Tutorial Solutions

Tutorial 1 Vectors:

PHY1010 Tutorial 1 (2018)

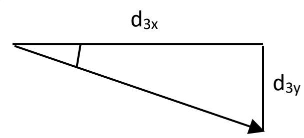
01. D = $\mathbf{d_1 + d_2 + d_3}$

$D_x = d_{1x} + d_{2x} + d_{3x}$ or 100 cos 37 = -100 + 200 cos 150 + d_{3x} giving

d_{3x} = 353m. $D_y = d_{1y} + d_{2y} + d_{3y}$ or 100 sin 37 = 0 + 200 sin 150 + d_{3y} giving

d_{3y} = -40m. Hence $d_3 = \sqrt{d_{3x}^2 + d_{3y}^2} = 355m$ The angle

$\phi = \tan^{-1}\dfrac{d_{3y}}{d_{3x}} = 6.5^0$ below the $+x-$axis



= 360 –6.5 = 353.5^0

02.

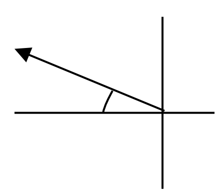
A + B = $\sqrt{-8.39^2 + 5.84^2}$ =10.22m

Tan θ = $\dfrac{5.84}{8.39}$ giving θ = 34.84^0

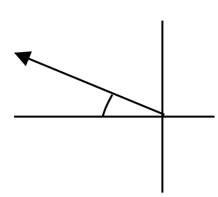
above the –x-axis = 145.16^0

	x	y
A	30 cos 155 = -27.19m	30 sin 155 = 12.68
B	20 cos 340 = 18.80	20 sin 340 = - 6.84

	x	y
A	30 cos 155 = -27.19	30 sin 155 =12.68
-B	= -18.80	= + 6.84



A - B = $\sqrt{-45.99^2 + 19.52^2}$ =49.56m

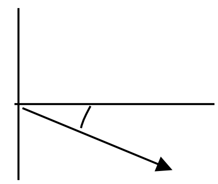


Tan $\theta = \frac{19.52}{45.99}$ giving $\theta = 23^0$ above the $-x$-axis = 157^0

B - A = $\sqrt{45.99^2 + (-19.52)^2}$ =49.56m

[Tan $\theta = \frac{19.52}{45.99}$ giving $\theta = 23^0$ below the $+x$-axis = 337^0

	x	y
-A	= 27.19	= - 12.68
B	= 18.80	= - 6.84



03* a particle undergoes three successive displacements in a plane as follows: 4.0m southwest, 5.0m east, and 6.0 m in a direction 60° north of east.

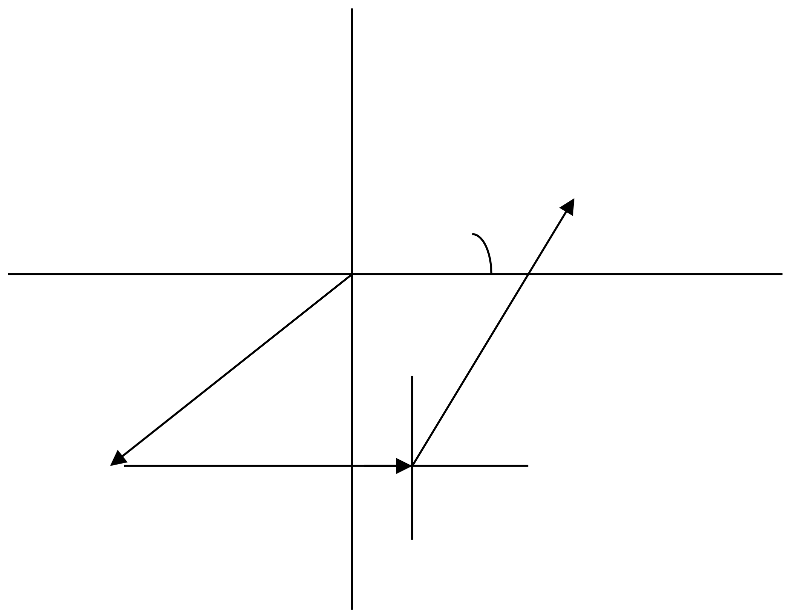
Choose the y-axis pointing north and the x-axis pointing east and find the components of each displacement,

i) The components of the resultant displacement,
ii) The magnitude and direction of the resultant displacement, and
iii) Graphically show the approximate displacement of the particle.

	x	*y*
A	$4\cos 225 = -2.83$	$4\sin 225 = -2.83$
B	+5	0
C	$6\cos 60 = 3$	$6\sin 60 = 5.20$
	$R_x = 5.17$	$R_y = 2.37$

Resultant $R = \sqrt{5.17^2 + 2.37^2} = 5.69$m. In the 1st quadrant.

$$\phi = \tan^{-1}\frac{2.37}{5.17} \rightarrow \phi = 24.63^0$$



Tutorial 2

PHY1010 2018 Tutorial Sheet 2

01. Consider the different time periods:

0 – 2 sec: $v_{avg} = 4m/s,\ x_{0\to 2} = v_{avg}t = 4\frac{m}{s}\times 2s = 8m$

2 – 10 sec: $v_{avg} = v = 8m/s(cons\tan t),\ x_{2\to 10} = 8\frac{m}{s}\times(10-2)s = 64m$

10 – 12 sec: $v_{avg} = 6m/s,\ x_{10\to 12} = 6\frac{m}{s}\times(12-10)s = 12m$

12 – 16 sec: $v_{avg} = 4m/s,\ x_{12\to 16} = 4\frac{m}{s}\times(16-12)s = 16m$

Hence $x_{0-16} = x_{0-2} + x_{2-10} + x_{10-12} + x_{12-16} = 100m$

(ii) acceleration at $t = 1\sec$

Period 0 – 2 sec shows constant acceleration

$$a_1 = \frac{v_f - v_0}{t_f - t_0} = \frac{(8-0)m/s}{(2-0)\mathrm{s}} = 4m/s^2$$

Period 10 – 12 sec also shows constant acceleration. At $t = 11\sec$ we have

$$a_{11} = \frac{v_f - v_0}{t_f - t_0} = \frac{(4-8)m/s}{(12-10)\mathrm{s}} = -2m/s^2\,(deceleration)$$

02. (a) The truck decelerates from $v_0 = 22.5m/s$ to $v_f = 0$ with an acceleration of $a = -2.27m/s^2$, so the time taken to stop is $t = \frac{v_f - v_0}{a} = \frac{0-22.5}{-2.27} = 9.91\sec$

(b) The distance travelled is $x = v_{avg}t = 22.5\times 9.91/2 = 112m$

(c) The distance moved after 3 sec is $x_3 = v_0 \times 3.0 + \frac{1}{2} a \times 3^2 = 57.3m,$ while the distance travelled after 2.0 sec is $x_2 = v_0 \times 2.0 + \frac{1}{2} a \times 2^2 = 40.5m.$ Therefore, the distance moved during the third second is $x_3 - x_2 = 57.3 - 40.5 = 16.8\,m$

03. Write the equations of motion for the two balls:

First ball: $S_1 = ut + \frac{1}{2} gt^2 \; with \; u = 0 \rightarrow S_1 = \frac{1}{2} gt^2 = 0.5 \times 9.8t^2 = 4.9t^2$

Second ball: $S_2 = u(t-2) + \frac{1}{2} g(t-2)^2 \; with \; u = 30m/s \rightarrow S_2 = 30(t-2) + 4.9(t^2 - 4t + 4)$

$= 30t - 60 + 4.9t^2 - 19.6t + 19.6$

Second ball catches up when

$S_1 = S_2 \rightarrow 4.9t^2 = 30t - 60 + 4.9t^2 - 19.6t + 19.6 \rightarrow 0 = 10.4t - 40.4$

Or $t = \frac{40.4}{10.4} = 3.88s$. Second ball will pass first ball after 3.88 seconds.

04. Take 'down' as the positive direction.

Vertical component of the initial velocity is $v_{0y} = -200\sin 35 = -115m/s.$

Find the projectile's time of flight $t.$ Vertical distance dropped by the projectile is $y = 300 = v_{oy}t + \frac{1}{2} gt^2 = -115t + 4.9t^2$. Solving this we get $t = 25.8\sec.$

Thus the range of the projectile is $x = v_{0x}t = 200\cos 35 \times 25.8 = 4.23 \times 10^3 m$

05. Take the origin of coordinates at the top of the ramp and take $+y$ to be upward. The object is displaced 40 m to the right when it is 15 m below the origin.

We have to find the time of flight, and the initial velocity. Write the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one uncommon.

Y-component: $y - y_0 = -15m \quad a_y = -9.8m/s^2 \quad v_{0y} = v_0 \sin 53$

$$y - y_0 = v_{oy}t + \frac{1}{2}a_y t^2 \rightarrow -15 = v_0 \sin 53 \times t - 4.9t^2$$

X-component: $x - x_0 = 40m \quad a_x = 0 \quad v_{0x} = v_0 \cos 53$

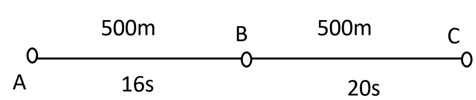
$$x - x_0 = v_{ox}t + \frac{1}{2}a_x t^2 \rightarrow 40 = v_0 \times t \times \cos 53$$

$$40 = v_0 \times t \times \cos 53 \rightarrow v_0 \times t = \frac{40}{\cos 53} = 66.47m \ -15 = v_0 \sin 53 \times t - 4.9t^2 = 66.47 \sin 53 - 4.9t^2$$

Hence $t = \sqrt{\frac{66.47 \sin 53 + 15}{4.9}} = 3.728 \sec$

Or $v_0 = \frac{40}{t \times \cos 53} = \frac{40}{3.728 \times \cos 53} = 17.8m/s$

06.



500 = v_A × 16 + ½ a × 16^2

= 16 (v_A + 8a) or 500/16 =31.25

= v_A + 8a

1000 = v_A × 36 + ½ a × 36^2 = 36 (v_A + 18a) or 1000/36 = 27.78 = v_A + 18*a*

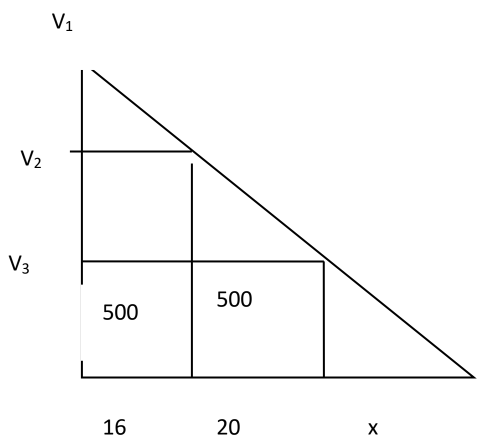
Hence 18*a* – 8*a* = 27.78 – 31.25 = -3.47 or 10 *a* = -3.47 hence *a* = - 0.347m/s^2

Then v_A = 27.78 - 18 × (-0.347) = 34.03m/s and v_C = v_A + 36 × (-0.347)

= 34.03 – 12.49 = 21.54 m/s

$$S = \frac{21.54^2}{2\times 0.347} = 669m$$

Q6.(2)



$$\frac{1}{2}(v_1+v_2)16 = 500 \rightarrow 8v_1 + 8v_2 = 500 \rightarrow v_1 + v_2 = 62.5$$

$$\frac{1}{2}(v_2+v_3)20 = 500 \rightarrow 10v_1 + 10v_2 = 500 \rightarrow v_1 + v_2 = 50$$

$$\frac{1}{2}(v_1+v_3)36 = 1000 \rightarrow v_1 + v_3 = 500/9 \text{ giving } v_3 = 21.65m/s \quad and \quad x =$$

07. Assume a period t_1 for acceleration from $v_0 = 0 \; to \; v_1$, reaching point x_1, and period t_2 decelerating from v_1 to $v_2 = 0$ to reach point x_2.

Total distance covered from $x_1 \; to \; x_2 = 500\,m.$

Use $v_f = v_0 + at$

For the first part: $v_1 = v_0 + a_1 t_1 \rightarrow v_1 = 6t_1$

For the second part $v_2 = v_1 + a_2 t_2 \rightarrow 0 = v_1 - 8t_2 \rightarrow v_1 = 8t_2$

Hence $6t_1 = 8t_2 \rightarrow t_2 = \frac{3}{4}t_1$

Now use $x = v_0 t + \frac{1}{2}at^2$

This gives for the first part: $x_1 = 0 + \frac{1}{2}a_1 t_1^2 = 3t_1^2$ (1)

For the second part: $500 - x_1 = v_1 t_2 + \frac{1}{2}a_2 t_2^2 = v_1 t_2 + 4t_2^2$

Substitute $v_1 = 6t_1 \; and \; t_3 = \frac{3}{4}t_1$ to obtain

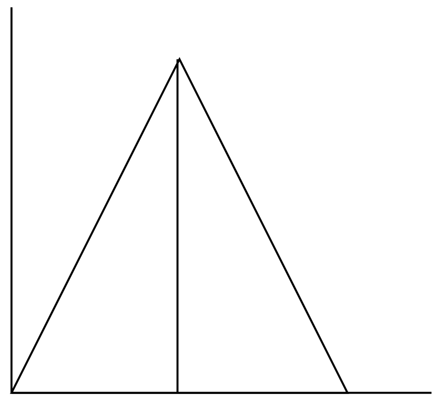
$$500 - x_1 = 6t_1 \times \frac{3}{4}t_1 - 4 \times \left(\frac{3}{4}t_1\right)^2 = \frac{18}{4}t_1^2 - \frac{9}{4}t_1^2 = \frac{9}{4}t_1^2 \quad (2)$$

(1)+(2) gives $500 = 3t_1^2 + \frac{9}{4}t_1^2 = \frac{21}{4}t_1^2 \rightarrow t_1 = \sqrt{\frac{2000}{21}} = 9.76\,\text{sec}$

And $t_2 = \frac{3}{4}t_1 = 7.32\,\text{sec}$

Minimum time $= t_1 + t_2 = 9.76 + 7.32 = 17.1\,\text{sec}$

Solution 2



$$\frac{1}{2}bh = 500 \quad \text{Also} \quad a = \frac{\Delta v - \Delta u}{t} \to 6 = \frac{\Delta v - 0}{t} \to 6t = \Delta v$$

$$a = \frac{\Delta v - \Delta u}{T - t} \to -8 = \frac{0 - 6}{T - t} \to T = \frac{7}{4}t$$

$$\frac{1}{2}bh = 500 \to \frac{1}{2} \times \frac{7}{4}t \times 6t = 500 \to 5.25t^2 = 500 \to t = 9.75s$$

$$T = \frac{7}{4} \times 9.75 = 17.1\text{sec}$$

Tutorial Sheet 3 2017/18 Newton's Laws of Motion

01. F = mg at rest, or at constant velocity.

When accelerating upward, $mg + ma = F \rightarrow 45(9.8 + 3.65) = 605.25N$

When accelerating downward, $mg - ma = F \rightarrow 45(9.8 - 3.65) = 276.75N$

02. Resolve 450N into components vertical and parallel to the inclined plane

Vertical component $a = 450\sin 45 = 318.2N$

Horizontal component $b = 450\cos 45 = 318.2N$

The force pushing up the incline is 318.2N, parallel to the incline. It is opposed by the force of friction, and $mg\sin\theta$

Force of friction $= (mg\cos 45 + 318.2)\mu = (138.6 + 318.2)\mu = 456.8N \times \mu$

And $mg\sin 45 = 20 \times 9.8 \times \sin 45 = 138.6N$

Hence 318.2 – 138.6 $- 456.8\mu = ma = 20 \times 0.7 = 14$

Or, 165.6 = $-456.8\mu \rightarrow \mu = 0.363$

Q3. Reaction force on the box in the backward direction $= ma = 20 \times 2 = 40N$

Friction force on the box in the opposite (forward) direction $= \mu mg = 0.15 \times 20 \times 9.8 = 29.4N$

So net force on the box in the backward direction

Acceleration of the box in the backward direction $a' = F/m = (10.6/20) = 0.53m/s^2$

Time taken by the box for falling off (while moving a distance of 4 m):

$$s = u't + \tfrac{1}{2}a't^2 = \tfrac{1}{2}a't^2 \rightarrow t = \sqrt{\frac{2s}{a'}} = \sqrt{\frac{2 \times 4}{0.53}} = 3.9s$$

Distance travelled by the truck during this time $x = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2 = \frac{1}{2}' \; 2' \; 3.9^2 = 15.2m$

Q4. Consider the two blocks as single system of mass $m = m_1 + m_2 = 3.2 + 4.1 = 7.30\,kg$

(a) Acceleration $a = f / m = 6.8 / 7.30 = 0.932 m / s^2$

(b) Force on $m_2 (push\ of\ m_1) is\, F_{m2} = m_2 a = 4.1' \; 0.932 = 3.82N$

(c) In this case, the acceleration is of the same magnitude, but in the opposite direction.

The push on m_2 is found from $6.80 - push = m_2 a = 4.1' \; 0.932 = 3.82$

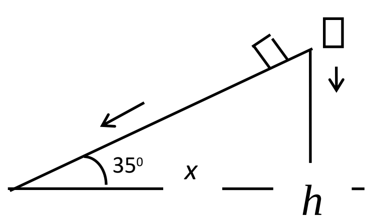
Hence $push = 6.80 - 3.82 = 2.98N$

Q5. Mass *M* pushes on mass *m* with a force *P* which is the normal force on *m*. Mass *m* will not fall provided that the friction force $mP = mg$. For mass *m*, $P = ma$ in the horizontal direction, giving $a = \frac{P}{m} = \frac{g}{m}$.

The equation of motion for the entire system is $F = (M + m)a = (M + m).\frac{g}{m}$

06.** A block takes twice as long to slide down an inclined plane that makes an angle of 35^0 with the horizontal as it does to fall freely through the same vertical distance h.

What is the coefficient of kinetic friction?



Let h be the vertical height of the incline and x be the length of the incline.

Then $\frac{h}{x} = \sin 35 \rightarrow h = x\sin 35 = 0.57x$

$mg\sin\theta - \mu mg\cos\theta = ma \rightarrow 9.8\sin 35 - \mu \times 9.8 \times \cos 35 = a$

Hence $a = 5.62 - 8.03\mu$

Let t_1 be the time to slide down, and t_2 be the time to fall down vertically.

$x = \frac{1}{2}at_1^2 \rightarrow t_1^2 = \frac{2x}{5.62 - 8.03\mu}$ and $s = ut_2 + \frac{1}{2}gt_2^2 \rightarrow 0.57x = 4.9t_2^2 \rightarrow t_2^2 = 0.116x$

Given $t_1^2 = 4t_2^2 \rightarrow \frac{2x}{5.62 - 8.03\mu} = 4 \times 0.116x = 0.464x$

$2 = 0.464(5.62 - 8.03\mu) = 2.61 - 3.73\mu \rightarrow 3.73\mu = 2.61 - 2 = 0.61$

Hence $\mu = \frac{0.61}{3.73} = 0.16$

07. The 2 boxes have identical masses of 45kg. Using *f* = μF_N, the friction forces on the 2 boxes are

$f_A = 0.15 \times mg = 0.15 \times 45 \times 9.8 = 66.15N$ and $f_B = 0.15 \times mg \times \cos 30 = 57.29\ N$.

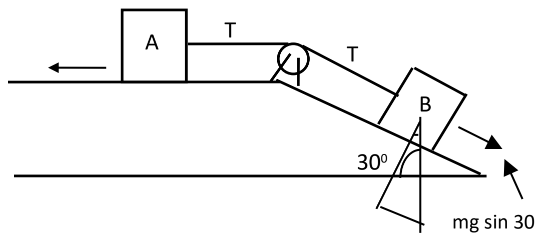
For each block, $\sum F_x = ma$, with the direction of motion as positive.

This gives: T – 66.15 = 45a and 0.5mg - T – 57.29N = 45 a.

Solve for T and a.

T = 45a + 66.15 and T = 163.21 – 45a giving 2T = 229.36 and T = 114.68N.

Hence a = 1.078m/s^2



Tutorial 4 Work and Energy

PHY1010 2018 Tutorial Sheet 4 Work and Energy.

01. Energy supplied by the pump in 60 seconds is $mgh = 0.600 \times g \times 3 = 17.64\,J$

Power = energy/time = $17.64\,J / 60\sec = 0.294\,watt\ \ or\ \ \dfrac{0.294}{746} = 3.94 \times 10^{-4}\,hp$

02. When the bead moves from *A* to *C*, it experiences a change in total energy.

Both K.E. and G.P.E. are lost, which equals the work done on the bead by the friction force.

$\Delta PE + \Delta KE =$ work done by friction

$$mg\left(h_C - h_A\right) + \tfrac{1}{2}m\left(v_c^2 - v_A^2\right) = f.s.\cos\theta$$
$$\cos\theta = -1, v_c = 0,\ v_A = 2m/s,\ \ h_c - h_A = -0.30\,\mathrm{m}$$

$$0.015 \times 9.8(-0.3) + \tfrac{1}{2} \times 0.015\left(0 - 2^2\right) = f \times 2.5 \times (-1)$$

$$-0.0441 - 0.03 = -2.50\,f \rightarrow 0.0741 = 2.50\,f \rightarrow f = \frac{0.0741}{2.50} = 0.0296\mathrm{N}$$

03. A steel block weighing 12N is pulled up an inclined plane 20^0 above the horizontal by a constant force of 7.35 N which makes an angle of 10^0 above the inclined plane.

The block starts from rest and is pulled 2.0 m along the inclined plane.

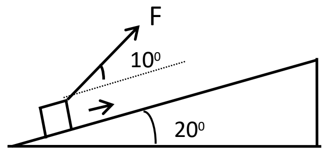
The coefficient of friction between the block and the inclined plane is 0.20.

Determine: (a) the work done on the block by the force,

(b) increase in potential energy of the block

(c) increase in kinetic energy of the block, and

(d) amount of work required to overcome the frictional force.



(a) Force is constant, displacement $s = 2m.$

Work $W = F.s = Fs\cos\theta = 7.35\times 2\times \cos 10 = 14.48J$

(b) To get PE, first calculate the vertical rise h. $\sin 20 = \frac{h}{s} \rightarrow h = s\sin 20 = 2\times \sin 20 = 0.684m$

Hence $\Delta PE = mgh = wh = 12\times 0.684 = 8.21J$

(c) *KE* depends on the final speed of the block. We can calculate the net force on the block in the direction of motion, and the acceleration it acquires.

$$\sum F_y = F\sin 10 - \mathrm{w}\cos 20 + \mathrm{F}_N \rightarrow \mathrm{F}_N = \mathrm{w}\cos 20 - F\sin 10$$

$$= 12\times 0.9396 - 7.35\times 0.1736 = 10N$$

Net force in the *x*-direction:

$$\sum F_x = F\cos 10 - \mathrm{w}\sin 20 - \mu_k \mathrm{F}_N = 7.35\times 0.985 - 12\times 0.342 - 0.2\times 10 = 1.134N$$

Hence $a = \frac{\sum F_x}{m} = \frac{\sum F_x}{w/g} = \frac{1.134\times 9.8}{12} = 0.926m/s^2$

Using $v^2 = u^2 + 2as$ to get the final velocity, $v^2 = 0^2 + 2\times 0.926\times 2 = 3.705m^2/s^2$

$$KE_{final} = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{1}{2}\left(\frac{12}{9.8}\right)3.705^2 = 2.27J$$

(d) Total mechanical energy acquired by the block is $E = PE + KE = 8.21 + 2.27 = 10.48J$

Hence work done to overcome friction is $W_f = W - E = 14.48 - 10.48 = 4J$

04. 203.1 km/h = 56.42 m/s. $mg \sin\theta = 75\times 9.8\times \sin 51 = 571.2$ N.

Friction force $f = \mu_k mg\cos\theta$ + air friction.

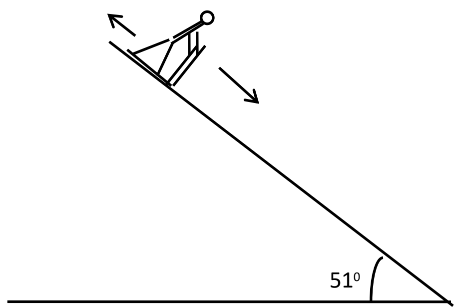
$\mu_k \, mg \cos\theta$ = 0.03 × 75 × 9.8 × cos 51 = 13.88 N

Work = force × distance. Rate of work = work/time = Force × velocity.

(a) Rate of work (gravity) = 571.2 × 56.42 = 32227 J/s.

(b) Hence rate of work (sliding friction) = 13.88 × 56.42 = 783 J/s.

(c) Hence rate of work (air friction) = (571.2 - 13.88) × 56.42 = 31444 J/s.



05. (a) A running man has one-half the kinetic energy that a running boy of half his mass has. The man speeds up by 1.0 m/s and then has the same kinetic energy as the boy.
What were the original speeds of the man and the boy?

Let *M* be the mass of the man and *m* that of the boy. Then $m = \frac{M}{2}$.

Let v_1 be the original speed of the man, and v_2 be the original speed of the boy.

Kinetic energy of the man = $\frac{1}{2} M v_1^2$, and kinetic energy of the boy $\frac{1}{2} m v_2^2 = \frac{1}{2}\left(\frac{M}{2}\right) v_2^2$.

Kinetic energy of man = $\frac{1}{2}$ kinetic energy of the boy.

$$\tfrac{1}{2} M v_1^2 = \tfrac{1}{2} \times \tfrac{1}{2}\left(\frac{M}{2}\right) v_2^2 \rightarrow v_1^2 = \frac{v_2^2}{4} \rightarrow v_1 = \frac{v_2}{2}$$

The man speeds up by 1 m/s, or new speed of the man = $(v_1 + 1)$.

New kinetic energy of the man $\tfrac{1}{2}M(v_1+1)^2$.

Now kinetic energy of man = kinetic energy of boy giving,

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}\left(\frac{M}{2}\right)v_2^2.$$

Hence $\tfrac{1}{2}(v_1+1)^2 = \dfrac{v_2^2}{2} \rightarrow v_1 + 1 = \dfrac{v_2}{\sqrt{2}} = 0.707v_2$

Or $v_1 = 0.707v_2 - 1 = \dfrac{v_2}{2} \rightarrow v_2 = 1.414v_2 - 2 \rightarrow 0.414v_2 = 2 \rightarrow v_2 = 4.83\text{m/s}$

Hence $v_1 = 2.42\,\text{m/s}$

06. First calculate the velocity of *m* when the small mass falls off using conservation of energy: *GPE* is positive as the mass goes up, *KE* is positive in both directions.

From energy conservation $\Delta PE + \Delta KE = 0$

$$\left[(1.0-1.3)mg\,(0.75-0)\right] + \left(\frac{1}{2}\times 2.3mv_{up}^2 - 0\right) = 0 \rightarrow v_{up} = 1.38\,m/s$$

Now solve the new problem using conservation of energy:

$$\left[0-(1.0-0.8)mg\,(0.75)\right] + \left(\frac{1}{2}\times 1.8mv_{down}^2 - \frac{1}{2}\times 1.8m(1.38)^2\right) = 0$$

$\rightarrow -1.47 + 0.9v_{down}^2 - 1.71 = 0$ or $0.9v_{down}^2 - = 1.47 + 1.71 = 3.18 \rightarrow v_{down}^2 = 3.53 \rightarrow v_{down} = 1.88\,m/s$

Tutorial 5 Linear Momentum and Collisions

Tutorial Sheet 5 (2017/18).

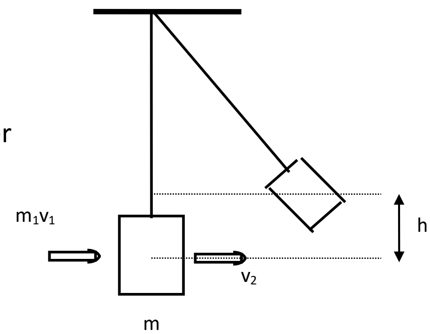
Linear Momentum & Collisions

01. Change in kinetic energy = work done gives ½ mv^2 = force × distance

½ × $2mv^2$ = 0.7 × 2m × g × 6 giving v = 9.07m/s = speed after the collision for both.

Momentum before = momentum after, giving 0 + $mv_2 = 2mv$ or

$v_2 = 2v$ = 2 × 9.07 = 18.14m/s.



02. Given v_1 = 500m/s, v_2 = ?. Let v_3 = speed of block just after the bullet has emerged.

Cons. of momentum gives $m_1v_1 + 0$

= $m_1v_2 + m_2v_3$ giving 0.01 × 500 =0.$01v_2 + 2v_3$

Also ½ $(m_2)v_3^2 = m_2gh$ giving ½v_3^2 = 9.8 × 0.1 or v_3^2 = 2 × 9.8 × 0.1

giving v_3 = 1.4m/s.

Hence 0.01 × 500 =0.01v_2 + 2v_3 =0.01v_2 + 2 × 1.4 or 5 – 2.8 = 0.01v_2

giving v_2 = 220m/s.

03. We have $m_1 = m_2 = m$. Also u_1 = 6m/s, v_2 =? and θ = 25^0.

From conservation of linear momentum $m_1u_1 = m_1v_1 + m_2v_2$

X-direction: $u_1 = v_1 \cos\theta + v_2 \cos\varphi$ giving 6 - $v_1 \cos\theta = v_2 \cos\varphi$ (Eqn.1)

Y-direction: 0 = $v_1\sin\theta - v_2\sin\varphi$ giving $v_1 \sin\theta = v_2 \sin\varphi$ (Eqn.2)

$(1)^2$ gives $(6 - v_1 \cos\theta)^2 = v_2^2 \cos^2\varphi$ or $36 - 12 v_1 \cos\theta + v_1^2 \cos^2\theta = v_2^2 \cos^2\varphi$ (Eqn.3)

$(2)^2$ gives $v_1^2 \sin^2\theta = v_2^2 \sin^2\varphi$ (Eqn.4). (3) + (4) gives $36 - 12 v_1 \cos\theta + v_1^2 = v_2^2$ (5)

From conservation of kinetic energy: $\frac{1}{2} mu_1^2 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$ giving

$u_1^2 = v_1^2 + v_2^2 = 36$ (Eqn.6). Putting (6) into (5) get $36 - 12 v_1 \cos\theta + v_1^2 = 36 - v_1^2$

Hence $2 v_1^2 - 12 v_1 \cos 25 = 0$ or $v_1^2 - 6v_1 \cos 25 = 0$ giving $v_1 = 0$ (discarded) or

$v_1 - 6\cos 25 = 0$ giving $v_1 = 5.44$m/s. This gives $v_2 = \sqrt{36 - 5.44^2} = 2.53\ m/s$.

Sin $\varphi = \dfrac{v_1 \sin\theta}{v_2} = \dfrac{5.44 \times 0.423}{2.53} = 0.91$ Giving $\varphi = 65.4$

04. All the pieces must move in the same *x*-*y* plane.

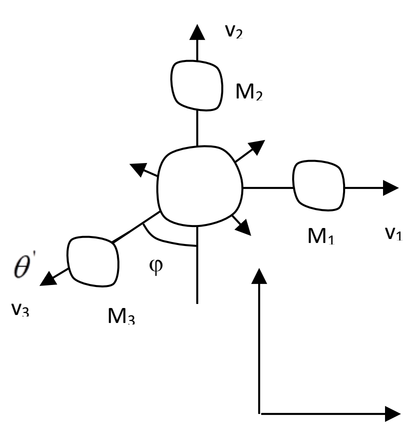
Given $M_1 = 2M_2$ and $M_3 = 3M_2$, $v_1 = v_2 = v = 30$m/s.

From momentum conservation, initial momentum = final momentum = 0.

Velocity $\mathbf{v_1}$ has components $v_{1x} = v$, and $v_{1y} = 0$; $v_{2x} = 0$, $v_{2y} = v$,

Hence $M_1 v_{1x} + M_2 v_{2x} + M_3 v_{3x} = 0$ giving $2M_2 v + 3M_2 v_{3x} = 0$

hence $v_{3x} = -\dfrac{2}{3} v = -20$m/s.



Also $M_1 v_{1y} + M_2 v_{2y} + M_3 v_{3y} = 0$ giving $M_2 v + 3M_2 v_{3y} = 0$ or $v_{3y} = -(1/3)v$

or $v_{3y} = -10$m/s.

Hence $v_3 = \sqrt{(-20)^2 + (-10)^2} = 22.4 m/s$.

For the angle, $\tan f = \left|\frac{v_{3x}}{v_{3y}}\right| = \frac{-20}{-10} = 2$ or $\phi = 63.4^0$ hence $\theta = 206.6^0$.

Alternatively: $\tan \theta' = \left|\frac{v_{3y}}{v_{3x}}\right| = \frac{10}{20} = 0.5$ giving $\theta' = 26.6^0$ or $\theta = 26.6 + 180 = 206.6^0$

05**. A particle of mass *m*, moving with a velocity *u* makes a head-on elastic collision with a particle of mass 2*m* initially at rest. Show that the particle of mass *m* loses (8/9)th of its initial kinetic energy in the collision.

The second particle is initially at rest.

Let v_1 and v_2 be the velocities of the particles after the collision respectively.

We have to relate v_1 to u in order to calculate the loss in *KE* of mass *m*.

Conservation of momentum gives: $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ giving

$mu + 0 = mv_1 + 2mv_2$ or $u - v_1 = 2v_2$ (Eqn 1).

Conservation of kinetic energy gives:

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2(= 0) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ giving $mu^2 = mv_1^2 + m_2v_2^2(= 2mv_2^2)$

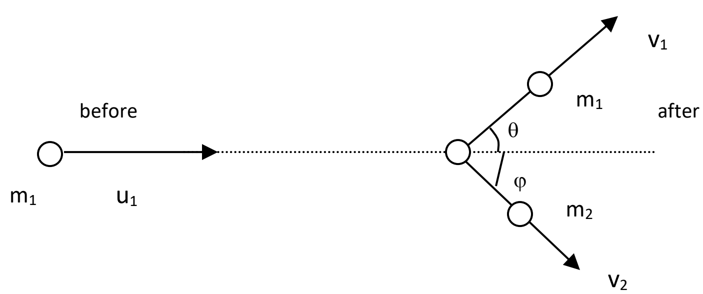
This gives $u^2 - v_1^2 = 2v_2^2$ or $(u - v_1)(u + v_1) = 2v_2^2$ (Eqn 2).

From the 2 equations we get $2v_2(u + v_1) = 2v_2^2$ Or $(u + v_1) = v_2$

Eliminating v_2 from Eqn 2 we get $v_1 = -\frac{1}{3}u$.

Hence fractional loss in kinetic energy of mass *m*

$$= \frac{(1/2)mu^2 - (1/2)mv_1^2}{(1/2)mu^2} = \left(1 - \frac{v_1^2}{u^2}\right) = \left(1 - \frac{1}{3^2}\right) = \frac{8}{9}$$

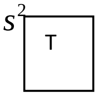


Tutorial 6 Motion in a Circle

PHY1010 Tutorial Sheet 6 2017/2018

01. The wheel turn through angle $\theta = \frac{s}{r} = \frac{29.5m}{0.44m} = 67\,rad = 10.7\,rev$

Angular acceleration $\alpha = -\frac{\omega_0^2}{2\theta} = -\frac{1.6^2}{2 \times 10.7} = -0.12\,rev/s^2$



02. Friction force keeps the carton moving in a circular path: $f = \frac{mv^2}{r}$ $\mu = \frac{f}{F_N} = \frac{f}{mg}$

Hence $\mu = \frac{v^2}{rg} = \frac{16.5^2}{26 \times 9.8} = 1.07$

03. Here $l = 30\text{cm}$. Angular velocity $\omega = 2\pi \frac{80}{60} = \frac{8}{3}\pi\ \text{rad/s}$

Let T be the tension in the string and θ the angle of inclination of the string with the vertical.

Radius of the circle $R = l\sin\theta$. Also $T\sin\theta = mR\omega^2 = 100 \times 300 \sin\theta \times \left(\frac{64\pi^2}{9}\right)$

Hence $T = 0.1 \times 0.30 \times \left(\frac{64\pi^2}{9}\right) = 2.10\text{N}$

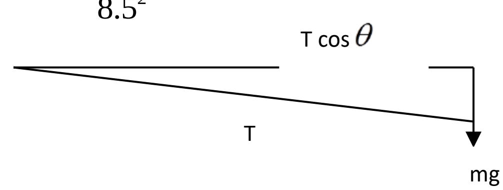
Again $T\cos\theta = mg \Rightarrow \cos\theta = \frac{mg}{T} = \frac{0.1 \times 9.8}{2.10} = 0.466$

$\theta = \cos^{-1} 0.466 = 62.18^0$

04. Vertical component of tension $T\sin\theta = mg$ (1)

Horizontal component is responsible for the centripetal force: $T\cos\theta = m\frac{v^2}{r}$ (2)

Dividing (1) by (2), $\tan\theta = \frac{gr}{v^2} \rightarrow \theta = \tan^{-1}\frac{gr}{v^2} = \tan^{-1}\frac{g \times 1.25}{8.5^2} = 9.62^0$



$$T = \frac{mg}{\sin\theta} = \frac{0.45 \times 9.8}{\sin 9.62} = 26.4\,N$$

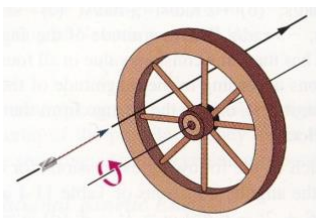
05. Angle θ between two spokes $= 360/8 = 45^0 = \pi/4\,rads$

Arrow goes through the wheel in time $t = 0.20/6 = 0.033\sec$

During this time the wheel rotates through an angle $\theta_0 = \omega t = 0.033\omega$

This is equal to $\pi/4\ rad$

Hence $0.033\omega = \pi/4 \rightarrow \omega = (\pi/4)/0.033 = 23.79\,rad/\sec = 3.79\,rev/\sec$



It does not matter where between the ream and the axle the arrow is aimed. Angular speed ω and angle θ between two spokes are the relevant quantities. Distance to axle and tangential speed do not appear in the above calculations.

06. We have 50 km/h = 50000/3600 =13.89 m/s

$R\cos\theta = mg$ *and* $R\sin\theta = m\dfrac{v^2}{R}$ where θ is the angle of banking.

Hence $\tan\theta = \dfrac{v^2}{rg} = \dfrac{13.89^2}{500\times 9.8} = 0.039$

If the outer rail is higher than the inner rail by $h,$ then $\sin\theta = \dfrac{h}{d}$, where d is the distance between the rails.

For very small angle $\sin\theta \approx \theta \approx \tan\theta$. Hence $\dfrac{h}{d} \approx 0.039$.

If $d = 1.5\text{m, h} = 1.5\times 0.039 = 0.0585\text{m} = 6\text{cm}$

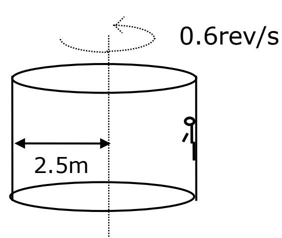
07. From $G\dfrac{mM}{r^2} = m\dfrac{v^2}{r}$ and the time for one revolution $t = \dfrac{2\pi r}{v}$,

we can solve to get $t^2 = \dfrac{4\pi^2 r^3}{GM}$. Then the mass of the earth is found to be

$$M = \frac{4p^2 r^3}{Gt^2} = \frac{4\times 3.14^2 \times \left(6.5\times 10^6\right)^3}{6.67\times 10^{-11}\times \left(87\times 60\right)^2} = \frac{4\times 9.8596\times 2.74625\times 10^{20}}{6.67\times 10^{-11}\times 2.7248\times 10^7}$$

$$= \frac{108.3077}{18.174416}\times 10^{24} = 5.96\times 10^{24}\ kg$$

08. He is held up against gravity by the static friction force exerted on him by the wall. The acceleration of the person, a_{rad} is toward the centre (axis of rotation).

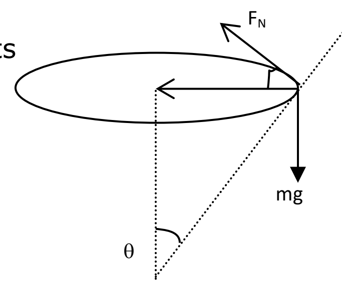


Take f_s to have max value; $f_s = \mu_s F_N$. $\sum F_y = ma_y$. $f_s - mg = 0$ or $f_s = mg = \mu_s F_N$

$\sum F_x = ma_x = mv^2/r$.

Hence $F_N = mv^2/r$ or $mg = \mu_s\, mv^2/r$ giving $\mu_s = gr/v^2 = (9.8 \times 2.5)/9.425^2 = 0.28$ as 0.6 rev/s = $0.6 \times 2\,\pi R = 0.6 \times 2\,\pi \times 2.5 = 9.425$m/s.

09. . The horizontal component of the normal force provides the centripetal force to keep the particle in its circular path; $F_N \cos\theta = \dfrac{mv^2}{r}$.



In the vertical direction $F_N \sin\theta = mg$.

Hence $\tan\theta = \dfrac{rg}{v^2}$ giving $v = \sqrt{\dfrac{rg}{\tan\theta}}$