(c) $F(x,y,z) = z^2(y-x)\ \mathbf{i} - \frac{4y^2}{z^3}\ \mathbf{j} + (x^2 - 3z)\ \mathbf{k}$

7. Determine if the following vector fields are conservative.
   (a) $F(x,y,z) = (2xy^2 - 16x)\ \mathbf{i} - 2y(x^2 - 1)\ \mathbf{j} + 9\ \mathbf{k}$
   (b) $F(x,y,z) = (y - 3z)\ \mathbf{i} + (x^2 + y^4)\ \mathbf{j} + 4z^2\ \mathbf{k}$
   (c) $F(x,y,z) = (18x^2 + 4z^3)\ \mathbf{i} - 12yz\ \mathbf{j} - (6y^2 - 12xz^2)\ \mathbf{k}$

8. Use Stoke's theorem to evaluate the following integrals.
   (a) $\displaystyle\int\int_S \text{Curl } F \cdot dS$ where $F(x,y,z) = x^3\ \mathbf{i} + (4y - z^3y^3)\ \mathbf{j} + 2x\ \mathbf{k}$ and S is the portion of $z = x^2 + y^2 - 3$ below $z = 1$ with orientation in the negative z-axis direction.
   (b) $\displaystyle\int\int_S \text{Curl } F \cdot dS$ where $F(x,y,z) = 2y\ \mathbf{i} + 3x\ \mathbf{j} + (z - x)\ \mathbf{k}$ and S is the portion of $y = 11 - 3x^2 - 3z^2$ in front of $y = 5$ with orientation in the positive y-axis direction.
   (c) $\displaystyle\int\int_S F \cdot dr$ where $F(x,y,z) = (zx^3 - 2z)\ \mathbf{i} + xz\ \mathbf{j} + xy\ \mathbf{k}$ and C is the triangle with vertices $(0,0,4)$, $(0,2,0)$ and $(2,0,0)$. Direction is $(2,0,0)$ to $(0,0,4)$, then $(0,0,4)$ to $(0,2,0)$ and then $(0,2,0)$ to $(2,0,0)$.

9. Use divergence theorem to evaluate the following surface integrals.
   (a) $\displaystyle\int\int_S F \cdot dS$ where $F(x,y,z) = (3x - zx^2)\ \mathbf{i} + (x^3 - 1)\ \mathbf{j} + (4y^2 + x^2z^2)\ \mathbf{k}$ and S is the surface of the box $0 \le x \le 1$, $-3 \le y \le 0$ and $-2 \le z \le 1$. Note that all six sides of the box are included in S.
   (b) $\displaystyle\int\int_S F \cdot dS$ where $F(x,y,z) = 4x\ \mathbf{i} + (1 - 6y)\ \mathbf{j} + z^3\ \mathbf{k}$ and S is the surface of the sphere of radius 2 with $z \ge 0$, $y \ge 0$ and $x \ge 0$. Note that all four surfaces of this solid are included in S.
   (c) $\displaystyle\int\int_S F \cdot dS$ where $F(x,y,z) = -xy\ \mathbf{i} + (z - 1)\ \mathbf{j} + z^3\ \mathbf{k}$ and S the surface of the solid bounded by $y = 4x^2 + 4z^2 - 1$ and the plane $y = 7$. Note that both of the surfaces of this solid are included in S.

The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

Tutorial Sheet 7 **September, 2018.**

Submit questions 8 and 9 for assessment: Submit on Friday, October 5, during class from 11-13hrs.

1. Write down a set of parametric equations for the given surfaces.
   (a) The plane containing the three points $(1,4,-2)$, $(-3,0,1)$ and $(2,4,-5)$.
   (b) The porion of the plane $x+9y3z=8$ that lies in the 1^{st} octant.
   (c) The portion of $y=10-3x^2-3z^2$ that is in front of the xz-plane.
   (d) The portion of the sphere of radius 3 with $y \geq 0$ and $z \geq 0$.

2. Determine the tangent plane to the surface given by

$$r(u,v) = (u+2v)\ \mathbf{i} + (u^2+3)\ \mathbf{j} - 3v^2\ \mathbf{k}$$

   at the point $(-5,4,-12)$.

3. Determine the surface area of the following surfaces.
   (a) The portion of $x = 6-y^2-z^2$ that is in front of $x=2$ with $y \geq 0$.
   (b) The portion of $x^2+y^2+z^2=11$ with $x \geq 0, y \geq 0$ and $z \geq 0$.
   (c) The portion of $x+4y+8z=4$ that is inside the cylinder $x^2+y^2=16$.

4. Evaluate the following surface integrals of scalar functions.
   (a) $\displaystyle\int\int_S 2x-3y+z\mathrm{d}S$ where S is the portion of $x+y+z=2$ that is in the 1^{st} octant.
   (b) $\displaystyle\int\int_S x+y^2+z^2\mathrm{d}S$ where S is the portion of $x=4-y^2-z^2$ that lies in front of $x=-2$.
   (c) $\displaystyle\int\int_S z+3\mathrm{d}S$ where S is the surface of the solid bounded by $z=2x^2+2y^2-3$ and $z=1$. Note that both surfaces of this solid are included in S.

5. Evaluate the following surface integrals of vector fields.
   (a) $\displaystyle\int\int_S F\cdot \mathrm{d}S$ where $F(x,y,z)=(z-y)\ \mathbf{i}+x\ \mathbf{j}+4y\ \mathbf{k}$ and S is the portion of $x+y+z=2$ that is in the 1^{st} octant oriented in the positive z-axis direction.
   (b) $\displaystyle\int\int_S F\cdot \mathrm{d}S$ where $F(x,y,z)=y\ \mathbf{i}-2\ \mathbf{k}$ and S is a closed surface consisting of sphere of radius 1 with $z \geq 0$ and $x \geq 0$ and the plane $x=0$ and $z=0$.
   (c) $\displaystyle\int\int_S F\cdot \mathrm{d}S$ where $F(x,y,z)=-x\ \mathbf{i}+(4+y)\ \mathbf{j}-z\ \mathbf{k}$ and S the portion of $x^2+z^2=9$ between $y=2$ and $y=10-x$ oriented inwards.

6. Compute the divergence and curl of the following vector fields.
   (a) $F(x,y,z)=(2y-\cos x)\ \mathbf{i}-z^2e^{3x}\ \mathbf{j}+(x^2-7z)\ \mathbf{k}$
   (b) $F(x,y,z)=-(4y-1)\ \mathbf{i}-xy^2\ \mathbf{j}+(x-3y)\ \mathbf{k}$