

The University of Zambia
Department of Mathematics and Statistics
Mat 3110 Test 1

Duration : 2 hours

May 4, 2018.

Instructions:

- This test contains 2 pages and 5 questions Total marks is 50.
- Answer all the questions.
- Any sort of malpractice will be severely punishment.

1. Solve the initial value problem below using Laplace transforms.

(10)

$$y'' + 2y' + 5y = 50t - 100, \quad y(2) = -4, \quad y'(2) = 14.$$

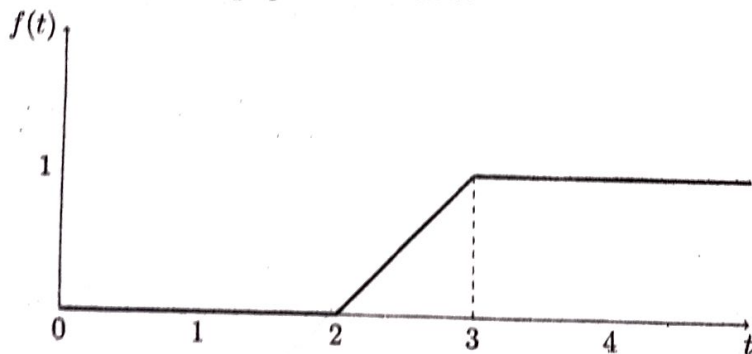
2. Determine the Laplace transforms of the functions below.

(a) $f(t) = t^2 \cos t$

(5)

(b) The function with graph shown below.

(5)



3. (a) Find the inverse Laplace transform of function

(5)

$$F(s) = \frac{3s - 2}{2s^2 - 6s + 3}$$

(b) Find and sketch the inverse Laplace transform of

$$F(s) = \frac{1}{e^{2s}s^2}$$

Handwritten work for part (a):

$$\frac{3s - 2}{2(s^2 - 3s + \frac{3}{2})} = \frac{\frac{3}{2}s - 1}{s^2 - 3s + \frac{3}{2}}$$

$$\left(\frac{s}{2}\right)^2 = \left(\sqrt{\frac{3}{2}}\right)^2$$

$$P: 6, \quad Q: -\frac{3}{2}$$

$$R: \frac{1}{2}, \quad S: -\frac{1}{2}$$

$$x = \frac{12}{50}$$

$$s^2 - 3s + \left(-\frac{3}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = 50$$

Handwritten work for part (b):

$$\frac{1-0}{3-2} = \frac{1}{1}$$

(5)

$$y = mx + c$$

$$y = x + c$$

$$0 = 2 + c$$

$$c = -2$$

$$y = mx$$

4. (a) Solve the following initial value problem (5)

$$\mathbf{x}' = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}.$$

- (b) Apply the power series method to find the solution of the first order linear differential equation (5)

$$(1+x)y' = y.$$

5. (a) Determine the constants α , β , y_0 and y'_0 so that $Y(s) = \frac{s}{(s+1)^2}$ is the Laplace transform of the solution to the initial value problem (5)

$$y'' + \alpha y' + \beta y = 0, \quad y(0) = y_0, \quad y'(0) = y'_0.$$

- (b) Find the general solution to the following differential equation. (5)

$$x^2 y'' - 7xy' + 16y = 0.$$

END OF TEST!