# EEE 5451 Power Electronics

## Lecture 6: DC Choppers

# 6. 1 Introduction

- A DC chopper converts directly from DC to DC voltage and is also known as a DC to DC converter.

- A chopper can be considered as a DC equivalent to an AC transformer with a continuously variable ratio. Like a transformer, it can be used to step-down or step-up a DC voltage source.

- A step-down chopper circuit is called a buck converter, while a step-up converter is referred to as a boost converter.

- Choppers are widely used for DC power supplies, DC motor drives, and so on.

- DC to DC conversion is often associated with stabilizing the output while the input varies.

- However the converse is also required in some applications, which is to produce a variable DC from a fixed source.

- The issues of selecting component parameters and calculating the performance of the system is the focus of this topic.

- Since these converters are switched mode systems, they are often referred to as choppers.

# 6.2 Buck Converter

- The chopper switch in Fig. 6.1 can be implemented by using a power BJT, power MOSFET, GTO, and so on.
- Analysis of the buck converter begins by making the following assumptions:

1. The circuit is operating in the steady state.
2. The inductor current is continuous (always positive)
3. The capacitor is large enough so that the output voltage is held constant at $V_o$.
4. The switching frequency is $f$ and the switching period is $T = 1/f$. The switch is closed for time $DT$ and open for time $(1 - D)T$ with $D$ duty cycle, which is defined as a ratio of the on-time to period.
5. The components are ideal.

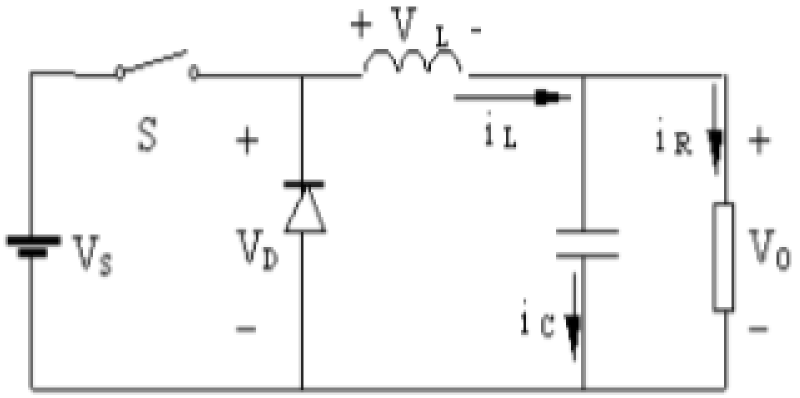


**Fig. 6.1 Buck Converter**

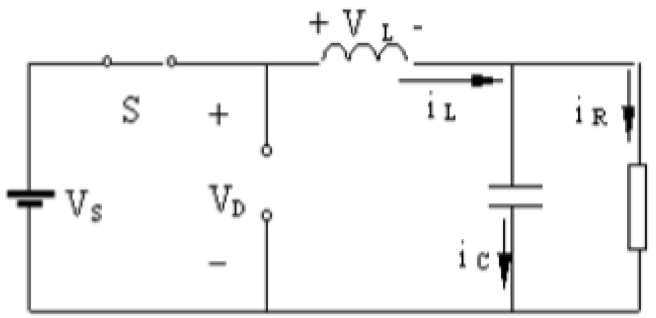


Fig. 6.2 Buck Converter: Mode 1

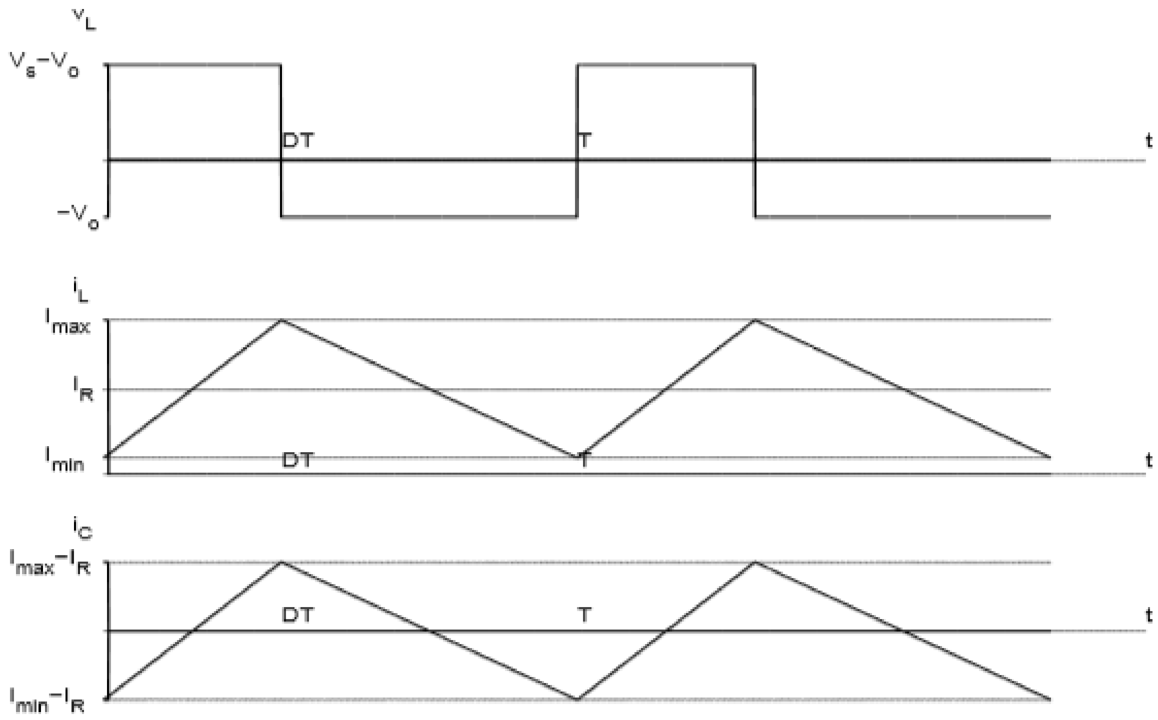


Fig. 6.4 Buck Converter output waveforms

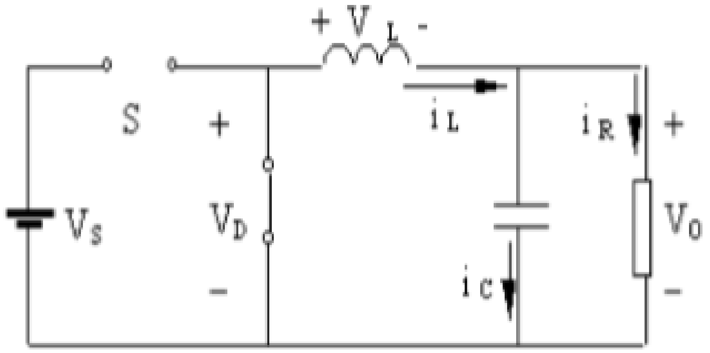


Fig. 6.3 Buck Converter: Mode 2

- The operation of the buck converter can be divided into two modes.

**Mode 1:** $0 \le t \le DT$

- In this mode, the switch is closed and the freewheeling diode is reverse biased. The equivalent circuit for this mode is given in Fig. 6.2.

- The inductor current $i_L$ is determined by

$$v_L = V_s - V_o = L\frac{\mathrm{d}i_L}{\mathrm{d}t}$$

That is $$\frac{\mathrm{d}i_L}{\mathrm{d}t} = \frac{V_s - V_o}{L}$$

- Suppose that $i_{L(0)} = I_{min}$. Then, integrating both sides of the above differential equation gives

$$\int_0^t \frac{\mathrm{d}i_L}{\mathrm{d}t}\mathrm{d}s = \int_0^t \frac{V_s - V_o}{L}\mathrm{d}s$$

As a result, $$i_L(t) = \frac{V_s - V_o}{L}t + I_{min}$$

which means that the inductor current increases linearly with time *t*.

**Mode 2:** $DT \le t \le T$

- In this mode, the switch is open and the freewheeling diode is forced to conduct by the inductor current. The equivalent circuit for this mode is given in Fig. 6.3.

- The inductor current $i_L$ is determined by

$$v_L = -V_o = L\frac{\mathrm{d}i_L}{\mathrm{d}t}$$

that is, $$\frac{\mathrm{d}i_L}{\mathrm{d}t} = -\frac{V_o}{L}$$

- Suppose that $i_L(DT) = I_{max}$. Then, integrating both sides of the above differential equation gives

$$\int_{DT}^{t} \frac{\mathrm{d}i_L}{\mathrm{d}t}\mathrm{d}s = -\int_{DT}^{t} \frac{V_o}{L}\mathrm{d}s$$

As a result, $$i_L(t) = -\frac{V_o}{L}(t-DT) + I_{max}$$

which means that the inductor current decreases linearly with time t.

- Under the steady state conditions, the inductor current at the end of Mode 1 is the same as that at the beginning of Mode 2, that is,

$$\frac{V_s - V_o}{L}DT + I_{min} = I_{max}$$

and the inductor current at the end of Mode 2 is the same as that at the beginning of Mode 1, that is,

$$-\frac{V_o}{L}(T-DT) + I_{max} = I_{min}$$

which implies that

$$I_{max} - I_{min} = \frac{V_o(1-D)T}{L}$$

- Adding the previous two equations, gives

$$\frac{V_s - V_o}{L} DT - \frac{V_o}{L}(T - DT) = 0$$

Solving this equation for $V_o$ yields $V_o = DV_s$

which means that the output voltage depends only on the input voltage and duty cycle.

- In order to determine the inductor current, we need to find $I_{min}$ and $I_{max}$. To this end, the average currents in the circuit satisfy the following equation:

$$I_L = I_C + I_R = \frac{V_o}{R}$$

- Note that the average capacitor current $I_C$ must be zero for the steady state.

- With $I_L$ determined, $I_{max}$ and $I_{min}$ are given by

$$\begin{aligned} I_{max} &= I_L + \frac{I_{max} - I_{min}}{2} = V_o\left[\frac{1}{R} + \frac{(1-D)T}{2L}\right] \\ I_{min} &= I_L - \frac{I_{max} - I_{min}}{2} = V_o\left[\frac{1}{R} - \frac{(1-D)T}{2L}\right] \end{aligned}$$

It is important to note that the above analysis is valid under the assumption of continuous inductor current, which means that $I_{min} \geq 0$.

*If* $I_{min} < 0$, then the freewheeling diode is off and so the inductor current will remain zero until the switch is closed again.

- In order to guarantee the continuity of the inductor current, the inductance *L* and the switching period *T* must be appropriately selected so that

$$I_{min} = V_o\left[\frac{1}{R} - \frac{(1-D)T}{2L}\right] \geq 0$$

that is,

$$L \geq \frac{R(1-D)T}{2}$$

or

$$T \leq \frac{2L}{R(1-D)}$$

which imply that

$$L_{min} = \frac{R(1-D)T}{2}$$

or

$$T_{max} = \frac{2L}{R(1-D)}$$

$$f_{min} = \frac{R(1-D)}{2L}$$

- The peak-to-peak ripple voltage at output is determined by

$$\Delta V_o = \frac{V_o(1-D)T^2}{8LC} = \frac{V_o(1-D)}{8LCf^2}$$

# Example:

A buck converter is supplied by a DC source of 48 V. It produces an output voltage of 18 V across a 10 Ω load resistor. Assume that the capacitor is large enough so that the output voltage is kept constant.

1. Determine the duty cycle D.
2. Find the minimum inductor size $L_{min}$ if the switching frequency is 40 kHz.
3. Calculate the inductor current if L =1 .25$L_{min}$ and f=40 kHz.
4. Select C so that the peak-to-peak output ripple voltage $\Delta V_o$ /$V_o$ does not exceed 0.5% if L =1 .25$L_{min}$ and f=40 kHz.
5. Compute the minimum switching frequency $f_{min}$ if L =1 .25$L_{min}$.

# Solution:

1. $D = \frac{V_o}{V_s} = \frac{18}{48} = 0.375$
2. $L_{min} = \frac{R(1-D)}{2f} = \frac{10\times(1-0.375)}{2\times40\times10^3} = 78\mu H$
3. 

$$
\begin{aligned}
L &= 1.25L_{min} = 1.25 \times 78 = 97.5\mu H \\
I_L &= \frac{V_o}{R} = \frac{18}{10} = 1.8A \\
I_{max} &= V_o\left[\frac{1}{R} + \frac{(1-D)}{2Lf}\right] = 18 \times \left[\frac{1}{10} + \frac{(1-0.375)}{2 \times 97.5 \times 10^{-6} \times 40 \times 10^3}\right] = 3.24A \\
I_{min} &= V_o\left[\frac{1}{R} - \frac{(1-D)}{2Lf}\right] = 18 \times \left[\frac{1}{10} - \frac{(1-0.375)}{2 \times 97.5 \times 10^{-6} \times 40 \times 10^3}\right] = 0.36A
\end{aligned}
$$

Note that

$$
\begin{aligned}
T &= \frac{1}{40 \times 10^3} = 0.000025s \\
DT &= 0.375 \times 0.000025 = 0.0000094s \\
\frac{V_s - V_o}{L} &= \frac{48-18}{97.5 \times 10^{-6}} = 307692 \\
\frac{V_o}{L} &= \frac{18}{97.5 \times 10^{-6}} = 184615
\end{aligned}
$$

So the inductor current is given by

$$i_L(t) = \begin{cases} \frac{V_s - V_o}{L} t + I_{min} & 0 \le t \le DT \\ -\frac{V_o}{L}(t - DT) + I_{max} & DT \le t \le T \end{cases}$$
$$= \begin{cases} 307692t + 0.36 & 0 \le t \le 0.0000094s \\ -184615(t - 0.0000094) + 3.24 & 0.0000094s \le t \le 0.000025s \end{cases}$$

4. Note that $\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} \le 0.005$. Then,

$$C_{min} = \frac{1-D}{0.005 \times 8Lf^2} = \frac{1 - 0.375}{0.005 \times 8 \times 97.5 \times 10^{-6} \times (40 \times 10^3)^2} = 100\mu F$$

5.

$$f_{min} = \frac{R(1-D)}{2L} = \frac{10(1-0.375)}{2 \times 97.5 \times 10^{-6}} = 32kHz$$

# 6.3 Boost Converters

- Fig. 6.5 shows a boost converter circuit. The chopper switch in this circuit can be implemented by using a power BJT, power MOSFET, GTO, and so on.

- Analysis of the boost converter begins by making the following assumptions:

1. The circuit is operating in the steady state.

2. The inductor current is continuous (always positive)

3. The capacitor is large enough so that the output voltage is held constant at $V_o$.

4. The switching frequency is f and the switching period is T =1/f. The switch is closed for time DT and open for time (1 − D)T with D duty cycle, which is defined as a ratio of the on-time to period.

5. The components are ideal.

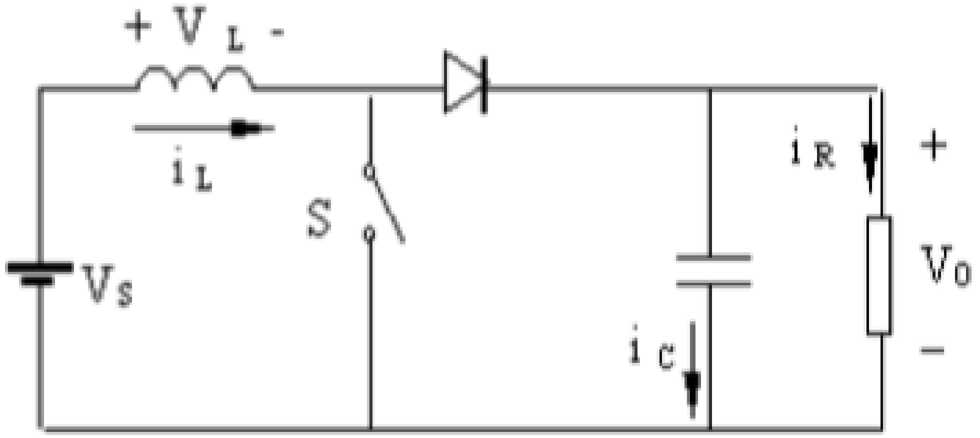


Fig. 6.5-Boost Converter

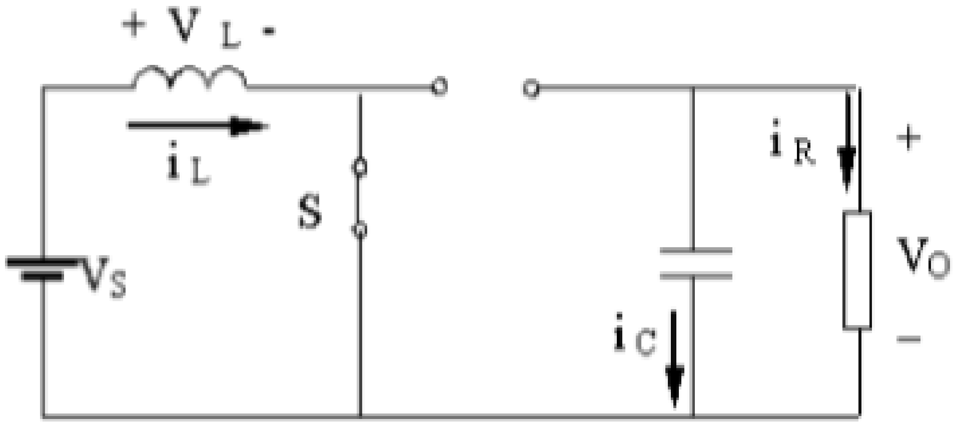


Fig. 6.6-Boost Converter: Mode 1

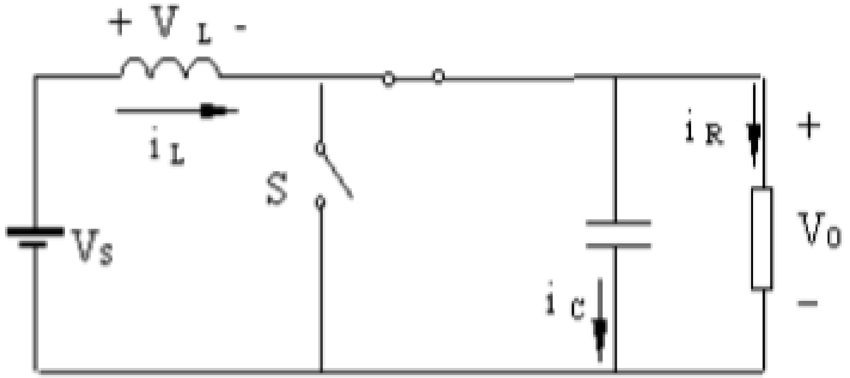


Fig. 6.7-Boost Converter: Mode 2

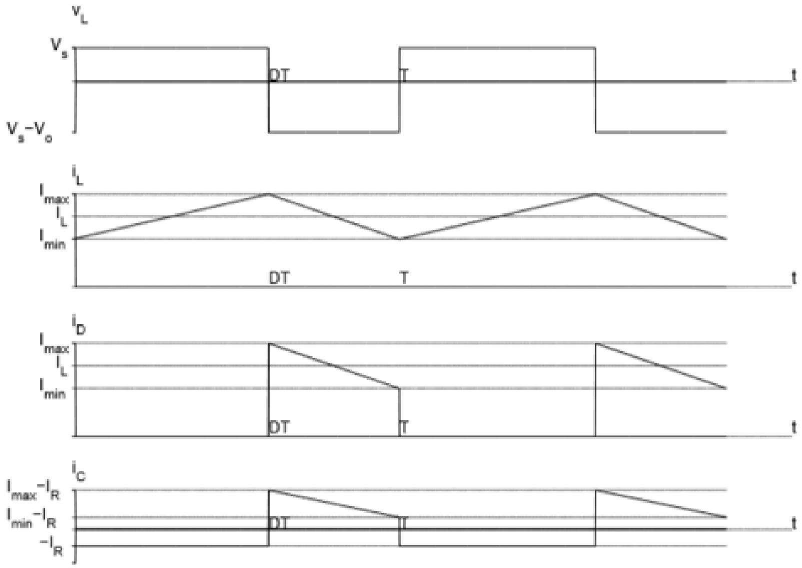


Fig. 6.8-Boost Converter Outputs

# Mode 1: $0 \le t \le DT$

- In this mode, the switch is closed and the freewheeling diode is reverse biased by $V_o$. The equivalent circuit for this mode is given in Fig. 6.6.

- The inductor current $i_L$ is determined by

$$v_L = V_s = L\frac{\mathrm{d}i_L}{\mathrm{d}t}$$

$$\frac{\mathrm{d}i_L}{\mathrm{d}t} = \frac{V_s}{L}$$

**Suppose that $i_L(0) = I_{min}$. Then**

$$\int_0^t \frac{\mathrm{d}i_L}{\mathrm{d}t}\mathrm{d}s = \int_0^t \frac{V_s}{L}\mathrm{d}s$$

$$i_L(t) = \frac{V_s}{L}t + I_{min}$$

which means that the inductor current increases linearly with time t.

# Mode 2: $DT \le t \le T$

- In this mode, the switch is open and the freewheeling diode is forward biased to provide a path for inductor current. The equivalent circuit for this mode is given in Fig. 6.7.

- The inductor current $i_L$ is determined by

$$v_L = V_s - V_o = L\frac{\mathrm{d}i_L}{\mathrm{d}t}$$

$$\frac{\mathrm{d}i_L}{\mathrm{d}t} = \frac{V_s - V_o}{L}$$

**Suppose that $i_L(DT)=I_{max}$**

$$\int_{DT}^t \frac{\mathrm{d}i_L}{\mathrm{d}t}\mathrm{d}s = \int_{DT}^t \frac{V_s - V_o}{L}\mathrm{d}s$$

- As a result,

$$i_L(t) = \frac{V_s - V_o}{L}(t - DT) + I_{max}$$

Under the steady state conditions, the inductor current at the end of Mode 1 is the same as that at the beginning of Mode 2, that is,

$$\frac{V_s}{L}DT + I_{min} = I_{max}$$

$$I_{max} - I_{min} = \frac{V_s DT}{L}$$

and the inductor current at the end of Mode 2 is the same as that at the beginning of Mode 1, that is,

$$\frac{V_s - V_o}{L}(T - DT) + I_{max} = I_{min}$$

- Adding gives

$$\frac{V_s}{L}DT + \frac{V_s - V_o}{L}(T - DT) = 0$$

$$V_o = \frac{V_s}{1 - D}$$

which means that the output voltage is higher than the input voltage.

- The average inductor current $I_L$ is determined as follows. Note that the power supplied by the source is the same as the power absorbed by the load due to the assumptions, that is,

$$P_o = \frac{V_o^2}{R} = \frac{\left(\frac{V_s}{1-D}\right)^2}{R} = P_s = I_L V_s$$

Solving this equation for $I_L$ gives

$$I_L = \frac{V_s}{(1 - D)^2 R}$$

With $I_L$ determined, $I_{max}$ and $I_{min}$ are given by

$$I_{max} = I_L + \frac{I_{max} - I_{min}}{2} = V_s \left[ \frac{1}{(1-D)^2 R} + \frac{DT}{2L} \right]$$
$$I_{min} = I_L - \frac{I_{max} - I_{min}}{2} = V_s \left[ \frac{1}{(1-D)^2 R} - \frac{DT}{2L} \right]$$

- Note that the above analysis is valid under the assumption of continuous inductor current, which means that $I_{min} \geq 0$.

- In order to guarantee the continuity of the inductor current, the inductance L and the switching period T must be appropriately selected so that

$$I_{min} = V_s \left[ \frac{1}{(1-D)^2 R} - \frac{DT}{2L} \right] \geq 0$$

that is,

$$\frac{1}{(1-D)^2 R} - \frac{D}{2Lf} \geq 0$$

or

$$L \geq \frac{D(1-D)^2 R}{2f}$$

which imply that

$$L_{min} = \frac{D(1-D)^2 R}{2f}$$

The peak-to-peak ripple voltage at output is determined by

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

# Example:

A boost converter is supplied by a DC source of 12 V. It produces an output voltage of 30 V across a 50 Ω load resistor. Assume that the capacitor is large enough so that the output voltage is kept constant.

1. Determine the duty cycle D.
2. Find the minimum inductor size $L_{min}$ if the switching frequency is 25 kHz.
3. Calculate the inductor current if $L$ = 120 µH and f=25 kHz.
4. Select C so that the peak-to-peak output ripple voltage $\Delta V_o/V_o$ does not exceed 1% if $L$ = 120 µH and f=25 kHz.

# Solution:

1. $D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$
2. $L_{min} = \frac{D(1-D)^2 R}{2f} = \frac{0.6 \times (1-0.6)^2 \times 50}{2 \times 25 \times 10^3} = 96\mu H$
3.

$$
\begin{aligned}
I_L &= \frac{V_s}{(1-D)^2 R} = \frac{12}{(1-0.6)^2 \times 50} = 1.5A \\
I_{max} &= V_s \left[\frac{1}{(1-D)^2 R} + \frac{D}{2Lf}\right] = 12 \times \left[\frac{1}{(1-0.6)^2 \times 50} + \frac{0.6}{2 \times 120 \times 10^{-6} \times 25 \times 10^3}\right] = 2.7A \\
I_{min} &= V_s \left[\frac{1}{(1-D)^2 R} - \frac{D}{2Lf}\right] = 12 \times \left[\frac{1}{(1-0.6)^2 \times 50} - \frac{0.6}{2 \times 120 \times 10^{-6} \times 25 \times 10^3}\right] = 0.3A
\end{aligned}
$$

Note that

$$
\begin{aligned}
T &= \frac{1}{25 \times 10^3} = 0.00004s \\
DT &= 0.6 \times 0.00004 = 0.000024s \\
\frac{V_s}{L} &= \frac{12}{120 \times 10^{-6}} = 100000 \\
\frac{V_s - V_o}{L} &= \frac{12 - 30}{120 \times 10^{-6}} = -150000
\end{aligned}
$$

So the inductor current is given by

$$
\begin{aligned}
i_L(t) &= \begin{cases} \frac{V_s}{L}t + I_{min} & 0 \le t \le DT \\ \frac{V_s - V_o}{L}(t - DT) + I_{max} & DT \le t \le T \end{cases} \\
&= \begin{cases} 100000t + 0.3 & 0 \le t \le 0.000024s \\ -150000(t - 0.000024) + 2.7 & 0.000024s \le t \le 0.00004s \end{cases}
\end{aligned}
$$

4. Note that $\frac{\Delta V_o}{V_o} = \frac{D}{RCf} \le 0.01$. Then,

$$C_{min} = \frac{D}{0.001Rf} = \frac{0.6}{0.01 \times 50 \times 25 \times 10^3} = 48\mu F$$

# 6.4 Buck-Boost Converter

- This converter has a schematic shown in fig. 6.9 and can provide output voltage that can be lower or higher than the input voltage.

- This converter is an inverting DC-to-DC converter i.e. polarity of the output voltage is reversed compared to the input supply. Thus, it is a negative-output buck-boost converter.

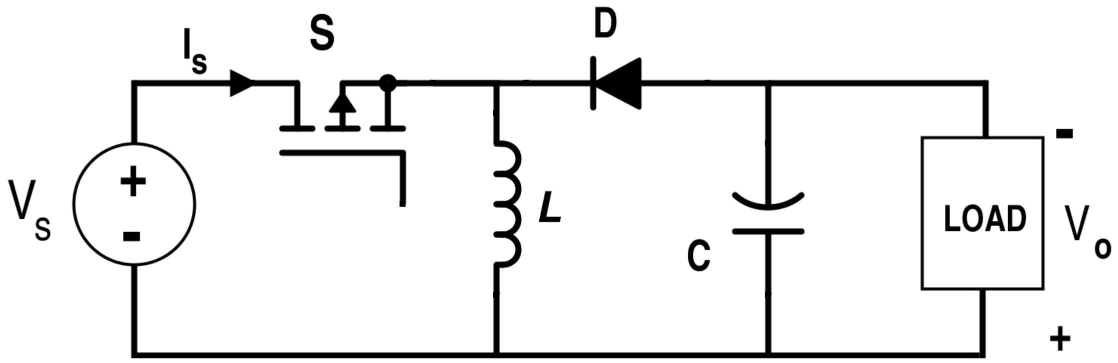


**Fig. 6.9-Buck-Booster Converter Circuit diagram.**

- Again the operation of the converter can be analyzed using the two topologies resulting from the operation of the switch as shown in Fig. 6.10 and Fig. 6.11.

- Let the capacitor be totally charged up before switching on the switch S. When the switch S is closed as shown in Fig. 6.10.

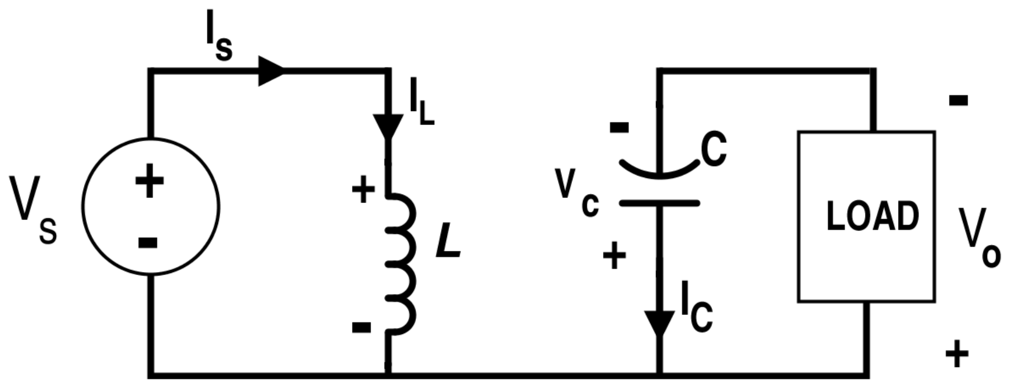


**Fig. 6.10- Buck-boost converter circuit when switch S is on (Mode-I)**

$-V_S + V_L = 0$

$$\Rightarrow V_S = V_L = L\frac{di}{dt}$$

Also,

$-V_C + V_O = 0$

$$\Rightarrow V_O = V_C$$

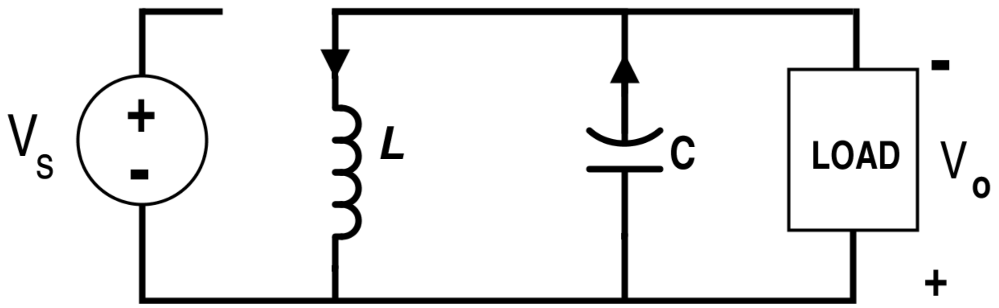


Fig. 6.11- Buck-boost converter circuit when switch S is off (Mode-II)

$$+V_L + V_C = 0$$

$$L\frac{di}{dt} + V_C = 0$$

$$\frac{di}{dt} = -\frac{V_C}{L}$$

- Waveforms for the voltage and current for buck-boost converter are shown in Fig. 6.12

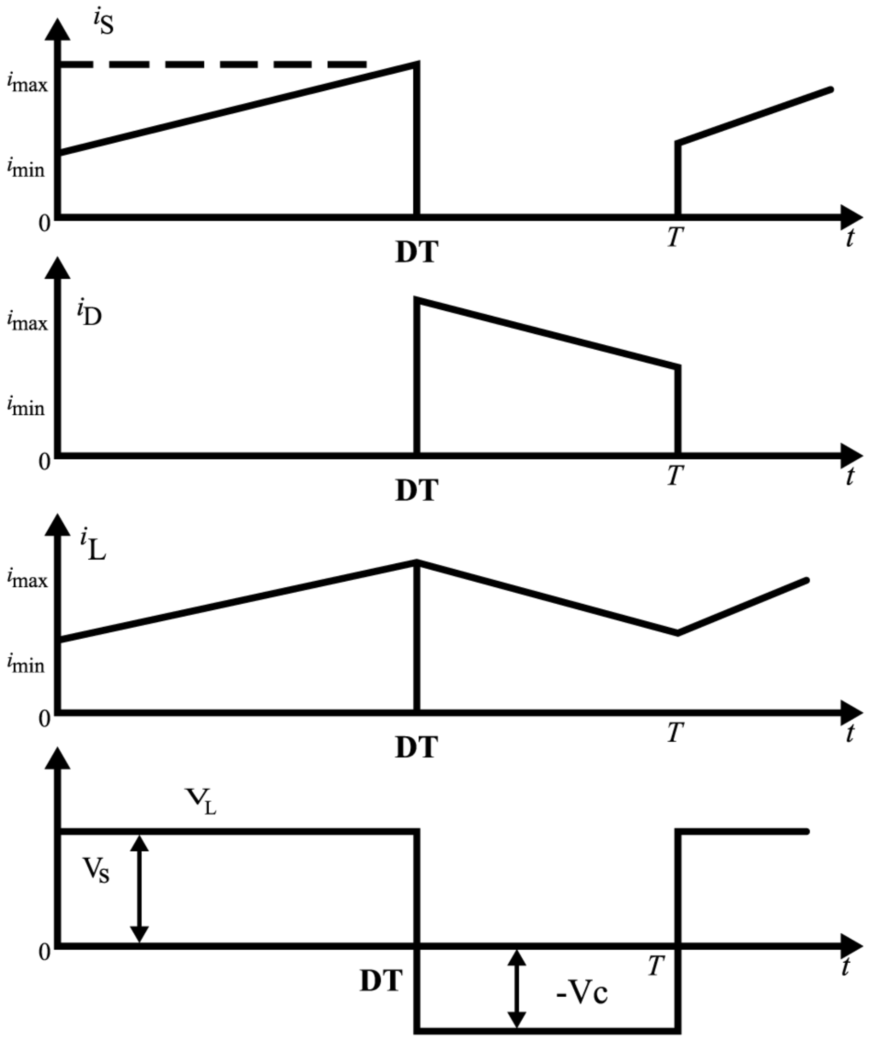


Fig. 6.12- Supply current, diode current, inductor current, and inductor voltage respectively

➢Now, from the waveforms given in Fig.6.12, when switch S is on,

$$Rise\,in\,the\,Inductor\,Current = I_{max} - I_{min} = \frac{V_S}{L}DT$$

And, when switch S is off,

$$Fall\,in\,the\,Inductor\,Current = I_{min} - I_{max} = -\frac{V_C}{L}(1-D)T$$

➢Equating the above two equations, we get

$$\frac{V_S}{L}DT = \frac{V_C}{L}(1-D)T$$

$$\Rightarrow V_O = V_C = \frac{D}{1-D}V_S$$

$$Average\,Inductor\,Current = \frac{I_{min} + I_{max}}{2}$$

➤As there is no supply current during the switch-off condition,

$$Input\ Power = P_{IN} = \frac{I_{max} + I_{min}}{2} DV_S$$

And,

$$Output\ Power = P_{OUT} = \frac{{V_O}^2}{R}$$

If there is no switching loss,

$P_{IN}=P_{OUT}$

$$\Rightarrow I_{max} + I_{min} = 2D\frac{V_S}{R(1-D)^2}$$

Combining the equations:

$$I_{min} = D\frac{V_S}{R(1-D)^2} - \frac{V_S}{2L}D$$

and

$$I_{max} = D\frac{V_S}{R(1-D)^2} + \frac{V_S}{2L}DT$$

For CCM, $I_{min}$=0

$$\Rightarrow Value\,of\,the\,Minimum\,Inductance\,for\,CCM = L_{min} = \frac{(1-D)^2}{2}TR$$

$$Ripple\,Voltage\,Across\,the\,Capacitor = \Delta V_C = \frac{\Delta Q}{C} = \frac{DTI_O}{C} = \frac{DTV_O}{RC} = \frac{D^2TV_S}{(1-D)RC}$$

as

$$V_O = \frac{D}{(1-D)}V_S$$

**Note:**

When D < 0.5, it acts as a step-down converter or a buck converter.

When D > 0.5, it acts as a step-up converter or a boost converter.

And when D = 0.5, input and output voltages are the same i.e. $V_O$=$V_S$.

➢Note that the basic circuit diagrams of all the fundamental converters are shown in Fig. 6.13. They consist of the same basic elements. The building blocks of these converters are DC supply $V_s$, load, diode D, power electronics switch S, inductor L, and capacitor C.

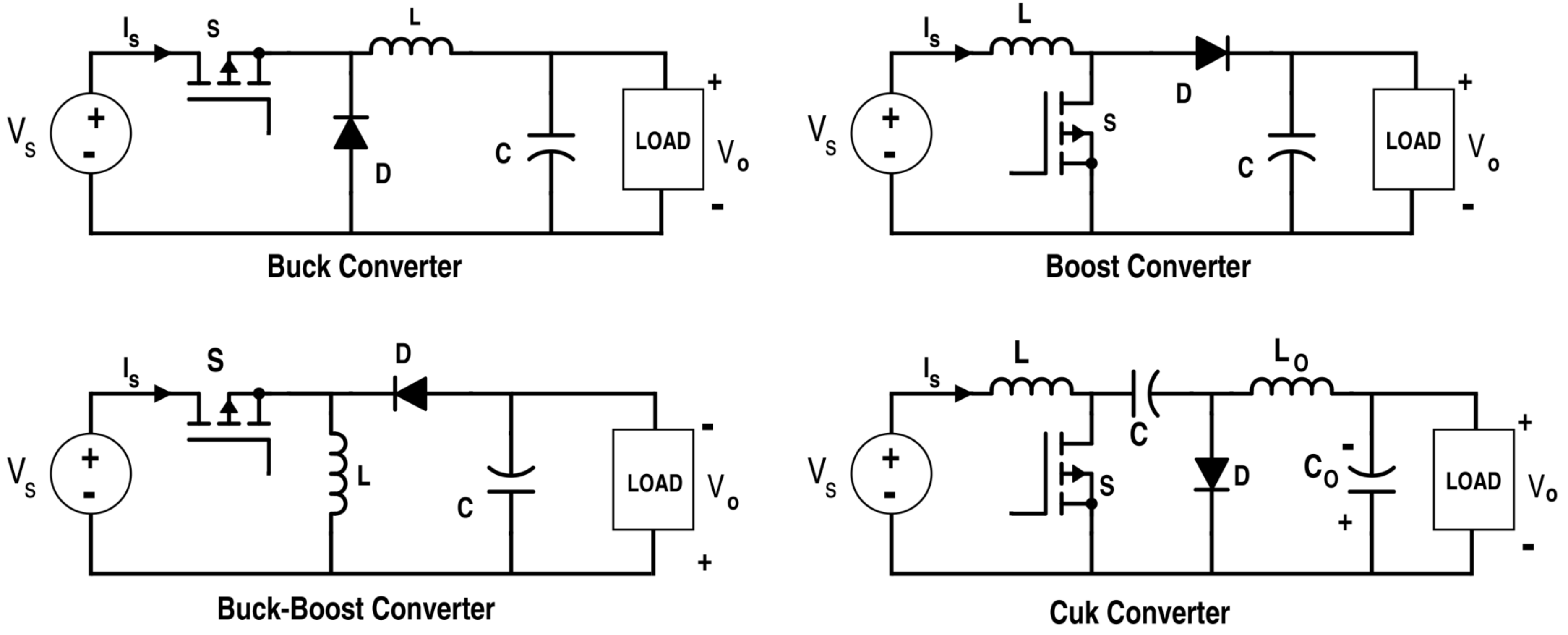


Fig. 6.13- Summary of Converters

# End of lecture 6!