The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

Tutorial Sheet 8 **October, 2018.**

Submit question 6 and 16 for assessment: Submit on Monday, November 5th, 2018 during class from 11-13hrs.

1. In how many ways can the letters of the word FACETIOUS be arranged in a line? What is the probability that an arrangement begins with F and end with S?

2. The letters of the word PROBABILITY are arranged at random. Find the probability that the two I's are separated.

3. A competition has first prize, a second prize, a third prize and a fourth prize. Ten competitors enter this competition and the prizes are awarded for the first, second, third and fourth competitors in order of merit.

   (a) Find the number of different ways in which these prizes could be won.

   Smith and Jones are two of the ten competitors. Find the number of different ways in which the prizes could be won if

   (b) neither Smith nor Jones wins a prize, (c) each of Smith and Jones wins a prize.

4. A coin is biased so that, on each toss, the probability of obtaining a head is 0.4. The coin is tossed twice.

   (a) Calculate the probability that at least one head is obtained.

   (b) Calculated the conditional probability that exactly one head is obtained, given that at least one head is obtained.

5. I travel to work by route A or route B. The probability that I choose route A is $\frac{1}{2}$. The probability that I am late for work if I go via route A is $\frac{2}{3}$ and the corresponding probability if I go via route B is $\frac{1}{3}$.

   (a) On a given day, what is the probability that I am late for work?

   (b) Given that I am late for work, what is the probability that I went via route B?

6. During an epidemic of a certain disease a doctor is consulted by 110 people suffering from symptoms commonly associated with the disease. Of the 110 people, 45 are female of whom 20 actually have the disease and 25 do not. Fifteen males have the disease and the rest do not.

   (a) A person is selected at random, The event that this person is female is denoted by A and the event that this person is suffering from the disease is denoted by B. Evaluate

   (i) $P(A)$ (ii) $P(AUB)$, (iii)$P(A \cap B)$, (iv)$P(A \mid B)$.

   (b) If three different people are selected at random without replacement, what is the probability of

   (i) all three having the disease, (ii) exactly one of the three having the disease,

   (iii) one of the three being a female with the disease, one a male with the disease and one a female without the disease?

   (c) Of people with the disease 96% react positively to a test for diagnosing the disease as do 8% of people without the disease. What is the probability of a person selected at random

   (i) reacting positively, (ii) having the disease given that he or she reacted positively?

7. In an experiment two bags A and B, containing red and green marbles are used. Bag A contains four red marbles and one green marble and bag B contains two red marbles and seven green marbles. An unbiased coin is tossed. If a head turns up, a marble is drawn at random from bag A while if a tail turns up, a marble is drawn at random from bag B. Calculate the probability that a red marble is drawn in a single trial. Given that a red marble is selected, calculate the probability that when the coin was tossed a head was obtained.

8. The random variable X has probability density function given by

$$f(x) = \begin{cases} kx(1-x^2), & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

   k being a constant. Find the value of k and find also the meand and variance of this distribution.

9. The probabilities of events A and B are $P(A)$ and $P(B)$ respectively.

   $P(A) = \frac{5}{12}, \quad P(A \cap B) = \frac{1}{6}, P(A \cup B) = q.$

   Find, in terms of q,

   (a) $P(B)$,

   (b) $P(A \mid B)$.

   Given that A and B are independent events,

   (c) find the value of q.

10. The number of customers entering a certain branch of a bank on a Monday lunchtime may be modelled by a Poisson distribution with mean 2.4 per minute.

    (a) Find the probability that, during a particular minute, four or more customers enter the branch.

    The probability that a customer, who enters the branch, intends to open a new account is 0.002 and is independent of the intentions of other customers. During a particular morning 450 customers enter the bank.

    (b) Use a suitable approximation to find the probability that three or fewer of these 450 customers intend to open new accounts

11. , In practising the high jump a certain athlete has five attempts at a particular height. The probability that she succeeds at any one attempt is p. Find an expression, in terms of p, for the probability that she succeeds

    (a) exactly four times,

    (b) exactly two times.

    The probability that she succeeds exactly four times is twice the probability that she succeeds exactly two times. Find the value of p.

12. Before starting to play the game Snakes and Ladders each player throws an ordinary unbiased die until a six is obtained. The number of throws before a player starts is the random variable Y, where Y takes the values 1, 2, 3,. . .

    (a) Name the probability distribution of Y, stating a necessary assumption.

    (b) Find Var(Y).

13. Gift invites 11 friends to a party. For each friend, the probability that he or she will accept the invitation may be taken to be $\frac{2}{3}$. Use a binomial distribution to calculate the probability that (a) exactly nine, (b) fewer than nine, of the friends will accept the invitation.

14. A random variable X has the distribution $B(12, p)$.

    (a) Given that $p = 0.25$ find

    (i) $P(X < 5)$

    (ii) $P(X \geq 7)$

    (b) Given that $P(X = 0) = 0.05$, find the value of p to 3 decimal places.

    (c) Given that the variance of X is 1.92, find the possible values of p.

15. A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random;

    (a) Find the probability that the box contains exactly one defective component.

    (b) Find the probability that there are at least 2 defective components in the box.

    (c) Using a suitable approximation, find the probability that a batch of 250 contains between 1 and 4 defective components.

16. Lemons are packed in boxes, each box containing 200. It is found that, on average, 0.45% of the lemons are bad when the boxes are opened. Use the Poisson distribution to find the probabilities of 0, 1, 2, and more than two bad lemons in a box.

    A buyer who is considering buying a consignment of several hundred boxes checks the quality of the consignment by having a box opened. If the box opened contains no bad lemons he buys the consignment. If it contains more than two bad lemons he refuses to buy, and if it contains one or two bad lemons he has another box opened and buys the consignment if the second box contains fewer than two bad lemons. What is the probability that he buys the consignment?

    Another buyer checks consignments on a different basis. He has one box opened; if that box contains more than one bad lemon he asks for another to be opened and does not buy if the second also contains more than one bad lemon. What is the probability that he refuses to buy the consignment?