UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

DEPARTMENT OF MATHEMATICS

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COURSE : MAT 3110

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ASSIGNMENT 1.

Question 1.

$\Rightarrow (1-x^2)y'' - 2xy' + 2y = 0$

$\Rightarrow (1-x^2)\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - 2x\sum_{n=1}^{\infty} n a_n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0.$

$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=2}^{\infty} n(n-1)a_n x^n - \sum_{n=1}^{\infty} 2n a_n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$

$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n - \sum_{n=2}^{\infty} n(n-1)a_n x^n - \sum_{n=1}^{\infty} 2n a_n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$

$\Rightarrow 2a_2 + 6a_3 x + \sum_{n=2}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=2}^{\infty} n(n-1)a_n x^n - 2a_1 x - \sum_{n=2}^{\infty} 2n a_n x^n + 2a_0 + 2a_1 x +$
$\sum_{n=2}^{\infty} 2a_n x^n = 0$

$\Rightarrow 2a_2 + 6a_3 x - 2a_1 x + 2a_0 + 2a_1 x + \sum_{n=2}^{\infty}\left[(n+2)(n+1)a_{n+2} - n(n-1)a_n - 2n a_n + 2a_n\right]x^n = 0$

$2a_2 + 2a_0 = 0$

$a_2 = -a_0.$

$6a_3 x - 2a_1 x + 2a_1 x = 0$

$6a_3 = 0$

$a_3 = 0$

$(n+2)(n+1)a_{n+2} - n(n-1)a_n - 2n a_n + 2a_n = 0$

$(n+2)(n+1)a_{n+2} = (n^2-n)a_n + 2n a_n - 2a_n$

$(n+2)(n+1)\ a_{n+2} = (n^2 - n + 2n - 2)a_n$

$a_{n+2} = \dfrac{(n^2+n-2)a_n}{(n+2)(n+1)}$

$a_{n+2} = \dfrac{(n+2)(n-1)a_n}{(n+2)(n+1)}$

$a_{n+2} = \dfrac{(n-1)}{(n+1)}a_n$

$n = 2, \quad a_4 = \cancel{\frac{8}{4 \times 3}} \cancel{a_2} \ \frac{1}{3} a_0 = -\frac{a_0}{3}.$

$n = 3, \quad a_5 = \frac{2}{4} a_3 = 0$

$n = 4, \quad a_6 = \frac{3}{5} a_4 = -\frac{a_0}{5}$

$n = 5, \quad a_7 = \frac{4}{6} a_5 = 0$

$n = 6, \quad a_8 = \frac{5}{7} a_6 = -\frac{a_0}{7}.$

$\therefore \ y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$

$y = a_0 + 0 - a_0 x^2 + 0 - \frac{a_0}{3} x^4 + 0 - \frac{a_0}{5} x^6 + 0 - \frac{a_0}{7} x^8 + 0 -$

$= a_0 \left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7} - \cdots \right)$

Question 2.

$$y'' + (1+x^2)y = 0$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + (1+x^2)\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} a_n x^{n+2} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^n + \sum_{n=0}^{\infty} a_n x^n + \sum_{n=2}^{\infty} a_{n-2} x^n = 0.$$

$$\Rightarrow 2a_2 + 6a_3 x + \sum_{n=2}^{\infty} (n+2)(n+1)a_{n+2} x^n + a_0 + a_1 x + \sum_{n=2}^{\infty} a_n x^n + \sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$$\Rightarrow 2a_2 + 6a_3 x + a_0 + a_1 x + \sum_{n=2}^{\infty}\left[(n+2)(n+1)a_{n+2} + a_n + a_{n-2}\right]x^n = 0$$

$$2a_2 + a_0 = 0$$

$$a_2 = -a_0/2 .$$

$$6a_3 x + a_1 x = 0$$

$$6a_3 = -a_1$$

$$a_3 = -a_1/6$$

$$(n+2)(n+1)a_{n+2} + a_n + a_{n-2} = 0.$$

$$a_{n+2} = \frac{-a_n + a_{n-2}}{(n+2)(n+1)}$$

$n=2$ $$a_4 = \frac{-a_2 - a_0}{4\times 3} = \frac{\frac{a_0}{2} - a_0}{12} = -\frac{a_0}{24}$$

$n=3$ $$a_5 = \frac{-a_3 - a_1}{5\times 4} = \frac{\frac{a_1}{6} - a_1}{5\times 4} = \frac{-5a_1}{6\times 5\times 4} = -a_1/24$$

$n=4$ $$a_6 = \frac{-a_4 - a_2}{6\times 5} = \frac{\frac{a_0}{24} + \frac{a_0}{2}}{6\times 5} = \frac{13a_0}{24\times 6\times 5} = \frac{13a_0}{720}$$

$n=5$ $$a_7 = \frac{-a_5 - a_3}{7\times 6} = \frac{a_1/24 + a_1/6}{7\times 6} = \frac{5a_1}{24\times 7\times 6} = \frac{5a_1}{1008}$$

$\Rightarrow y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$

$\Rightarrow y = a_0 + a_1 x - \frac{a_0 x^2}{2} - \frac{a_1 x^3}{6} - \frac{a_0 x^4}{24} - \frac{5a_1 x^5}{120} + \cdots$

$\Rightarrow y = a_0 \left(1 - \frac{x^2}{2} - \frac{x^4}{24} - \cdots\right) + a_1 \left(x - \frac{x^3}{6} - \frac{x^5}{24} - \cdots\right)$

Question 3.

$$y'' - 4xy' + (4x^2-2)y = 0$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - 4x\sum_{n=1}^{\infty} na_n x^{n-1} + (4x^2-2)\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\Rightarrow \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=1}^{\infty} 4na_n x^n + \sum_{n=0}^{\infty} 4a_n x^{n+2} - \sum_{n=0}^{\infty} 2a_n x^n = 0.$$

$$\Rightarrow \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n - \sum_{n=1}^{\infty} 4na_n x^n + \sum_{n=2}^{\infty} 4a_{n-2} x^n - \sum_{n=0}^{\infty} 2a_n x^n = 0.$$

$$\Rightarrow 2a_2 + 6a_3 x + \sum_{n=2}^{\infty}(n+2)(n+1)a_{n+2}x^n - 4a_1 x - \sum_{n=2}^{\infty} 4na_n x^n + \sum_{n=2}^{\infty} 4a_{n-2}x^n - 2a_0 - 2a_1 x - \sum_{n=2}^{\infty} 2a_n x^n = 0$$

$$\Rightarrow 2a_2 - 2a_0 + 6a_3 x - 4a_1 x - 2a_1 x + \sum_{n=2}^{\infty}\left[(n+2)(n+1)a_{n+2} - 4na_n + 4a_{n-2} - 2a_n\right]x^n = 0$$

$$\Rightarrow 2a_2 - 2a_0 = 0$$

$$a_2 = a_0$$

$$6a_3 - 4a_1 - 2a_1 = 0$$

$$a_3 = a_1.$$

$$(n+2)(n+1)a_{n+2} - 4na_n + 4a_{n-2} - 2a_n = 0.$$

$$a_{n+2} = \frac{(4n+2)a_n - 4a_{n-2}}{(n+2)(n+1)}$$

$n=2$, $\quad a_4 = \dfrac{10a_2 - 4a_0}{4\times 3} = \dfrac{6a_0}{12} = \dfrac{a_0}{2}.$

$n=3$, $\quad a_5 = \dfrac{14a_3 - 4a_1}{5\times 4} = \dfrac{10a_1}{20} = a_1/2$

$n=4$ $\quad a_6 = \dfrac{18a_4 - 4a_2}{6\times 5} = \dfrac{18(a_0/2) - 4a_0}{30} = \dfrac{a_0}{6}$

$n=5$ $\quad a_7 = \dfrac{22a_5 - 4a_3}{7\times 6} = \dfrac{22(a_1/2) - 4a_1}{42} = a_1/6$

$$\Rightarrow y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

$$y = a_0 + a_1 x + a_0 x^2 + a_1 x^3 + a_0/2\, x^4 + a_1/2\, x^5 + a_0/6\, x^6 + a_1/6\, x^7 + \cdots$$

$$y = a_0\left(1 + x^2 + x^4/2 + x^6/6 + \cdots\right) + a_1\left(x + x^3 + x^5/2 + x^7/6 + \cdots\right)$$