

The University of Zambia
 Department of Mathematics and Statistics
 Mat 3110-Engineering Mathematics II

Tutorial Sheet 4

August, 2018.

Submit questions 2(g), 2(h), 2(i), 3(d) and 3(e) for assessment: Submit on Monday, August 13 , 2018 during class from 11-13hrs.

1. Evaluate the following triple integrals.

(a) $\int_1^2 \int_0^2 \int_{-1}^1 2 + z^2 - xy \, dz \, dx \, dy$

(b) $\int_0^2 \int_{x^2}^2 \int_0^{xz} y^2 - 6z \, dz \, dx \, dy$

(c) $\int_{-1}^2 \int_0^1 \int_0^{2z} 3x - \sqrt{1 + z^2} \, dz \, dx \, dy$

In each of the above, can you tell over which solid we are integrating?

2. Evaluate the following triple integrals over the given solids.

(In as much as you are not required to sketch the solids over which you are integrating, you are encouraged to do so as it will enhance your understanding.)

(a) Evaluate $\int \int \int_E 12y \, dV$ where E is the region below $6x + 4y + 3z = 12$ in the first octant.

(b) Evaluate $\int \int \int_E 5x^2 \, dV$ where E is the region below $x + 2y + 4z = 8$ in the first octant.

(c) Evaluate $\int \int \int_E 10z^2 - x \, dV$ where E is the solid below $z = 8 - y$ and above the region in the xy -plane bounded by $y = 2x$, $x = 3$ and $y = 0$.

(d) Evaluate $\int \int \int_E 4y^2 \, dV$ where E is the solid below $z = -3x^2 - 3y^2$ and above $z = -12$.

(e) Evaluate $\int \int \int_E 2y - 9z \, dV$ where E is the solid behind the plane $6x + 3y + 3z = 15$ and in front of the triangle in the xz -plane with vertices, in (x, z) form : $(0, 0)$, $(0, 4)$ and $(2, 4)$.

(f) Evaluate $\int \int \int_E 18x \, dV$ where E is the solid behind the surface $y = 4 - x^2$ that is in front of the solid in the xz -plane bounded by $z = -3x$, $z = 2x$ and $z = 2$.

(g) Evaluate $\int \int \int_E 20x^3 \, dV$ where E is the solid bounded by $x = 2 - y^2 - z^2$ and $x = 5y^2 + 5z^2 - 6$.

(h) Evaluate $\int \int \int_E 6z^2 \, dV$ where E is the solid behind $x + 6y + 2z = 8$ that is in front of the region in the yz -plane bounded by $z = 2y$ and $z = \sqrt{4y}$.

(i) Evaluate $\int \int \int_E 8y \, dV$ where E is the solid between $x + y + z = 6$ and $z + y + z = 10$ above the triangle in the xy -plane with vertices, in (x, y) form : $(0, 0)$, $(1, 2)$ and $(1, 4)$.

3. Use triple integrals to find the volumes of the following solids.

- (a) The volume of the solid is below the plane $z = 8 - y$ and above the region in the xy -plane bounded by $y = 2x$, $x = 3$ and $y = 0$.
- (b) The volume of the solid in the 1st octant that is below $4x + 8y + z = 16$.
- (c) The volume of the solid behind $6x + 3y + 3z = 15$ and in front of the triangle in the xz -plane with vertices, in (x, z) form : $(0, 0)$, $(0, 4)$ and $(2, 4)$.
- (d) The solid enclosed by $y = x^2 + z^2$ and $y = \sqrt{x^2 + z^2}$.
- (e) The solid behind $x + 6y + 2z = 8$ that is in front of the region in the yz -plane bounded by $z = 2y$ and $z = \sqrt{4y}$.