UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

DEPARTMENT OF ELECTRICAL ENGINEERING

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Course	:	MAT 3110
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Assignment 3.

Question 1.

$$\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$$

$A - rI = 0.$

$\det(A - rI) = 0$

$$\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix} - \begin{pmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{pmatrix} =$$

$$\begin{vmatrix} 3-r & 1 & -1 \\ 2 & 2-r & -1 \\ 2 & 2 & -r \end{vmatrix}.$$

$= (3-r)\left[(2-r)(-r)+2\right] - 1\left[2(-r)+2\right] + (-1)\left[2(2) - 2(2-r)\right] = 0.$

$= (3-r)\left[-2r+r^2+2\right] - 1\left[-2r+2\right] - 1\left[4-4+2r\right] = 0$

$= -6r + 3r^2 + 6 + 2r^2 - r^3 - 2r + 2r - 2 - 2r = 0$

$= -r^3 + 5r^2 - 8r + 4 = 0.$

$= r^3 - 5r^2 + 8r - 4 = 0$

$(r-1)(r^2 + tr + 4).$

$-r^2 + tr^2 = -5r^2$

$-1 + t = -5$

$t = -4$

$\Rightarrow (r-1)(r^2 - 4r + 4)$

$(r-1)(r^2 - 2r - 2r + 4)$

$(r-1)\, r(r-2) - 2(r-2)$

$(r-1)(r-2)^2.$

$= r = 1$

$r = 1$

$\Rightarrow (A - rI)x = 0.$

$$\begin{pmatrix} 2 & 1 & -1 \\ 2 & 1 & -1 \\ 2 & 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$2x_1 + x_2 - x_3 = 0$

$2x_1 + x_2 - x_3 = 0.$ $\qquad x_3 = 2x_1 + x_2$

$2x_1 + 2x_2 - x_3 = 0$

$x_1 = -\frac{1}{2}x_2$

$2x_1 + x_2 + x_3 = 0$

$+\ 2x_1 + x_2 - x_3 = 0$

$4x_1 + 2x_2 = 0$

$2x_1 = -x_2.$

$x_1 = -\frac{1}{2}x_2.$

$$\begin{pmatrix} -\frac{1}{2}x_2 \\ x_2 \\ 2x_1 + x_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}x_2 \\ x_2 \\ 2(-\frac{1}{2}x_2) + x_1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}x_2 \\ x_2 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix}.$$

$x_2 = 1$

$2x_1 + x_2 - x_3 = 0$

$-\ 2x_1 + 2x_2 - x_3 = 0$

$2x_1 + x_2 + x_3 = 0$

$+\ 2x_1 + 2x_2 - x_3 = 0$

$4x_1 + 3x_2 = 0$

$2x_1 + x_2 - x_3 = 0$

$-\ 2x_1 + x_2 - x_3 = 0$

$$\begin{pmatrix} x_1 \\ x_2 \\ 2x_1 + x_2 \end{pmatrix}. \qquad x_2 = 0, \quad x_1 = 1$$

$2x_1 + 2x_2 - x_3 = 0$

$-\ 2x_1 + x_2 - x_3 = 0$

$x_2 = 0.$

$x_3 = 2x_1 + x_2.$

$$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$$

$\Rightarrow r = 1 \qquad \vec{x}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}.$

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$r = 2$.

$(A - rI)x = 0$

$$\begin{pmatrix} 3-2 & 1 & -1 \\ 2 & 2-2 & -1 \\ 2 & 2 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & -1 \\ 2 & 2 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$x_1 + x_2 - x_3 = 0$;

$2x_1 + 0 - x_3 = 0$.

$2x_1 + 2x_1 - 2x_3 = 0$.

$\Rightarrow$ $x_3 = 2x_1$.

$x_2 = x_3 - x_1$

$= 2x_1 - x_1$

$x_2 = x_1$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_1 \\ 2x_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad x_1 = 1$$

$$\therefore r = 2 \quad x^{(2)} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

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(a) $x' = x - 3y$.

$y' = -2x + 2y$.

$$\Rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

$$\det(A - rI) = \begin{vmatrix} 1-r & -3 \\ -2 & 2-r \end{vmatrix}$$

$(1-r)(2-r) - 6 = 0$

$r^2 - 2r - r + 2 - 6$

$-4r-4$ $\quad r^2 - 3r - 4 = 0$.

$-4(r+1)$ $\quad (r+1)(r-4) = 0$

$r = -1$ and $r = 4$.

$\Rightarrow$ $r = -1$.

$(A - rI)\eta = 0$.

$$\begin{pmatrix} 2 & -3 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$\Rightarrow$ $2\eta_1 - 3\eta_2 = 0$.

$\eta_1 = \frac{3}{2}\eta_2$.

$$\begin{pmatrix} \frac{3}{2}\eta_2 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \eta_2 = 2.$$

$r = 4$.

$(A - rI)\eta = 0$.

$$\begin{pmatrix} -3 & -3 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} \eta_3 \\ \eta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$-2\eta_3 - 2\eta_4 = 0$.

$\eta_3 = -\eta_4$.

$$\begin{pmatrix} -\eta_4 \\ \eta_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad \eta_4 = 1.$$

$$\Rightarrow \underline{x}(t) = C_1 e^{-t} \begin{pmatrix} 3 \\ 2 \end{pmatrix} + C_2 e^{4t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

(a) $x' = x - 2y$.

$y' = -2x + 2y$

$y = \frac{1}{2}(x - x')$ $\quad x_1(t) = y \quad x_1'(t) = y' = x_2$.

$y = \frac{x}{2} - \frac{x'}{2}$, $\quad x_2(t) = y' \quad x_2'(t) = y'' =$

$y' = -2x + 2y$

$x_2 = -2$

(b) $x' = \begin{pmatrix} 2 & 5 \\ -\frac{1}{2} & -\frac{3}{2} \end{pmatrix} x$.

$\det(A - rI) = \begin{vmatrix} 2-r & 5 \\ -\frac{1}{2} & -\frac{3}{2} - r \end{vmatrix}$

$= (2-r)(-\frac{3}{2} - r) + \frac{5}{2} = 0$

$= -3 - 2r + \frac{3}{2}r + r^2 + \frac{5}{2} = 0$

$= r^2 - \frac{1}{2}r - \frac{1}{2} = 0$

$= (r - \frac{1}{4})^2 - (\frac{1}{4})^2 - \frac{1}{2} = 0$

$= (r - \frac{1}{4})^2 - \frac{1}{16} - \frac{1}{2} = 0$

$(r - \frac{1}{4})^2 = \frac{9}{16}$

$r - \frac{1}{4} = \pm \frac{3}{4}$.

$r = \frac{1}{4} \pm \frac{3}{4}$.

$r = 1$ or $-\frac{1}{2}$.

$\therefore \underline{x}(t) = C_1 e^{t} \begin{pmatrix} -5 \\ 1 \end{pmatrix} + C_2 e^{-\frac{1}{2}t} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

$r = 1$

$(A - rI)\eta = 0$.

$\begin{pmatrix} 1 & 5 \\ -\frac{1}{2} & -\frac{5}{2} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\eta_1 + 5\eta_2 = 0$.

$\eta_1 = -5\eta_2$

$\begin{pmatrix} 1 & 5 \\ -\frac{1}{2} & -\frac{5}{2} \end{pmatrix} \begin{pmatrix} -5\eta_2 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\eta_2 = 1 = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$

$r = -\frac{1}{2}$

$(A - rI)\eta = 0$

$\begin{pmatrix} \frac{5}{2} & 5 \\ -\frac{1}{2} & -1 \end{pmatrix} \begin{pmatrix} \eta_3 \\ \eta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\frac{5}{2}\eta_3 + 5\eta_4 = 0$

$\frac{5}{2}\eta_3 = -5\eta_4$.

$\eta_3 = -2\eta_4$.

$\begin{pmatrix} \frac{5}{2} & 5 \\ -\frac{1}{2} & -1 \end{pmatrix} \begin{pmatrix} -2\eta_4 \\ \eta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\eta_4 = 1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

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(c) $x' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x$, $x(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

$\det(A - rI) = 0$

$\begin{vmatrix} -r & 1 \\ 1 & -r \end{vmatrix}$

$r^2 - 1 = 0$.

$r^2 = 1$.

$r = \pm 1$.

$\Rightarrow$ For $r = 1$.

$(A - rI)\eta = 0$.

$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

$-\eta_1 + \eta_2 = 0$.

$\eta_1 = \eta_2$.

$\begin{pmatrix} \eta_2 \\ \eta_2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\eta_2 = 1$.

$r = -1$.

$(A - rI)\eta = 0$

$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \eta_3 \\ \eta_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

$\eta_3 + \eta_4 = 0$.

$\eta_3 = -\eta_4$.

$\begin{pmatrix} -\eta_4 \\ \eta_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $\eta_4 = 1$.

$x(t) = c_1 e^{t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

$x(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

$0 = c_1 - c_2 \Rightarrow c_1 = c_2$.

$2 = c_1 + c_2$

$2 = 2c_2$

$c_2 = 1$ and $c_1 = 1$

$\therefore x(t) = e^{t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + e^{-t} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ ans.