

The University of Zambia
 Department of Mathematics and Statistics
 Mat 3110-Engineering Mathematics II

Tutorial Sheet 2 - Laplace transforms

March, 2019.

Submit question 7 for assessment: Submit on 15th March, 2019 during class from 11-13hrs.

1. Find the Laplace transforms of the following functions. Assume that a, b, ω and θ are constants.

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|------------------------------------|----------------------|-------------------------------|
| (a) $(a - bt)^2$ | (b) $3t + 12$ | (c) $\cos^2 \omega t$ |
| (d) $e^{2t} \sinh t$ | (e) $e^{-t} \sin 4t$ | (f) $\sin(\omega t + \theta)$ |
| (g) $1.5 \sin(3t - \frac{\pi}{2})$ | (h) $t^2 e^{-3t}$ | (i) $ke^{-at} \cos \omega t$ |
| (j) $\sinh t \cos t$ | | |

2. Find the inverse Laplace transforms of the following functions. Assume that a, b, n and L are constants.

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|---------------------------------------|---------------------------------------|--|
| (a) $\frac{5s+1}{s^2-25}$ | (b) $\frac{s^2}{L^2 s^2 + n^2 \pi^2}$ | (c) $\frac{12}{s^4} - \frac{228}{s^6}$ |
| (d) $\frac{4s+32}{s^2-16}$ | (e) $\frac{s+10}{s^2-s-2}$ | (f) $\frac{1}{(s+a)(s+b)}$ |
| (g) $\frac{\pi}{(s+\pi)^2}$ | (h) $\frac{6}{(s+1)^3}$ | (i) $\frac{4}{s^2-2s-3}$ |
| (j) $\frac{\pi}{s^2+10\pi s+24\pi^2}$ | (k) $\frac{2s-1}{s^2-6s+18}$ | |

3. Solve the following initial value problems by using Laplace transforms.

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| (a) $y' + 2y = 0, \quad y(0) = \frac{3}{2}$ | (b) $y'' - y' - 6y = 0, \quad y(0) = 11, \quad y'(0) = 28$ |
| (c) $y'' + 9y = 10e^{-t}, \quad y(0) = 0, \quad y'(0) = 0$ | (d) $y'' - 4y' + 3y = 6t - 8, \quad y(0) = 0, \quad y'(0) = 0$ |

4. Solve the following shifted data initial value problems by using Laplace transforms.

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| (a) $y' - 6y = 0, \quad y(-1) = 4.$ | (b) $y'' - 2y' - 3y = 0, \quad y(4) = -3, \quad y'(4) = -17$ |
| (c) $y'' + 2y' + 5y = 10t - 100, \quad y(2) = -4, \quad y'(2) = 14$ | |

5. Sketch or graph the given function, which is assumed to be zero outside the given interval. Represent it, using unit step functions. Find its Laplace transform.

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|---|---------------------------------------|--|
| (a) $t \quad (0 < t < 2)$ | (b) $t - 2 \quad (t < 2)$ | (c) $\cos 4t \quad (0 < t < \pi)$ |
| (d) $e^t \quad (0 < t < \frac{\pi}{2})$ | (e) $\sin \pi t \quad (2 < t < 4)$ | (f) $e^{-\pi t} \quad (2 < t < 4)$ |
| (g) $t^2 \quad (1 < t < 2)$ | (h) $t^2 \quad (0 < t < \frac{3}{2})$ | (i) $\sin t \quad (\frac{\pi}{2} < t < \pi)$ |

6. Find and sketch the inverse Laplace transforms of the following functions.

$$\begin{array}{lll}
 \text{(a)} \frac{e^{-3s}}{(s-1)^3} & \text{(b)} \frac{6(1-e^{-})}{(s^2+9)} & \text{(c)} \frac{4(e^{-2s}-2e^{-5s})}{s} \\
 \text{(d)} \frac{e^{-3s}}{s^4} & \text{(e)} \frac{2(e^{-s}-e^{-3s})}{(s^2-4)} & \text{(f)} \frac{(1+e^{-2\pi(s+1)})(s+1)}{((s+1)^2+1)}
 \end{array}$$

7. Solve the following initial value problems with a discontinuous force.

$$\text{(a)} \quad y'' + 9y = \begin{cases} 8 \sin t, & : 0 < t < \pi \\ 0, & : t \geq \pi \end{cases} \quad y(0) = 0, \quad y'(0) = 4$$

$$\text{(b)} \quad y'' + 3y' + 2y = \begin{cases} 4t, & : 0 < t < 1 \\ 8, & : t \geq 1 \end{cases} \quad y(0) = 0, \quad y'(0) = 0$$

$$\text{(c)} \quad y'' + y' - 2y = \begin{cases} 3 \sin t - \cos t, & : 0 < t < 2\pi \\ 3 \sin 2t - \cos 2t, & : t \geq 2\pi \end{cases} \quad y(0) = 1, \quad y'(0) = 0$$

$$\text{(d)} \quad y'' + 3y' + 2y = \begin{cases} 1, & : 0 < t < 1 \\ 0, & : t \geq \pi \end{cases} \quad y(0) = 0, \quad y'(0) = 0$$

$$\text{(e)} \quad y'' + 2y' + 5y = \begin{cases} 10 \sin t, & : 0 < t < 2\pi \\ 0, & : t \geq 2\pi \end{cases} \quad y(\pi) = 1, \quad y'(\pi) = 2e^{-\pi} - 2$$

$$\text{(f)} \quad y'' + 4y = \begin{cases} 8t^2, & : 0 < t < 5 \\ 0, & : t \geq 5 \end{cases} \quad y(1) = 1 + \cos 2, \quad y'(1) = 4 - 2 \sin 2$$

8. Find

$$\begin{array}{lll}
 \text{(a)} 1 * \sin \omega t & \text{(b)} e^t * e^{-t} & \text{(c)} (\cos \omega t) * (\cos \omega t) \\
 \text{(d)} (\sin \omega t) * (\cos \omega t) & \text{(e)} t * e^t &
 \end{array}$$

ω is a constant.

9. Solve the following integral equations using Laplace transforms.

$$\begin{array}{ll}
 \text{(a)} y(t) + 4 \int_0^t y(\tau) (t - \tau) d\tau = 2t & \text{(b)} y(t) - \int_0^t y(\tau) d\tau = 1 \\
 \text{(c)} y(t) - \int_0^t y(\tau) \sin 2(t - \tau) d\tau = \sin 2t & \text{(d)} y(t) + \int_0^t y(\tau) \cosh(t - \tau) d\tau = t + e^t \\
 \text{(e)} y(t) + 2e^t \int_0^t y(\tau) e^{-\tau} d\tau = te^t & \text{(f)} y(t) - \int_0^t y(\tau) (t - \tau) d\tau = 2 - \frac{1}{2}t^2
 \end{array}$$

10. Find the inverse Laplace transform of the following functions. ω is a constant.

$$\begin{array}{lll}
 \text{(a)} \frac{2\pi s}{(s^2+\pi^2)^2} & \text{(b)} \frac{\omega}{s^2(s^2+\omega^2)} & \text{(c)} \frac{18s}{(s^2+36)^2}
 \end{array}$$