



# EEE 5451 Power Electronics

## Lecture 4: Three-Phase DC Drives

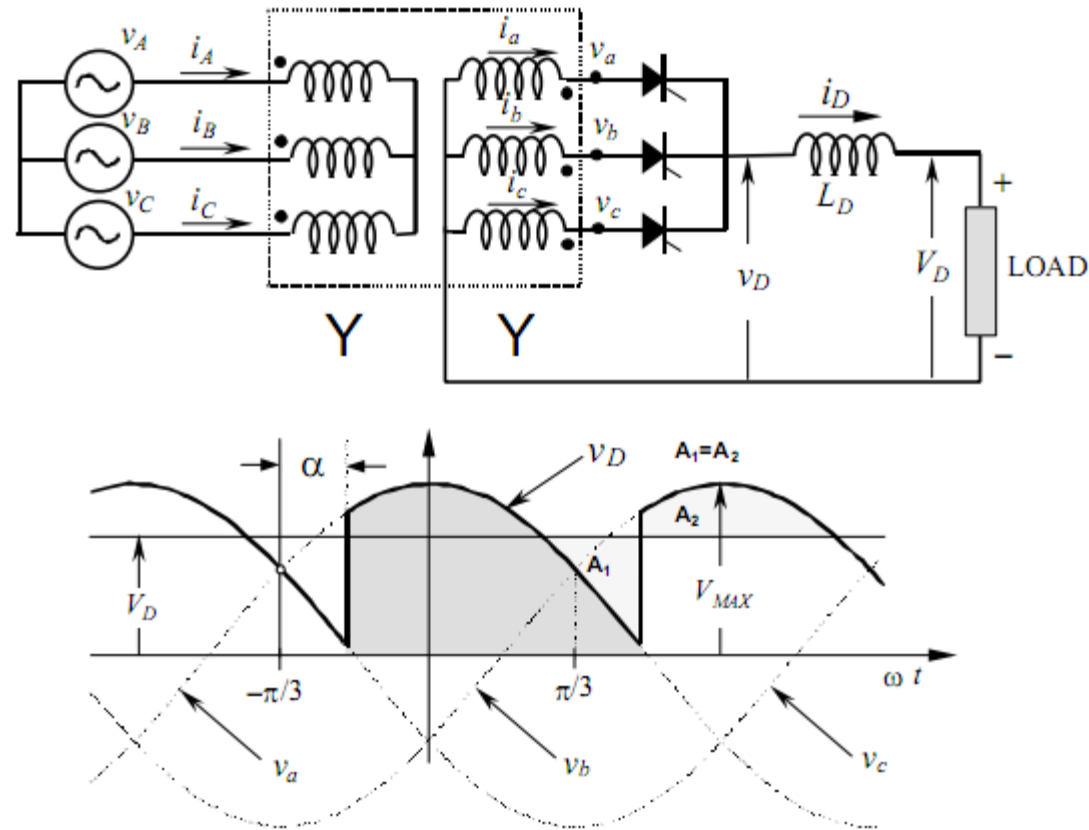
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# 4.1 Introduction

- Three-phase converters are commonly used in adjustable speed drives from about 15 kW up to several thousand kilowatts ratings. The output voltage of a three-phase converter has less ripple contents than the single-phase converter, and therefore, the armature current will be smoother and mostly continuous.
- The three-phase half-wave circuit is only of theoretical importance and is generally not used in industrial applications because of the d.c. components inherent in its line currents. For medium size motors, in the range 15 – 120 kW, either the full-converters or semiconverter are used.

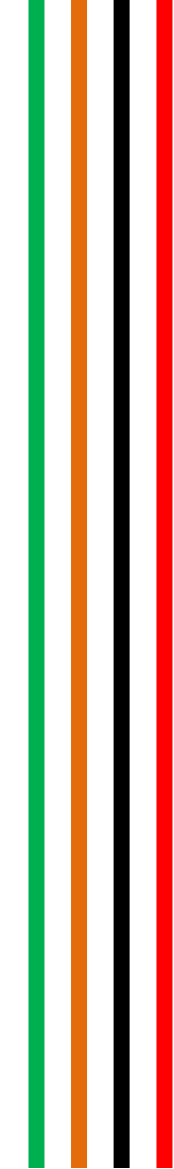


# 4.2 Line Commutated Controlled Rectifiers

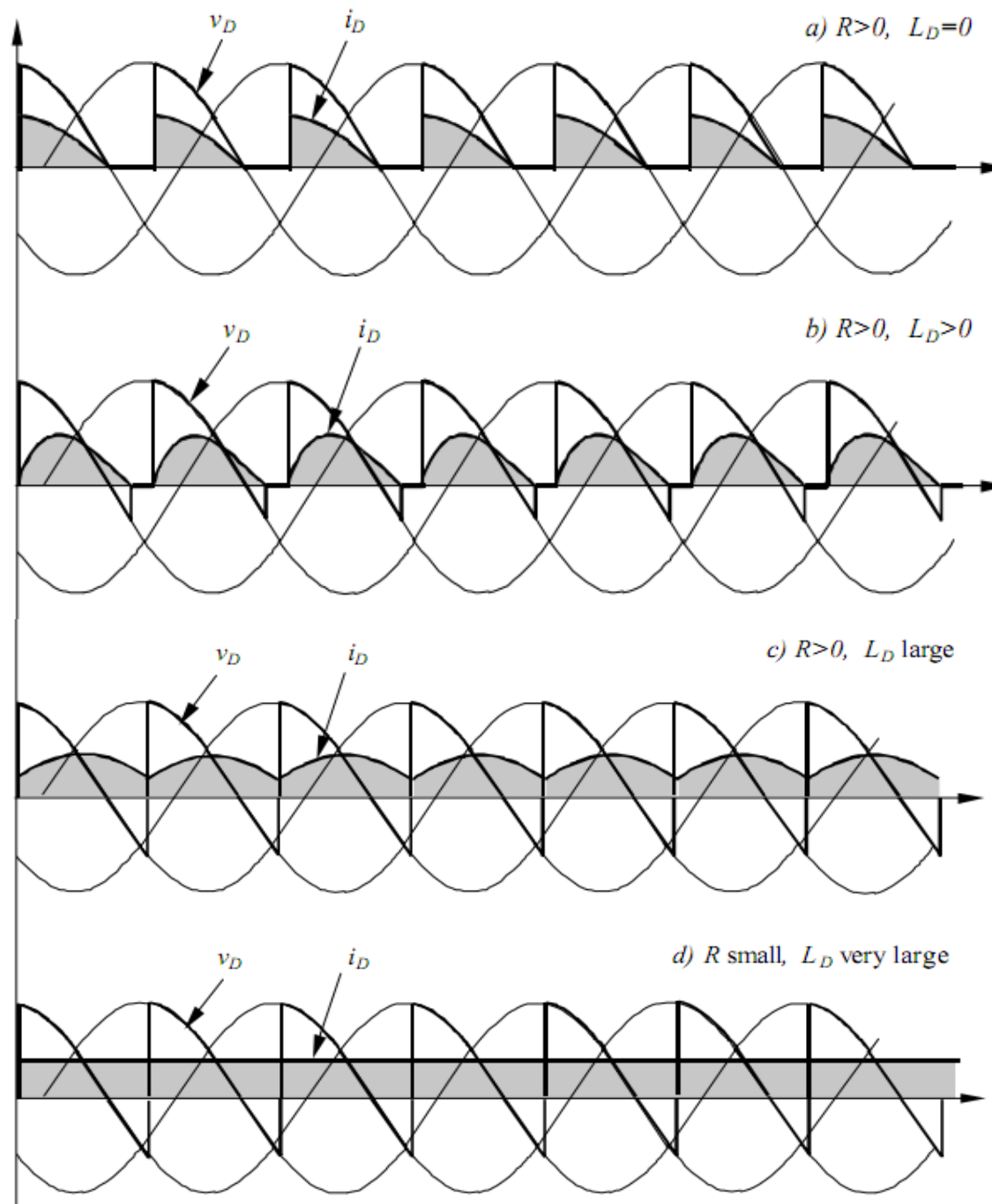


$$V_D = 1.17V_{LN} \cos \alpha$$

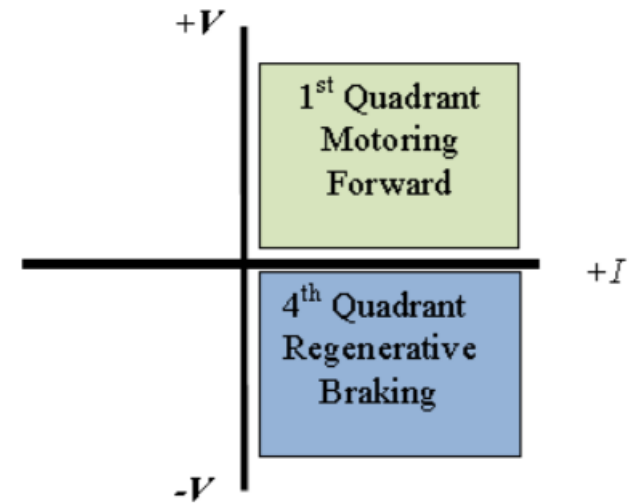
Fig. 4.1 Three-phase half-wave rectifier with output waveforms



**Fig. 4.2.** Output waveforms from Fig. 4.1



- A firing angle of zero degree produces the maximum output d.c. voltage for all ac-to-dc converter circuits. For continuous current conduction, each thyristor carries current for  $120^\circ$ , followed by  $240^\circ$  of non-conduction.
- The firing angle  $\alpha$  can be varied in the range of  $0 < \alpha < 180^\circ$ . For  $\alpha > 90^\circ$ , the output d.c. voltage becomes negative, whilst the motor current is positive and continuous. This implies operation of the converter in the fourth quadrant of the  $V-I$  plane as shown in Fig.4.3 where the converter operates in the inversion mode.
- In this mode of operation the motor supplies power to the a.c. source through the converter steadily. This mode of operation is called regenerative conversion. For example, an overhauling motor can supply its energy to the a.c. mains in this way.

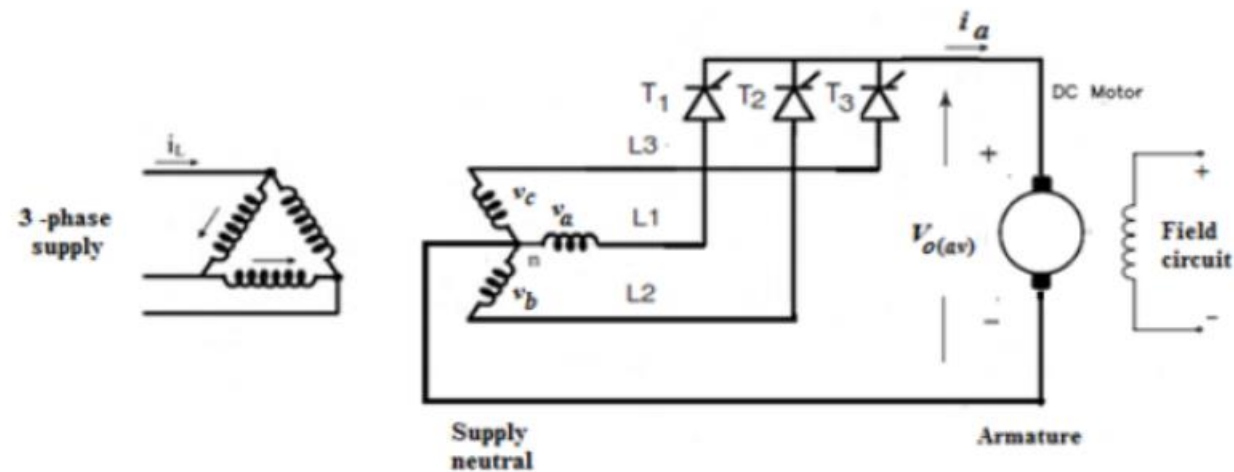


**Fig. 4.3** Two-quadrant operation of three-phase half-wave converter drive.

## Example 4.1

A 100 hp, 1750 rpm, d.c. shunt motor has an armature inductance of 1.15 mH, a resistance of  $0.0155 \Omega$  and an armature voltage constant of  $1.3 \text{V}\cdot\text{s}/\text{rad}$ . The motor is operated from a three-phase half-wave controlled-rectifier at rated armature current of 35 A. Find the firing angle  $\alpha$ , assuming that the supply voltage is 400 V and the motor speed is 1750 rpm. Consider the thyristors to have a forward voltage drop of 1.5 Volt and assume continuous conduction.

**Fig. 4.4** Three-phase half-wave rectifier



### Solution

Speed of the motor in rpm:  $n = 1750$

Change the speed from rpm to rad/s:

$$\omega = \frac{2\pi n}{60} = \frac{2\pi}{60} \times 1750 = 183.25 \text{ rad/s}$$

Armature voltage constant  $K\Phi = 1.3$

$$\therefore E_a = K\Phi\omega = 1.3 \times 183.25 \text{ V}$$

$$V_m = \sqrt{2} \times \frac{400}{\sqrt{3}} = 326.2 \text{ V}$$

$$V_{o(av)} = \frac{3\sqrt{3}V_m}{2\pi} \cos\alpha = \frac{3\sqrt{3} \times 326.2}{2\pi} \cos\alpha = 270.08 \cos\alpha$$

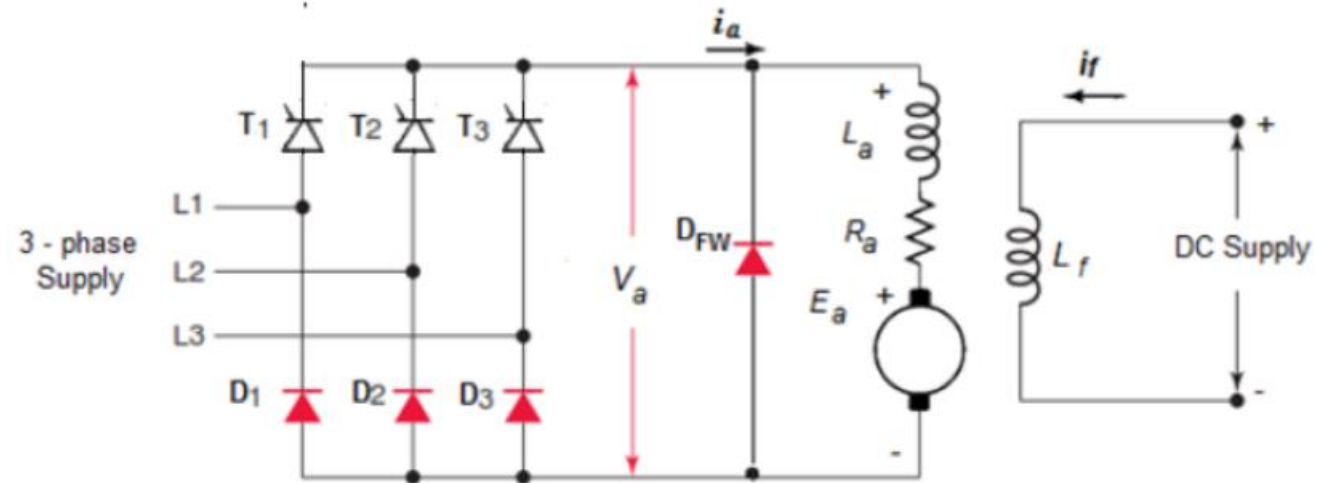
$$V_{o(av)} = E_a + I_a R_a + V_T \leftarrow \text{Thyristor drop}$$

$$270.08 \cos\alpha = 238.22 + 15.5 \times 10^{-3} \times 35 + 1.5 = 240.26$$

$$\therefore \alpha = \cos^{-1} \frac{240.26}{270.08} = \cos^{-1} 0.8895 = 27.2^\circ$$



**Fig. 4.5** Three-phase semiconverter drive.



- The three-phase semiconverter is a one-quadrant drive. Its circuit includes a freewheeling diode  $D_{FW}$  to maintain continuous load current. It uses three thyristors and three diodes; hence a cost advantage is obtained compared with the full-converter.
- This converter is used for motor ratings from 15 to 150 hp. The field converter may be single-phase or three-phase semiconverter with firing angle of  $\alpha_f$ . Assuming continuous current operation, the average value of the armature voltage at the motor terminals is a contribution from the upper half-bridge plus a contribution from the uncontrolled lower bridge. Hence for all firing angles we can write:

$$V_{o(av)} = \frac{3\sqrt{3}}{2\pi} V_m \cos\alpha + \frac{3\sqrt{3}}{2\pi} V_m$$

$$= \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos\alpha)$$

The average armature current is:

$$I_{a(av)} = \frac{V_{a(av)} - E_a}{R_a}$$

$$= \frac{3\sqrt{3}V_m}{2\pi R_a} (1 + \cos\alpha) - \frac{E_a}{R_a}$$

- For discontinuous current operation, the above equations are not valid.

### Features:

- ✓ Since only three thyristors are used, the circuit is not expensive and a simple control circuitry is required.
- ✓ Dynamic braking can be performed by switching armature connection to an external resistance.
- ✓ Operation is in the first quadrant only. However, bi-directional rotation can be obtained by reversing field current or armature terminals when the motor has been stopped.

## Example 4.2

A Three-phase half-controlled thyristor bridge with 400 V, three-phase, 50 Hz supply is feeding a separately-excited d.c. motor. Armature resistance is  $0.2 \Omega$ , armature rated current is 100 A and back *emf* constant is 0.25 V/rpm. Determine the no-load speed if the no-load armature current is 5 A and firing angle is  $45^\circ$ . Also determine the firing angle to obtain a speed of 1500 rpm at rated current.

### Solution

Armature voltage at no-load,

$$\begin{aligned}V_{a(o)} &= \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos \alpha) \\ &= \frac{3\sqrt{3}}{2\pi} \times \frac{400\sqrt{2}}{\sqrt{3}} (1 + \cos 45^\circ) = 461 \text{ V}\end{aligned}$$

Back *emf* at no-load,  $E_{ao} = V_{a(o)} - I_{ao} R_a = 461 - 5 \times 0.2 = 460 \text{ V}$



Speed at no-load

$$n_o = \frac{E_{ao}}{K_e \Phi} = \frac{460}{0.25} = 1840 \text{ rpm}$$

Back *emf* developed at speed of 1500 rpm

$$E_{ao} = 1500 \times 0.25 = 375 \text{ V}$$

Armature voltage will be:

$$V_o = E_a + I_a R_a = 375 + 100 \times 0.2 = 395 \text{ V}$$

Also

$$V_o = \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos \alpha)$$

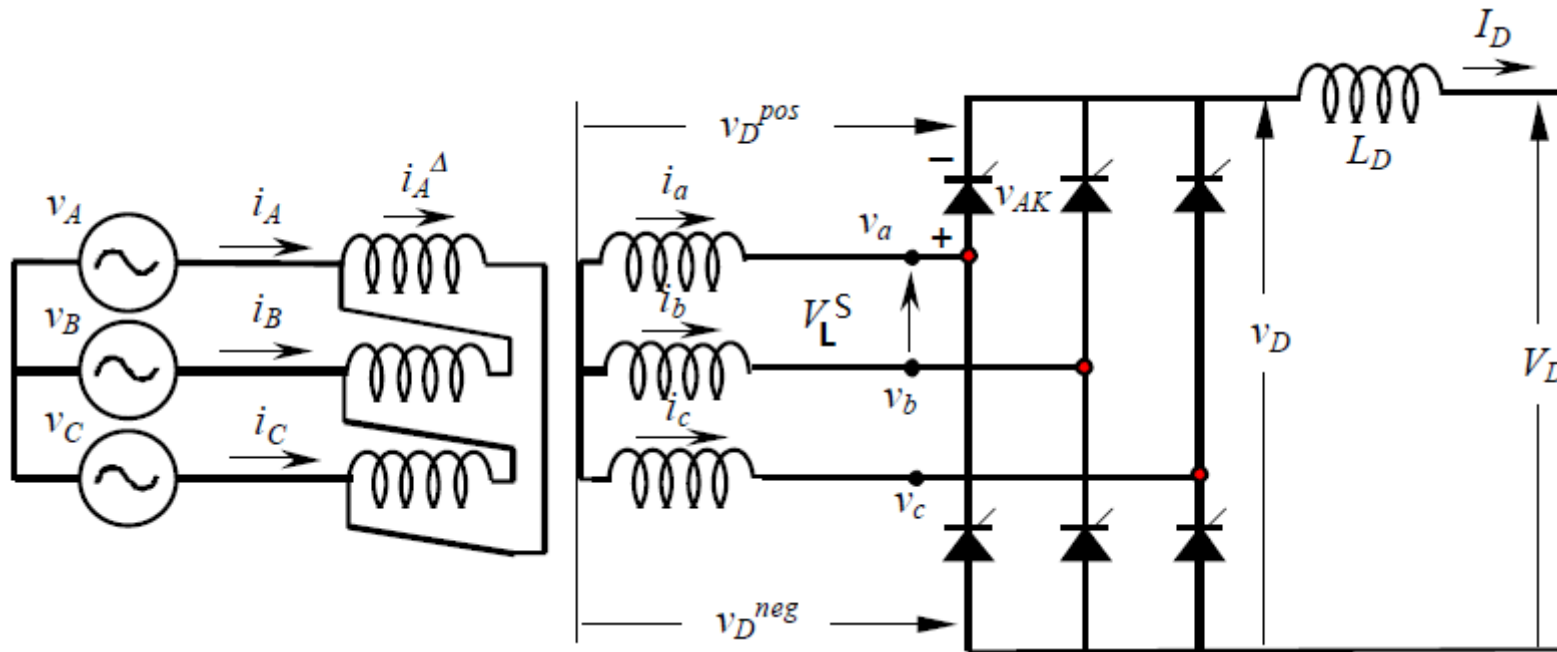
$$395 = \frac{3\sqrt{3}}{2\pi} \times \frac{400\sqrt{2}}{\sqrt{3}} (1 + \cos \alpha)$$

$$\alpha = 62.45^\circ$$



## 4.2.1 - Three-Phase Full-Converter

- The circuit diagram for a three-phase a.c. supply through a three-phase full-converter is shown in Fig. 4.6.

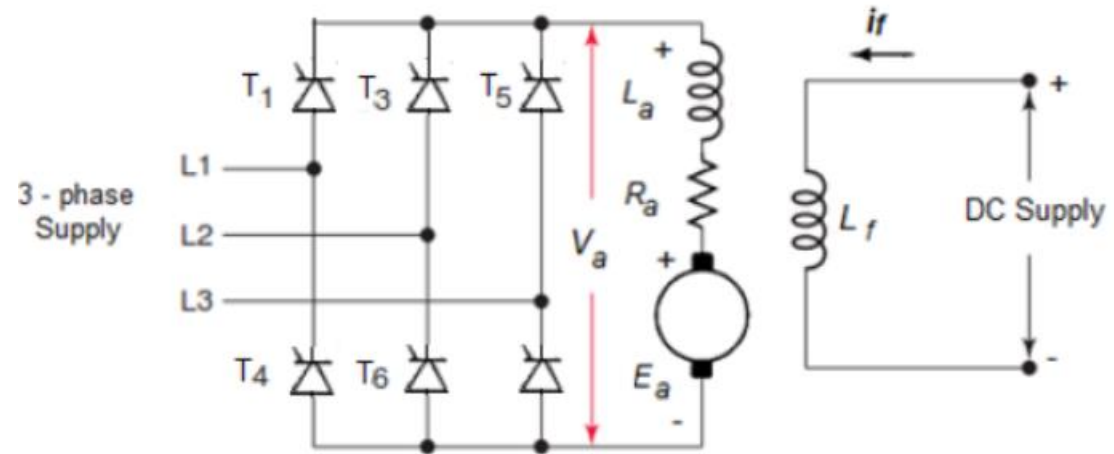


$$\begin{aligned}
 V_D &= \frac{3\sqrt{2} \cdot V_L^s}{\pi} \cos \alpha \\
 &= 1.35 V_L^s \cos \alpha \\
 &= 2.34 V_{LN}^s \cos \alpha
 \end{aligned}$$

Fig. 4.6 Graetz Bridge (Three-phase full-wave rectifier)

- The circuit diagram for a separately-excited d.c. motor supplied from a three-phase a.c. supply through a three-phase full-converter is shown in Fig. 4.7.

**Fig. 4.7** Three-phase full-converter with a separately-excited d.c. motor load.



- If the motor armature inductance is large and the firing angle is small then the armature current is likely to be continuous. However, with small armature inductance and large firing angles the armature current may become discontinuous particularly when the back *emf* is relatively high.

- For continuous current operation, the armature voltage has an average value shown below:

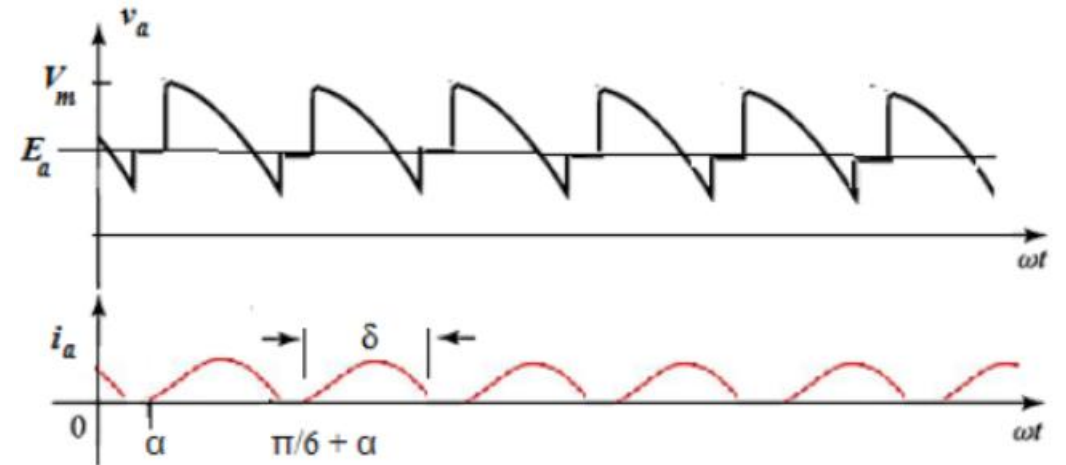
$$V_{a(av)} = \frac{3\sqrt{3}}{\pi} V_m \cos \alpha$$

Thus, the average armature current is:

$$I_{a(va)} = \frac{V_{a(av)} - E_a}{R_a}$$

$$= \frac{3\sqrt{3}V_m}{\pi R_a} \cos \alpha - \frac{E_a}{R_a}$$

- For discontinuous current operation of the full-converter, the waveforms of the voltage and currents are shown in Fig. 4.8.



**Fig. 4.8** Discontinuous current operation waveforms of the full-converter.

- The differential equations describing the motor system, during the period the thyristors conduct, are

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + e_a$$

$$\sqrt{3}V_m \sin(\omega t + 30^\circ) = i_a R_a + L_a \frac{di_a}{dt} + K\Phi\omega_m$$

$$T_m = K\Phi I_a = J \frac{d\omega_m}{dt} + B \cdot \omega_m + T_L$$

If it is assumed that the inertia of the rotating system is large then speed fluctuations will be negligible. If each term of  $v_a$  is integrated from  $\alpha$  to  $(\alpha + \pi/6)$  and then divided by  $\pi/3$ , the instantaneous voltage, current and speed will be converted to their respective average values,

$$V_{a(av)} = \frac{3}{\pi} \int_{\alpha}^{\alpha+\pi/6} \sqrt{3}V_m (\sin \omega t + 30^\circ) d\omega t$$



$$= \frac{1}{\pi} \int_{\alpha}^{\alpha+\pi/6} L_a \frac{di_a}{dt} d\omega t + \frac{1}{\pi} \int_{\alpha}^{\alpha+\pi/6} R_a i_a d\omega t$$

$$+ \frac{1}{\pi} \int_{\alpha}^{\alpha+\pi/6} K\Phi \omega_m d\omega t$$

Thus

$$V_{a(av)} = I_{a(av)} R_a + K\Phi \omega_{m(av)}$$

where

$$V_{a(av)} = \frac{3\sqrt{3}V_m}{\pi} [\cos(\alpha + 30^\circ) - \cos(\alpha + 90^\circ)]$$

and the average voltage across  $L_a$  is zero.

Similarly,

$$T_{m(av)} = K\Phi I_{a(av)} = B \cdot \omega_{m(av)} + T_L$$



## 4.2.2 - Three-Phase Dual Converter Drive

- Four-quadrant operation of a medium and large size d.c. motor drive (200-2000 hp) can be obtained by the three-phase dual converter shown in Fig.4.9. The average motor voltage is required to be equal for both converters, which requires that the firing angles of the two sets of the thyristors should sum to  $180^\circ$ .

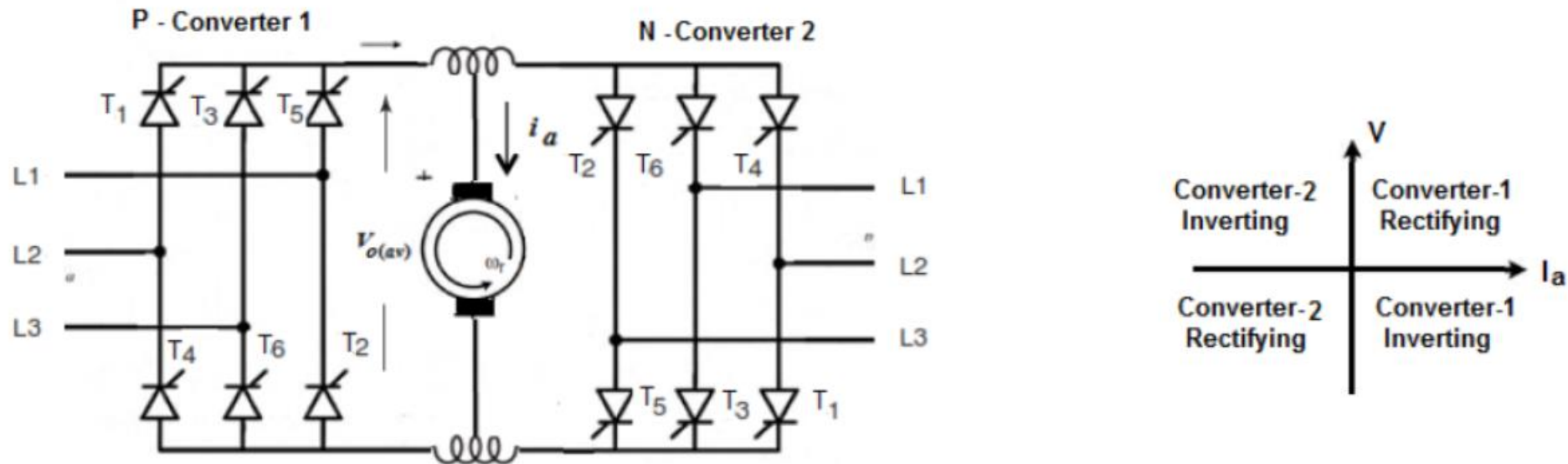


Fig. 4.9 Four-quadrant three-phase d.c. drive.

- The armature voltage supplied by converter-1 (for continuous current operation) is

Bridge – I operating:

$$V_1 = V_{a(av)1} = \frac{3 \sqrt{3} V_m}{\pi} \cos \alpha_1$$
$$= V_{do} \cos \alpha_1 = E_a + I_1 R_a$$

Bridge – II operating:


$$V_2 = V_{a(av)2} = - \left( \frac{3 \sqrt{3} V_m}{\pi} \cos \alpha_2 \right)$$
$$= V_{do} \cos \alpha_2 = E_a - I_1 R_a$$

where  $V_{do} = \frac{3 \sqrt{3} V_m}{\pi}$

and  $\alpha_2 = \pi - \alpha_1$  .

Two modes of operation can be achieved with this circuit:





**(a) Circulating current operating mode:**

Here, instantaneous values of circulating current are limited by use of reactors and mean level is controlled by current loop. Circulating current may be constant giving linear characteristic or it may be reduced to zero giving higher gain portion of overall characteristic.

Advantage: Continuous bridge current maintain armature current at all times, no discontinuity occurs.

Disadvantage: Presence of circulating current reduces efficiency.

**(b) Circulating current-free operation mode:**

In this mode only one converter operates at a time. Logic used to prevent the two bridges being turn on at the same time. Reactors or inductors used to maintain continuous current down to acceptable low levels. Discontinuity occurs at zero and also a time delay (*ms*) introduced at the zero current level.

Advantage: Higher efficiency than circulating current schemes, hence used more widely.

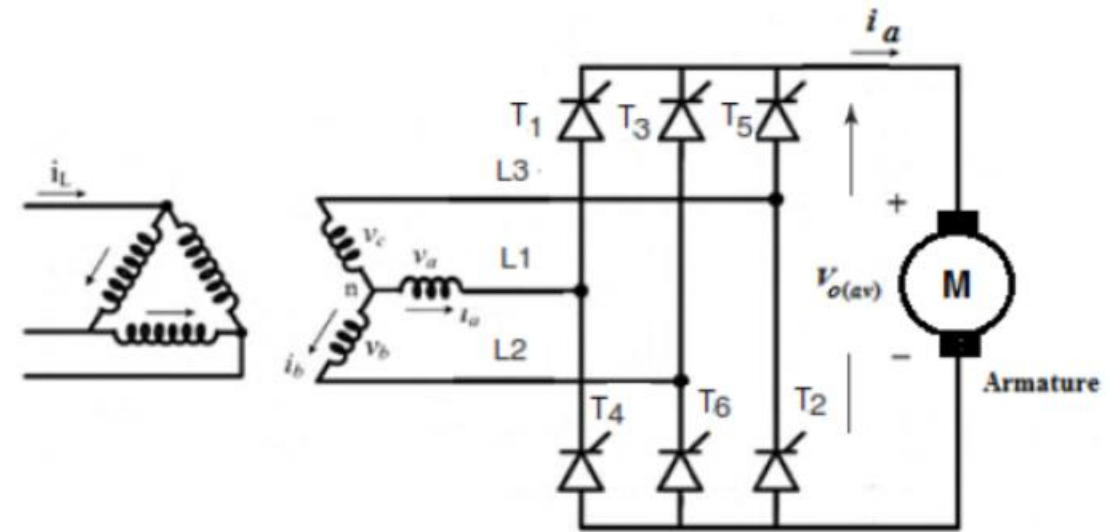
Disadvantage: Dead time, discontinuity in zero current regions.



## Example 4.3

A three-phase full-converter, shown in Fig. 4.10, is used to control the speed of a separately-excited d.c. motor rated at 100 kW, 600 V, 2000 rpm. The converter is connected to a three-phase 400 V, 50 Hz supply.

**Fig. 4.10** Three-phase full-converter d.c. drive.



The armature resistance  $R_a = 0.051 \Omega$  and the armature circuit inductance is  $L_a = 10\text{mH}$ . The motor voltage constant is  $K_e \Phi = 0.25 \text{ V/rpm}$ . The rated armature current is 100 A and the no-load current is 10 A.

With the converter operating as a rectifier, and assuming that the motor current is continuous and ripple-free, determine:

- (a) The no load speed when the firing angles:  $\alpha = 0^\circ$  and  $\alpha = 60^\circ$ .
- (b) The firing angle to obtain the rated speed of 2000 rpm at rated motor current.

**Solution**

(a) At no-load condition

$$V_m = \frac{400}{\sqrt{3}} \times \sqrt{2} = 325.22 \text{ V}$$

Let the converter output voltage =  $V_{o(av)}$  = armature terminal voltage  $V_a$  :

$$V_{o(av)} = \frac{3\sqrt{3}}{\pi} V_m \cos \alpha$$

$$V_{o(av)} = \frac{3\sqrt{3}}{\pi} \times 325.22 \cos \alpha$$

$$V_{o(av)} = 538 \cos \alpha$$

For  $\alpha = 0^\circ$  ,  $\longrightarrow V_{o(av)} = 538 \text{ V}$

$$E_a = V_{o(av)} - I_a R_a = 538 - 10 \times 0.051 = 537.5 \text{ V}$$



No-load speed:

$$\text{since } E_a = K_e \phi n$$

Hence

$$n_o = \frac{E_a}{K_e \phi} = \frac{537.5}{0.25} = 2145 \text{ rpm}$$

For  $\alpha = 60^\circ$ :

$$V_{o(av)} = 538 \cos 60^\circ = 269 \text{ V}$$

$$E_a = V_{a(av)} - I_a R_a = 269 - 10 \times 0.051 = 268.4 \text{ V}$$

$$n_o = \frac{E_a}{K_e \phi} = \frac{268.41}{0.25} = 1073.64 \text{ rpm}$$

(b) At full-load condition

$$E_a = K_e \phi n = 0.25 \times 2000 = 500 \text{ V}$$

$$V_t = E_a + I_a R_a = 500 + 100 \times 0.051 = 505 \text{ V}$$

$$505 = 538 \cos \alpha$$

$$\text{Hence } \alpha = 20.14^\circ$$



# 4.3 Commutation Process

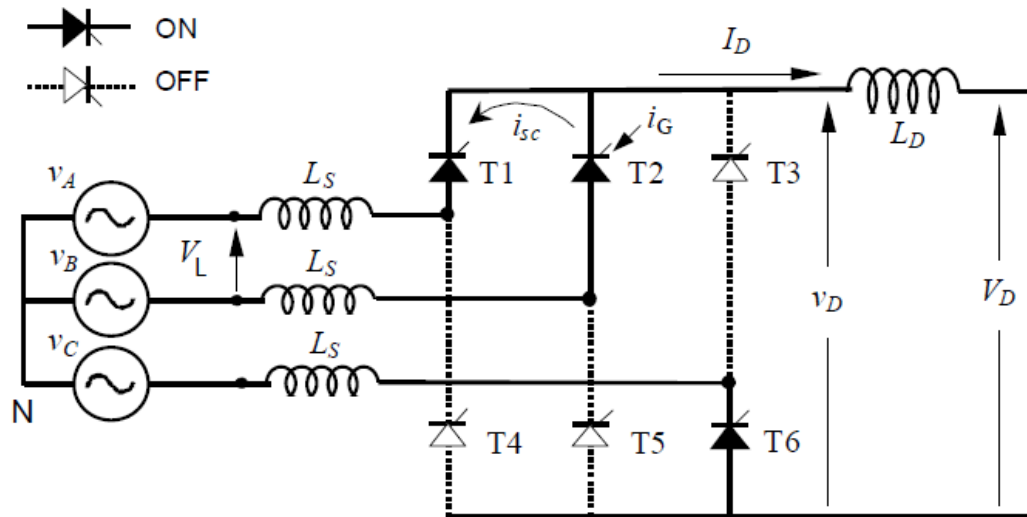


Fig. 4.11 Commutation Circuit

$$2L_s \frac{di_{sc}}{dt} = \sqrt{2}V_L \sin \omega t = v_A - v_B$$

$$i_{sc} = -\frac{\sqrt{2}}{2L_s} V_L \frac{\cos \omega t}{\omega} + C$$

When  $\omega t = \alpha$ ,  $i_{sc} = 0$ ,  $\therefore C = \frac{V_L}{\sqrt{2}\omega L_s} \cos \alpha$

Thus;  $i_{sc} = \frac{V_L}{\sqrt{2}\omega L_s} (\cos \alpha - \cos \omega t)$



When  $\omega t = \alpha + \mu$ ,  $i_{sc} = I_D$ ;  $\mu$  is overlap angle

$$I_D = \frac{V_L}{\sqrt{2}\omega L_s} [\cos \alpha - \cos(\alpha + \mu)]$$

Voltage drop during commutation :

$$\Delta v = \frac{v_A - v_B}{2} = \frac{\sqrt{2}V_L \sin \omega t}{2}$$

$$\Delta V_{med} = \frac{3}{\pi} \cdot \frac{1}{2} \int_{\alpha}^{\alpha+\mu} \sqrt{2}V_L \sin \omega t \cdot d\omega t$$

$$\Delta V_{med} = \frac{3V_L}{\pi\sqrt{2}} [\cos \alpha - \cos(\alpha + \mu)]$$

$$V_D = \frac{3\sqrt{2}V_L}{\pi} \cos \alpha - \Delta V_{med}$$

$$V_D = \frac{3\sqrt{2}V_L}{2\pi} [\cos \alpha + \cos(\alpha + \mu)]$$



## 4.3.1 - Equivalent Circuit of a Converter

$$V_D = \frac{3\sqrt{2}V_L}{\pi} \cos\alpha - \frac{3I_D\omega L_s}{\pi}$$

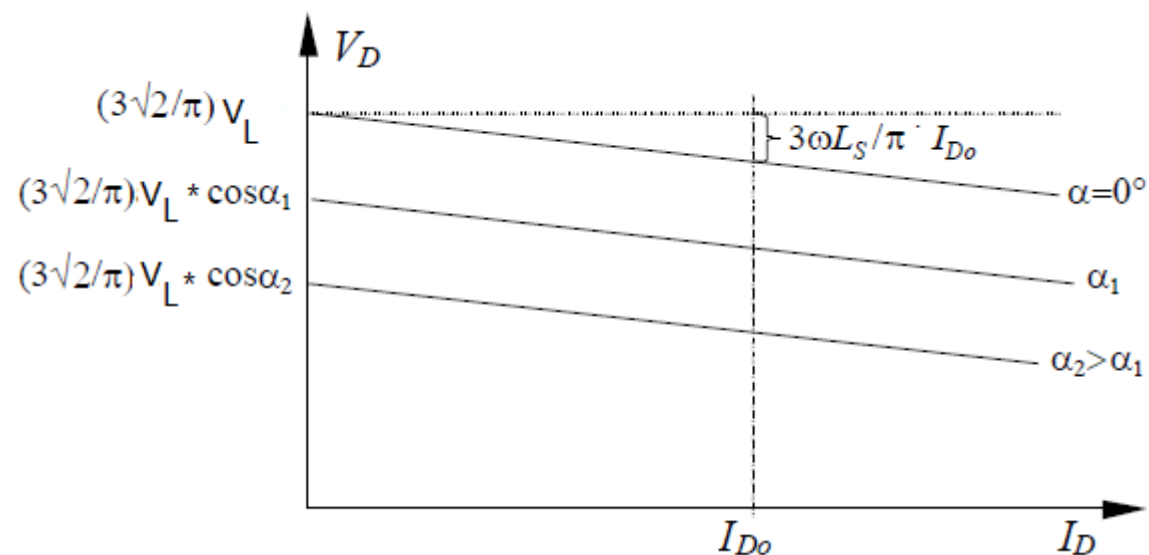
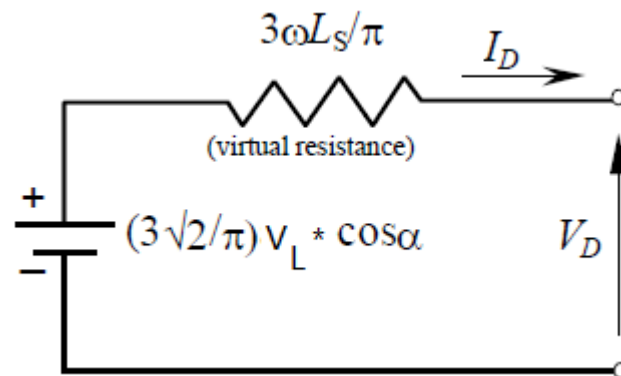


Fig. 4.12 Converter Equivalent Circuit & Output Characteristics

## Example 4.4

A three-phase full-wave converter is supplied from the 415 V ac, 50 Hz mains with phase source inductance of 0.1 mH. If the average load current is 100 A continuous, determine the supply reactance voltage drop, the overlap angle, and the mean output voltage for phase delay angles of (a) 0° (b) 60°. Ignoring thyristor forward blocking time requirements, determine the maximum delay angle.

Solution:

$$V_{drop} = \frac{3\omega L_s}{\pi} I_D = \frac{3 \times 2\pi \times 50 \times 10^{-4} \times 100}{\pi} = 3[\text{V}]$$



(i) for  $\alpha = 0^\circ$

$$V_D = \frac{3\sqrt{2} \times 415 \times \cos 0}{\pi} - 3 = 557.65 \text{ [V]}$$

$$V_D = \frac{3\sqrt{2}V_L}{2\pi} [\cos \alpha + \cos(\alpha + \mu)]$$

$$557.65 = \frac{3\sqrt{2} \times 415}{2\pi} [\cos 0 + \cos \mu]$$

$$\Rightarrow \mu = 8.4^\circ$$

(ii) for  $\alpha = 60^\circ$

$$V_D = \frac{3\sqrt{2} \times 415}{\pi} \times \cos 60^\circ - 3 = 277.32 \text{ [V]}$$

$$277.32 = \frac{3\sqrt{2} \times 415}{2\pi} [\cos 60^\circ + \cos(60^\circ + \mu)]$$

$$\Rightarrow \mu = 0.71^\circ$$

$$\hat{\alpha} = \cos^{-1} \left\{ \frac{\omega L_s I_D}{\sqrt{2} V_L \sin(\pi/n)} - 1 \right\}$$

$$\hat{\alpha} = \cos^{-1} \left\{ \frac{2\pi \times 50 \times 10^{-4} \times 100}{\sqrt{2} \times 415 \times 0.5} - 1 \right\} = 171.56^\circ \text{ and } V_D = -557.41 \text{ [V]}$$

**End of Lecture 4!**

