UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

DEPARTMENT OF MATHEMATICS

Name	:	SUWILANJI NSANGWE
Computer #	:	2018236865
Course	:	MAT 3110
Lecturer	:	Mr. Trevor Chilombo
Due Date	:	April 02, 2021

QUESTION 1.

$$x^2y'' + 3xy' + y = 0.$$

$$y = x^r$$

$$y' = rx^{r-1}$$

$$y'' = r(r-1)x^{r-2}.$$

$$\Rightarrow x^2\left[r(r-1)x^{r-2}\right] + 3x\left[rx^{r-1}\right] + x^r = 0$$

$$\Rightarrow r(r-1)x^r + 3rx^r + x^r = 0$$

$$\Rightarrow \left[r(r-1) + 3r + 1\right]x^r = 0$$

$$\Rightarrow r(r-1) + 3r + 1 = 0$$

$$\Rightarrow r^2 - r + 3r + 1 = 0$$

$$\Rightarrow r^2 + 2r + 1 = 0$$

$$\Rightarrow r^2 + r + r + 1 = 0$$

$$\Rightarrow r(r+1) + 1(r+1) = 0$$

$$\Rightarrow (r+1)(r+1) = 0$$

$$\Rightarrow (r+1)^2 = 0$$

$$\Rightarrow r = -1.$$

$$\Rightarrow y(x) = C_1x^r + C_2x^r\ln x$$

$$y(x) = C_1x^{-1} + C_2x^{-1}\ln x.$$

QUESTION 2.

$\Rightarrow x^2y'' + 4y = 0$

$y = x^r$

$y'' = r(r-1)x^{r-2}$

$\Rightarrow x^2[r(r-1)x^{r-2}] + 4(x^r) = 0$

$\Rightarrow r(r-1)x^r + 4x^r = 0$

$\Rightarrow [r(r-1) + 4]x^r = 0$

$\Rightarrow r(r-1) + 4 = 0$

$\Rightarrow r^2 - r + 4 = 0$

$\Rightarrow (r - 1/2)^2 - 1/2^2 + 4 = 0$

$\Rightarrow (r - 1/2)^2 - 1/4 + 4 = 0$

$\Rightarrow (r - 1/2)^2 + 3\frac{3}{4} = 0$

$\Rightarrow (r - 1/2)^2 = -15/4$

$\Rightarrow \sqrt{(r - 1/2)^2} = \sqrt{-15/4}$

$\Rightarrow r - 1/2 = \sqrt{-15/4}$

$\Rightarrow r = 1/2 + \sqrt{15/4}\, i$

$\Rightarrow r = 1/2 + 1/2\sqrt{15}\, i$

$r = \lambda + \mu i$

$\therefore \lambda = 1/2, \quad \mu = 1/2\sqrt{15}$

$\therefore y(x) = C_1 x^{\lambda} \cos(\mu \ln x) + C_2 x^{\lambda} \sin(\mu \ln x)$

$= C_1 x^{1/2} \cos(1/2\sqrt{15} \ln x) + C_2 x^{1/2} \sin(1/2\sqrt{15} \ln x)$ ans.

QUESTION 3.

$$x^2y'' + 5xy' - 21y = 0$$

$$y = x^r$$

$$y' = rx^{r-1}$$

$$y'' = r(r-1)x^{r-2}$$

$$\Rightarrow x^2\left[r(r-1)x^{r-2}\right] + 5x\left[rx^{r-1}\right] - 21x^r = 0$$

$$\Rightarrow r(r-1)x^r + 5rx^r - 21x^r = 0$$

$$\Rightarrow \left[r(r-1) + 5r - 21\right]x^r = 0$$

$$\Rightarrow r(r-1) + 5r - 21 = 0$$

$$\Rightarrow r^2 - r + 5r - 21 = 0$$

$$\Rightarrow r^2 + 4r - 21 = 0$$

$$\Rightarrow r^2 - 3r + 7r - 21 = 0$$

$$\Rightarrow r(r-3) + 7(r-3) = 0$$

$$\Rightarrow (r-3)(r+7) = 0$$

$$\Rightarrow r_1 = 3, \quad r_2 = -7$$

$$\therefore y(x) = C_1x^{r_1} + C_2x^{r_2}$$

$$y(x) = C_1x^3 + C_2x^{-7}$$