QUESTION 1

How many 11 letter words can you make from the letters of the word PROBABILITY ?

$$\frac{11!}{2!\,2!} = 9,979,200 \quad (3)$$

How many such 11 letter words are such the I's are together

$$\frac{10!}{2!} = 1,814,400 \quad (3)$$

Therefore, the words with the I's separated are

$$9,979,200 - 1,814,400 = 8,164,800. \quad (4)$$

QUESTION 2

$$P(\mu - 3\sigma \leq X \leq \mu + 3\sigma)$$

$$= P\left(\frac{\mu - 3\sigma - \mu}{\sigma} \leq \frac{X - \mu}{\sigma} \leq \frac{\mu + 3\sigma - \mu}{\sigma}\right)$$

$$= P(-3 \leq Z \leq 3)$$

$$= P(Z \leq 3) - P(Z \leq -3)$$

$$= P(Z \leq 3) - P(Z \geq 3)$$

$$= P(Z \leq 3) - [1 - P(Z \leq 3]$$

$$= 2P(Z \leq 3) - 1$$

$$= 2(0.99865) - 1$$

$$= 0.9973.$$

QUESTION 3

Let

R - be the event that a red marble is drawn

G - be the event that a red marble is drawn

H - be the event that heads turns up

$\overline{T}$ = be the event that tails turns up

Using total probability theorem

$$P(R) = P(R \cap H) + P(R \cap \overline{T})$$

$$= P(R/H)\,P(H) + P(R/\overline{T})\,P(\overline{T})$$

$$= \frac{4}{5} \times \frac{1}{2} + \frac{2}{9} \times \frac{1}{2}$$

$$= \frac{2}{5} + \frac{1}{9} \quad (5)$$

$$= \frac{18+5}{45}$$

$$= \frac{23}{45}$$

$$P(H/R) = \frac{P(R/H)\,P(H)}{P(R)}$$

$$= \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{23}{45}}$$

$$= \frac{\cancel{13}}{\cancel{10}} \times \frac{\cancel{45}}{23}$$

$$= \frac{2}{5} \times \frac{45}{23}$$

$$= \frac{90}{115}$$

$$= \frac{30}{37}$$

(5)

QUESTION 4

The is a Geometric distribution

$$P(Y = x) = \left(\frac{5}{6}\right)^{x-1} \frac{1}{6}$$

$$P(Y = 2) = \frac{5}{6} \times \frac{1}{6}$$

$$= \frac{5}{36}.$$

QUESTION 5

~~(a) $P(A \cup B) = P(A) + P(B) - P(A \cup B)$~~

~~$P(B) =$~~

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(B) = P(A \cup B) + P(A \cap B) - P(A)$

$P(B) = \frac{4}{6}q + \frac{1}{6} - \frac{5}{12}$

$P(B) = \frac{12q + 2 - 5}{12}$

$P(B) = \frac{12q - 3}{12} = q - \frac{1}{4}$ (3)

(b) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{q - \frac{1}{4}}$

$= \frac{1}{6q - \frac{3}{2}}$ (3)

(6) $P(A/B) = P(A)$

$$\frac{1}{6q - \frac{3}{2}} = \frac{5}{12}$$

$$30q - \frac{15}{2} = 12$$

$$30q = 12 + \frac{15}{2}$$

$$30q = \frac{39}{2}$$

$$q = \frac{39}{60}$$ (4)

QUESTION 6

$X \sim P(2.4)$

$\lambda = 2.4$

$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$ $\quad x = 0, 1,$

$P(X \geq 4) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3)$

$= 1 - \frac{(2.4)^0 e^{-2.4}}{0!} - \frac{(2.4) e^{-2.4}}{1!} - \frac{(2.4)^2 e^{-2.4}}{2!} - \frac{(2.4)^3 e^{-2.4}}{3!}$

$= 1 - 0.0907 - 0.2177 - 0.2613 - 0.2090$

$= 0.2213$

(6) $P(A/B) = P(A)$

$$\frac{1}{6q - \frac{3}{2}} = \frac{5}{12}$$

$$30q - \frac{15}{2} = 12$$

$$30q = 12 + \frac{15}{2}$$

$$30q = \frac{39}{2}$$

$$q = \frac{39}{60}$$

(4)

QUESTION 7

(a) This is a binomial distribution

$n = 5$ and $p = p$

$$P(X = x) = \binom{5}{x} p^x (1-p)^{5-x}$$

(a) $P(X = 4) = \binom{5}{4} p^4 (1-p) = 5p^4(1-p)$

(b) $P(X = 2) = \binom{5}{2} p^2 (1-p)^3 = 10p^2(1-p)^3$

$$5p^4(1-p) = 20p^2(1-p)^3$$

$$p^2 = 4(1-p)^2$$

$$p^2 = 4 - 8p + 4p^2$$

$$0 = 3p^2 - 8p + 4$$

$$0 = (3p-2)(p-2)$$

$p = 3/3$ or $p = 2$

$p = \frac{2}{3}$

QUESTION 8

$X \sim N(65, 16)$

$P(X > a) = 0.15$

$P(X < a) = 0.85$

$P\left(z < \frac{a-\mu}{\sigma}\right) = 0.85$

$\frac{a-65}{4} = 1.04$

$a = 69.16$

(5)

$P(X < b) = 0.2$

$P\left(z < \frac{b-\mu}{\sigma}\right) = 0.2$

$P\left(z > \frac{-b+\mu}{\sigma}\right) = 0.2$

$P\left(z < \frac{-b+\mu}{\sigma}\right) = 0.8$

$\frac{-b+65}{4} = 0.84$

$b = 61.64$

(5)

$\geq$ 69.16 high intelligence

between 61.64 and 69.16 Averagean intelligence

$\leq$ 61.64 low intelligence.