EEE 5451 Power Electronics

Lecture 3: DC Drives Using Controlled Rectifiers

By Shadreck Mpanga
shadreck.mpanga@unza.zm

3.1 Introduction

- The speed of a d.c. motor can be controlled very easily by means of regulating its supply voltage by the use of phase controlled rectifiers. This control can be applied to either the field or the armature circuit.

- The motor response with armature control is faster than that with field control since the time constant of the field is very much larger than that of the armature. Generally, field control is used for speeds above rated value and armature control for speeds below rated value.

- Controlled rectifiers are widely used in d.c. drive applications where a.c. source is available. The single-phase or multi-phase a.c. is converted to d.c. by a controlled converter to give a variable d.c. source, by varying the triggering angle of the thyristor or any other power semiconductor device, that could be supplied to a d.c. motor and thus the speed of the motor can be controlled.

- Controlled rectifiers used in d.c. drives can be classified as follows:

1- Single-phase controlled rectifiers

(i) Single-phase half-wave converter drives.
(ii) Single-phase full-wave half-controlled converter drives.
(iii) Single-phase full-wave fully-controlled converter drives.
(iv) Single-phase dual converter drives.

2. Multi-phase controlled rectifiers

(i) Three-phase half-wave converter drives.
(ii) Three-phase full-wave fully-controlled converter drives.
(iii) Three-phase full-wave half-controlled converter drives.
(iv) Three-phase full-wave dual converter drives.

3.2 Single-Phase Converter Dives

- These are used for small and medium power motors up to 75-kW (100-hp) ratings.

- Fig. 3.1 shows a single-phase half-wave converter drive used to control the speed of a separately-excited d.c. motor.

- This d.c. drive is very simple, needs only one power switch and one freewheeling diode connected across the motor terminals for the purpose of dissipation of energy stored in the inductance of the motor and to provide an alternative path for the motor current to allow the power switch to commutate easily.

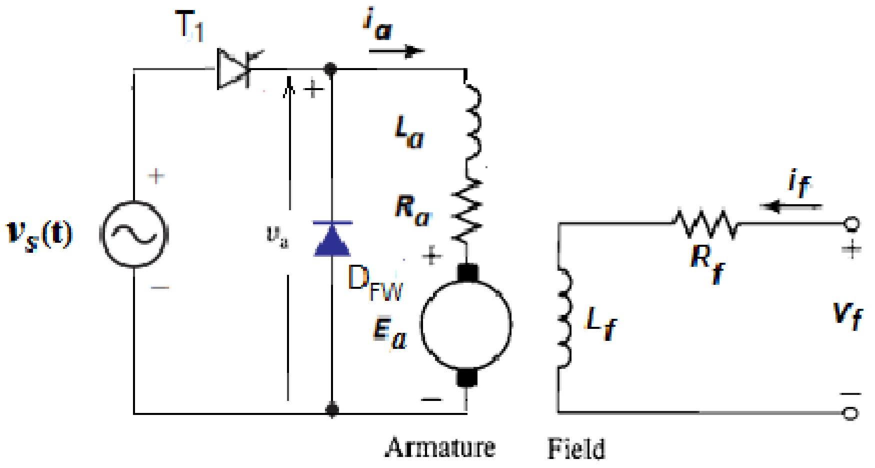


Fig.3.1 Single-phase half-wave converter drive.

- Waveforms for steady-state operation of the converter with the motor load is shown in Fig.3.2 for the case α ≡ 80˚.

- It is clear that during the interval β<ωt<2π , the armature current is zero, hence the torque developed by the motor is zero, the speed of the motor will be reduced.

- Since the mechanical time constant of the motor is larger than its electrical time constant, the inertia of the motor will maintain the speed, but its value will fluctuate resulting in poor motor performance.

- Therefore, this type of drive is rarely used; it is only used for small d.c. motors below 500 W ratings.

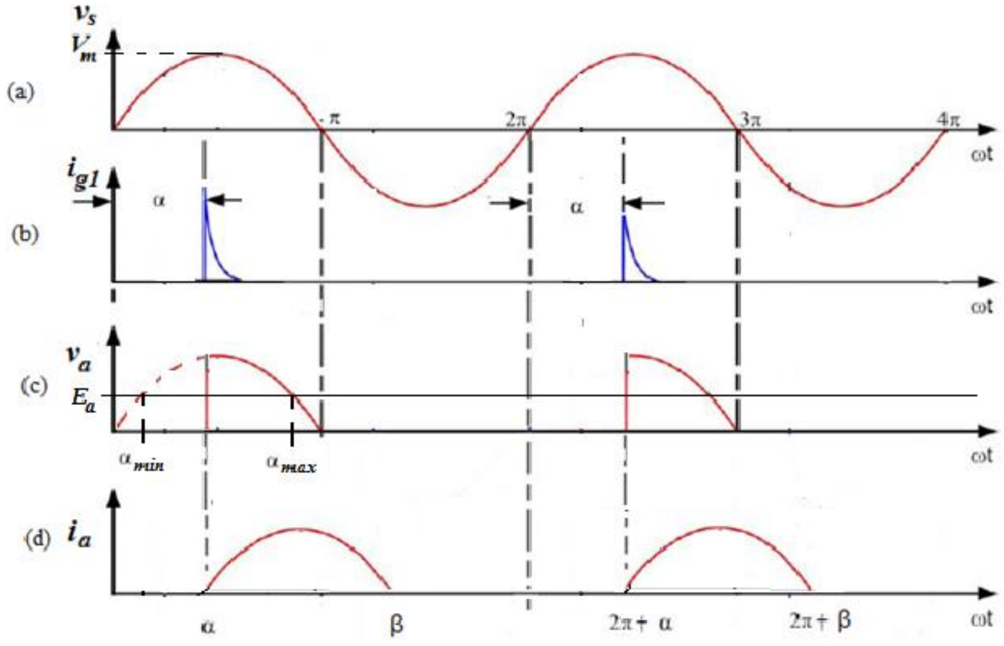


Fig.3.2 Waveforms for steady-state operation of the single-phase half-wave rectifier with motor load

The average value of the armature voltage can be evaluated as follows:

Assuming the supply voltage $v_s(\omega t) = V_m \sin \omega t$, thus in the positive half-cycle, T_1 will conduct from α to π , where α is the firing angle, and D_{FW} will conduct from π to β, where β is the extinction angle of the current. Hence the average value of the armature current will be,

$$V_{a(av)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_s(\omega t)\ d\omega t = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin \omega t\ d\omega t$$
$$= \frac{V_m}{2\pi}(1 + \cos\alpha) \qquad (3.1)$$

- It must be noted that the thyristor T1 only conducts when supply voltage exceeds back *emf* E_a , therefore, referring to Fig. 3.2(c), we define two triggering angles α_{min} and α_{max} as

α_{min} is the minimum firing angle below which the thyristor cannot be triggered. i.e. when the supply voltage $V_m\ sin\alpha > E_a$. This angle can be calculated as,

$$V_m \sin \alpha_{min} = E_a \quad \rightarrow \quad \alpha_{min} = \sin^{-1}\left(\frac{E_a}{V_m}\right) \qquad (3.2)$$

Similarly α_{max} is the maximum firing angle above which the thyristor cannot be triggered. Its value is given by

$$\alpha_{max} = \pi - \alpha_{min} = \pi - \sin^{-1}\left(\frac{E_a}{V_m}\right) \qquad (3.3)$$

The speed of the motor can be calculated from the general equation of the speed of d.c. motor as,

$$n = \frac{V_a}{K_e\emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_L$$

Substituting for $V_{a(av)}$ from Eq.(3.1) into the above equation we get,

$$n = \frac{V_m}{2\pi K_e \emptyset}(1+\cos\alpha) - \frac{R_a}{K_T K_e \emptyset^2} T_L \qquad (3.4)$$

or in terms of the angular velocity ω

$$\omega = \frac{V_{a(av)}}{K\emptyset} - \frac{R_a}{K^2\emptyset^2} T_L$$

Substituting for $V_{a(av)}$ we get,

$$\omega = \frac{V_m}{2\pi K\emptyset}(1+\cos\alpha) - \frac{R_a}{K^2\emptyset^2} T_L \qquad (3.5)$$

The starting torque can also be calculated from Eq.(3.4) or Eq.(3.5) by setting n or ω equal zero and calculate the torque, (using Eq.(3.5) for example), as

$$0 = \frac{V_m}{2\pi K\emptyset}(1+\cos\alpha) - \frac{R_a}{K^2\emptyset^2}T_{st}$$

From which,

$$T_{st} = \frac{K\emptyset V_m}{2\pi R_a}(1+\cos\alpha) \qquad (3.6)$$

And the no load speed is calculated from Eq.(3.5) by setting $T_L = 0$ to give,

$$\omega_o = \frac{V_m}{2\pi K\emptyset}(1+\cos\alpha) \qquad (3.7)$$

By knowing ω_o and T_{st} the mechanical characteristics of the motor can be obtained for various values of the triggering angle α as illustrated in the following example:

Example 3.1

For the d.c. drive circuit of a separately-excited motor shown in Fig. 3.1, it is required to calculate and draw the speed-torque characteristics of the motor for firing angles $\alpha = 0°$, $\alpha = 45°$, and $\alpha = 90°$. The supply voltage is 60V (rms) , the motor armature resistance $R_a = 0.5\ \Omega$ and the motor voltage constant $K\emptyset = 1$ V.s / rad.

Solution

Using equations (3.5) , (3.6) and (3.7),

$$T_{st} = \frac{\sqrt{2}K\emptyset V_{rms}}{2\pi R_a}(1 + \cos\alpha)$$

$$\omega_o = \frac{\sqrt{2}V_{rms}}{2\pi K\emptyset}(1 + \cos\alpha)$$

Solution

Firing angle α	Starting torque T_{st} (Nm)	No load speed ω_o (rad/s)
0°	54	27
45°	46.1	23
90°	27	13.5

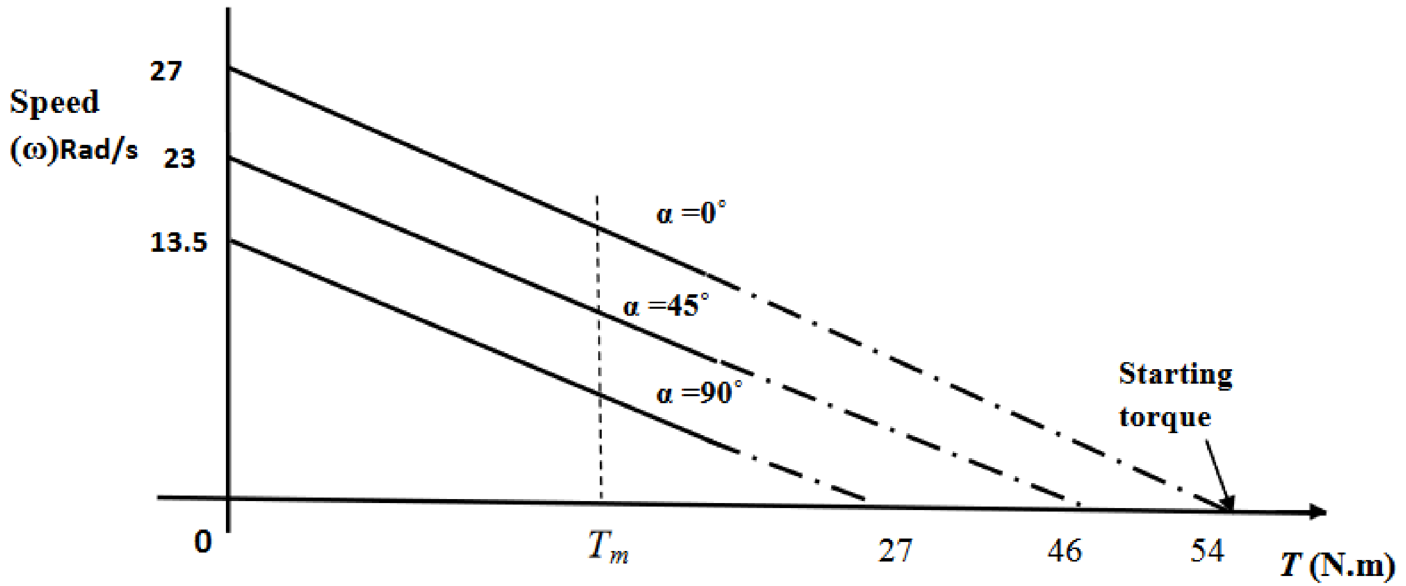


Fig. 3.3 Speed-torque characteristics of a separately-excited d.c. motor controlled by single-phase half-wave rectifier drive

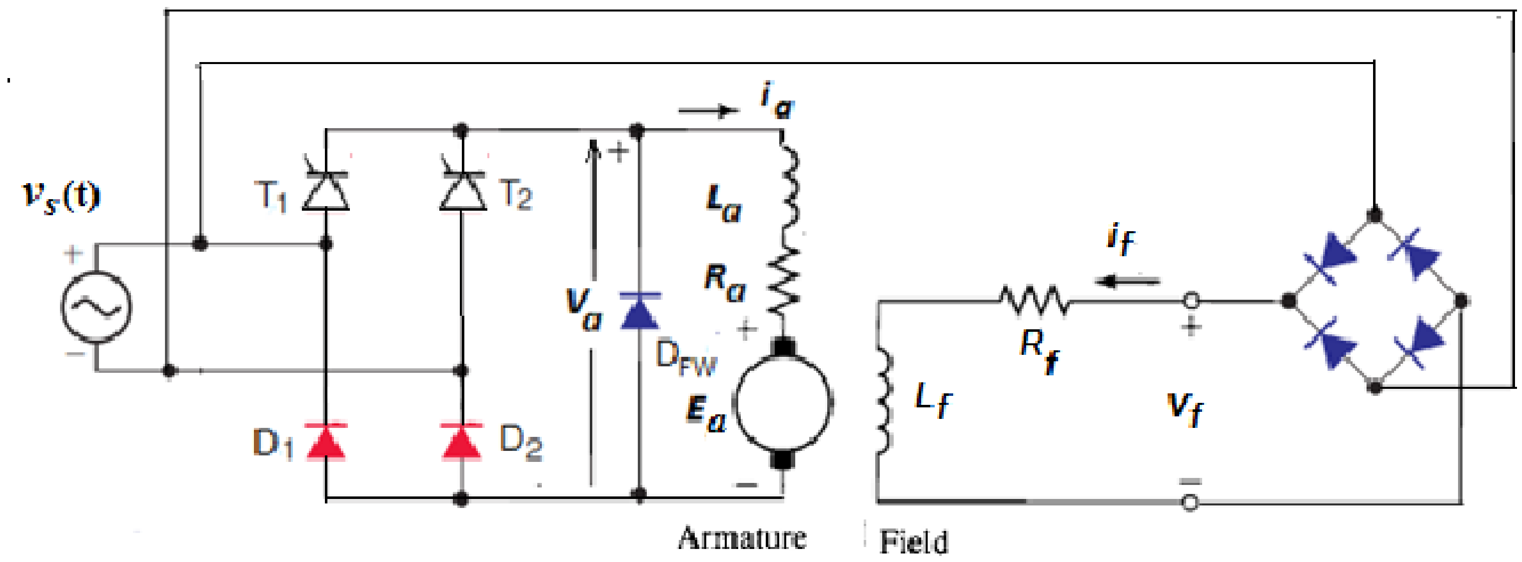


Fig.3.4 Single-phase semiconverter with d.c. motor load.

- The circuit diagram of a single-phase full-wave half-controlled (semiconverter) drive for controlling a separately-excited d.c. motor is depicted in Fig. 3.4.

- Here a full wave rectifier bridge supplies the field circuit, and a half-controlled bridge supplies the armature circuit. The vast majority of shunt motors are controlled in this manner.

Assuming the supply voltage $v_s(t) = V_m \sin \omega t$, in the positive half-cycle, T_1 and D_2 will conduct from α to (α + δ), where α is the firing angle and δ is the conduction angle. Generally, for medium and large motors the inductance of the armature is small and hence, for the separately-excited motor, the armature current falls to zero at the instant when the back *emf* E_a is equal to the supply voltage . i.e.

$$\alpha + \delta = \pi - sin^{-1}\frac{E_a}{V_m} \qquad (3.8)$$

The waveforms of the voltage v_a across the armature and the current i_a through the armature are shown in Fig. 3.5. It is obvious that the armature current is discontinuous.

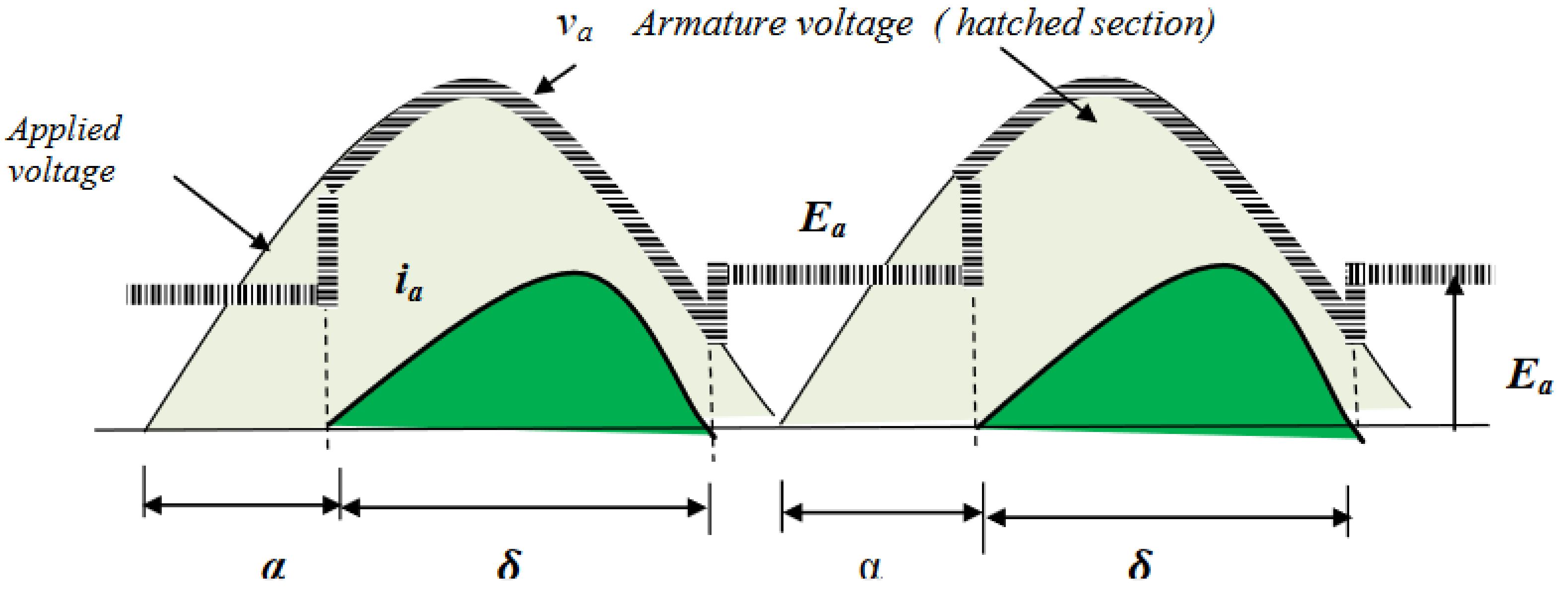


Fig. 3.5 Waveforms for single-phase semiconverter operation with discontinuous armature current.

(A) Discontinuous armature current operation

The differential equations describing the motor system, during the period the thyristors conduct, are

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + e_a = i_a R_a + L_a \frac{di_a}{dt} + K\emptyset\omega_m \qquad (3.9)$$

$$T_m = K\emptyset\, I_a = J\,\frac{d\omega_m}{dt} + B.\omega_m + T_L \qquad (3.10)$$

If we assume that the inertia of the rotating system is large then speed fluctuations will be negligible. If each term of v_a is integrated from α to (α + δ) and then divided by π , the instantaneous voltage, current and speed will be converted to their respective average values,

$$V_{a(av)} = \frac{1}{\pi}\int_{\alpha}^{\alpha+\delta} V_m \sin\omega t\ d\omega t$$

$$= \frac{1}{\pi}\int_{\alpha}^{\alpha+\delta} L_a \frac{di_a}{dt}\ d\omega t + \frac{1}{\pi}\int_{\alpha}^{\alpha+\delta} R_a\, i_a\ d\omega t + \frac{1}{\pi}\int_{\alpha}^{\alpha+\delta} K\emptyset\,\omega_m\, d\omega t$$

Thus

$$V_{a(av)} = I_{a(av)}R_a + K\emptyset\,\omega_{m(av)} \qquad (3.11)$$

where

$$V_{a(av)} = \frac{V_m}{\pi}[\cos\alpha - \cos(\alpha+\delta)\,] \qquad (3.12)$$

and the average voltage across L_a is zero.

Similarly,

$$T_{m(av)} = K\emptyset\, I_{a(av)} = B.\omega_{m(av)} + T_L \qquad (3.13)$$

Example 3.2

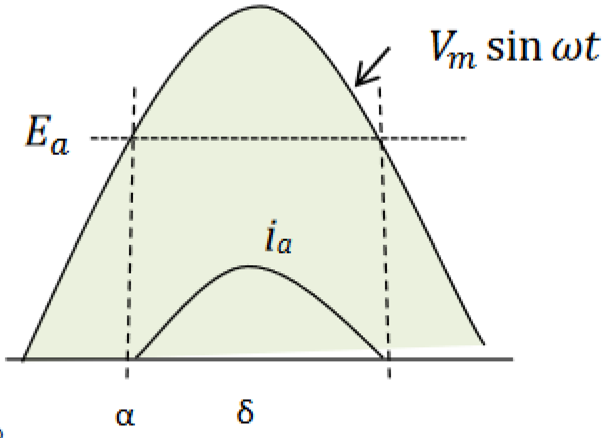
- A d.c. shunt motor , operating from a single-phase half-controlled bridge at speed of 1450 rpm, has an input voltage v_s = 330 sinωt and a back emf of 75 V. The thyristors are fired symmetrically at α = π / 4 in every half-cycle, and the armature resistance is 5 Ω. Neglecting the armature inductance, calculate the average armature current and load torque.

Solution

$$\alpha = \pi/4 = 45^\circ$$

$$(\alpha + \delta) = \pi - sin^{-1}\left(\frac{E_a}{V_m}\right)$$

$$= \pi - sin^{-1}\left(\frac{75}{330}\right) = 166.9^\circ$$



$$\omega_{av} = \frac{2\pi n}{60} = \frac{2\pi \times 1450}{60} = 151.8 \text{ rad/s}$$

$$V_{a(av)} = \frac{1}{\pi}\int_{45°}^{166.9°} V_m \sin\omega t \ d\omega t = \frac{V_m}{\pi}[\cos 45° - \cos 166.9°]$$

$$= \frac{330}{\pi} \times 1.68 = 176.6 \text{ V}$$

$$E_a = K\,\emptyset\,\omega_{m(av)}$$

$$75 = 151.8\,K\emptyset \quad \rightarrow \quad K\emptyset = \frac{75}{151.8} = 0.494 \text{ V.s/rad}$$

Now use the general equation of the shunt motor (3.11),

$$176.6 = 5\,I_{a(av)} + 75 \qquad \therefore \quad I_{a(av)} = 20.32 \text{ A}$$

From Eq.(3.10) , $T_m = K\emptyset\, I_a$

$$\therefore T_{av} = K\emptyset\, I_{a(av)} = 0.494 \times 20.32 = 10.04 \text{ Nm}$$

The torque can also be evaluated as,

$$T_{av} = \frac{P}{\omega_{av}} = \frac{E_a I_{a(av)}}{\omega_{av}} = \frac{75 \times 20.32}{151.8} = 10.04 \text{ Nm}$$

(B) Analysis with Continuous Armature Current Operation

If the armature inductance is large then conduction will continue, even after the supply voltage has reversed, for which typical waveforms are shown in Fig. 3.6. Hence assume continuous current operation, the average value of the armature voltage is:

$$V_{a(av)} = \frac{1}{\pi}\int_0^{2\pi} V_a(\omega t)d\omega t = \frac{1}{\pi}\int_\alpha^{\pi} V_m \sin(\omega t)\, d\omega t$$

$$V_{a(av)} = \frac{V_m}{\pi}(1 + cos\alpha) \qquad (3.14)$$

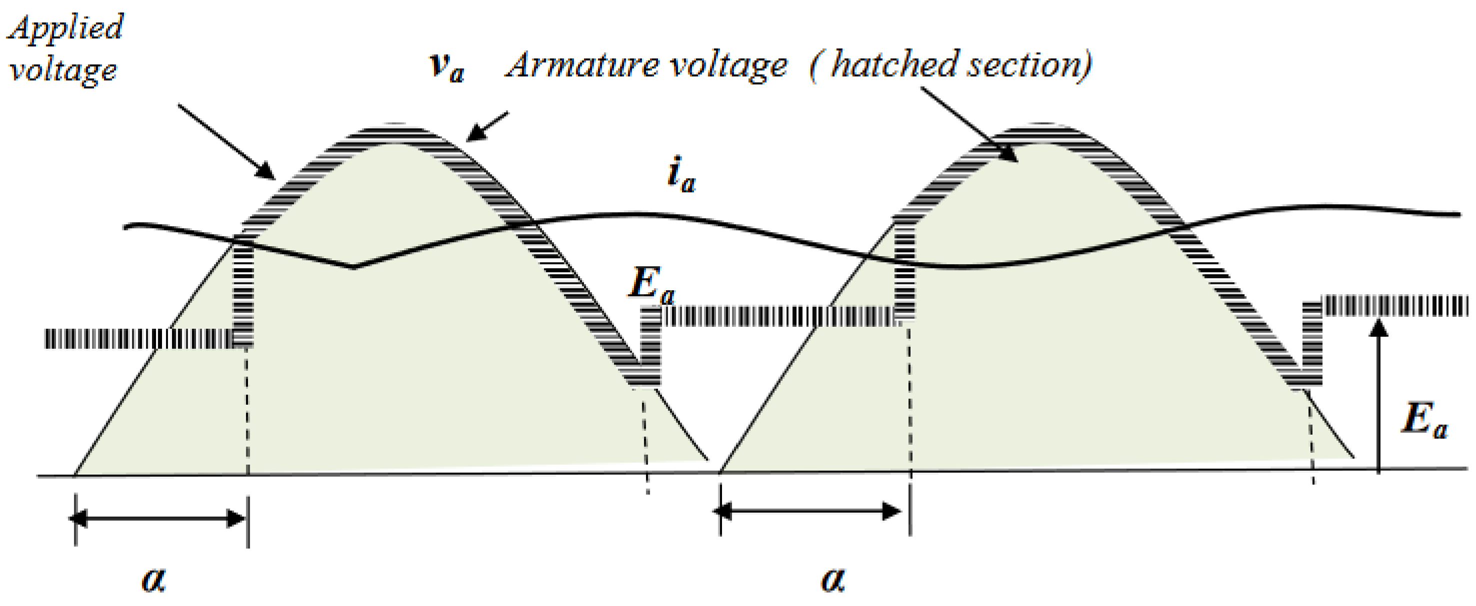


Fig.13.6 Waveforms for single-phase semiconverter operation with continuous armature current.

The average load current is:

$$I_{a(av)} = \frac{V_{a(av)} - E_a}{R_a}$$

$$I_{a(av)} = \frac{V_m}{\pi R_a}(1 + cos\alpha) - \frac{E_a}{R_a} \qquad (3.15)$$

Example 3.3

A series d.c. motor is to be controlled by a single-phase, half-controlled, full-wave rectifier bridge as shown in Fig. 3.7. The a.c. input voltage has an *rms* value of 240V at 50Hz. The combined armature and field resistance is 2.5Ω and $K_{af} = 300$ mH. If the load torque is 30 Nm and damping is neglected, calculate the average current and the speed for α = 60˚.

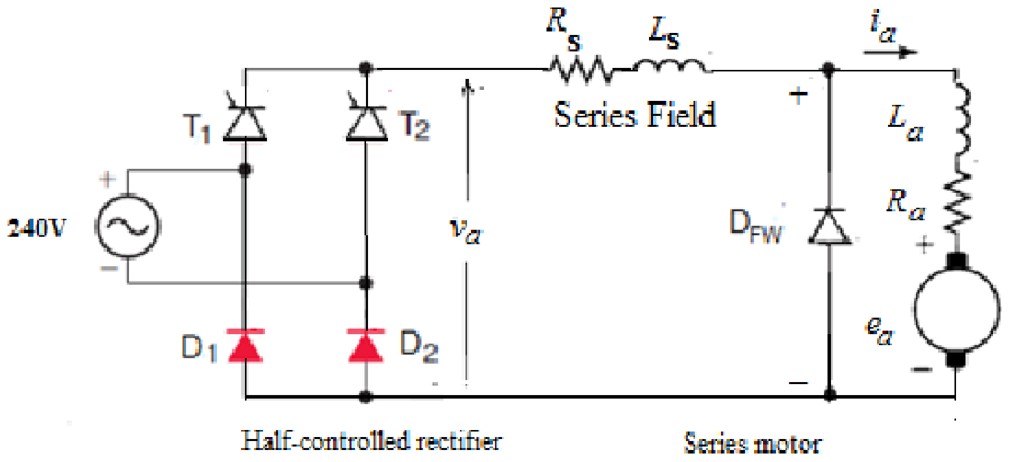


Fig. 3.7 Series motor drive.

$$V_{a(av)} = \frac{V_m}{\pi}(1 + \cos\alpha) = \frac{\sqrt{2}}{\pi} \times 240\,(1+\cos 60^\circ) = 162.11\text{ V}$$

$$T_{av} = K_{af} I_{a(av)}{}^2 = 30 = 300 \times 10^{-3} I_{a(av)}{}^2$$

$$\therefore I_{a(av)} = 10\text{ A}$$

$$V_{a(av)} = R_T\, I_{a(av)} + K_{af} I_{a(av)}\, \omega_{m(av)}$$

$$162.11 = 2.5 \times 10 + 0.3 \times 10\, \omega_{m(av)}$$

$$\therefore \omega_{m(av)} = 45.7\text{ rad/s} \longrightarrow n = 436.6\text{ rpm}$$

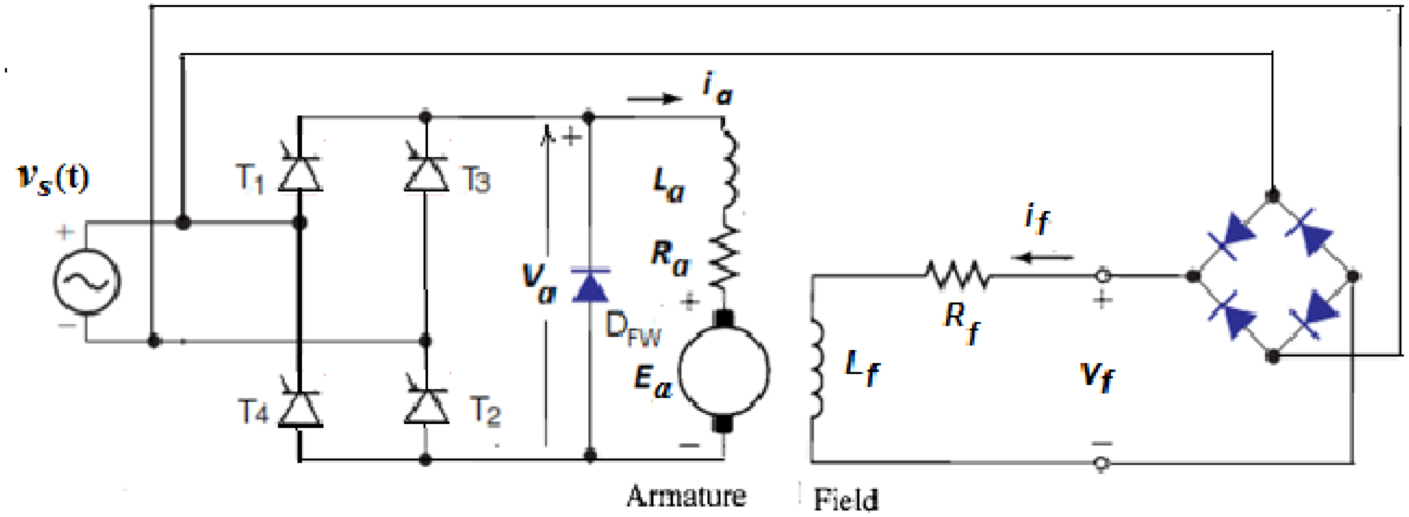


Fig. 3.8 Single-phase full-converter with d.c. motor load.

Here a full-wave rectifier bridge supplies the field circuit, while the full-converter supplies the armature circuit. The converter has four thyristors that need alternate switching of the pairs of these thyristors T_1 ,T_2 or T_3 , T_4 . The converter provides + Va or – Va depending on the value of the triggering angle α of the thyristors, thus two quadrant operation is possible. Armature current remains unidirectional due to the converter configuration. The vast majority of shunt motors are also controlled in this manner.

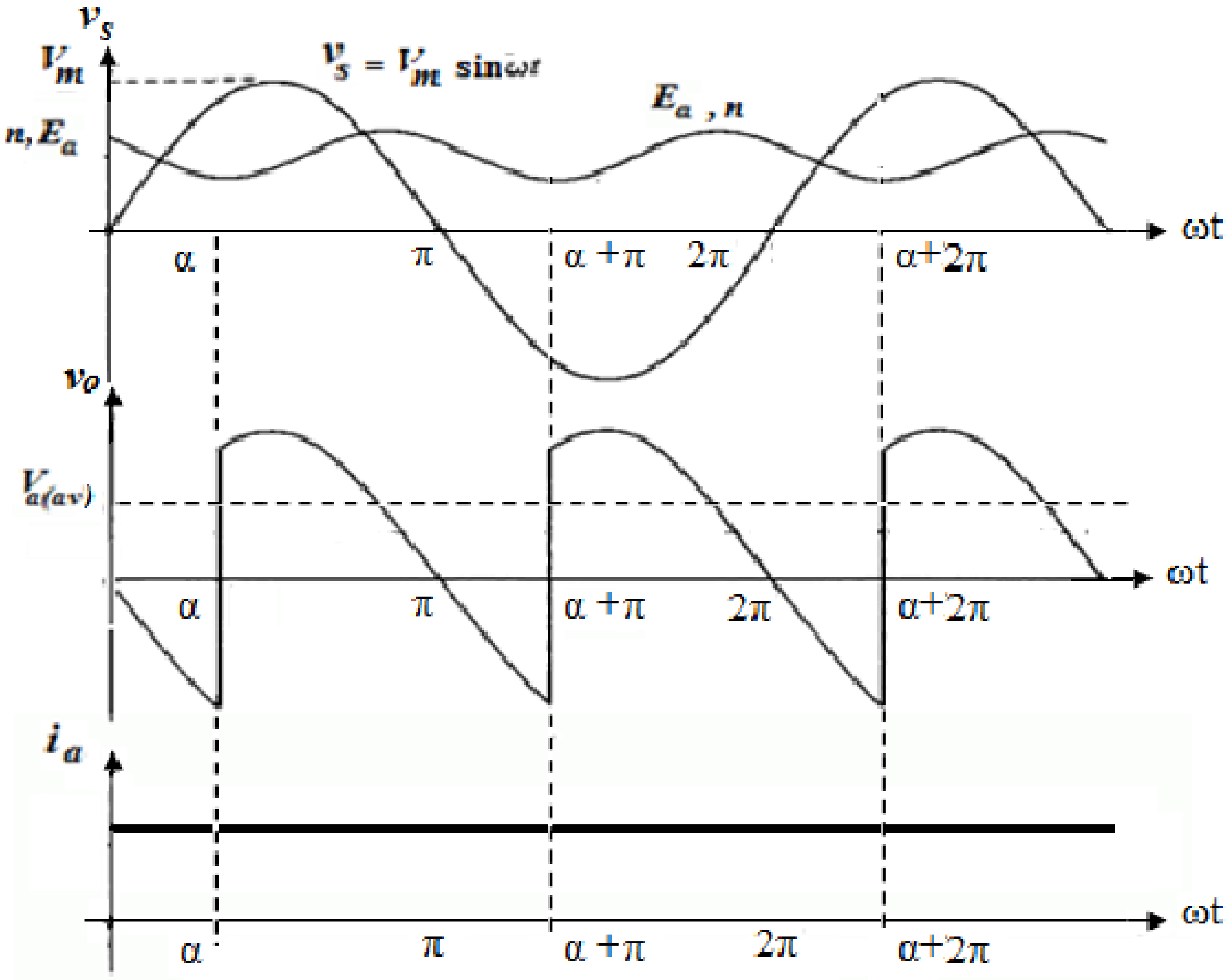


Fig. 3.9 Waveforms of the armature voltage and the current for continuous current operating mode.

$$V_{a(av)} = \frac{1}{2\pi}\int_{0}^{2\pi} v_a(\omega t)d\omega t = \frac{2}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)\, d\omega t$$

$$V_{a(av)} = \frac{2V_m}{\pi}\cos\alpha$$

The average current

$$I_{a(va)} = \frac{V_{a(av)} - E_a}{R_a}$$

$$= \frac{2V_m}{\pi R_a}(cos\alpha) - \frac{E_a}{R_a}$$

(B) Power and power factor

The power taken by the motor can be calculated as

$$P_{in} = P_a = I_a^2 R_a + E_a I_a$$

$E_a I_a$ represents the output power plus the motor friction and windage losses. If the mechanical losses in the motor and electrical losses in the rectifier switches are neglected, the output power and the operating efficiency are

$$P_{out} = E_a I_a = T\omega_m$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{E_a I_a}{P_a}$$

Since the input current has *rms* value equal to that of the motor current, thus the operating power factor is

$$\text{Power factor} = \frac{P_a}{\frac{E_m}{\sqrt{2}} . I_L}$$

Example 3.4

A separately-excited d.c. motor has the following parameters:

$R_a = 0.25\ \Omega$, $K_e = 0.62$ V/ rpm.Wb , Φ (flux per pole) = 175 mWb.

The motor speed is controlled by a single-phase, full-wave bridge rectifier. The firing angle α is set at 45°, and the average speed is 1300 rpm. The applied a.c. voltage to the bridge is 230 V at 50Hz. Assuming the motor current is continuous; calculate the armature current drawn by the motor and the steady-state torque for the cases of:

(a) Fully-controlled bridge shown in Fig. 3.10 (a).
(b) Half-controlled (semiconverter) bridge shown in Fig. 3.10 (b).

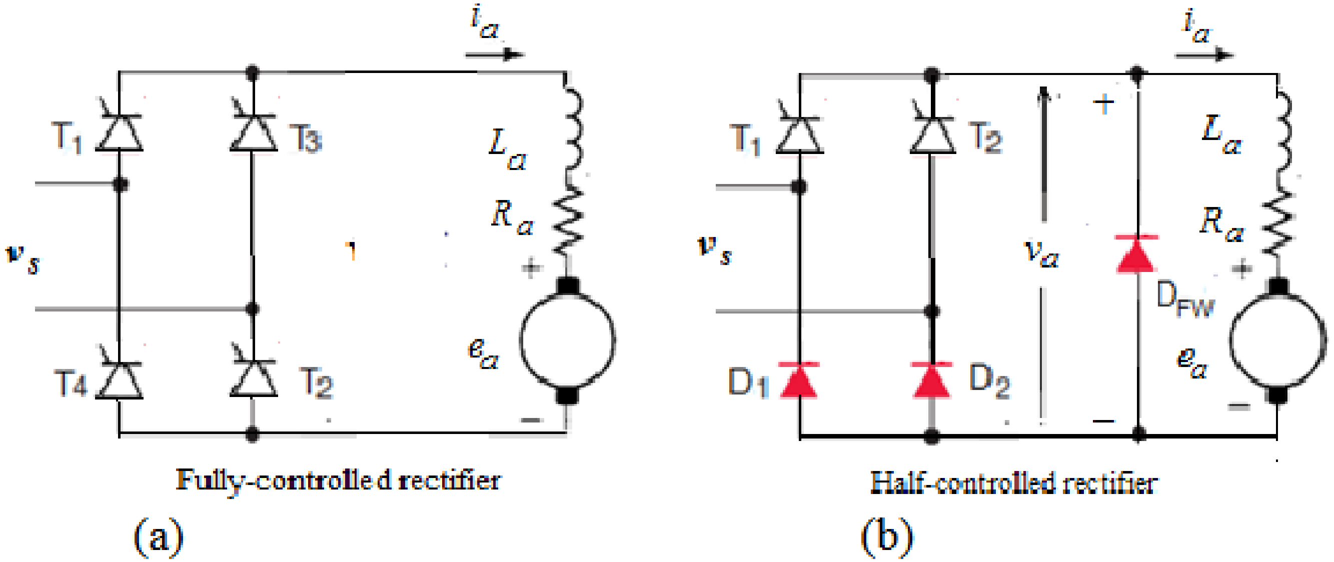


Fig. 3.10.

Solution

(a) For fully-controlled bridge with continuous current operating mode:

$$V_{a(av)} = \frac{2V_m}{\pi}\cos\alpha$$

$$V_m = \sqrt{2} \times 230 = 325.2\text{V}$$

$$V_{a(av)} = \frac{2 \times 325.2}{\pi}\cos 45° = 146.44 \text{ V}$$

$$E_a = K_e\,\varphi\, n = 0.62 \times 175 \times 10^{-3} \times 1300 = 141.05 \text{ V}$$

$$V_t = V_a = E_a + I_a R_a$$

$$\therefore I_a = \frac{V_a - E_a}{R_a} = \frac{146.44 - 141.05}{0.25} = 21.56 \text{ A}$$

Since $T_d = K_T I_a \varphi$

K_T = Torque constant = 9.55 K_e = 9.55 × 0.62 = 6.2

$T_d = 6.2 \times 21.56 \times 175 \times 10^{-3} = 23.4$ Nm

(b) For half-controlled (semiconverter) bridge.

$$V_{a(av)} = \frac{V_m}{\pi}(1 + cos\alpha)$$

$$V_{a(av)} = \frac{325.2}{\pi}(1 + cos45°)$$

$$V_{a(av)} = 176.78 \text{ V}$$

$E_a = K_e\, \varphi\, n = 0.62 \times 175 \times 10^{-3} \times 1300 = 141.05$V

$V_t = V_{a(av)} = E_a + I_a R_a$

$$I_a = \frac{V_{a(av)} - E_a}{R_a} = \frac{176.78 - 141.05}{0.25} = 142.95 \text{ A}$$

$T_d = 6.2 \times 142.95 \times 175 \times 10^{-3} = 155.1$ Nm

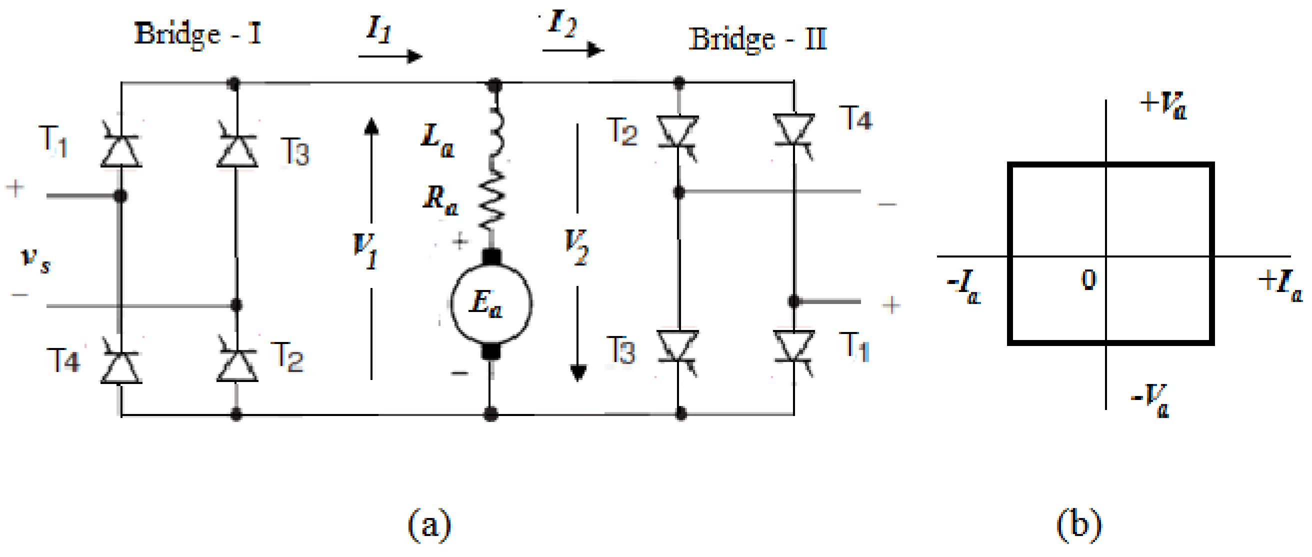


Fig.3.11 Dual converter drive: (a) Circuit diagram, and (b) Quadrants of operation.

- Bridge-I provides operation in the first and fourth quadrants while bridge-II provides operation in second and third quadrants.
- Therefore, the dual converter is a four quadrant drive which allows four quadrant of machine operation without a switching changeover.

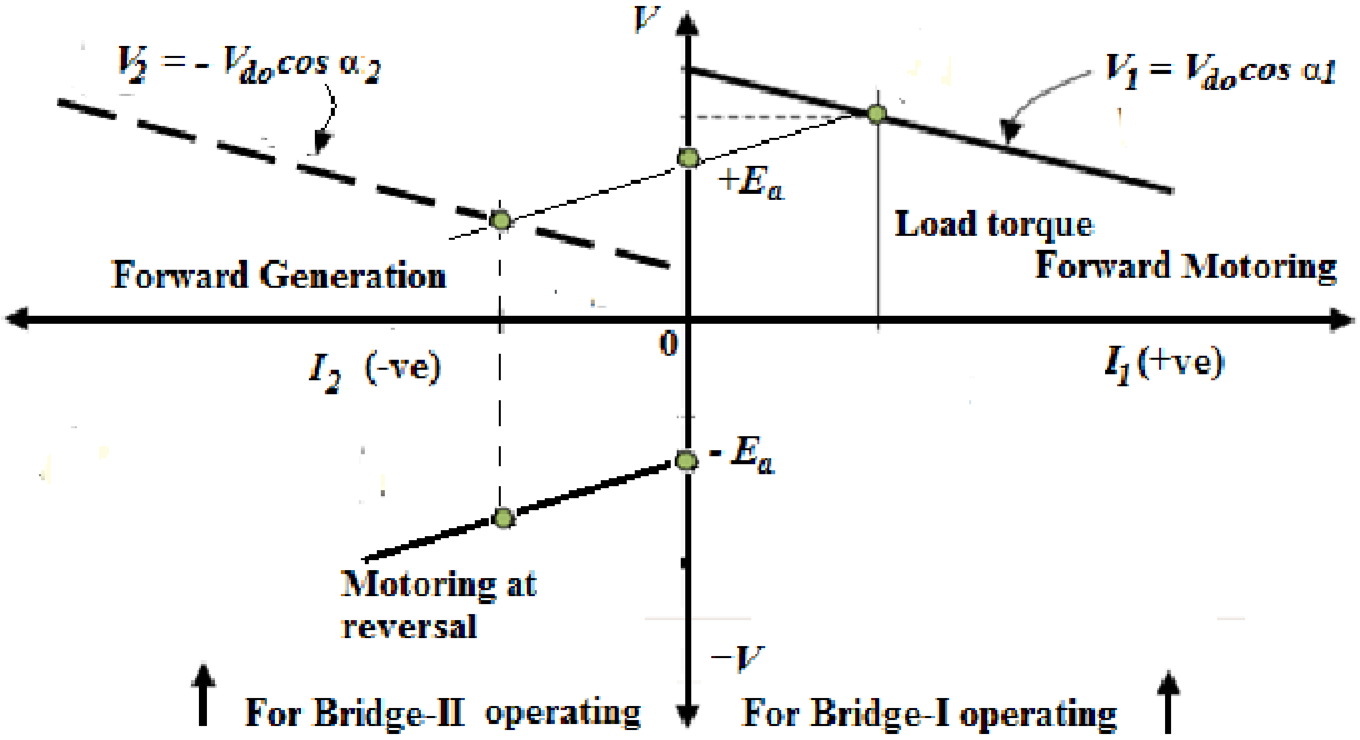


Fig.3.12 Dual converter.

- This drive is employed for motors of rating up to 15 kW.

$$V_1 = V_{a(av)\,1} = \frac{2V_m}{\pi}\cos\alpha_1 = V_{do}\cos\alpha_1 = E_a + I_1 R_a$$

Bridge – II operating:

$$V_2 = V_{a(av)\,2} = -\left(\frac{2V_m}{\pi}\cos\alpha_2\right) = V_{do}\cos\alpha_2 = E_a - I_1 R_a$$

where

$$V_{do} = \frac{2V_m}{\pi}$$

Which is the output voltage of the converter when $\alpha = 0^\circ$.

- The two equations above are shown as straight lines in Fig. 3.12, the intersection of the machine and bridge characteristics giving the operating points.

Example 3.5

A d.c. separately-excited motor rated at 10 kW, 200 V is to be controlled by dual converter. The armature circuit resistance is 0.2 Ω and the machine constant $K_e\emptyset$ is 0.35 V/ rpm. For the following conditions, determine the firing angles of the converter, the back *emf* and the machine speed given that for the converter system V_{do} = 250 V. Neglect any losses in the converter circuit.

(a) Machine operates in a forward motoring mode at rated current and with terminal voltage of 200 V.

(b) Machine operates at forward generation mode at rated current and with terminal voltage of 200 V.

Solution

(a) For the motoring case,

$$V = V_{do}\ \cos\alpha \quad \rightarrow \quad 200 = 250\cos\alpha \quad \rightarrow \quad \cos\alpha = \frac{200}{250} = 0.8$$

$$\therefore\ \alpha = cos^{-1}(0.8) = 36.8^{o}$$

The rated current of the machine $I_a = 10000 / 200 = 50$ A.

$$V = E_a + I_a\ R_a \quad \rightarrow \quad 200 = E_a + 50 \times 0.2$$

$$\therefore \quad E_a = 200 - 10 = 190\ \text{V}$$

The speed of the motor can be calculated as,

$$E_a = K_e\emptyset\, n \quad \rightarrow \quad n = \frac{E_a}{K_e\emptyset} = \frac{190}{0.35} = 542.85\ \text{rpm}$$

Solution

(b) For generating mode,

$$-V = V_{do}\ \cos\alpha \quad \rightarrow \quad -200 = 250\cos\alpha$$

$$\therefore\ \alpha = cos^{-1}(-0.8) = 143.13^{\circ}$$

$$E_a = V + I_a\ R_a \quad \rightarrow \quad E_a = \quad 200 + \ 50 \times 0.2 \quad = 210\ \text{V}$$

$$E_a = \ K_e\emptyset\, n \quad \rightarrow \quad n = \frac{E_a}{K_e\emptyset} = \frac{210}{0.35} = 600\ \text{rpm}$$

End of Lecture 3!