# The University of Zambia
# Department of Mathematics and Statistics
# Mat 3110-Engineering Mathematics II

**Tutorial Sheet 2** **May, 2018.**

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**Submit the whole of question 7 and 8 for assessment: Submit on Friday, 1st June, 2018 during class from 11-13hrs.**

1. Find the Laplace transforms of the following functions. Assume that $a, b, \omega$ and $\theta$ are constants.

   (a) $(a - bt)^2$ (b) $3t + 12$ (c) $\cos^2 \omega t$
   (d) $e^{2t} \sinh t$ (e) $e^{-t} \sin 4t$ (f) $\sin(\omega t + \theta)$
   (g) $t^2\ e^{-3t}$ (h) $k\ e^{at} \cos \omega t$ (i) $1.5 \sin(3t - \frac{\pi}{2})$
   (j) $t \cos 4t$ (k) $\sin^4 t$ (l) $te^{-at}$
   (m) $\cosh^2 t$ (n) $\cos^2 2t$ (o) $\sin^2 \omega t$
   (p) $t \sin \omega t$ (q) $t \cos \omega t$ (r) $t \sinh at$
   (s) $t \cosh at$

2. Find the inverse Laplace transforms of the following functions.

   (a) $\frac{1}{s^n}$ (b) $\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}$ (c) $\frac{0.2s+18}{s^2+324}$
   (d) $\frac{12}{s^4} - \frac{228}{s^6}$ (e) $\frac{5s+1}{s^2-25}$ (f) $\frac{4s+32}{s^2-16}$
   (g) $\frac{s}{L^2s^2-a^2\pi^2}$ (h) $\frac{s+10}{s^2-s-2}$ (i) $\frac{\pi}{(s+\pi)^2}$
   (j) $\frac{6}{(s+1)^3}$ (k) $\frac{21}{(s+\sqrt{2})^4}$ (l) $\frac{4}{s^2-2s-3}$
   (m) $\frac{\pi}{s^2+10\pi s+24\pi^2}$ (n) $\frac{a_0}{(s+1)} + \frac{a_1}{(s+1)^2} + \frac{a_2}{(s+1)^3}$ (o) $\frac{2s-1}{s^2-6s+8}$
   (p) $\frac{a(s+k)+b\pi}{(s+k)^2+\pi^2}$ (q) $\frac{k_0(s+a)+k}{(s+a)^2}$

3. Solve the following initial value problems using Laplace transforms.

   (a) $y'' - y' - 2y = e^{2t}, \quad y(0) = 0,\ y'(0) = 1.$

   (a) $y'' - 2y' - 2y = e^{-t}, \quad y(0) = 0,\ y'(0) = 1.$

   (a) $y'' + y' = \cos 2t, \quad y(0) = 2,\ y'(0) = 1.$

   (a) $y'' + y'9y = 0, \quad y(0) = 0.16,\ y'(0) = 0.$

   (a) $y'' - y' - 6y = 0, \quad y(0) = 11,\ y'(0) = 28.$

4. Solve the following initial value problems using Laplace transforms.

   (a) $y'' - 2y' - 3y = 0, \quad y(4) = -3,\ y'(4) = -17.$

   (b) $y' - 6y = 0, \quad y(-1) = 4.$

(c) $y'' + 2y' + 5y = 50t - 100, \quad y(2) = -4,\ y'(2) = 14.$

5. Find the Laplace transforms of the following functions

(a) $f(t) = \begin{cases} t, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$

(b) $f(t) = \begin{cases} 0, & 0 \le t < 2 \\ t-2, & t \ge 2 \end{cases}$

(c) $f(t) = \begin{cases} \cos 4t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$

(d) $f(t) = \begin{cases} e^t, & 0 \le t < \frac{\pi}{2} \\ 0, & t \ge \frac{\pi}{2} \end{cases}$

(e) $f(t) = \begin{cases} 0, & 0 \le t < 2 \\ \sin \pi t, & 2 \le t < 4 \\ 0, & t \ge 4 \end{cases}$

(f) $f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t^2, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$

6. Find and sketch $f(t)$ if $\mathcal{L}\{f(t)\}$ equals

(a) $\frac{e^{-3s}}{s^2-s-12}$ (b) $\frac{se^{-s}}{s^2+4s+5}$ (c) $\frac{e^{-3s}}{(s-1)^3}$

(d) $\frac{6(1-e^{-\pi s})}{s^2 9}$ (e) $\frac{4(e^{-2s}-2e^{-5s})}{s}$ (f) $\frac{2(e^{-s}-e^{-3s})}{s^2-4}$

(g) $\frac{1+e^{-2\pi(s+1)}(s+1)}{(s+1)^2+1}$

7. Solve the following initial value problems using Laplace transforms.

(a) $y'' + 4y' = \begin{cases} 0, & 0 \le t < 5 \\ \frac{t-5}{5}, & 5 \le t < 10 \\ 1, & t \ge 10 \end{cases}, \quad y(0) = 0,\ y'(0) = 0.$

(b) $y'' + 9y' = \begin{cases} 8\sin t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}, \quad y(0) = 0,\ y'(0) = 4.$

(c) $y'' + 3y' + 2y = \begin{cases} 4t, & 0 \le t < 1 \\ 8, & t \ge 1 \end{cases}, \quad y(0) = 0,\ y'(0) = 0.$

(d) $y'' + 2y' + 5y = \begin{cases} 10\sin t, & 0 \le t < 2\pi \\ 0, & t \ge 2\pi \end{cases}, \quad y(\pi) = 1,\ y'(\pi) = 2e^{-\pi} - 2.$

8. Using the convolution theorem, find the inverse Laplace transforms of the following functions.

(a) $\frac{20}{(s^2+1)(s^2+25)}$ (b) $\frac{\omega}{s^2(s^2+\omega^2)}$ (c) $\frac{2\pi s}{(s^2+\pi^2)^2}$

9. Solve the following integral equations using Laplace transforms.

(a) $y(t) + 4\int_0^t y(\tau)(t-\tau)\ \mathrm{d}\tau = 2t$

(b) $y(t) - \int_0^t y(\tau)\ \mathrm{d}\tau = 1$

(c) $y(t) - \int_0^t y(\tau)\sin 2(t-\tau)\ \mathrm{d}\tau = \sin 2t$