$

time of flight
It applies to a perfect parabola

$a = -g$

$u_y = u \sin \theta$

$y = u_y t + \frac{1}{2}at^2$

$y = u_y t - \frac{1}{2}gt^2$

disp = 0

$0 = u_y t - \frac{1}{2}gt^2$

95

Special cases

$\theta = 0$

$\theta = 90$

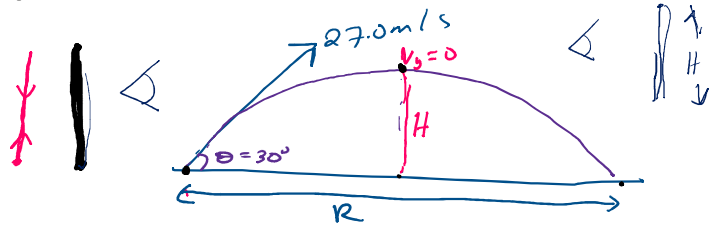
vertical motion

$\theta = 0$

96

A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal. Find each of the following. Assume that air resistance is negligible.

- the ball's hang time
- the ball's maximum height
- the ball's range



Horizontal : $a=0$

$$\checkmark U_x = U \cos \theta$$

$$= 27 \cos 30 = 23.4 \text{ m/s}$$

$$R = U_x t = 23.4 (2.76)$$

$$= \underline{64.6 \text{ m}}$$

Vertical : $a=-g$

$$\checkmark U_y = U \sin 30$$

$$= 27 \sin 30 = 13.5 \text{ m/s}$$

$$y = U_y t - 4.9 t^2 ; y=0$$

$$0 = 13.5 t - 4.9 t^2 ; t = 13.5 / 4.9 = 2.76 \text{ s}$$

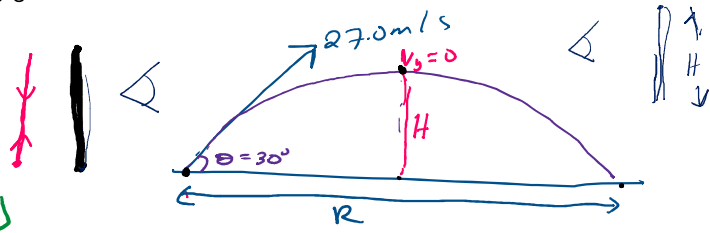
$$U_y^2 = U_y^2 - 2g y \quad (v_y^2 = U_y^2 - 2g y)$$

$$0 = 13.5^2 - 2g y , y = \frac{13.5^2}{19.6} = 9.3 \text{ m}$$

97

A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal. Find each of the following. Assume that air resistance is negligible.

- the ball's hang time
- the ball's maximum height
- the ball's range



Horizontal : $a=0$

$$\checkmark U_x = U \cos \theta$$

$$= 27 \cos 30 = 23.4 \text{ m/s}$$

Vertical : $a=-g$

$$\checkmark U_y = U \sin 30$$

$$= 27 \sin 30 = 13.5 \text{ m/s}$$

$$S_y = U_y t - 4.9 t^2$$

$$0 = 13.5 t - 4.9 t^2$$

$$t = \underline{2.76 \text{ s}}$$

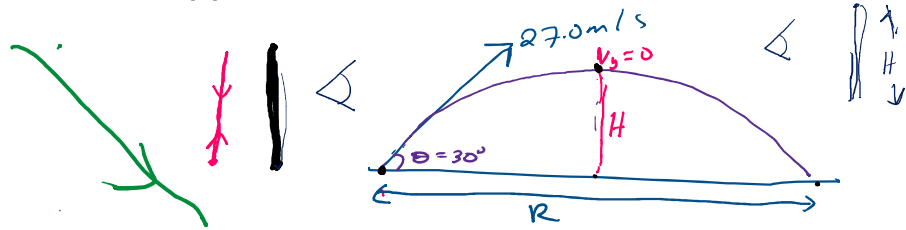
$$t = \frac{2u \sin \theta}{g}$$

$$= \frac{2(27) \sin 30}{9.8}$$

98

A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal. Find each of the following. Assume that air resistance is negligible.

- the ball's hang time
- the ball's maximum height
- the ball's range



Horizontal : $a=0$

$$\checkmark U_x = U \cos \theta$$

$$= 27 \cos 30 = 23.4 \text{ m/s}$$

Vertical : $a=-g$

$$\checkmark U_y = U \sin 30$$

$$= 27 \sin 30 = 13.5 \text{ m/s}$$

$$v_y^2 = u_y^2 + 2 a s_y$$

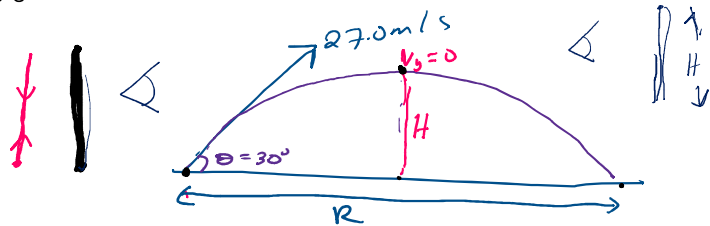
$$0 = 13.5^2 - 2(9.8) s_y$$

$$s_y = \frac{13.5^2}{19.6} = 9.3 \text{ m}$$

99

A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30.0° above the horizontal. Find each of the following. Assume that air resistance is negligible.

- the ball's hang time
- the ball's maximum height
- the ball's range



Horizontal : $a=0$

$$\checkmark U_x = U \cos \theta$$

$$= 27 \cos 30 = 23.4 \text{ m/s}$$

$$\text{Range} = U_x t$$

$$= 23.4 (2.76)$$

$$= 64.6 \text{ m}$$

Vertical : $a=-g$

$$\checkmark U_y = U \sin 30$$

$$= 27 \sin 30 = 13.5 \text{ m/s}$$

100

$u_x = u$
 $u_y = 0$

Projectile fire horizontally

$u_x = u$
 $u_y = 0$

$\theta = 0$

$u_x = u \cos \theta = u \cos 0 = u$
 $u_y = u \sin \theta = u \sin 0 = 0$

Diagram (a) shows a horizontal path with $a_x = 0$ and $a_y = -g$.
 Diagram (b) shows the parabolic path with velocity components u_x and v_y at various points.

101

$u_x = u$
 $u_y = 0$

or
 $u_x = u \cos 0 = u$
 $u_y = u \sin 0 = 0$

H	V
$a = 0$ $u_x = u$ $R = u_x t$	$a = -g$ $u_y = 0$ $v_y = u_y - g t, y = u_y t - \frac{1}{2} g t^2$ $v_y^2 = u_y^2 - 2 g y$

$v_y = u_y + a t$
 $v_y = u_y - g t$

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► PROBLEM-SOLVING Strategies

Motion in Two Dimensions

Projectile motion in two dimensions can be determined by breaking the problem into two connected one-dimensional problems.

1. Divide the projectile motion into a vertical motion problem and a horizontal motion problem.
2. The vertical motion of a projectile is exactly that of an object dropped or thrown straight up or straight down. A gravitational force acts on the object and accelerates it by an amount, g . Review Section 3.3 on free fall to refresh your problem-solving skills for vertical motion.
3. Analyzing the horizontal motion of a projectile is the same as solving a constant velocity problem. No horizontal force acts on a projectile when drag due to air resistance is neglected. Consequently, there are no forces acting in the horizontal direction and therefore, no horizontal acceleration; $a_x = 0.0$ m/s. To solve, use the same methods that you learned in Section 2.4.
4. Vertical motion and horizontal motion are connected through the variable of time. The time from the launch of the projectile to the time it hits the target is the same for both vertical motion and horizontal motion. Therefore, solving for time in one of the dimensions, vertical or horizontal, automatically gives you time for the other dimension.

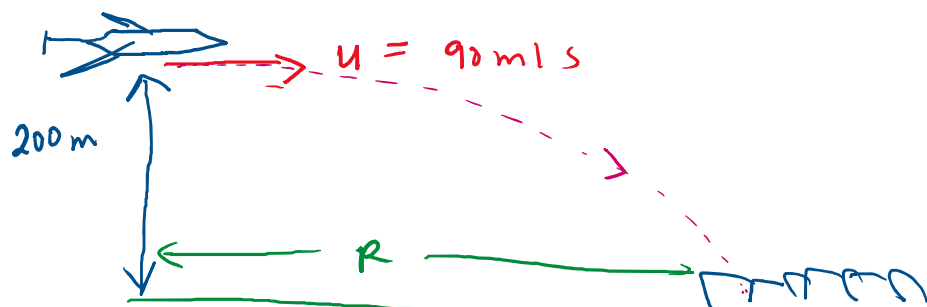
- In (Problems) # Since the two motions (horizontal and vertical) are independent but occur simultaneously, they are dealt with separately. In any problem, connect the two to time the key factor of the two time.

Horizontal	Vertical
$a = 0$	$a = -g$

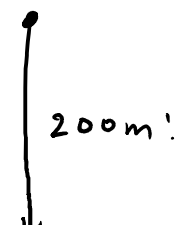
103

An aeroplane carrying out a parcel drop releases a parcel while travelling at a steady speed of 90 m/s at an altitude of 200 m.

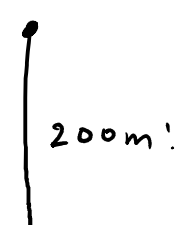
- a) time b/w the parcel leaving plane & it striking the ground **6.45**
- b) the horizontal distance travelled
- c) the speed at which the parcel strikes the ground.




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Horizontal	Vertical
$a = 0$ $u_x = 90 \text{ m/s}$	$a = -g$ $u_y = 0$ a)  $s_y = u_y t - 4.9 t^2$ $-200 = -4.9 t^2$ $t = 6.39 \text{ s}$

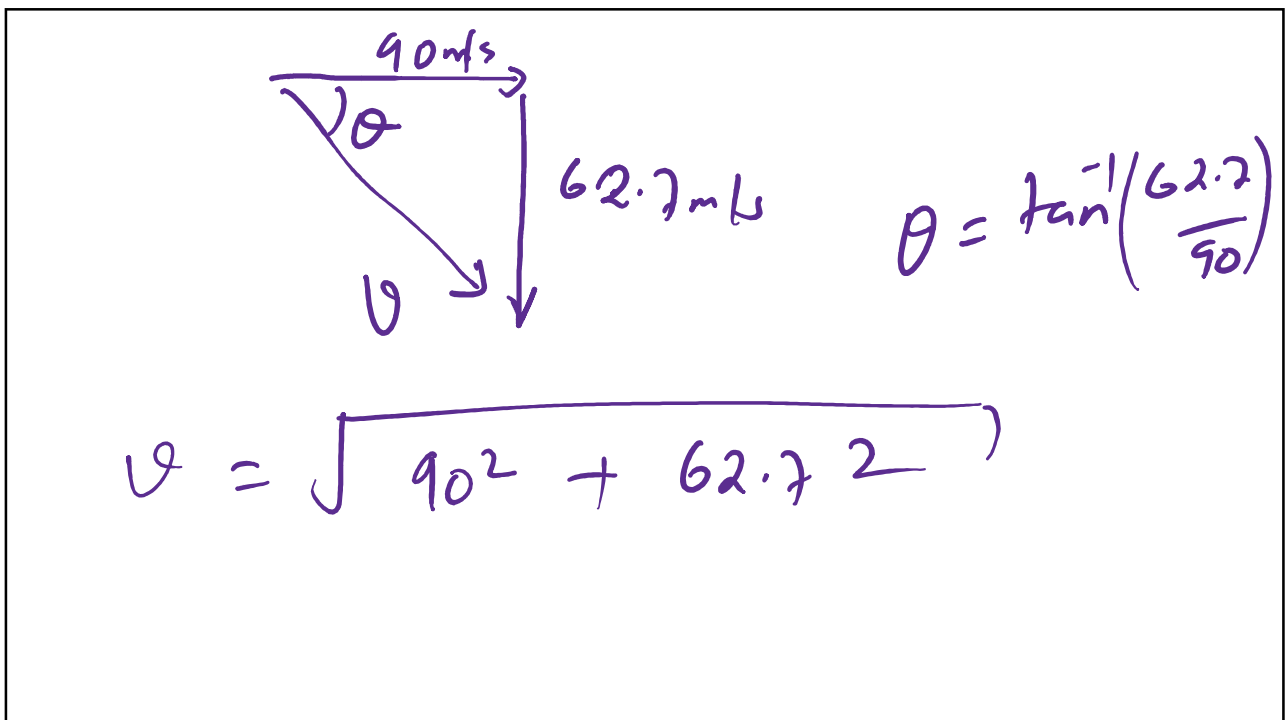
105

Horizontal	Vertical
$a = 0$ $u_x = 90 \text{ m/s}$ b) $R = u_x t$ $= 90 (6.39)$ $\approx \underline{575 \text{ m}}$	$a = -g$ $u_y = 0$ a)  $s_y = u_y t - 4.9 t^2$ $-200 = -4.9 t^2$ $t = 6.39 \text{ s}$

106

Horizontal	Vertical
$a = 0$ $u_x = 90 \text{ m/s}$ $u_x = 90 = v_x$	$a = -g$ $u_y = 0$ a)  $v_y = u_y + at$ $= 0 + (9.8)6.39$ $= 62.7 \text{ m/s}$

107



108

Monkey - Hunter Puzzle

CONCEPTUAL EXAMPLE 3-7 The wrong strategy. A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance d away, Fig. 3-24. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.) Ignore air resistance.

RESPONSE Both the water balloon and the boy in the tree start falling at the same instant, and both have the same initial downward acceleration, g . (See Fig. 3-24.) At the time t a certain distance y below the initial horizontal direction, the balloon will have the same y position as the falling boy. If the boy had stayed in the tree, he would have avoided the water balloon.

109

Monkey - Hunter Puzzle (Project Tranquilize)

<https://ophysics.com/k10.html>



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