The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

Tutorial Sheet 1 - Laplace transforms **March, 2020.**

---

1. Find the Laplace transforms of the following functions. Assume that a, b, ω and θ are constants.

   (a) $(a-bt)^2$ (b) $3t+12$ (c) $\cos^2 \omega t$
   (d) $e^{2t}\sinh t$ (e) $e^{-t}\sin 4t$ (f) $\sin(\omega t+\theta)$
   (g) $1.5\sin(3t-\frac{\pi}{2})$ (h) t^2e^{-3t} (i) $ke^{-at}\cos\omega t$
   (j) $\sinh t\cos t$

2. Find the inverse Laplace transforms of the following functions. Assume that a, b, n and L are constants.

   (a) $\frac{5s+1}{s^2-25}$ (b) $\frac{s^2}{L^2s^2+n^2\pi^2}$ (c) $\frac{12}{s^4}-\frac{228}{s^6}$
   (d) $\frac{4s+32}{s^2-16}$ (e) $\frac{s+10}{s^2-s-2}$ (f) $\frac{1}{(s+a)(s+b)}$
   (g) $\frac{\pi}{(s+\pi)^2}$ (h) $\frac{6}{(s+1)^3}$ (i) $\frac{4}{s^2-2s-3}$
   (j) $\frac{\pi}{s^2+10\pi s+24\pi^2}$ (k) $\frac{2s-1}{s^2-6s+18}$

3. Solve the following initial value problems by using Laplace transforms.

   (a) $y'+2y=0, \quad y(0)=\frac{3}{2}$ (b) $y''-y'-6y=0, \quad y(0)=11,\ y'(0)=28$
   (c) $y''+9y=10e^{-t}, \quad y(0)=0,\ y'(0)=0$ (d) $y''-4y'+3y=6t-8, \quad y(0)=0,\ y'(0)=0$

4. Solve the following shifted data initial value problems by using Laplace transforms.

   (a) $y'-6y=0, \quad y(-1)=4.$ (b) $y''-2y'-3y=0, \quad y(4)=-3,\ y'(4)=-17$
   (c) $y''+2y'+5y=10t-100,\ y(2)=-4,\ y'(2)=14$

5. Sketch or graph the given function, which is assumed to be zero outside the given interval. Represent it, using unit step functions. Find its Laplace transform.

   (a) $t \quad (0<t<2)$ (b) $t-2 \quad (t<2)$ (c) $\cos 4t \quad (0<t<\pi)$
   (d) $e^t \quad (0<t<\frac{\pi}{2})$ (e) $\sin\pi t \quad (2<t<4)$ (f) $e^{-\pi t} \quad (2<t<4)$
   (g) $t^2 \quad (1<t<2)$ (h) $t^2 \quad (0<t<\frac{3}{2})$ (i) $\sin t \quad (\frac{\pi}{2}<t<\pi)$

6. Find and sketch the inverse Laplace transforms of the following functions.

   (a) $\frac{e^{-3s}}{(s-1)^3}$ (b) $\frac{6(1-e^{-\pi s})}{(s^2+9)}$ (c) $\frac{4(e^{-2s}-2e^{-5s})}{s}$
   (d) $\frac{e^{-3s}}{s^4}$ (e) $\frac{2(e^{-s}-e^{-3s})}{(s^2-4)}$ (f) $\frac{(1+e^{-2\pi(s+1)}(s+1)}{((s+1)^2+1)}$

7. Solve the following initial value problems with a discontinuous force.

   (a) $y'' + 9y = \begin{cases} 8\sin t, & : 0 < t < \pi \\ 0, & : t \geq \pi \end{cases}$ $\quad y(0) = 0,\ y'(0) = 4$

   (b) $y'' + 3y' + 2y = \begin{cases} 4t, & : 0 < t < 1 \\ 8, & : t \geq 1 \end{cases}$ $\quad y(0) = 0,\ y'(0) = 0$

   (c) $y'' + y' - 2y = \begin{cases} 3\sin t - \cos t, & : 0 < t < 2\pi \\ 3\sin 2t - \cos 2t, & : t \geq 2\pi \end{cases}$ $\quad y(0) = 1,\ y'(0) = 0$

   (d) $y'' + 3y' + 2y = \begin{cases} 1, & : 0 < t < 1 \\ 0, & : t \geq \pi \end{cases}$ $\quad y(0) = 0,\ y'(0) = 0$

   (e) $y'' + 2y' + 5y = \begin{cases} 10\sin t, & : 0 < t < 2\pi \\ 0, & : t \geq 2\pi \end{cases}$ $\quad y(\pi) = 1,\ y'(\pi) = 2e^{-\pi} - 2$

   (f) $y'' + 4y = \begin{cases} 8t^2, & : 0 < t < 5 \\ 0, & : t \geq 5 \end{cases}$ $\quad y(1) = 1 + \cos 2,\ y'(1) = 4 - 2\sin 2$

8. Find

   (a) $1 * \sin \omega t$ (b) $e^t * e^{-t}$ (c) $(\cos \omega t) * (\cos \omega t)$
   (d) $(\sin \omega t) * (\cos \omega t)$ (e) $t * e^t$

   ω is a constant.

9. Solve the following integral equations using Laplace transforms.

   (a) $y(t) + 4\int_0^t y(\tau)\ (t-\tau)\mathrm{d}\tau = 2t$ (b) $y(t) - \int_0^t y(\tau)\mathrm{d}\tau = 1$
   (c) $y(t) - \int_0^t y(\tau)\ \sin 2(t-\tau)\mathrm{d}\tau = \sin 2t$ (d) $y(t) + \int_0^t y(\tau)\ \cosh(t-\tau)\mathrm{d}\tau = t + e^t$
   (e) $y(t) + 2e^t \int_0^t y(\tau)\ e^{-\tau}\mathrm{d}\tau = te^t$ (f) $y(t) - \int_0^t y(\tau)\ (t-\tau)\mathrm{d}\tau = 2 - \frac{1}{2}t^2$

10. Find the inverse Laplace transform of the following functions. ω is a constant.

    (a) $\frac{2\pi s}{(s^2+\pi^2)^2}$ (b) $\frac{\omega}{s^2(s^2+\omega^2)}$ (c) $\frac{18s}{(s^2+36)^2}$