The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

Tutorial Sheet 6 **September, 2018.**

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1. Evaluate the following line integrals.

   (a) $\int_C x + 12xz \, \mathrm{d}s$ where C is given by

   $$r(t) = t\ i + \frac{t^2}{2}\ j + \frac{t^4}{4}\ k, \quad 0 \leq t \leq 1.$$

   (b) $\int_C y^2 - 10xy \, \mathrm{d}s$ where C is the left half of the circle centered at the origin of radius 6 with counter clockwise rotation.

   (c) $\int_C z^3 - 4x + 2y \, \mathrm{d}s$ where C s the line segment from $(2,4,1)$ to $(1,1,0)$.

2. Evaluate the following line integrals.

   (a) $\int_C F \cdot \mathrm{d}r$ where C is the portion of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ that is in the 1st, 4th and 3rd quadrants in the clockwise direction.

   $$F(x,y) = 2x^2\ i + (y^2 - 1)\ j.$$

   (b) $\int_C F \cdot \mathrm{d}r$ where C is the portion of $y = x^3 + 2$ from $x = -1$ to $x = 2$.

   $$F(x,y) = (x^3 - y)\ i + (x^2 + 7x)\ j.$$

   (c) $\int_C F \cdot \mathrm{d}r$ where C is the line segment from $(1,4,-2)$ to $(3,4,6)$.

   $$F(x,y,z) = (3x - 3y)\ i + (y^3 - 10)\ j + yz\ k.$$

3. Evaluate the following integrals

   (a) $\int_C \bigtriangledown f \cdot \mathrm{d}r$ where $f(x,y) = 4x + 3xy^2 - \ln(x^2 + y^2)$ and C is the upper half of $x^2 + y^2 + 1$ with clockwise rotation followed by the right half of $(x-1)^2 + \frac{(y-2)^2}{4} = 1$ with counter clockwise rotation.

   (b) $\int_C \bigtriangledown f \cdot \mathrm{d}r$ where $f(x,y,z) = zx^2 + x(y-2)^2$ and C is the line segment from $(1,2,0)$ to $(-3,10,9)$ followed by line segment from $(-3,10,9)$ to $(6,0,2)$.

   (c) $\int_C \bigtriangledown f \cdot \mathrm{d}r$ where $f(x,y) = 20y\cos(x+3) - yx^3$ and C is the right of $(x+31)^2 + \frac{(y-1)^2}{4} = 1$ with clockwise rotation.

   (d) $\int_C \bigtriangledown f \cdot \mathrm{d}r$ where $f(x,y,z) = \frac{3x-8y}{z-6}$ and C is given by

   $$r(t) = 6t\ i + 4\ j + (9 - t^3)\ k$$

   with $-1 \leq t \leq 3$.

(e) $\int_C \nabla f \cdot \mathrm{d}r$ where $f(x,y) = 5x - y^2 + 10xy + 9$ and C is given by

$$r(t) = \frac{2t}{t^2+1}\ i + (1-8t)j$$

with $-2 \le t \le 0$.

4. For the following vector fields, determine if the vector fields are conservative. If a vector field is conservative, find its potential function.
   (a) $F(x,y) = (6x - 5y^2 + 2xy^3 - 10)\ i + (3x^2y^2 - 10xy)\ j$
   (b) $F(x,y) = (8 - 14xy^2 + 2ye^{2x})\ i + (e^{2x} - 14x^2y)\ j$
   (c) $F(x,y) = -(3 - (1+2y)e^{x-1}\ i + (3y^2 + 2e^{x-1})\ j$
   (d) $F(x,y) = (y^2 - 4y + 5)\ i + (2xy - 4x - 9)\ j$

5. Evaluate the following line integrals.
   (a) $\int_C F \cdot \mathrm{d}r$ where $F(x,y) = (6x - 5y^2 + 2xy^3 - 10)\ i + (3x^2y^2 - 10xy)\ j$ and C is the upper half of $x^2 + y^2 = 1$ in the clockwise direction, followed by $y = \sin(\pi x)$ from $(1,0)$ to $(2,0)$, followed by the lower half of $(x-3)^2y^2 = 1$ in the counter clockwise direction.
   (b) $\int_C F \cdot \mathrm{d}r$ where $F(x,y) = (8 - 14xy^2 + 2ye^{2x})\ i + (e^{2x} - 14x^2y)\ j$ and C is a sequence of line segments: $(-1,2)$ to $(0,0)$ then $(0,0)$ to $(2,1)$ and then $(2,1)$ to $(4,-2)$.
   (c) $\int_C F \cdot \mathrm{d}r$ where $F(x,y) = -(3 - (1+2y)e^{x-1}\ i + (3y^2 + 2e^{x-1})\ j$ and C is the portion of $y = x^3 + 1$ from $x = -2$ to $x = 1$.
   (d) $\int_C F \cdot \mathrm{d}r$ where $F(x,y) = (y^2 - 4y + 5)\ i + (2xy - 4x - 9)\ j$ and C is the upper half of $\frac{x^2}{36} + \frac{y^2}{16} = 1$ with clockwise direction.

6. The following three-dimensional vector fields are conservative. Find their potential functions.
   (a) $F(x,y,z) = (2z^4 - 2y - y^3)\ i + (z - 2x - 3xy^2)\ j + (6 + y + 8xz^3)$
   (b) $F(x,y,z) = \left(\frac{2xy}{z^3}\right)\ i + \left(2y - z^2 + \frac{x^2}{z^3}\right)\ j - \left(4z^3 + 2yz + \frac{3x^2y}{z^4}\right)\ k$
   (c) $F(x,y,z) = 2xy^3z^4\ i + 3x^2y^2z^4\ j + 4x^2y^3z^3\ k$
   (d) $F(x,y,z) = \left(2x\cos y - 2z^3\right)\ i + \left(3 + 2ye^z - x^2\sin y\right)\ j + \left(y^2e^z - 6xz^2\right)\ k$

7. Evaluate the following line integrals.
   (a) $\int_C F \cdot \mathrm{d}r$ where C is a the following sequence of straight lines: $(0,0)$ to $(1,3)$, then $(1,3)$ to $(1,5)$ and lastly $(1,5)$ to $(0,0)$.

   $$F(x,y) = (6y - 3y^2 + x)\ i + yx^3\ j.$$

   (b) $\int_C F \cdot \mathrm{d}r$ where C is a the path consisting of the straight line from $(0,0)$ to $(0,-4)$, then along $x^2 + y^2 = 16$ from $(0,-4)$ to $(4,0)$ and lastly the straight line from $(4,0)$ to $(0,0)$.

   $$F(x,y) = (y^3 - xy^2)\ i + (2 - x^3)\ j.$$

   (c) $\int_C F \cdot \mathrm{d}r$ where C is the parallelogram with vertices $(1,1)$, $(-1,2)$, $(1,4)$ and $(-1,-1)$.

   $$F(x,y) = xy^2\ i + (1 - xy^3)\ j.$$