EEE 5451 Power Electronics

Lecture 7: Inverters

7.1 Introduction

- The function of an inverter is to convert a DC input voltage to a symmetrical AC output voltage of desired magnitude and frequency.

- The output voltage could be at a fixed or variable frequency. A variable voltage can be obtained by varying the gain of the inverter, which is accomplished by pulse-width-modulation (PWM) control within the inverter,

- $$\text{Inverter gain} = \frac{AC\ Output\ Voltage}{DC\ Input\ Voltage}$$

- The output waveforms are non-sinusoidal for practical inverters. Ideal inverters will produce sinusoidal voltages. Square wave voltages may be acceptable for low and medium power applications, and low distorted sinusoidal waveforms for high power applications.

- The harmonic contents of output voltages can be minimized by switching techniques with the use of high-speed power semiconductor devices such as GTOs, BJTs, IGBTs, and MOSFETs.

- Inverters are widely used in industrial applications, for example, variable speed AC motor drives, induction heating, standby power supplies, un-interruptible power supplies, and so on.

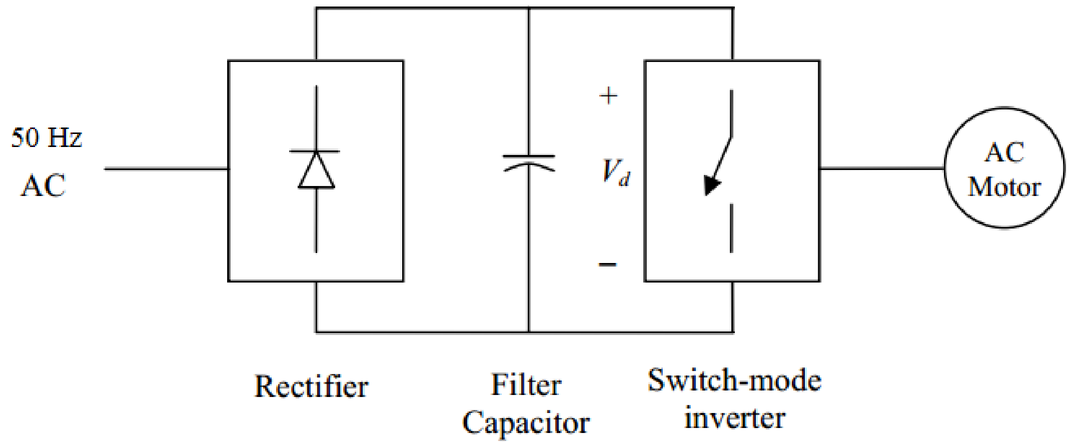

Fig. 7.1-Typical variable voltage and variable frequency system.

7.1.1 H-Bridges

- An ***H bridge*** is an electronic circuit that enables a voltage to be applied across a load in either direction.
- These circuits are often used in robotics and other applications to allow DC motors to run forwards or backwards.
- Most DC-to-AC converters (power inverters), AC/AC converters, the DC-to-DC push–pull converter, motor controllers, and many other kinds of power electronics use H bridges.
- In particular, a bipolar stepper motor is almost invariably driven by a motor controller containing two H bridges.

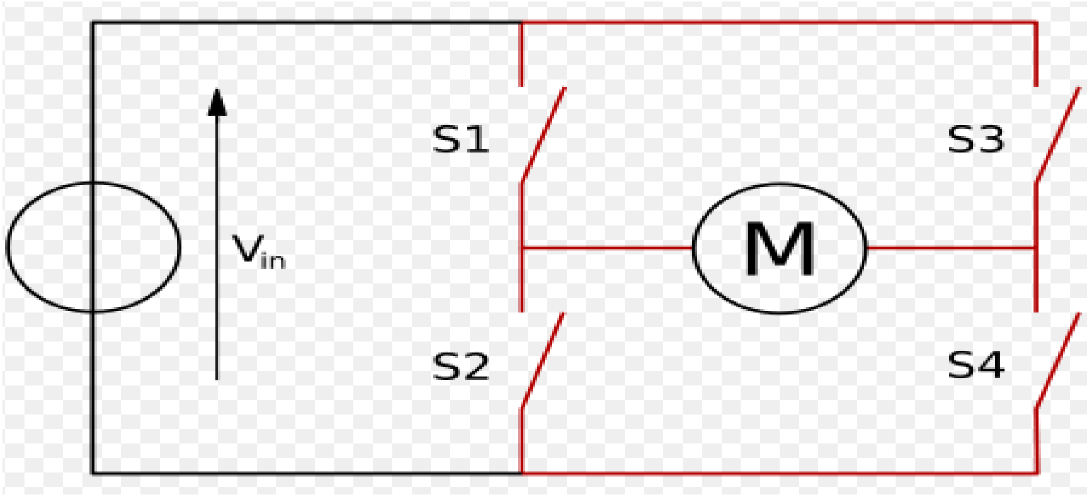

Fig. 7.2 Structure of an H bridge (highlighted in red)

- H bridges are available as integrated circuits, or can be built from discrete components.

- The switches S1 and S2 should never be closed at the same time, as this would cause a short circuit on the input voltage source. The same applies to the switches S3 and S4. This condition is known as shoot-through.

➢ The following table summarizes the operation of the H-Bridge.

S1	S2	S3	S4	Result
1	0	0	1	Motor moves right
0	1	1	0	Motor moves left
0	0	0	0	Motor coasts
0	1	0	1	Motor brakes
1	0	1	0	Motor brakes
1	1	0	0	Short circuit
0	0	1	1	Short circuit
1	1	1	1	Short circuit

➢ A common use of the H bridge is an inverter. The arrangement shown in Fig. 7.2 is sometimes known as a single-phase bridge inverter.

➢ The H bridge with a DC supply will generate a square wave voltage waveform across the load.

❖ **For a purely inductive load, the current waveform would be a triangle wave, with its peak depending on the inductance, switching frequency, and input voltage.**

7.2 Half-Bridge Inverter

- A pulse with a period of T and duty cycle of 0.5, $i_g(t)$, is applied to the base of Q_1, while it is applied to the base of Q_2 through a NOT gate.

- So, Q_1 and Q_2 are turned on and off alternately, each 50% of the period, with one transistor on while the other is off.

- **Mode 1**: $0 \le t \le 0.5T$

  In this mode, either Q_1 or D_1 is turned on and Q_2 and D_2 are off. The output voltage is

$$v_o = V_{dc}$$

- Now assume that $i_o(0) = I_{min} < 0$. Then, D_1 is turned on and Q_1 is still off at t = 0. The load current is determined by

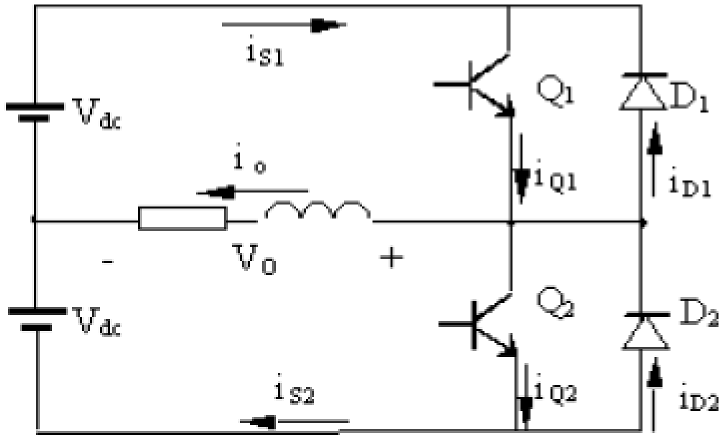

Fig. 7.3 Half-Bridge Inverter

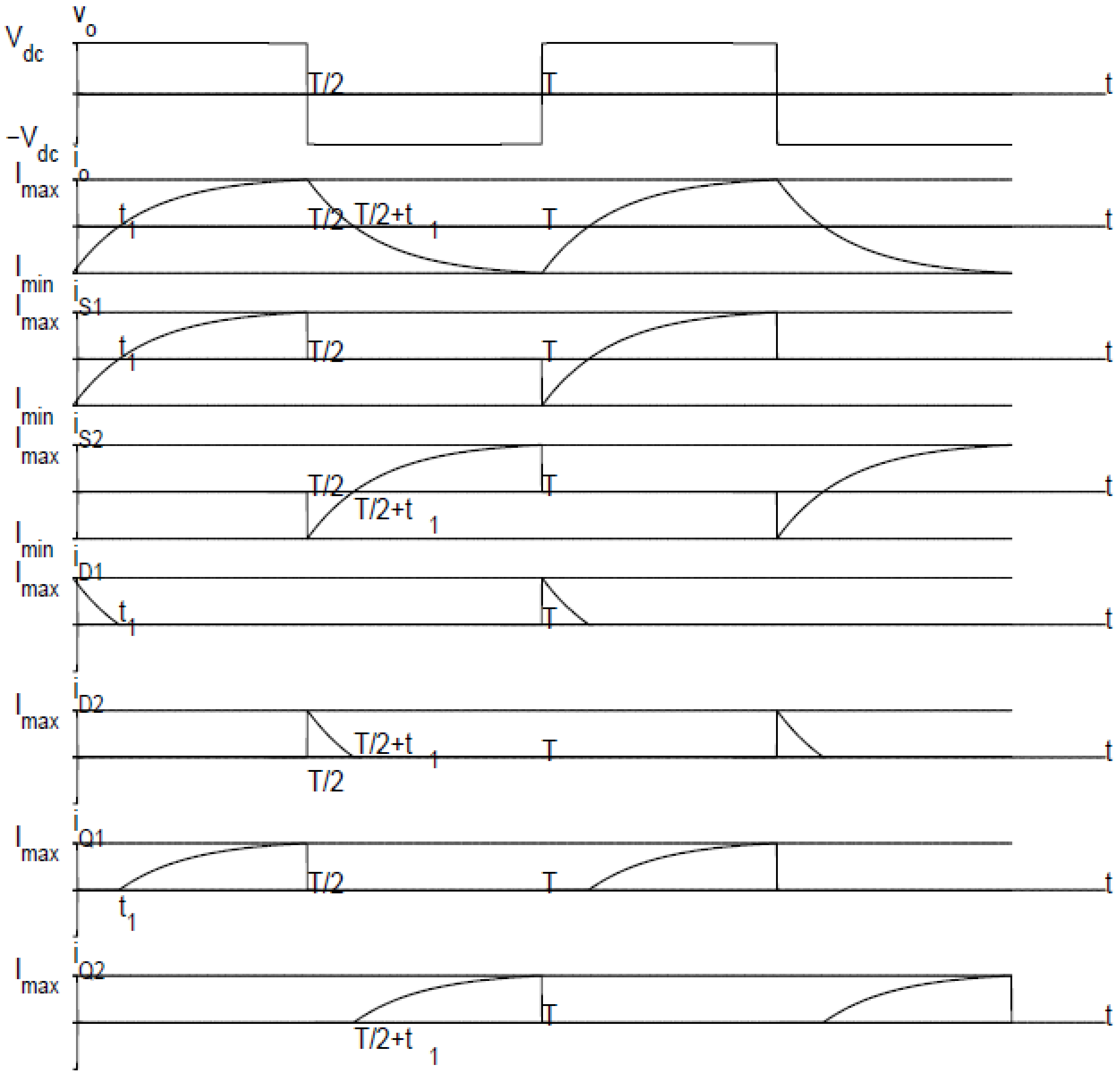

Fig. 7.4 Half-Bridge Inverter Waveforms

$$v_o = V_{dc} = Ri_o + L\frac{di_o}{dt}$$

- Solution to this equation takes the form of

$$i_o(t) = \frac{V_{dc}}{R} + Ae^{-\frac{t}{\tau}}$$

- with $\tau = L/R$. The constant A is determined by the initial condition $i_o(0) = I_{min}$, that is,

$$i_o(0) = \frac{V_{dc}}{R} + A = I_{min}$$

- Solving the above equation for A gives; $A = I_{min} - \frac{V_{dc}}{R}$

- Therefore, the load current in this mode is

$$i_o(t) = \frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right] e^{-\frac{t}{\tau}}$$

- This current increases from a negative value, I_{min}, to the maximum value I_{max}, which is reached at t =0 .5T, that is

$$I_{max} = i_o(0.5T) = \frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right] e^{-\frac{0.5T}{\tau}} = \frac{V_{dc}}{R}\left[1 - e^{-\frac{0.5T}{\tau}}\right] + I_{min} e^{-\frac{0.5T}{\tau}}$$

- When it goes through zero and becomes positive, D_1 is cut off, Q_1 is turned on and begins to conduct. The time when $i_o(t)$ becomes zero is determined by

$$0 = i_o(t_1) = \frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right] e^{-\frac{t_1}{\tau}}$$

Mode 2: 0.5T≤t≤T

- In this mode, either Q_2 or D_2 is turned on and Q_1 and D_1 are off. The output voltage is

$$v_o = -V_{dc}$$

- Note that $i_o(0.5T)=I_{max} > 0$. Then, D_2 is turned on and Q_2 is still off at t =0 .5T. The load current is determined by

$$v_o = -V_{dc} = Ri_o + L\frac{di_o}{dt}$$

- Solution to this equation takes the form of

$$i_o(t) = -\frac{V_{dc}}{R} + Be^{-\frac{t-0.5T}{\tau}}$$

- The constant B is determined by the initial condition $i_o(0.5T)=I_{max}$, that is,
- $i_o(0.5T) = -\frac{V_{dc}}{R} + B = I_{max}$
- Solving the above equation for B gives
- $B = I_{max} + \frac{V_{dc}}{R}$
- Therefore, the load current in this mode is
- $i_o(t) = -\frac{V_{dc}}{R} + \left[I_{max} + \frac{V_{dc}}{R}\right] e^{-{(t-0.5T)}/{\tau}}$
- This current decreases from a positive value, I_{max}, to the minimum value I_{min}, which is negative and reached at t = T, that is,

$$I_{min} = i_o(T) = -\frac{V_{dc}}{R} + \left[\mathrm{I}_{\max} + \frac{V_{dc}}{R}\right] e^{-\frac{0{\cdot}5T}{\tau}} = -\frac{V_{dc}}{R}\left[1 - e^{-\frac{0{\cdot}5T}{\tau}}\right] + I_{max} e - {}^{0{\cdot}5T}/_{\tau}$$

- When it goes through zero and become negative, D_2 is cut off, Q_2 is turned on and begins to conduct. The time when $i_o(t)$ becomes zero is determined by

$$0 = i_o(t_2) = -\frac{V_{dc}}{R} + \left[I_{max} + \frac{V_{dc}}{R}\right] e^{-\frac{t_2 - 0{\cdot}5T}{\tau}}$$

- Adding I_{max} to I_{min} gives:

$$I_{max} + I_{min} = (I_{max} + I_{min}) e^{-\frac{0{\cdot}5T}{\tau}}$$

- So $I_{min} = -I_{max}$. Substituting it into I_{max}-expression yields:

$$I_{max} = \frac{V_{dc}}{R} \frac{1 - e^{-T/2\tau}}{1 + e^{-T/2\tau}}$$

- **With I_{max} and I_{min}, it follows that**

$$
\begin{aligned}
t_1 &= \tau \ln\left(\frac{2}{1+e^{-\frac{T}{2\tau}}}\right)\\
t_2 &= 0.5T + t_1
\end{aligned}
$$

In summary, currents through the devices in the circuit are given as:

$$
i_o(t) = \begin{cases} \frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right] e^{-\frac{t}{\tau}} & 0 \le t \le 0.5T \\ -\frac{V_{dc}}{R} + \left[I_{max} + \frac{V_{dc}}{R}\right] e^{-\frac{t-0.5T}{\tau}} & 0.5T \le t \le T \end{cases}
$$

$$
i_{s1}(t) = \begin{cases} i_o(t) & 0 \le t \le 0.5T \\ 0 & 0.5T \le t \le T \end{cases}
$$

$$
i_{s2}(t) = \begin{cases} 0 & 0 \le t \le 0.5T \\ -i_o(t) & 0.5T \le t \le T \end{cases}
$$

$$
i_{D1}(t) = \begin{cases} -i_o(t) & 0 \le t \le t_1 \\ 0 & t_1 \le t \le T \end{cases}
$$

$$
i_{D2}(t) = \begin{cases} i_o(t) & 0.5T \le t \le 0.5T + t_1 \\ 0 & otherwise \end{cases}
$$

$$
i_{Q1}(t) = \begin{cases} i_o(t) & t_1 \le t \le 0.5T \\ 0 & otherwise \end{cases}
$$

$$
i_{Q2}(t) = \begin{cases} -i_o(t) & 0.5T + t_1 \le t \le T \\ 0 & otherwise \end{cases}
$$

The rms output voltage is

$$
V_{rms} = \sqrt{\frac{1}{0.5T}\int_0^{0.5T} V_{dc}^2 \mathrm{d}t} = V_{dc}
$$

and the rms output current is

$$I_{rms} = \sqrt{\frac{1}{0.5T}\int_0^{0.5T}\left\{\frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right]e^{-\frac{t}{\tau}}\right\}^2 dt}$$

- Both average output current and voltage are zero.

Example 1:

A half-bridge inverter has the following data: V_{dc} = 300 V , R = 10 Ω, L =0 .05 H, and f = 60 Hz. Find

1. peak load current
2. time of current zero crossing
3. average transistor current
4. average diode current

Solutions:

1.

$$\begin{aligned}\tau &= \frac{L}{R} = \frac{0.05}{10} = 0.005s \\ T &= \frac{1}{f} = \frac{1}{60} = 0.01667s \\ I_{max} &= \frac{V_{dc}}{R}\frac{1-\mathrm{e}^{-\frac{T}{2\tau}}}{1+\mathrm{e}^{-\frac{T}{2\tau}}} \\ &= \frac{300}{10}\frac{1-\mathrm{e}^{-\frac{0.01667}{0.01}}}{1+\mathrm{e}^{-\frac{0.01667}{0.01}}} \\ &= 20.47A\end{aligned}$$

2.

$$t_1 = \tau \ln\left(\frac{2}{1+\mathrm{e}^{-\frac{T}{2\tau}}}\right) = 0.005 \times \ln\left(\frac{2}{1+\mathrm{e}^{-\frac{0.01667}{0.01}}}\right) = 0.0026s$$

3. The load current during the first half cycle is given by

$$i_o(t) = \frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right]\mathrm{e}^{-\frac{t}{\tau}} = \frac{300}{10} + \left[-20.47 - \frac{300}{10}\right]\mathrm{e}^{-\frac{t}{0.005}} = 30 - 50.47\mathrm{e}^{-\frac{t}{0.005}}$$

So, the average transistor current is

$$
\begin{aligned}
I_C &= \frac{1}{T}\int_{t_1}^{0.5T} i_o(t)\mathrm{d}t = \frac{1}{0.01667}\int_{0.0026}^{0.00833}[30 - 50.47\mathrm{e}^{-\frac{t}{0.005}}]\mathrm{d}t \\
&= \frac{1}{0.01667}\left[30t - 54.47\times(-0.005)\mathrm{e}^{-\frac{t}{0.005}}\right]_{0.0026}^{0.00833} \\
&= \frac{1}{0.01667}\left[30(0.00833-0.0026) - 54.47\times(-0.005)\left(\mathrm{e}^{-\frac{0.00833}{0.005}} - \mathrm{e}^{-\frac{0.0026}{0.005}}\right)\right] \\
&= 4.18A
\end{aligned}
$$

4. The average diode current is calculated as follows:

$$
\begin{aligned}
I_D &= \frac{1}{T}\int_{0}^{t_1} -i_o(t)\mathrm{d}t = -\frac{1}{0.01667}\int_{0}^{0.0026}[30 - 50.47\mathrm{e}^{-\frac{t}{0.005}}]\mathrm{d}t \\
&= -\frac{1}{0.01667}\left[30t - 54.47\times(-0.005)\mathrm{e}^{-\frac{t}{0.005}}\right]_{0}^{0.0026} \\
&= -\frac{1}{0.01667}\left[30(0.0026-0) - 54.47\times(-0.005)\left(\mathrm{e}^{-\frac{0.0026}{0.005}} - \mathrm{e}^{-\frac{0}{0.005}}\right)\right] \\
&= 1.46A
\end{aligned}
$$

7.3 Full-Bridge Inverter

- A pulse with a period of T and duty cycle of 0.5, $i_g(t)$, is applied to the bases of Q_1 and Q_2, while it is applied to the bases of Q_3 and Q_4 through a NOT gate.

- So, Q_1 and Q_2 are turned on and Q_3 and Q_4 off during the first half cycle, while Q_1 and Q_2 are off and Q_3 and Q_4 on during the second half cycle.

Mode 1: $0 \leq t \leq 0.5T$

- In this mode, Q_1 or D_1 and Q_2 or D_2 are turned on and Q_3, D_3, Q_4 and D_4 are off. The output voltage is

$$v_o = V_{dc}$$

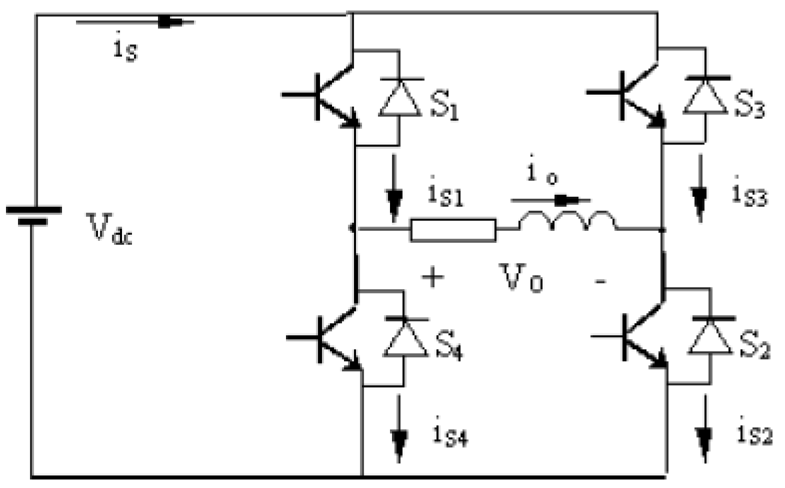

Fig. 7.5: Full-bridge inverter

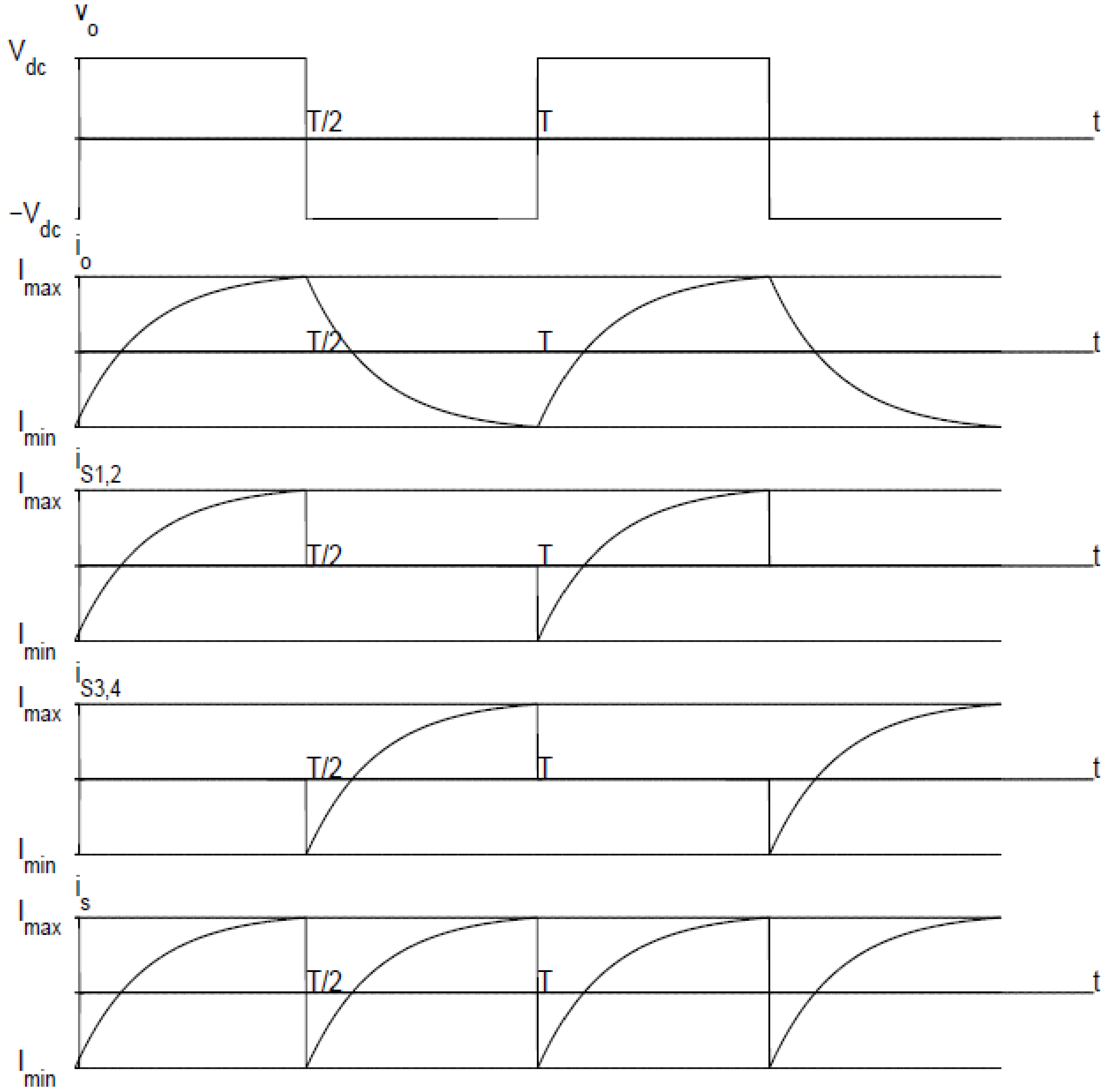

Fig. 7.6: Full-bridge inverter waveforms

➢Similar to the half-bridge inverter, the load current is determined by

$$v_o = V_{dc} = Ri_o + L\frac{\mathrm{d}i_o}{\mathrm{d}t}$$

Solution to this equation is given by

$$i_o(t) = \frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right]\mathrm{e}^{-\frac{t}{\tau}}$$

with $\tau = \frac{L}{R}$ and

$$I_{min} = -\frac{V_{dc}}{R}\frac{1-\mathrm{e}^{-\frac{T}{2\tau}}}{1+\mathrm{e}^{-\frac{T}{2\tau}}}$$

➢When the current $i_o(t)$ is negative, both D_1 and D_2 conduct, otherwise both Q_1 and Q_2 conduct.

Mode 2: 0.5*T*≤*t*≤*T*

In this mode, Q_3 or D_3 and Q_4 or D_4 are turned on and Q_1, D_1, Q_2 and D_2 are off. The output voltage is

$$v_o = -V_{dc}$$

Similar to the half-bridge inverter, the load current is determined by

$$v_o = -V_{dc} = Ri_o + L\frac{\mathrm{d}i_o}{\mathrm{d}t}$$

Solution to this equation is given by

$$i_o(t) = -\frac{V_{dc}}{R} + \left[I_{max} + \frac{V_{dc}}{R}\right]\mathrm{e}^{-\frac{t-0.5T}{\tau}}$$

with

$$I_{max} = \frac{V_{dc}}{R}\frac{1-\mathrm{e}^{-\frac{T}{2\tau}}}{1+\mathrm{e}^{-\frac{T}{2\tau}}}$$

➢When the current $i_o(t)$ is positive, both D_3 and D_4 conduct, otherwise both Q_3 and Q_4 conduct. In summary, currents through the devices in the circuit is given below.

$$
\begin{aligned}
i_o(t) &= \begin{cases} \frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right] e^{-\frac{t}{\tau}} & 0 \le t \le 0.5T \\ -\frac{V_{dc}}{R} + \left[I_{max} + \frac{V_{dc}}{R}\right] e^{-\frac{t-0.5T}{\tau}} & 0.5T \le t \le T \end{cases} \\
i_s(t) &= \begin{cases} i_o(t) & 0 \le t \le 0.5T \\ -i_o(t) & 0.5T \le t \le T \end{cases} \\
i_{s1}(t) = i_{s2}(t) &= \begin{cases} i_o(t) & 0 \le t \le 0.5T \\ 0 & 0.5T \le t \le T \end{cases} \\
i_{s3}(t) = i_{s4}(t) &= \begin{cases} 0 & 0 \le t \le 0.5T \\ -i_o(t) & 0.5T \le t \le T \end{cases}
\end{aligned}
$$

The rms output voltage is $V_{rms} = \sqrt{\frac{1}{0.5T}\int_0^{0.5T} V_{dc}^2 \mathrm{d}t} = V_{dc}$

and the rms output current is $I_{rms} = \sqrt{\frac{1}{0.5T}\int_0^{0.5T} \left\{\frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right] e^{-\frac{t}{\tau}}\right\}^2 \mathrm{d}t}$

❖**Both average output current and voltage are zero.**

Example 2:

A full-bridge inverter has the following data: V_{dc} = 100 V, R = 10 Ω, L = 0.025 H, and f = 60 Hz. Determine

1. an expression for the load current
2. the power absorbed by the load
3. average current through the source

Solutions:

1.

$$\begin{aligned}
\tau &= \frac{L}{R} = \frac{0.025}{10} = 0.0025s \\
T &= \frac{1}{f} = \frac{1}{60} = 0.01667s \\
I_{max} &= \frac{V_{dc}}{R}\frac{1 - e^{-\frac{T}{2\tau}}}{1 + e^{-\frac{T}{2\tau}}} \\
&= \frac{100}{10}\frac{1 - e^{-\frac{0.01667}{0.005}}}{1 + e^{-\frac{0.01667}{0.005}}} \\
&= 9.31A
\end{aligned}$$

$$
i_o(t) = \begin{cases} \frac{V_{dc}}{R} + \left[I_{min} - \frac{V_{dc}}{R}\right] e^{-\frac{t}{\tau}} & 0 \le t \le 0.5T \\ -\frac{V_{dc}}{R} + \left[I_{max} + \frac{V_{dc}}{R}\right] e^{-\frac{t-0.5T}{\tau}} & 0.5T \le t \le T \end{cases}
$$

$$
= \begin{cases} \frac{100}{10} + \left[-9.31 - \frac{100}{10}\right] e^{-\frac{t}{0.0025}} & 0 \le t \le 0.00833 \\ -\frac{100}{10} + \left[9.31 + \frac{100}{10}\right] e^{-\frac{t-0.00833}{0.0025}} & 0.00833 \le t \le 0.01667 \end{cases}
$$

$$
= \begin{cases} 10 - 19.31 e^{-\frac{t}{0.0025}} & 0 \le t \le 0.00833 \\ -10 + 19.31 e^{-\frac{t-0.00833}{0.0025}} & 0.00833 \le t \le 0.01667 \end{cases}
$$

2. The rms load current is

$$
I_{rms} = \sqrt{\frac{2}{T}\int_0^{0.5T} [i_o(t)]^2 \mathrm{d}t} = \sqrt{\frac{2}{0.01667}\int_0^{0.00833} [10 - 19.31 e^{-\frac{t}{0.0025}}]^2 \mathrm{d}t}
$$

$$
= \sqrt{\frac{2}{0.01667}\left[100 - 38.62 e^{-\frac{t}{0.0025}} + 19.31^2 e^{-\frac{2t}{0.0025}}\right] \mathrm{d}t}
$$

$$
= \sqrt{\frac{2}{0.01667}\left[100t + 38.62 \times 0.0025 e^{-\frac{t}{0.0025}} - 19.31^2 \times \frac{0.0025}{2} e^{-\frac{2t}{0.0025}}\right]_0^{0.00833}}
$$

$$
= \sqrt{\frac{2}{0.01667}\left[0.83 + 38.62 \times 0.0025 \left(e^{-\frac{0.00833}{0.0025}} - 1\right) - 19.31^2 \times \frac{0.0025}{2}\left(e^{-\frac{2\times 0.00833}{0.0025}} - 1\right)\right]}
$$

$$
= 6.64A
$$

$$
P_o = I_{rms}^2 R = 6.64^2 \times 10 = 441W
$$

3. The average source current is calculated as follows:

$$
\begin{aligned}
I_s &= \frac{2}{T}\int_0^{0.5T} i_o(t)\mathrm{d}t = \frac{2}{0.01667}\int_0^{0.00833}[10 - 19.31\mathrm{e}^{-\frac{t}{0.0025}}]\mathrm{d}t \\
&= \frac{2}{0.01667}\left[10t - 19.13\times(-0.0025)\mathrm{e}^{-\frac{t}{0.0025}}\right]_0^{0.00833} \\
&= \frac{2}{0.01667}\left[10(0.00833-0) - 19.13\times(-0.0025)\left(\mathrm{e}^{-\frac{0.00833}{0.0025}} - 1\right)\right] \\
&= 4.41A
\end{aligned}
$$

Note: Inverters can be single-phase or three-phase as follows:

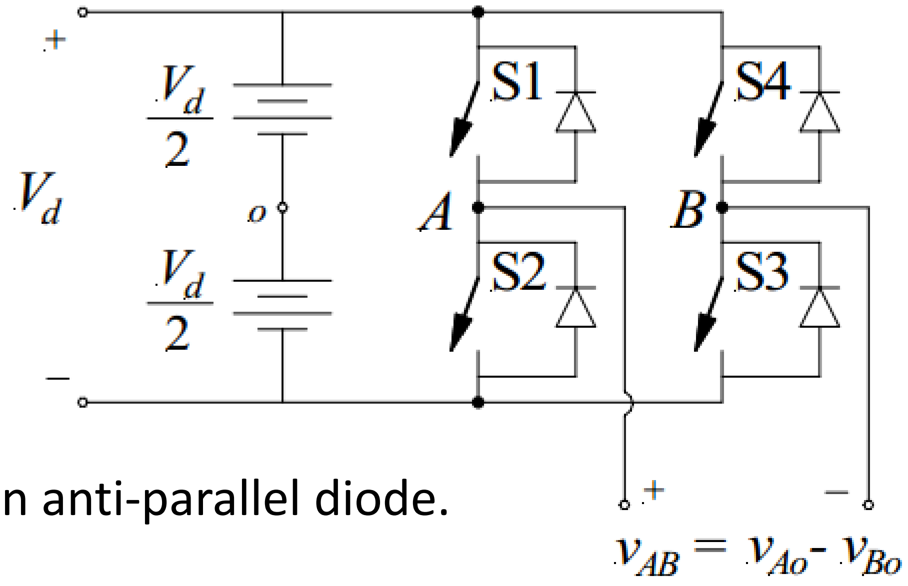

Fig. 7.7 One-phase full wave inverter.

Note the four controlled switches, each with an anti-parallel diode.

- For three-phase loads it makes more sense to use a three-phase inverter shown in Fig. 7.8, rather than using three one-phase inverters.

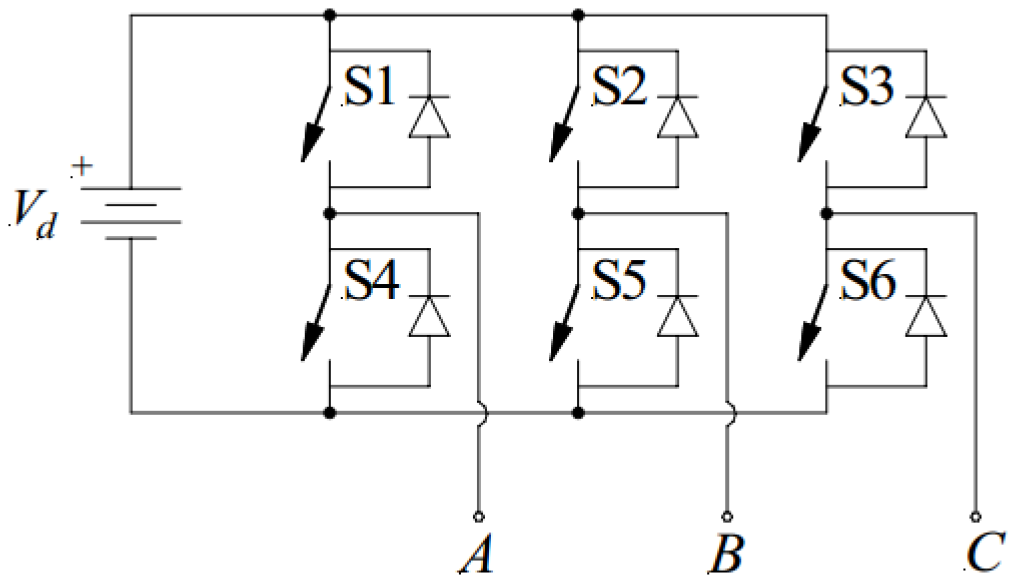

Fig. 7.8-Three-phase, full wave inverter.

End of Lecture 7!