EEE 5451 Power Electronics

Lecture 5: AC-DC Converter Operation

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5.0 Converter Classifications

1. According to the number of phases of the input supply

- Single-phase Converter
- Three-phase converter

2. Based on the number of output voltage pulses per cycle of the converter

- Single-pulse Converter
- Two-pulse converter
- Three-pulse converter
- Six-pulse converter

3. Based on the converter circuit configuration

- Midpoint type converter
- Bridge type converter

4. Based on the modes of operation

- Single quadrant converter
- Two quadrant converter
- Four quadrant converter

5. Based on the control over output voltage pulses

- Full wave controlled rectifier
- Half wave controlled rectifier
- Uncontrolled rectifier

6. Converters can also be classified as:

- Full converter or fully controlled rectifier-a two quadrant converter
- Semi-converter or half-controlled converter-a single quadrant converter

5.1 Single-Phase Controlled Thyristor Converter Circuits

5.1.1 Half-wave circuit with an R-L load

- The output voltage is controlled by the thyristor trigger angle, *α*. The output voltage ripple is at the supply frequency. Circuit waveforms are shown in figure 5.1b.

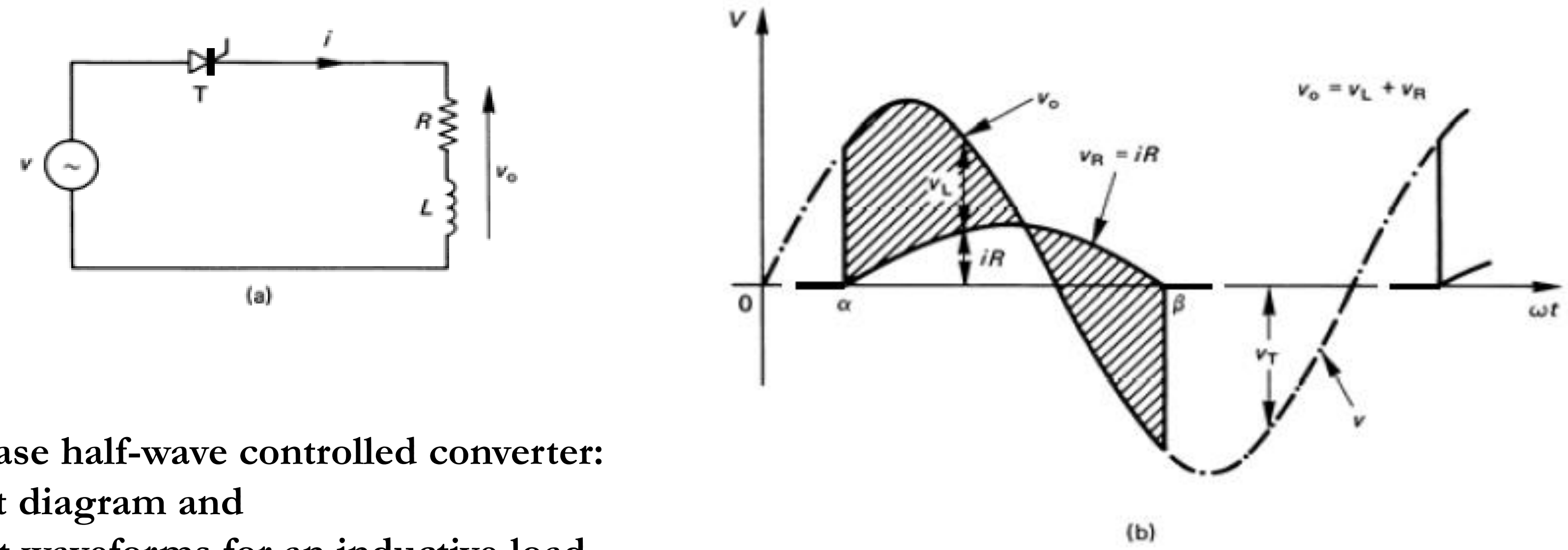


Fig. 5.1 Single-phase half-wave controlled converter:
(a) circuit diagram and
(b) circuit waveforms for an inductive load.

- The output current, hence output voltage, for the circuit is given by

$$L\frac{di}{dt}+Ri=\sqrt{2}V\sin\omega t\ [\mathrm{V}],\ \alpha\le\omega t\le\beta\ [\mathrm{rad}]$$

where α is the phase delay angle and β is the extinction angle.

- Solving this equation yields the load and supply current

$$i(\omega t)=\frac{\sqrt{2}V}{Z}\left\{\sin(\omega t-\phi)-\sin(\alpha-\phi)e^{(\alpha-\omega t)/\tan\phi}\right\}[\mathrm{A}]$$

$$\text{Where } Z=\sqrt{R^2+(\omega L)^2}\ [\mathrm{Ohms}],\ \tan\phi=\frac{\omega L}{R},\alpha\le\omega t\le\beta\ [\mathrm{rad}]$$

- The current extinction angle *β* is dependent on the load impedance and trigger angle *α,* and can be determined by solving the above equation with *ωt = β* when *i(β)* = 0, that is

$$\sin(\beta-\phi)=\sin(\alpha-\phi)e^{(\alpha-\beta)/\tan\phi}$$

- A family of curves of current conduction angle versus delay angle, that is β - α versus α, is shown in figure 5.2. The plot for ϕ =½π is for a purely inductive load, whereas ø = 0 is a purely resistive load.

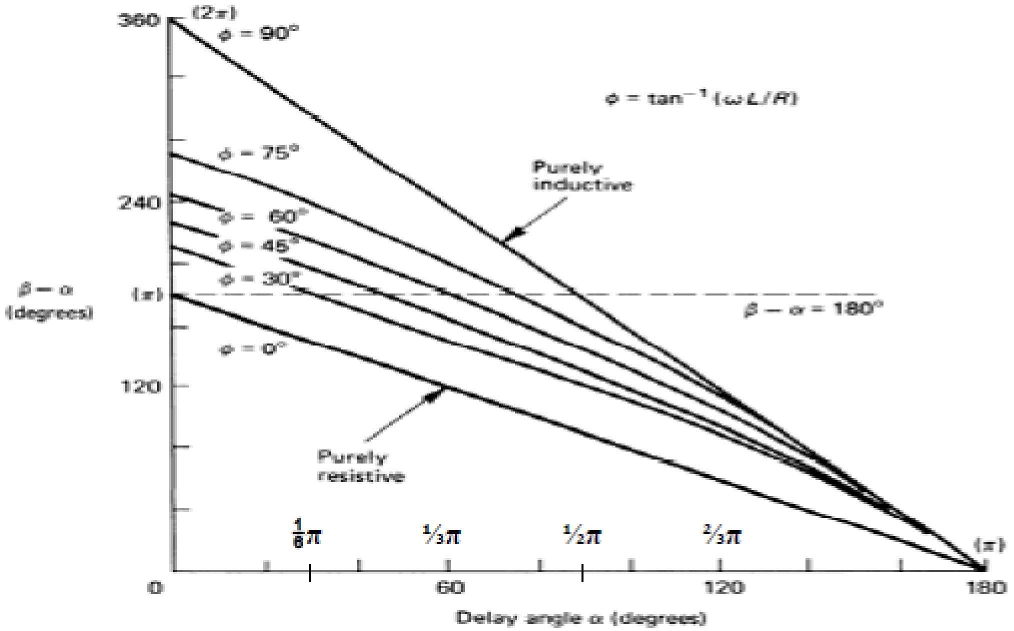


Fig. 5.2 Half-wave, controlled converter thyristor trigger delay angle α versus thyristor conduction angle, β-α

- The mean load voltage, whence the mean load current, is given by

$$V_O = \frac{1}{2\pi}\int_{\alpha}^{\beta} \sqrt{2}V \sin \omega t d\omega t$$

$$V_O = I_O R = \frac{\sqrt{2}V}{2\pi}(\cos\alpha - \cos\beta)[\mathrm{V}]$$

where the angle β can be extracted from Fig. 5.2. The rms load voltage is

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\beta}\left(\sqrt{2}V\right)^2 \sin^2 \omega t d\omega t\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2}V}{2}\left[\frac{1}{\pi}\left\{(\beta-\alpha) - \frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right\}\right]^{\frac{1}{2}}$$

- The rms current involves integration of the load current and the solution is found as:

$$I_{rms} = \frac{V}{2\pi R}\left[\cos\phi\cos(\beta-\alpha) - \sin\phi\cos(\alpha+\phi+\beta)\right]^{\frac{1}{2}}$$

Thus, $\mathrm{P} = \mathrm{I}_{\mathrm{rms}}^2 \mathrm{R}$

Case I: Purely resistive load

$Z = R, \phi = 0$, and the current is given by

$$i(\omega t) = \frac{\sqrt{2}V}{R}\sin(\omega t)\,[\text{A}],\ \alpha \le \omega t \le \pi \text{ and } \beta = \pi \forall\, \alpha$$

The average load voltage, hence average load current is

$$V_O = \frac{1}{2\pi}\int_{\alpha}^{\pi} \sqrt{2}V \sin \omega t d\omega t$$

$$V_O = I_O R = \frac{\sqrt{2}V}{2\pi}\left(1 + \cos\alpha\right)[\text{V}]$$

The rms output voltage is

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi}\left(\sqrt{2}V\right)^2 \sin^2 \omega t d\omega t\right]^{\frac{1}{2}} = \frac{\sqrt{2}V}{2}\left[\frac{1}{\pi}\left\{(\pi - \alpha) + \frac{1}{2}\sin 2\alpha\right\}\right]^{\frac{1}{2}}$$

Since the load is purely resistive, $\text{I}_{\text{rms}} = \text{V}_{\text{rms}}/\text{R}$ and the power delivered to the load is $\text{P}_\text{o} = I_{rms}^2 R$. The supply power factor, for a resistive load, is $\text{P}_{\text{out}} / V_{rms} I_{rms}$, that is

$$\text{pf} = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}}$$

Case II: Purely inductive load

$Z = \omega L,\ \phi = \frac{\pi}{2}$, and the current is given by

$$i(\omega t) = \frac{\sqrt{2}V}{\omega L}\left(\sin(\omega t - \pi/2) - \sin(\alpha - \pi/2)\right)$$

$$= \frac{\sqrt{2}V}{\omega L}(\cos\alpha - \cos\omega t)\ [\mathrm{A}],\ \alpha \le \omega t \le \beta \text{ and } \beta = 2\pi - \alpha$$

The average load voltage, based on the equal area criterion, is zero

Thus, $V_O = \frac{1}{2\pi}\int_{\alpha}^{2\pi-\alpha} \sqrt{2}V \sin\omega t\, d\omega t = 0$

- **The average output current is**

$$I_O = \frac{1}{2\pi}\int_{\alpha}^{2\pi-\alpha} \frac{\sqrt{2}V}{\omega L}\{\cos\alpha - \cos\omega t\}d\omega t$$

$$= \frac{\sqrt{2}V}{\pi\omega L}[(\pi - \alpha)\cos\alpha + \sin\alpha]$$

The rms output current is derived from

$$I_{rms} = \frac{\sqrt{2}V}{\omega L}\left[\frac{1}{2\pi}\int_{\alpha}^{2\pi-\alpha}(\cos\alpha - \cos\omega t)^2 d\omega t\right]^{\frac{1}{2}}$$

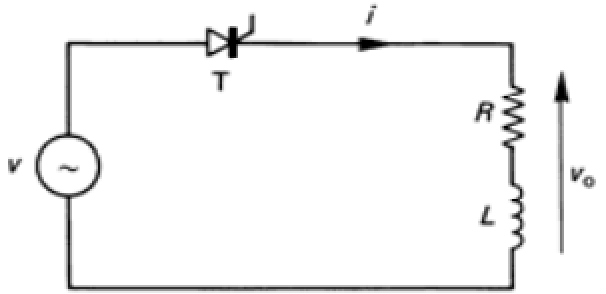
The rms output voltage is

$$V_{rms} = \left[\frac{1}{2\pi}\int_{\alpha}^{2\pi-\alpha}\left(\sqrt{2}V\right)^2 \sin^2\omega t\, d\omega t\right]^{\frac{1}{2}}$$

$$= \sqrt{2}V\left[\frac{1}{2\pi}\left\{(\pi - \alpha) + \frac{1}{2}\sin 2\alpha\right\}\right]^{\frac{1}{2}}$$

- Since the load is purely inductive, P_o=0.
- Note that when α=0, these equations become valid for an uncontrolled rectifier

Practice Questions:



1. The ac supply of the half-wave controlled single-phase converter shown above is $v = \sqrt{2}.240 \sin \omega t$.
 For the following loads:

 Load 1: R=10 Ω, ωL=0 Ω

 Load 2: R=0 Ω, ωL=10 Ω

 Load 3: R=7.1 Ω, ωL=7.1 Ω

 Determine in each load case, for a firing delay angle α=π/6

 (a) the conduction angle γ, hence the extinction angle β

 (b) the dc output voltage and the average output current

 (c) the power dissipated in the load and power factor for the first two loads.

Solutions:

LOAD1: $R = 10\,\Omega,\ \ \omega L = 0\,\Omega$

Thus, $Z = 10\,\Omega$ and $\phi = 0^\circ$

$\beta = \pi$ for all α, $\alpha = \pi/6$, thus $\gamma = \beta - \alpha = 5\pi/6$.

$$V_O = I_O R = \frac{\sqrt{2}V}{2\pi}(1+\cos\alpha)$$

$$= \frac{\sqrt{2}V}{2\pi}(1+\cos\pi/6) = 100.9\,[\mathrm{V}]$$

The average load current is

$$I_O = \frac{V_O}{R} = \frac{\sqrt{2}V}{2\pi R}(1+\cos\alpha) = 100.9/10 = 10.1\,[\mathrm{A}]$$

The rms load voltage is given as

$$V_{rms} = \frac{\sqrt{2}V}{2}\left[\frac{1}{\pi}\left\{(\pi-\alpha)+\frac{1}{2}\sin 2\alpha\right\}\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2}\times 240}{2}\left[\frac{1}{\pi}\left\{(\pi-\pi/6)+\frac{1}{2}\sin\pi/3\right\}\right]^{\frac{1}{2}} = 167.71\,[\mathrm{V}]$$

Since the load is purely resistive, the power delivered to the load is

$$\mathrm{P_O} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$$

$$= \frac{167.71^2}{10} = 2797\,[\mathrm{W}]$$

The power factor is

$$\mathrm{P.F.} = \frac{2797}{240\times 16.71} = 0.697$$

LOAD 2: $R = 0\ \Omega,\ \omega L = 10\ \Omega$.

Thus, $Z = 10\ \Omega,\ \phi = \frac{\pi}{2}$ and $\beta = 2\pi - \alpha = 2\pi - \pi/6 = 11\pi/6$

Whence the conduction angle, $\gamma = \beta - \alpha = 5\pi/3$

$V_O = 0\,[\mathrm{V}]$

The load average current is

$$I_O = \frac{\sqrt{2}V}{\pi\omega L}\left[(\pi - \alpha)\cos\alpha + \sin\alpha\right]$$

$$= \frac{\sqrt{2}\times 240}{\pi \times 10}\left[(5\pi/6)\cos\pi/6 + \sin\pi/6\right] = 29.9\,[\mathrm{A}]$$

- Since the load is purely inductive, P_o=0.
- The power factor is also zero.

LOAD 3: $R = 7.1\ \Omega,\ \omega L = 7.1\ \Omega$.

Thus, $Z = 10.04\ \Omega,\ \phi = \frac{\pi}{4},\ \gamma = \beta - \alpha = 195°$ (from Fig. 5.2)

Whence, $\beta = 225°$

$$V_O = I_O R = \frac{\sqrt{2}V}{2\pi}\left(\cos\alpha - \cos\beta\right)$$

$$= \frac{\sqrt{2}\times 240}{2\pi}\left(\cos 30° - \cos 225°\right) = 85.0\,[\mathrm{V}]$$

The average load current is

$$I_O = \frac{V_O}{R} = \frac{85.0}{7.1} = 12.0\,[\mathrm{A}]$$

2. A sinusoidal voltage $v(t) = V_m \sin \omega t$ is passed through a half-wave rectifier that clips the negative portion of the wave. Find the harmonic function of the voltage output waveform.

Solution:

$$v(t) = \begin{cases} 0 & \text{if } -\pi/\omega < t < 0 \\ V_m \sin \omega t & \text{if } \quad 0 < t < \pi/\omega \end{cases}$$

$$a_o = \frac{\omega}{2\pi}\int_0^{\pi/\omega} V_m \sin \omega t dt = \frac{V_m}{\pi}$$

$$a_n = \frac{\omega}{\pi}\int_0^{\pi/\omega} V_m \sin \omega t \cos n\omega t dt = \frac{\omega V_m}{2\pi}\int_0^{\pi/\omega} [\sin(1+n)\omega t + \sin(1-n)\omega t] dt$$

$$= \frac{\omega V_m}{2\pi}\left[\frac{-\cos(1+n)\omega t}{(1+n)\omega} - \frac{\cos(1-n)\omega t}{(1-n)\omega}\right]_0^{\pi/\omega}$$

$$= \frac{V_m}{2\pi}\left[\frac{-\cos(1+n)\pi + 1}{(1+n)} - \frac{\cos(1-n)\pi + 1}{(1-n)}\right]$$

If n is odd, this is equal to zero, and for even n we have

$$a_n = \frac{V_m}{2\pi}\left(\frac{2}{1+n} + \frac{2}{1-n}\right) = -\frac{2V_m}{(n-1)(n+1)\pi} \quad (\text{n} = 2, 4, \ldots.)$$

In a similarfashion;

$$b_1 = \frac{V_m}{2} \text{ and } b_n = 0 \text{ for n} = 2, 3, \ldots.$$

$$\text{Thus, } v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin\omega t - \frac{2V_m}{\pi}\left(\frac{1}{1.3}\cos 2\omega t + \frac{1}{3.5}\cos 4\omega t + \ldots.\right)$$

5.1.2 Full-wave circuit with an R-L load

- Full-wave voltage control is possible with the circuits shown in Fig. 5.3a and b.

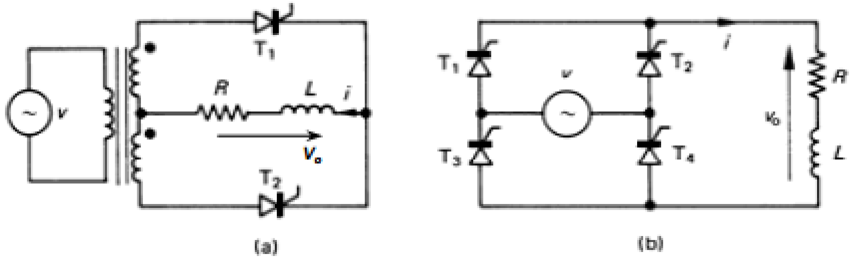


Fig. 5.3 Full-wave controlled converter: (a) and (b) circuit diagrams

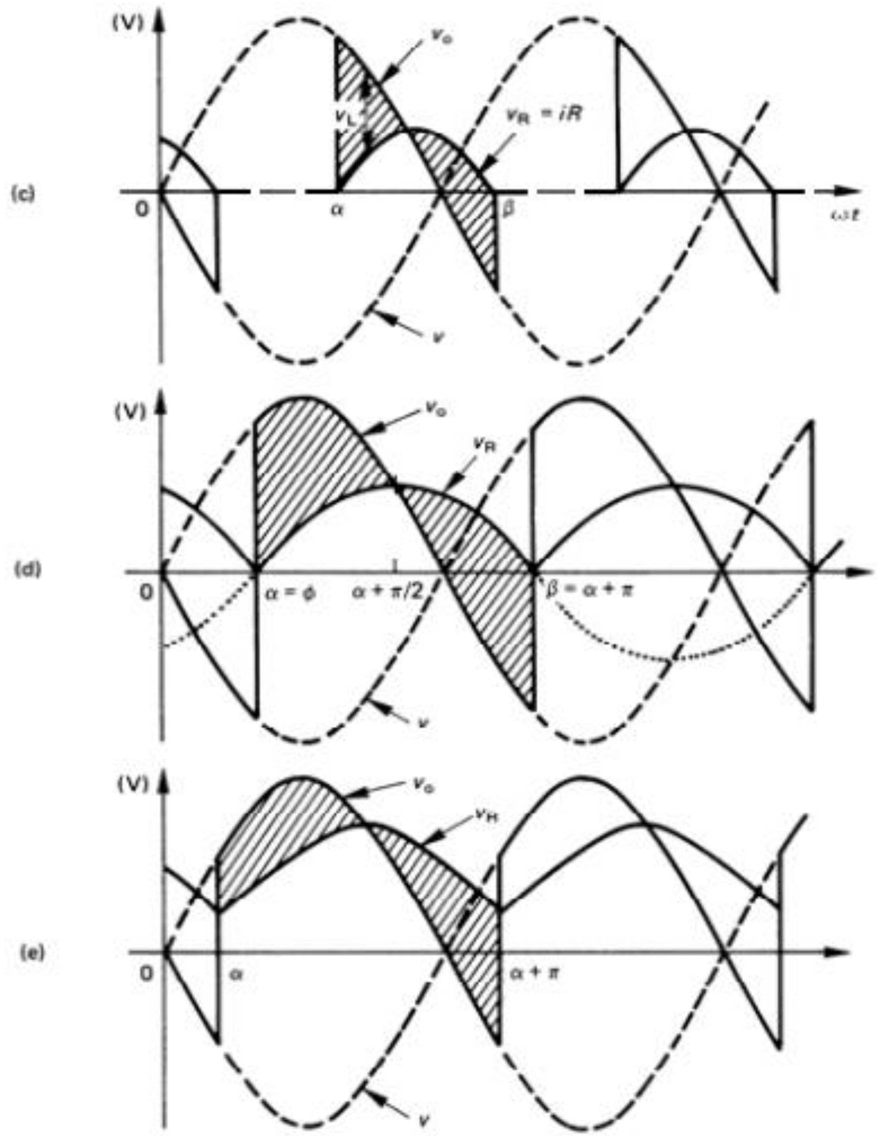


Fig. 5.3 Full-wave controlled converter: (c) discontinuous load current; (d) verge of continuous load current, when α= ø; and (e) continuous load current.

- The circuit in figure 5.3a uses a centre-tapped transformer and two thyristors which experience a reverse bias of twice the supply.

- At high powers where a transformer may not be applicable, a four-thyristor configuration as in figure 5.3b is suitable. The voltage ratings of the thyristors in figure 5.3b are half those of the devices in figure 5.3a, for a given input voltage.

- Load voltage and current waveforms are shown in figure 5.3 parts c, d, and e for three different phase control angle conditions.

- The load current waveform becomes continuous when the phase control angle α is given by the following expression, at which angle the output current is a rectified sine wave.

$$\alpha = \tan^{-1}\frac{\omega L}{R} = \phi$$

- For α > ø, discontinuous load current flows as shown in figure 5.3c. At α= ø the load current becomes continuous as shown in figure 5.3d, whence β = α + π. Further decrease in α, that is α < ø, results in continuous load current that is always greater than zero, as shown in figure 5.3e.

Case I: α>ϕ, β-α<π, discontinuous load current

- The load current waveform is the same as for the half-wave situation considered earlier, given as

$$i(\omega t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\{(\alpha - \omega t)/\tan\phi\}}\right][\mathrm{A}],\ \alpha \le \omega t < \beta\ [\mathrm{rad}]$$

The mean output voltage for this full-wave circuit will be twice that of the half-wave case considered earlier. That is

$$V_O = I_O R = \frac{1}{\pi}\int_{\alpha}^{\beta}\sqrt{2}V\sin\omega t\, d\omega t = \frac{\sqrt{2}V}{\pi}(\cos\alpha - \cos\beta)[\mathrm{V}]$$ where β can be extracted from Fig. 5.2.

- The average output current is given as

$$I_O = \frac{V_O}{R}$$

The rms load voltage is

$$V_{rms} = \sqrt{2}V\left[\frac{1}{\pi}\int_{\alpha}^{\beta}\sin^2 \omega t d\omega t\right]^{\frac{1}{2}}$$

$$= \sqrt{2}V\left[\frac{1}{2\pi}\left\{(\beta-\alpha)-\frac{1}{2}(\sin 2\beta - \sin 2\alpha)\right\}\right]^{\frac{1}{2}}$$

The rms load current is

$$I_{rms} = \frac{\sqrt{2}V}{2\pi R}\left[\cos\phi\cos(\beta-\alpha)-\sin\phi\cos(\alpha+\phi+\beta)\right]^{\frac{1}{2}}$$

The load power is therefore $P = I_{rms}R$.

Case II: $\alpha = \phi$, $\beta - \alpha = \pi$, verge of continuous load current

When $\alpha = \phi = \tan^{-1}\dfrac{\omega L}{R}$, the load current expression reduces to

$$i(\omega t) = \frac{\sqrt{2}V}{Z}\sin(\omega t - \phi)\ \text{[A]}\ \text{ for } \phi \le \omega t \le \phi + \pi\ \ \text{[rad]}$$

and the mean output voltage, on reducing the mean output voltage expression of Case I using $\beta = \alpha + \pi$, is given by

$$V_O = \frac{2\sqrt{2}V}{\pi}\cos\alpha\quad \text{[V]}$$

which is dependent on the load such that $\alpha = \phi = \tan^{-1}\omega L / R$.

From the V_{rms} expression of Case I, with $\beta - \alpha = \pi$, the rms output voltage is V, I = V/Z, and P = VIcosϕ.

Case III: α <ϕ , β- π = α, continuous load current

- Under this condition, a thyristor is still conducting when another is forward-biased and is turned on.

- The first device is instantaneously reverse-biased by the second device which has been turned on.

- The first device is commutated and load current is instantaneously transferred to the oncoming device.

- The load current is given by

$$i(\omega t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t - \phi) - \frac{2\sin(\alpha - \phi)}{1 - e^{-\pi/\tan\phi}} e^{\{(\alpha - \omega t)/\tan\phi\}}\right] \text{[A]}$$

The mean output voltage, whence mean output current, are defined as for Case II. Thus, we have

$$V_O = I_O R = \frac{2\sqrt{2}V}{\pi}\cos\alpha \ \ [\mathrm{V}]$$

which is uniquely defined by α. The maximum mean output voltage occurs at $\alpha = 0$ and is given by

$$\hat{V}_O = \frac{2\sqrt{2}V}{\pi} \ \ [\mathrm{V}]$$

The normalised mean output voltage V_n is given as

$$V_n = \frac{V_O}{\hat{V}_O} = \cos\alpha$$

The rms output voltage is equal to the rms input supply voltage:

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\left(\sqrt{2}V\right)^2 \sin^2\omega t \, d\omega t} = V$$

The ac component harmonic magnitudes in the load are given by

$$V_n = \frac{\sqrt{2}V}{2\pi}\times\left(\frac{1}{(2n-1)^2}+\frac{1}{(2n+1)^2}-\frac{2\cos 2\alpha}{(2n-1)(2n+1)}\right) \ \text{for n} = 1, 2, 3\ldots$$

The current harmonics are obtained by division of the voltage harmonic by its load impedance at that frequency, that is

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}}$$

Critical load inductance

- The critical inductance, to prevent the current falling to zero, is given by

$$\frac{\omega L_{crit}}{R} = \frac{\pi}{2\cos\alpha}\left(\cos\theta + \frac{2}{\pi}\sin\alpha - \frac{2}{\pi}\cos\alpha\left(\frac{\pi}{2} + \alpha + \theta\right)\right)$$

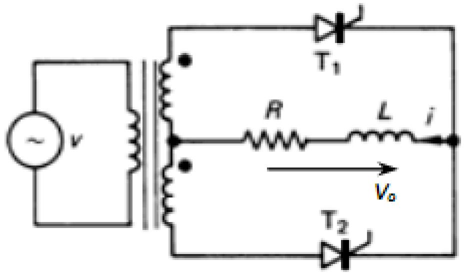
for $\alpha \leq \theta$ where

$$\theta = \sin^{-1}\frac{V_O}{\sqrt{2}V} = \sin^{-1}\frac{2\cos\alpha}{\pi}$$

The minimum current occurs at the angle θ, where the mean output voltage V_O equals the instantaneuos load voltage v_o. When the phase delay angle α is greater than the critical angle θ, substituting $\alpha = \theta$ in the equation for critical load inductance gives

$$\frac{\omega L_{crit}}{R} = -\tan\alpha$$

Practice Question:



The fully controlled full-wave converter above has a source of 240 V rms, 50 Hz, and a 10 Ω, 50 mH series load. The delay angle is 45°. Determine (i) the average output voltage and current, hence thyristor mean current (ii)the rms load voltage and current, hence thyristor rms current (iii) the power absorbed by the load and the power factor.

Solution:

The load natural power factor angle is given by

$$\phi = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\left(\frac{2\pi \times 50 \times 50 \times 10^{-3}}{10}\right) = 57.3^{\circ}$$

Since $\alpha < \phi\left(45^{\circ} < 57.3^{\circ}\right)$, continuous load current flows.

(i). The average output current and voltage are given as

$$V_o = I_o R = \frac{2\sqrt{2}V}{\pi}\cos\alpha = \frac{2\sqrt{2}\times 240}{\pi}\cos 45^{o} = 152.8\ [\mathrm{V}]$$

$$I_o = \frac{V_o}{R} = \frac{152.8}{10} = 15.3\ [\mathrm{A}]$$

Each thyristor conducts for 180°, hence the thyristor mean current

is half of 15.3 A, that is, $I_{to} = \frac{1}{2}\times 15.3 = 7.65\ [\mathrm{A}]$

(ii). The rms load current is determined by harmonic analysis.

The volatge harmonics are given by the equation below :

$$V_n = \frac{\sqrt{2}V}{2\pi} \times \left(\frac{1}{(2n-1)^2} + \frac{1}{(2n+1)^2} - \frac{2\cos 2\alpha}{(2n-1)(2n+1)} \right) \text{ for n} = 1, 2, 3\ldots$$

The corresponding current is given by the following expression :

$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{\sqrt{R^2 + (n\omega L)^2}}$$

The dc output voltage is already found in (i). From the calculations in the table on the next slide, the rms load current is

$$I_{rms} = \sqrt{I_o^2 + \frac{1}{2}\sum I_n^2} = \sqrt{238.705} = 15.45 \text{ [A]}$$

Since each thyristor conducts for 180 deg, the thyristor rms current is

$$I_{Trms} = \frac{1}{\sqrt{2}} \times 15.45 = 10.9 \text{ [A]}$$

harmonic n	V_n	$Z_n = \sqrt{R^2 + (n\omega L)^2}$	$I_n = \frac{V_n}{Z_n}$	$½I_n^2$
0	***152.79***	10.00	15.28	***233.44***
1	60.02	18.62	3.22	5.19
2	8.16	32.97	0.25	0.03
3	3.26	48.17	0.07	0.00
4	1.77	63.62	0.03	0.00
			$I_o^2 + \sum ½I_n^2 =$	238.705

The rms load voltage, as shown earlier, is 240 [V]

(iii). The power absorbed by the load is

$$P_L = I_{rms}^2 R = 15.45^2 \times 10 = 2387 \ [W]$$

The power factor PF is given as

$$PF = \frac{P_L}{V_{rms} I_{rms}} = \frac{2387}{240 \times 15.45} = 0.64$$

5.2 Full-wave circuit with R-L and emf load

- An emf source and *R-L* load can be encountered in dc machine modelling. The emf represents the machine speed back emf, defined by $E = k\varphi\omega$.
- These machines can be controlled by a fully controlled converter configuration as shown in figure 5.4 (a).
- If in each half sine period the thyristor firing delay angle occurs after the rectified sine supply has fallen below the emf level *E,* then no load current flows since the bridge thyristors will always be reverse-biased.
- Thus the zero current firing angle α_0, for $\alpha_0 > \frac{1}{2}\pi$ is given by

$$\alpha_o = \sin^{-1}\left(\frac{E}{\sqrt{2}V}\right) \text{ [rad]}$$

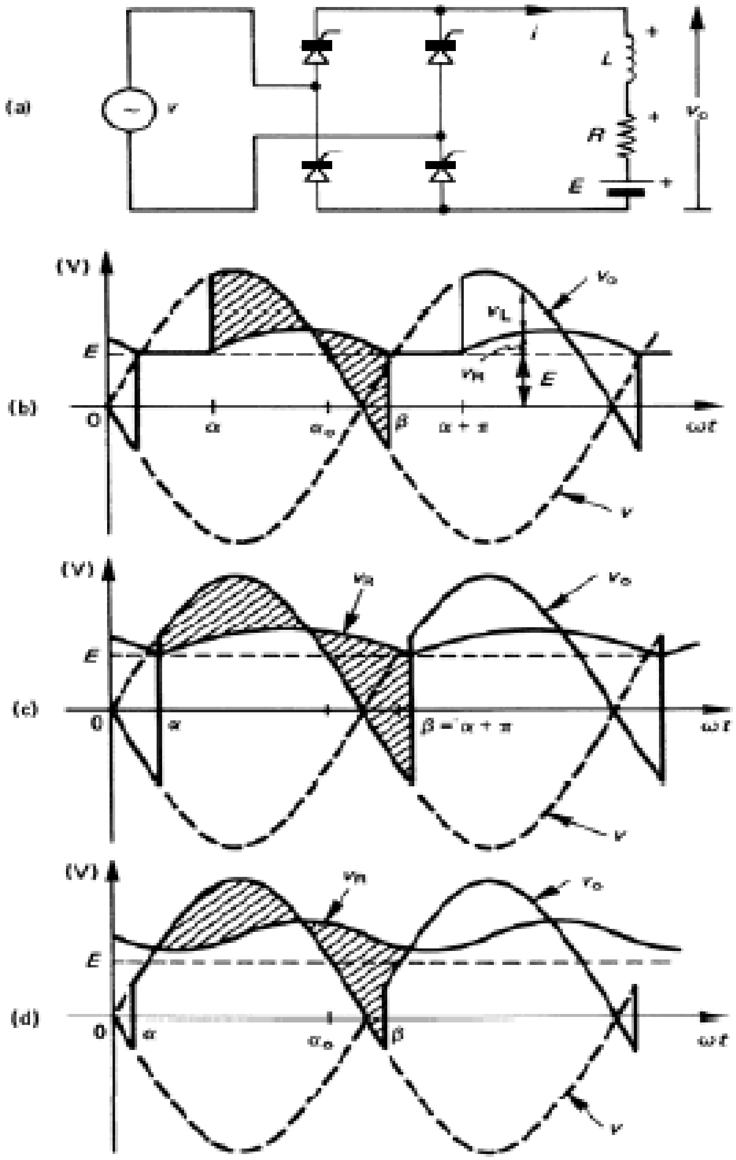


Fig. 5.4 A full-wave fully controlled converter with an inductive load which includes an emf source: (a) circuit diagram; (b) voltage waveforms with discontinuous load current; (c) verge of continuous load current; and (d) continuous load current.

- Note that in the equation just before Fig. 5.4, it has been assumed the emf has the polarity shown in figure 5.4a. Load current can flow with a firing angle defined by

$$0 \le \alpha \le \alpha_o \quad \text{[rad]}$$

whence $\hat{\alpha} = \pi - \alpha_o$

The load circuit current can be evaluated by using

$$\sqrt{2}V \sin \omega t = L\frac{di}{dt} + Ri + E \quad \text{[V]}$$

- For continuous load current conditions, as shown in figures 5.4c and 5.4d, the mean output voltage is given by

$$V_o = \frac{2\sqrt{2}V}{\pi}\cos\alpha \quad \text{[V]}$$

- The average output voltage is dependent only on the phase delay angle *α*. The mean load current is given by

$$I_o = \frac{V_o - E}{R} = \frac{\sqrt{2}V}{R}\left(\frac{2}{\pi}\cos\alpha - \frac{E}{\sqrt{2}V}\right) \text{ [A]}$$

- The power absorbed by the emf source in the load is $P = I_o E$. The output voltage harmonic magnitudes for continuous conduction, are given by

$$V_n = \frac{\sqrt{2}V}{2\pi} \times \left(\frac{1}{(2n-1)^2} + \frac{1}{(2n+1)^2} - \frac{2\cos 2\alpha}{(2n-1)(2n+1)}\right) \text{ for n} = 1, 2, 3\ldots$$

- The dc component across the R-L part of the load is

$$V_{oR-L} = V_o - E$$

$$= \frac{2\sqrt{2}V}{\pi}\cos\alpha - E \quad \text{[V]}$$

5.3 Generation of Harmonics

- The ***n^th^ order harmonic*** (commonly referred to as simply the n^{th} harmonic) in a signal is the sinusoidal component with a frequency that is n times the fundamental frequency.

- Harmonics distort current and/or voltage waves, disturbing the electrical distribution system and degrading power quality.

- Example of signals (current and voltage waves) on the Zambian electrical distribution system:
  1. The value of the fundamental frequency (or first order harmonic) is 50 hertz (Hz)
  2. The second (order) harmonic has a frequency of 100 Hz
  3. The third harmonic has a frequency of 150 Hz, etc

- A distorted signal is the sum of a number of superimposed harmonics.
- Figure 5.5 shows an example of a current wave affected by harmonic distortion.

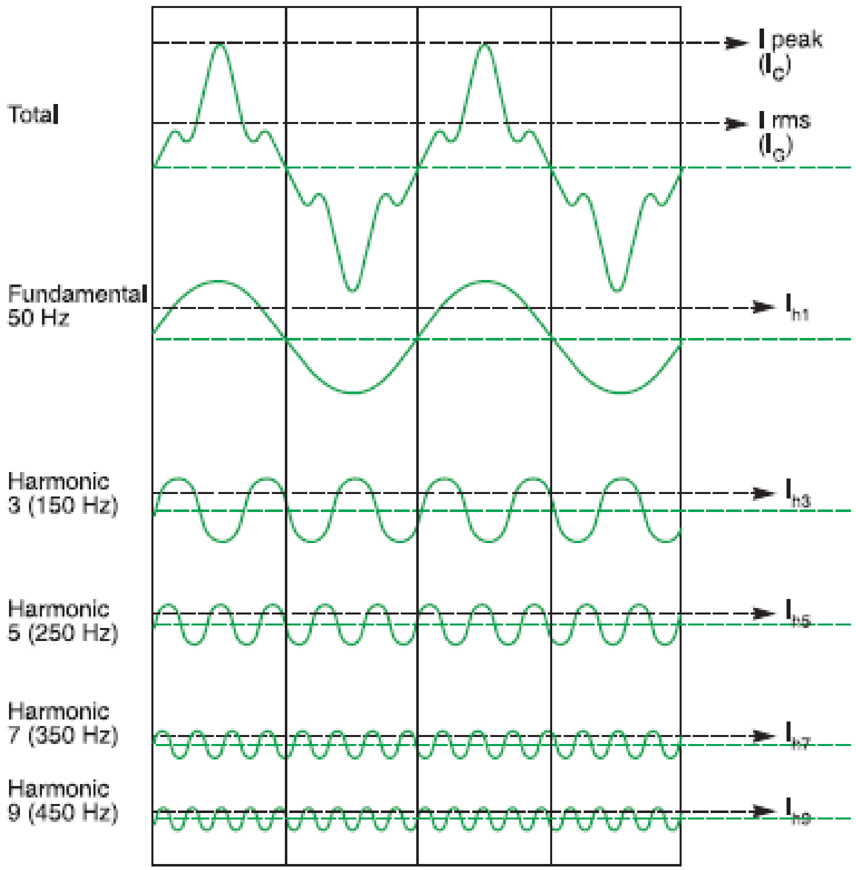


Figure 5.5 *- Example of a current containing harmonics and expansion of the overall current into its harmonic orders 1 (fundamental), 3, 5, 7 and 9.*

5.3.1 Representation of harmonics: the frequency spectrum

- The frequency spectrum is a practical graphical means of representing the harmonics contained in a periodic signal.
- The graph indicates the amplitude of each harmonic order.
- This type of representation is also referred to as spectral analysis.
- The frequency spectrum indicates which harmonics are present and their relative importance.
- Figure 5.6 shows the frequency spectrum of the signal presented in figure 5.5.

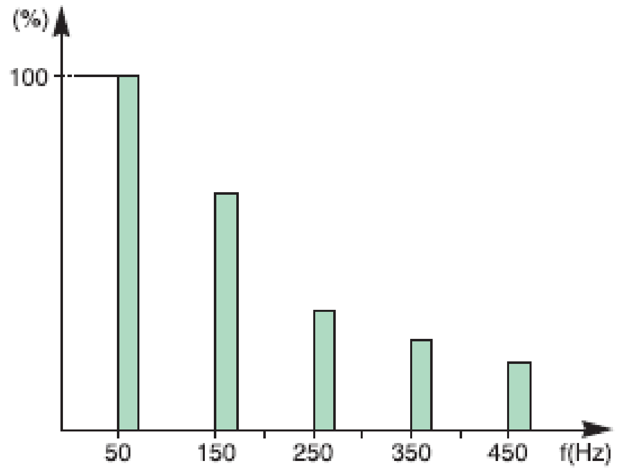


***Figure 5.6** - Spectrum of a signal comprising a 50 Hz fundamental and harmonic orders 3 (150 Hz), 5 (250 Hz), 7 (350 Hz) and 9 (450 Hz).*

- Devices causing harmonics are present in all industrial, commercial and residential installations. Harmonics are caused by ***non-linear loads.***

- A load is said to be **non-linear when the current it draws does not have the same** wave form as the supply voltage.

5.3.2 Examples of non-linear loads

Devices comprising **power electronics circuits are typical non-linear loads:**

- industrial equipment (welding machines, arc furnaces, induction furnaces, rectifiers)
- variable-speed drives for asynchronous and DC motors
- office equipment (PCs, photocopy machines, fax machines, etc.)
- household appliances (television sets, microwave ovens, fluorescent lighting, etc.)
- UPSs.

Note: Saturation of equipment (essentially transformers) may also cause non-linear currents.

5.3.3 Influence of Harmonics

The following two points must be considered regarding the influence of inverter's power supply harmonics:

1. **Influence of the inverter's power supply harmonics on other peripheral devices**

2. **Suppression of the outgoing harmonic current to the power receiving point for the consumer**

- Since harmonics are generated in the converter circuit (rectifying circuit) of the inverter, the inverter can be represented as a power supply in an equivalent circuit with regard to harmonics as shown in figures 5.7 and 5.8.

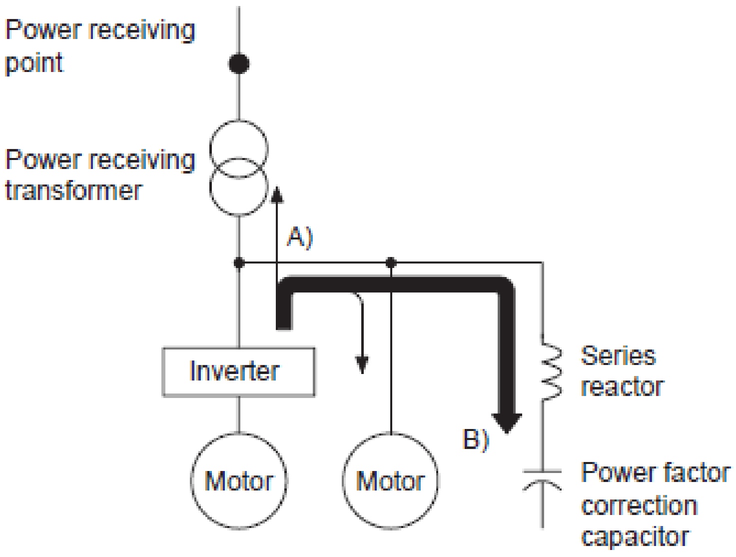


Fig. 5.7 System Diagram Example

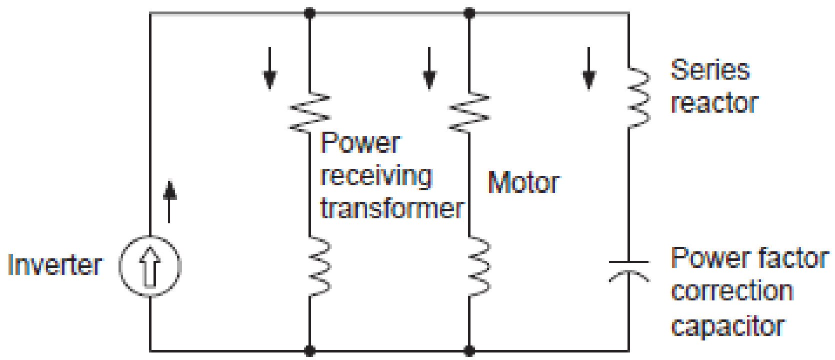


Fig. 5.8 Equivalent Circuit

Influence of Harmonics (Excerpts from Electricity Joint Research Vol. 46, No. 2)

Affected Equipment		Conditions	Ratio (%)
Power Capacitor	Equipment itself, series reactor	Occurrence of burnout, overheat, vibration and/or noise due to excessive current	75
	Fuse	Fusing or malfunction due to excessive current	1
Breaker for motor Earth leakage circuit breaker		Malfunction	3
Household appliances	Stereo Television	Noise / inteference Video jitter / picture deterioration	3
Others	Motor Elevator Various control equipment Harmonic filter	Vibration, noise Vibration, stop Malfunction Failure due to excessive current	18

5.3.4 Harmonic Distortion

- Thus, the currents of the line-commutated rectifiers are far from being sinusoidal.

- The currents from the Graetz bridge have the following harmonic content:

$$i_A = \frac{2\sqrt{3}}{\pi} I_D \left(\cos \omega t - \frac{1}{5}\cos 5\omega t + \frac{1}{7}\cos 7\omega t - \frac{1}{11}\cos 11\omega t + \ldots \right)$$

- Some characteristics of the currents in this equation are: (i) the absence of tripple harmonics; (ii) the presence of harmonics of order $6k \pm 1$ for integer values of k; (iii) those of order $6k+1$ are of positive sequence, and those of order $6k-1$ are of negative sequence.

- The rms magnitude of the fundamental frequency is:

$$I_1 = \frac{\sqrt{6}}{\pi} I_D$$

- The rms magnitude of the k^{th} harmonic is:

$$I_k = \frac{I_1}{k}$$

- The delta side of a rectifier transformer has the following harmonic content in phase a:

$$i_A = \frac{2\sqrt{3}}{\pi} I_D \left(\cos \omega t + \frac{1}{5} \cos 5\omega t - \frac{1}{7} \cos 7\omega t - \frac{1}{11} \cos 11\omega t + \ldots \right)$$

- This series differs from that of the star connected transformer by the sequence of rotation of harmonic orders $6k \pm 1$ for odd values of k, i.e. the 5^{th}, 7^{th}, 17^{th}, 19^{th}, etc.

5.3.5 Special Configurations for Harmonic Reduction

1. PASSIVE FILTERS

A common solution for harmonic reduction is through the connection of passive filters (shown below), which are tuned to trap a particular harmonic frequency.

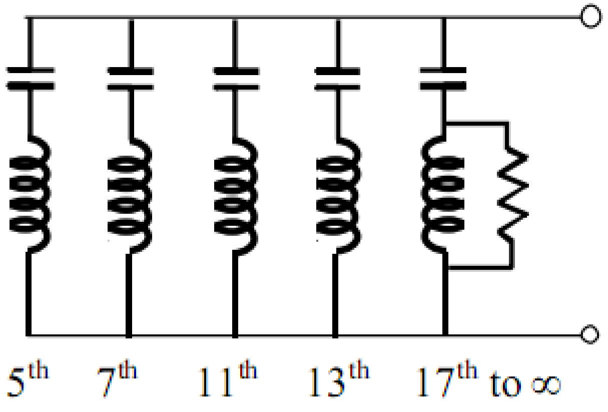


Fig. 5.9 (a) Typical passive filter for one phase

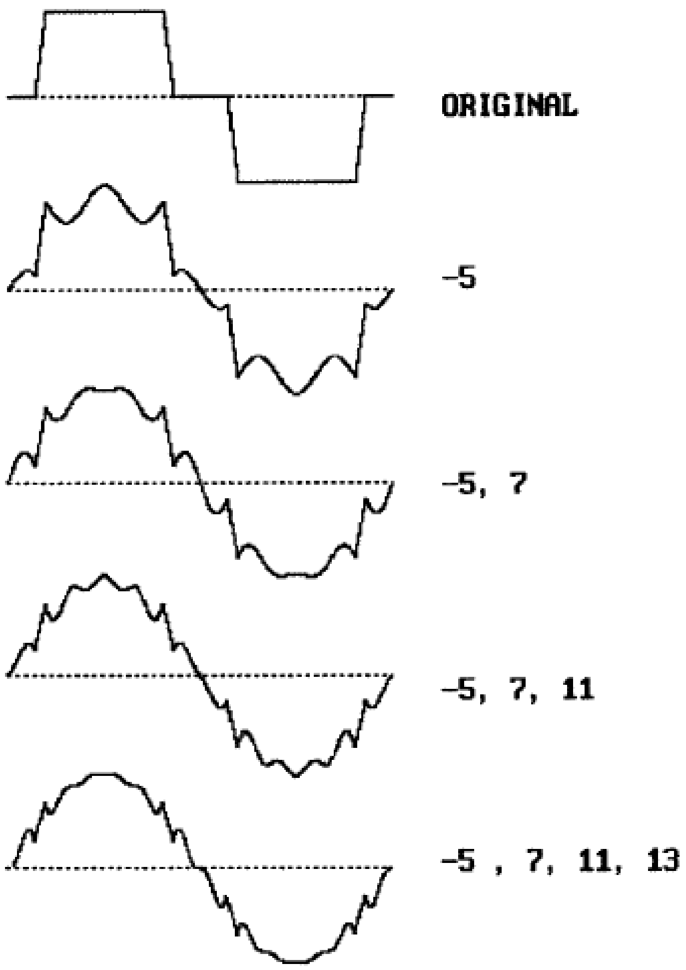


Fig. 5.9 (b) Harmonic trap results.

2. 12-PULSE CONFIGURATION

Harmonics can also be eliminated using a twelve-pulse configuration consisting of two sets of converters connected as shown below:

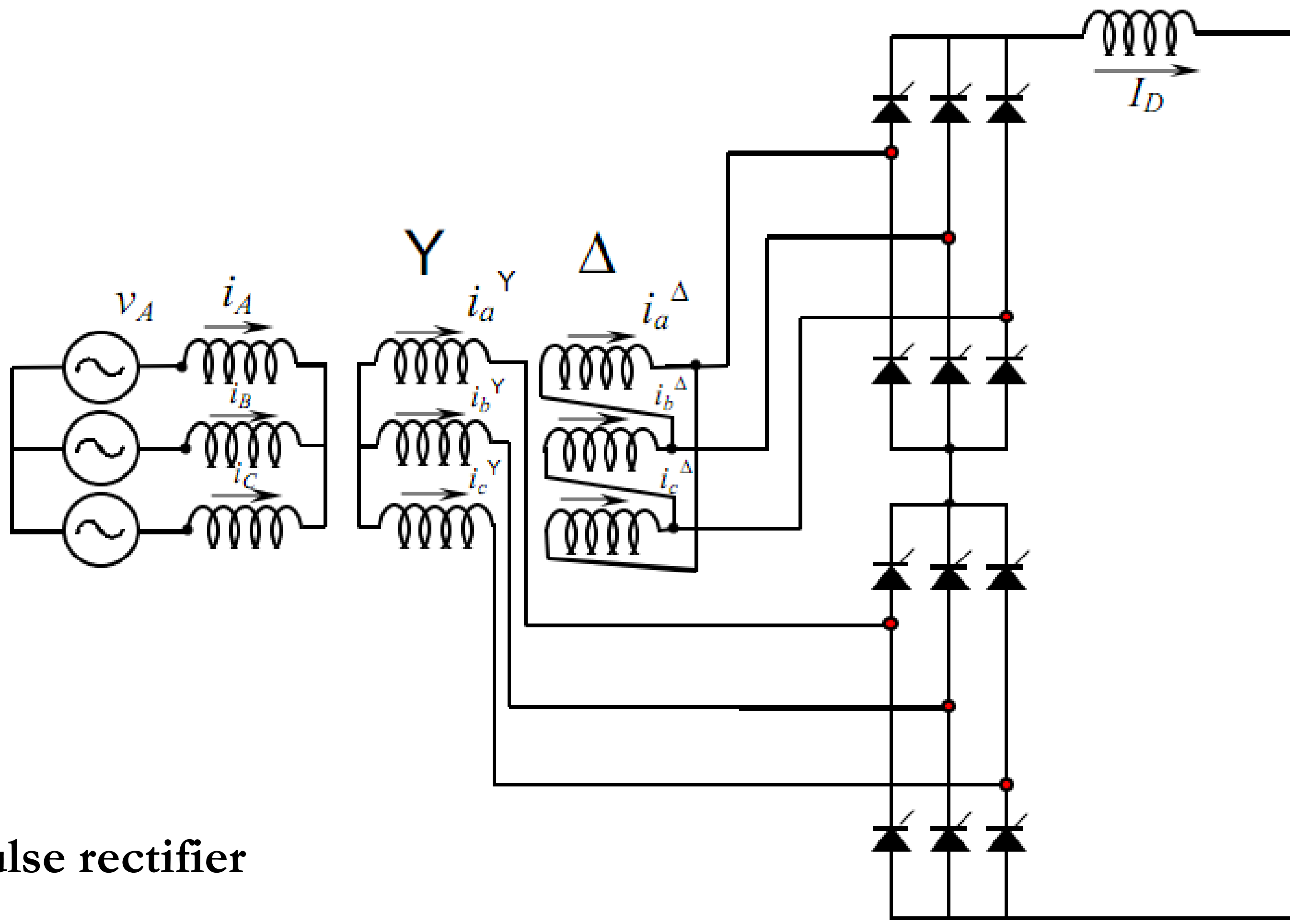


Fig. 5.10 12-pulse rectifier

- Note that the resultant ac current is given by the sum of the two Fourier series of the star connection and the delta connection transformers:

$$i_A = 2\frac{2\sqrt{3}}{\pi}I_D\left(\cos\omega t - \frac{1}{11}\cos 11\omega t + \frac{1}{13}\cos 13\omega t - \frac{1}{23}\cos 23\omega t + \ldots\right)$$

- This new series only contains harmonics of order $12k \pm 1$.

- The harmonic currents of order $6k \pm 1$ with k odd, i.e. 5^{th}, 7^{th}, 17^{th}, 19^{th}, etc., circulate between the two converter transformers but do not penetrate the ac network.

- The 12-pulse is achieved with a 30^{o} phase-shift between the two secondary transformers.

3. HIGH-POWER FACTOR CONVERTER

- High power factor converters perform switching operation with transistors in the rectifying circuit (converter circuit) in order to shape a current waveform to a sine wave. Harmonics can be reduced most significantly with this method.

- An inverter can satisfy the requirements of the guideline on harmonics without any other suppression techniques. This is the best technique to suppress inverter-generated harmonics.

- However, the switching operation in the rectifying circuit at a high-frequency range may increase noise.

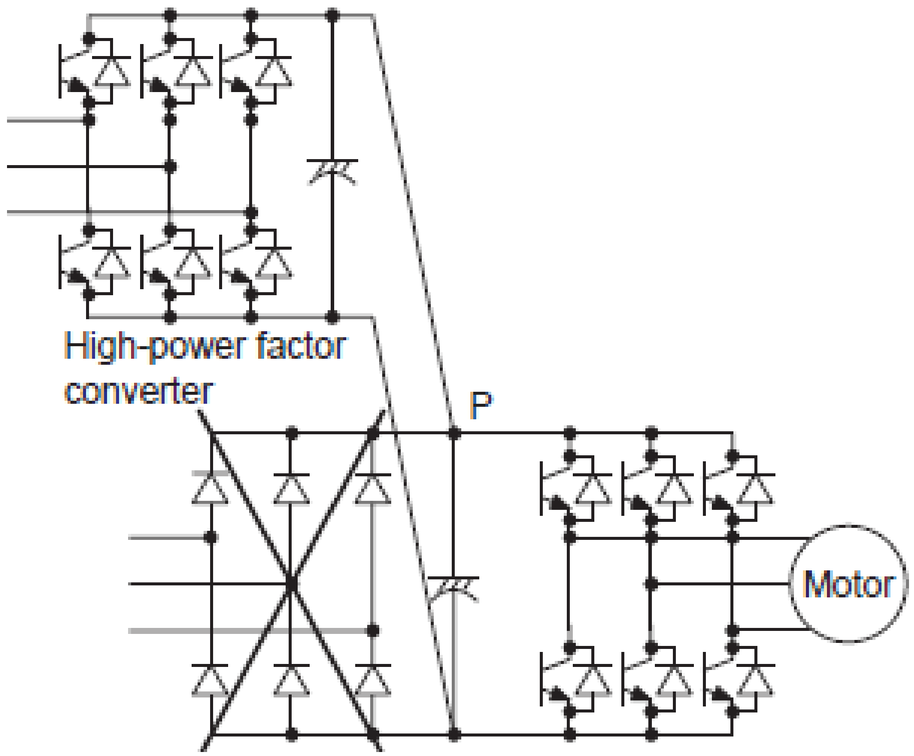


Fig. 5.11 High power factor converter

4. AC REACTOR

- Install an AC reactor to the power supply side of the inverter to increase line impedance, suppressing harmonics.

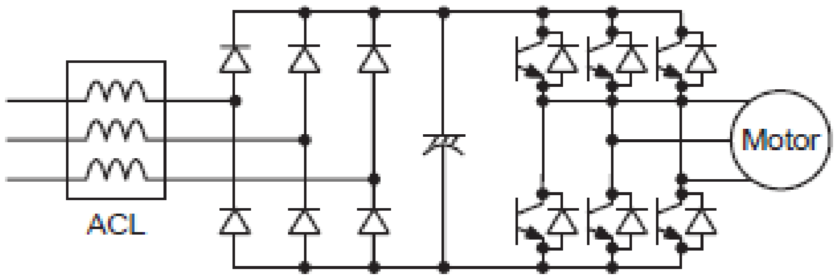


Fig. 5.12 ACL connection example

5. DC REACTOR

- Install a DC reactor in the DC circuit of the inverter to increase impedance, suppressing harmonics.

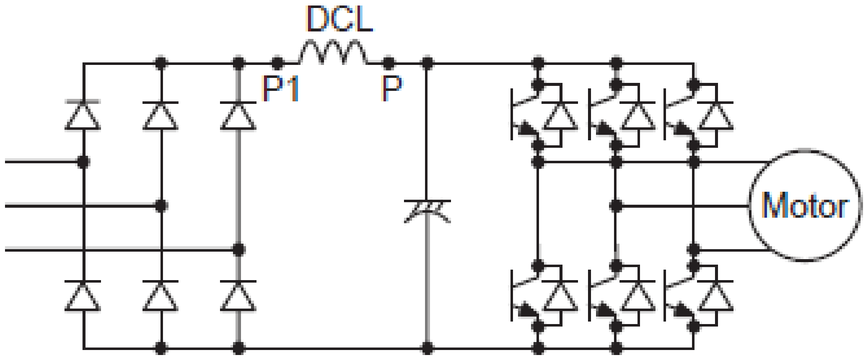


Fig. 5.13 DCL connection example

6. AC AND DC REACTORS USED TOGETHER

- Install an AC reactor in the power supply side and a DC reactor in the DC circuit to increase impedance, suppressing harmonics.

- Using the AC and DC reactors together increases the harmonic suppression effect.

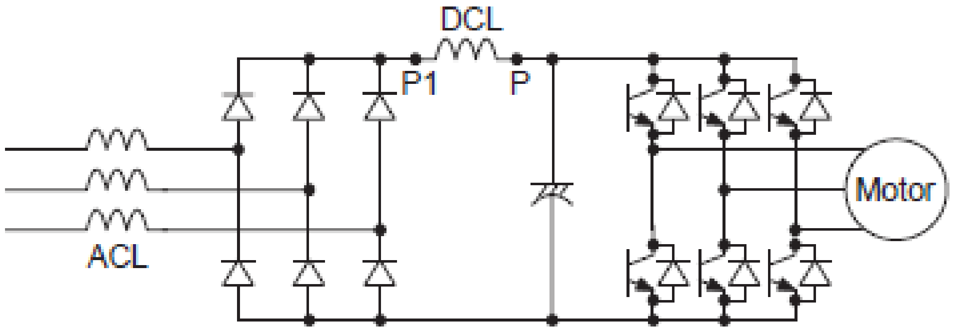


Fig. 5.14 Example of using ACL and DCL together

7. POWER FACTOR IMPROVING CAPACITOR

- A power factor improving capacitor has small impedance for harmonics, so harmonic currents concentrate on that capacitor.

- Using a power factor improving capacitor with a series reactor absorbs harmonic current escaping to a power receiving point.

- A power factor correction capacitor may either be installed in the high or low voltage side.

- A power factor correction capacitor installed in the low voltage side has a higher (about twice as large) absorption effect than a power factor correction capacitor of the same capacity installed on the high voltage side.

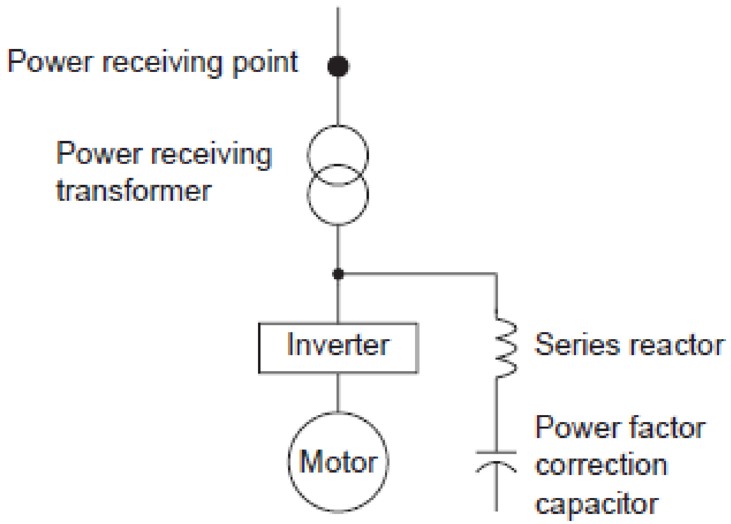


Fig. 5.15 System Example

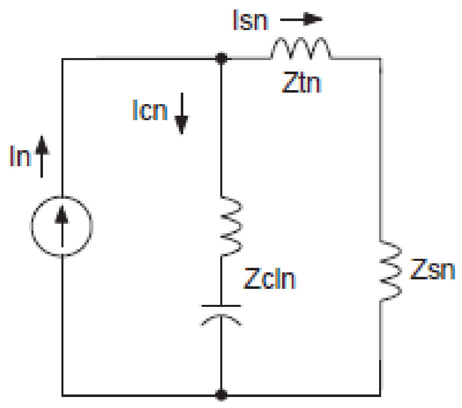


Fig. 5.16 Equivalent Circuit

8. MULTI-PHASE TRANSFORMERS

- Use two or more transformers at 30° phase-angle differences, such as Y-Δ and Δ-Δ.
- Different peak current timings of the transformers produce a harmonic suppression effect equivalent to that of a 12-pulse converter.

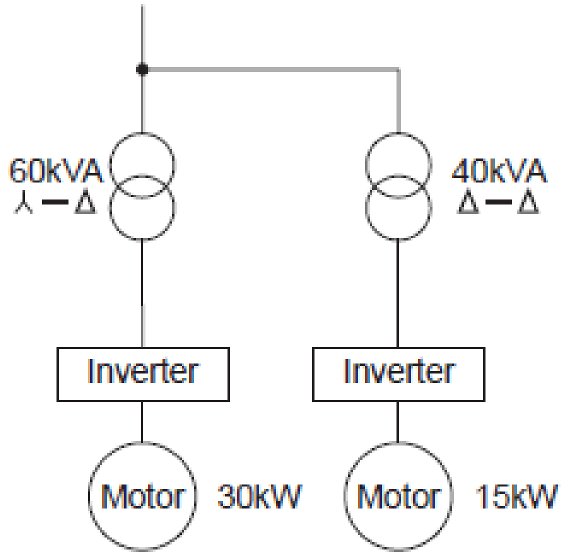


Fig. 5.17 Multi-Phase Operation Example

Notes:

- When the two lines have the same capacity, outgoing harmonic currents are suppressed to ½.

- The third harmonics of a Y-Y Transformer escape to the system, it is wiser to use a Δ connection transformer for either of the primary or secondary winding, with the exception for small capacities.

9. ACTIVE FILTER

- An active filter detects the current of a harmonic current generating circuit and generates a harmonic current equivalent to a difference between the detected harmonic current and fundamental wave current to suppress the harmonic current at the detection point.

- Compensating for a whole waveform, a single filter can be used for suppression of more than one harmonic degree.

- An active filter's harmonic absorption capacity decreases at an inflow of excessive harmonic current. However, it is resistant to overheating and burning because it is equipped with the protective function.

- Fig. 5.18 shows an active filter model.

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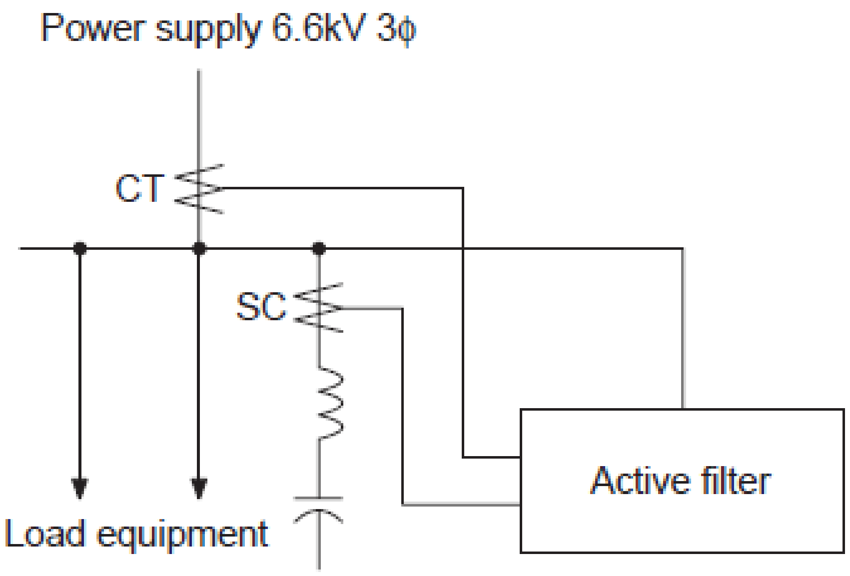

Fig. 5.18 Active Filter Connection Example

Notes:

1) The AC filter (passive filter) can be made more compact by using ACL or DCL with the inverter.

2) To prevent a leading power factor, it is desirable to switch the AC filter ON/OFF according to inverter operation.

5.3.6 Harmonic measurement

- Harmonics in circuits may be measured in any of the following methods:

(1) FFT analyzer

Use an FFT (Fast Fourier Transform) analyzer with a current detection circuit to measure the current of each frequency component, compare it with the fundamental wave component, and find a content.

(2) Harmonic monitoring system

Monitoring system using harmonic transducer

(3) Simple measuring instrument

A portable harmonic measuring instrument which can easily measure and record a harmonic content is available on the market.

Example: Harmonic monitor HM2300: made by Shizuki Electric

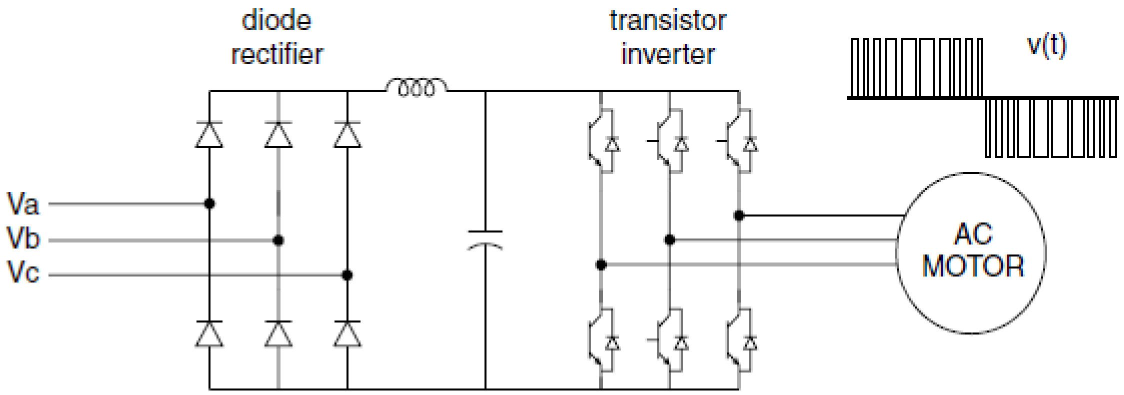

Fig. 5.19 PWM ASD

Notes:

- AC drives generally use standard squirrel cage induction motors. These motors are rugged, relatively low in cost, and require little maintenance.
- Synchronous motors are used where precise speed control is critical.

5.4 Voltage Conditioning in a Process-Based Plant

- Electrical power supply voltage disturbances, particularly voltage sags, have been identified as a major cause of unplanned production stoppages in continuous process-based industries.

- Power electronic technology has allowed the development of new high performance power conditioning equipment that can mitigate problems with the incoming power supply at the point of interface with the utility or within the plant internal distribution system.

- The replacement of conventional motor starters by VSDs and the wide spread use of power electronic devices has provided the industry with much greater process control.

- The balance of production stoppages is due largely to voltage variations and of these by far the most common are voltage sags.

5.3.1 Effects of Voltage Sags

- Sensitive electronic equipment such as VSDs can stop.
- Sensing devices such as photo-electric or proximity sensors cease to operate.
- Contactor and relay coils drop out.
- UPS systems will transfer to battery.

5.3.2 Active Voltage Conditioning

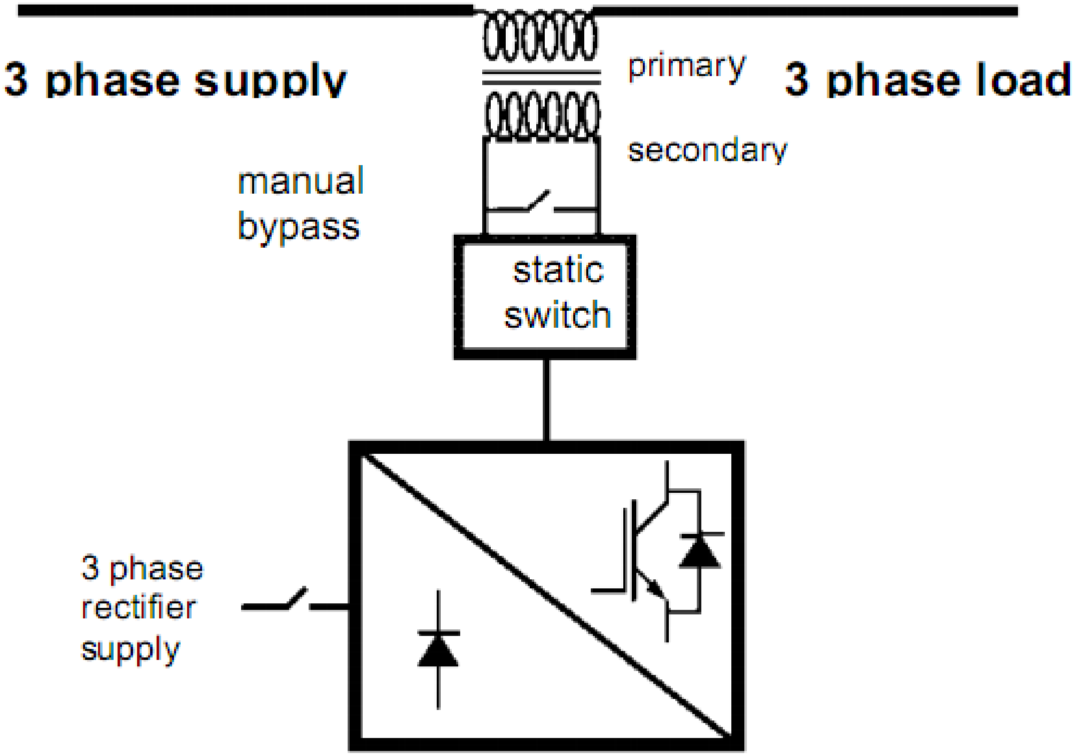

Fig. 5.20 AVC system one-line diagram

Basic AVC Concept

- The transformer primary is connected in series with the load as shown in fig. 5.20.
- The secondary windings are connected to the power electronics as shown.
- When a voltage sag occurs, it is detected by sensors on the supply side of the transformer primary.
- Current is then drawn from the line, converted to DC by the rectifier, the inverter draws power from the rectifier and will inject voltage into the transformer secondary at the correct amplitude and phase angle to correct the voltage sag.
- Typical response times for correction are of the order of 2-10 ms.

End of Lecture 5!