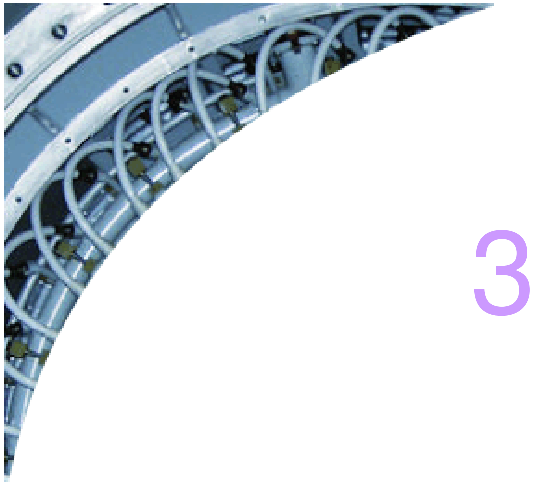
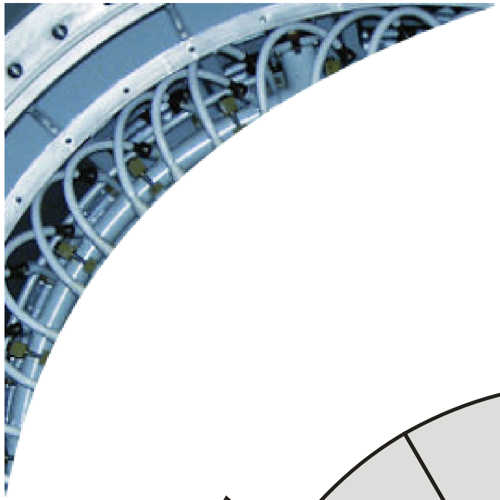
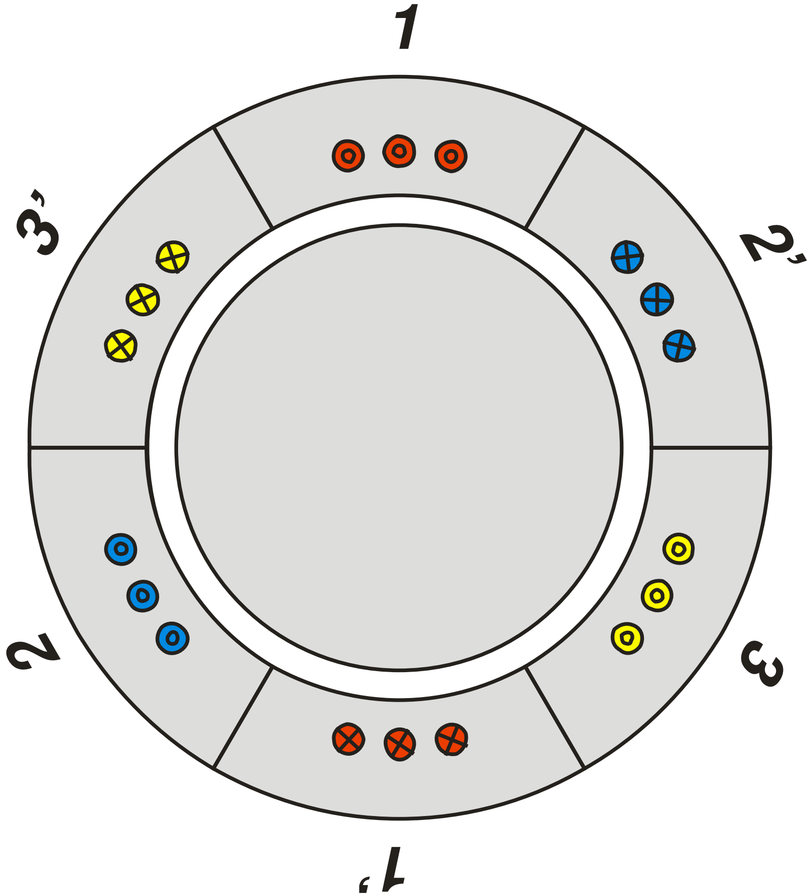


3-phase AC Machines

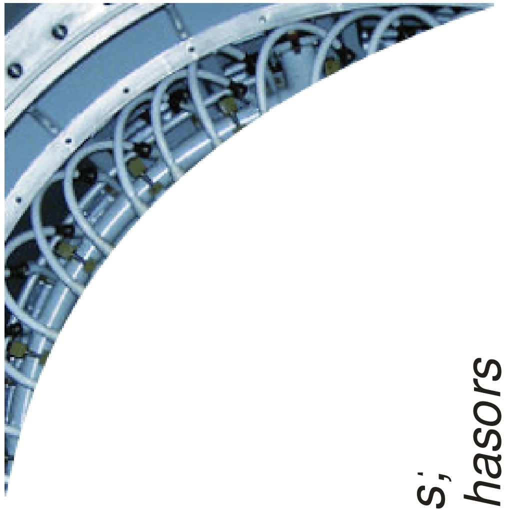
- Production of rotating magnetic field
- Induction machine
- Synchronous machine

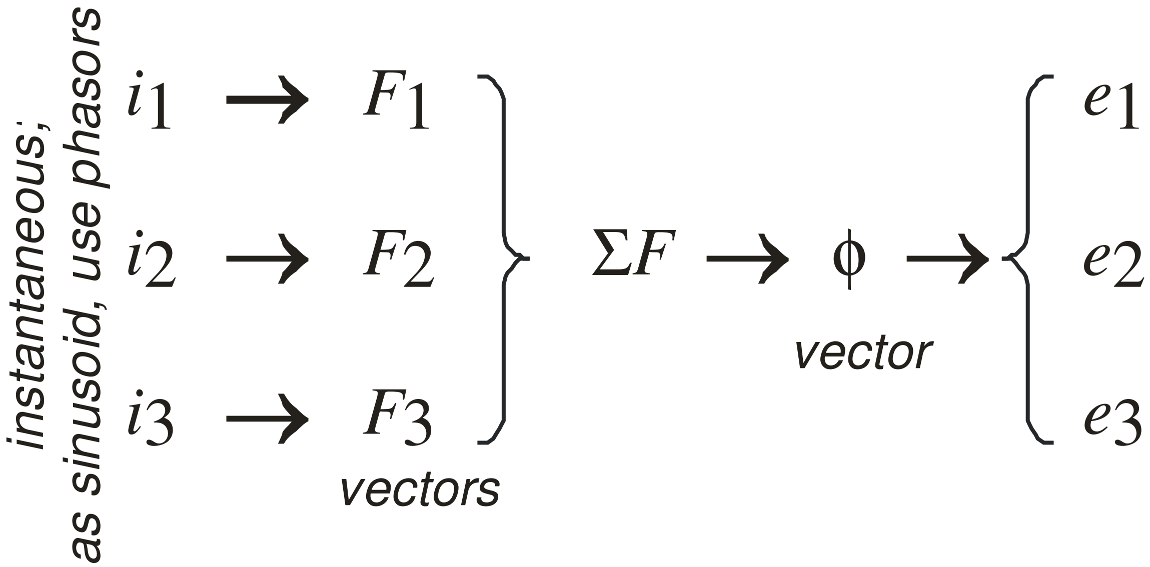


Rotating magnetic field



- ⇒ balanced 3-phase supply
- ⇒ 3 identical coils 120° from each other
- ⇒ each coil has *N* -turns



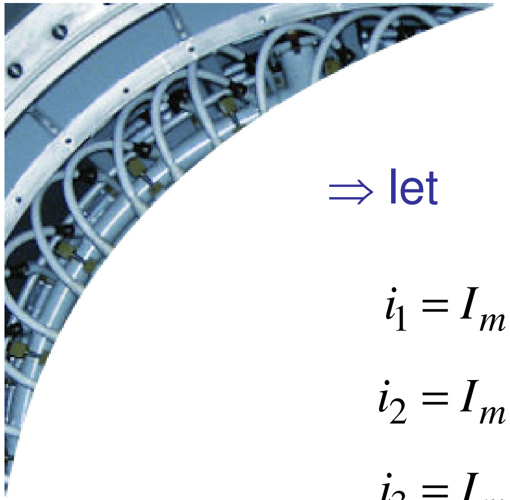


instantaneous;
as sinusoid, use phasors

$$\left.\begin{matrix} i_1 \rightarrow F_1 \\ i_2 \rightarrow F_2 \\ i_3 \rightarrow F_3 \end{matrix}\right\} \Sigma F \rightarrow \phi \rightarrow \left\{\begin{matrix} e_1 \\ e_2 \\ e_3 \end{matrix}\right.$$

vectors

vector

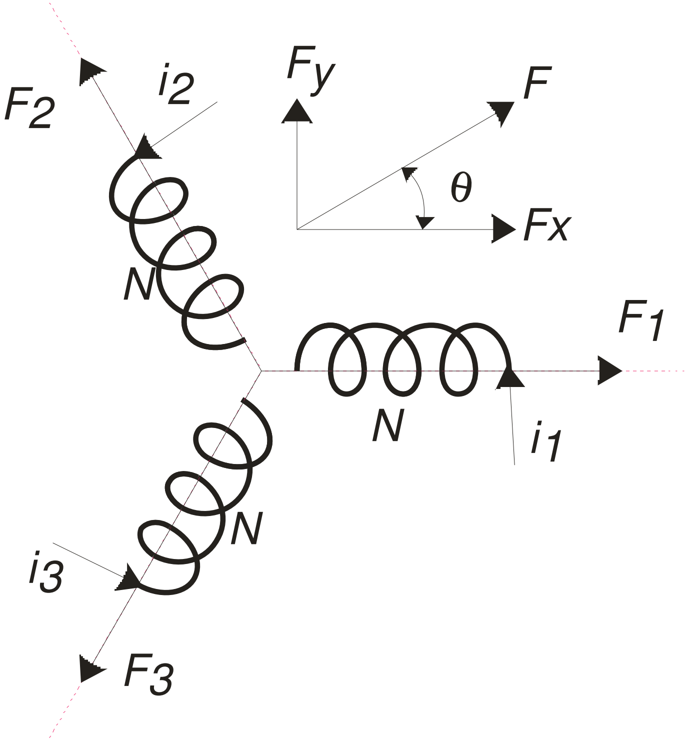


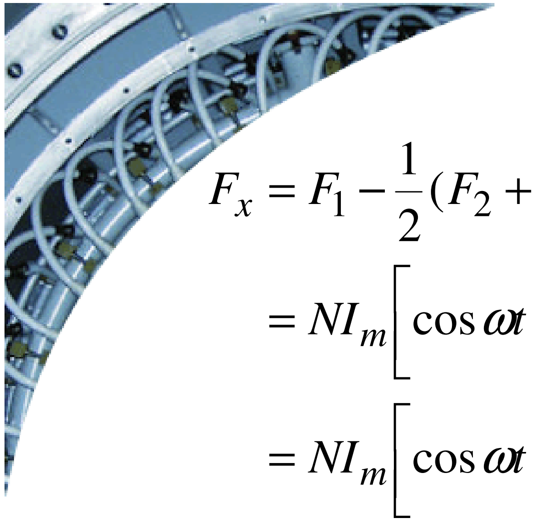
⇒ let

$$i_1 = I_m \cos \omega t$$
$$i_2 = I_m \cos(\omega t - 120^\circ)$$
$$i_3 = I_m \cos(\omega t + 120^\circ)$$

⇒

$$F_1 = NI_m \cos \omega t$$
$$F_2 = NI_m \cos(\omega t - 120^\circ)$$
$$F_3 = NI_m \cos(\omega t + 120^\circ)$$





$$
\begin{aligned}
F_x &= F_1 - \frac{1}{2}(F_2 + F_3) \\
&= NI_m\left[\cos\omega t - \frac{1}{2}(\cos\overline{\omega t - 120^\circ} + \cos\overline{\omega t + 120^\circ})\right] \\
&= NI_m\left[\cos\omega t - \frac{1}{2}2\cos\omega t\cos 120^\circ\right] \\
F_x &= \frac{3}{2}NI_m\cos\omega t
\end{aligned}
$$

$$
\begin{aligned}
F_y &= \frac{\sqrt{3}}{2}(F_2 - F_3) \\
&= \frac{\sqrt{3}}{2}NI_m\left[\cos\overline{\omega t - 120^\circ} - \cos\overline{\omega t + 120^\circ}\right] \\
&= \frac{\sqrt{3}}{2}NI_m 2\sin\omega t\sin 120^\circ \\
F_y &= \frac{3}{2}NI_m\sin\omega t
\end{aligned}
$$

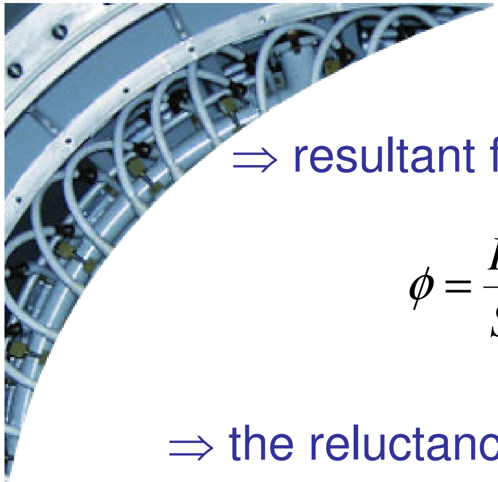
$$|F| = \sqrt{F_x^2 + F_y^2} = \frac{3}{2}NI_m$$

⇒ the magnitude of the resultant mmf in space is independent of time and is constant

$$\tan\theta = \frac{F_y}{F_x} = \tan\omega t$$

$$\theta = \omega t$$

⇒ the direction of the resultant mmf in space is varying continuously in time with angular speed ω

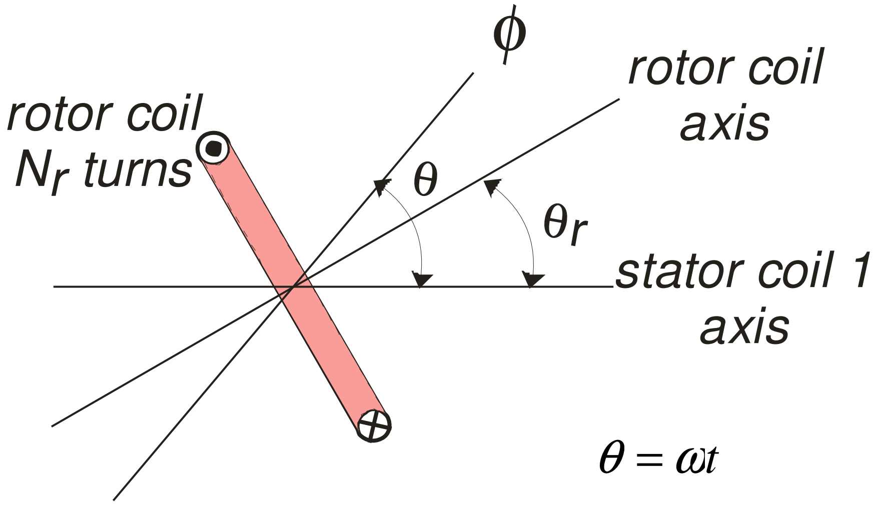


⇒ resultant flux

$$\phi = \frac{F}{S} = \frac{3}{2}\frac{NI_m}{S}$$

⇒ the reluctance *S*, though difficult to evaluate, is at least constant

⇒ ϕ has constant value and rotates in space at constant speed



$$\theta = \omega t$$

$$\theta_r = \omega_r t + \beta$$

⇒ flux linking stator coil 1 is

$$\phi_r = \phi\cos\overline{\theta-\theta_r} = \phi\cos(\overline{\omega-\omega_r}t-\beta)$$

⇒ let $\frac{\omega_r}{\omega} = m$

$$\phi_r = \phi\cos(\overline{1-m}\omega t-\beta)$$

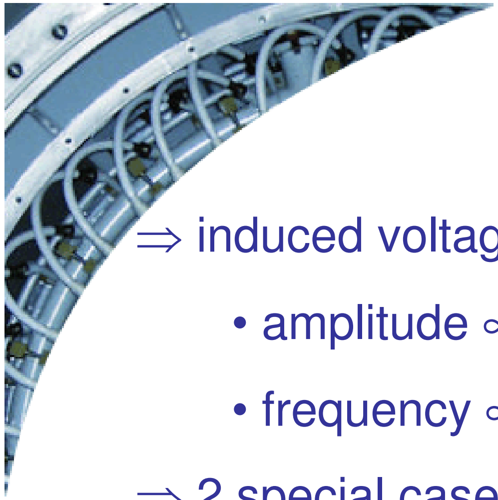
⇒ define slip, *s* $s = \frac{\omega-\omega_r}{\omega} = 1-m$

⇒ induced voltage in rotor

$$e_r = N_r\frac{d\phi_r}{dt}$$

$$= N_r\phi\left[\sin(\overline{1-m}\omega t-\beta\right](1-m)\omega$$

$$e_r = sN_r\phi\omega\sin(s\omega t-\beta)$$



⇒ induced voltage in rotor coil has

- amplitude ∝ s
- frequency ∝ s

⇒ 2 special cases:

1) at synchronous speed, $m = 1$ or $s = 0$

$$e_r = 0$$

2) at standstill, $m = 0$ or $s = 1$

$$e_r = N_r \phi \omega \sin(\omega t - \beta)$$

voltages in stator & rotor windings

⇒ for stator coil 1

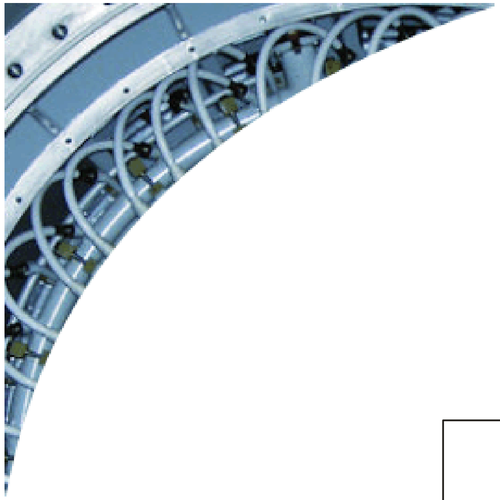
$$e_1 = N \frac{d(\phi \cos \theta)}{dt} = -N \phi \omega \sin \omega t$$

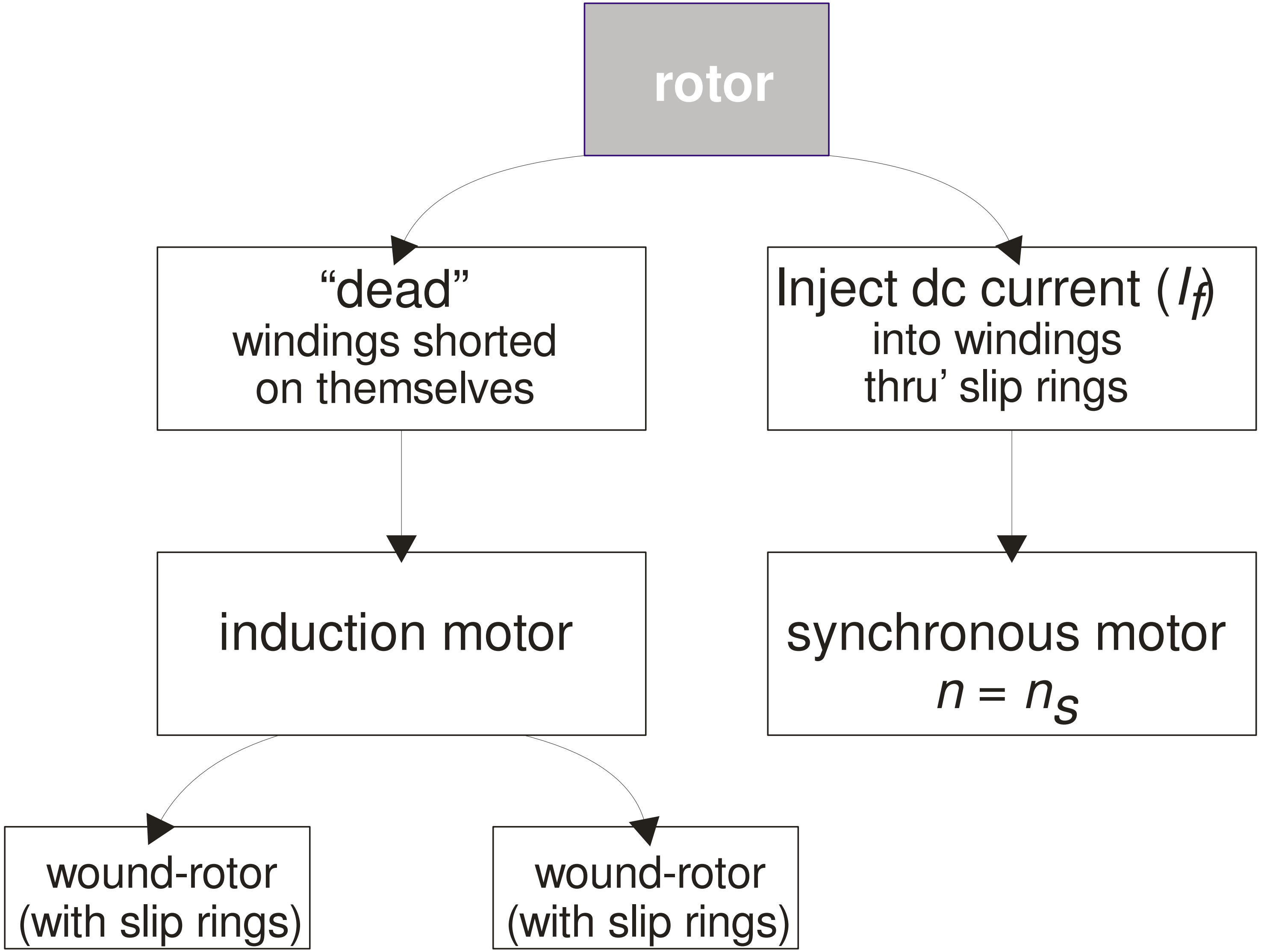
⇒ rms values

$$E_r = s \frac{N_r}{\sqrt{2}} \phi \omega$$

$$E_1 = \frac{N}{\sqrt{2}} \phi \omega$$

$$\frac{E_r}{E_1} = s \frac{N_r}{N}$$





rotor

“dead”
windings shorted
on themselves

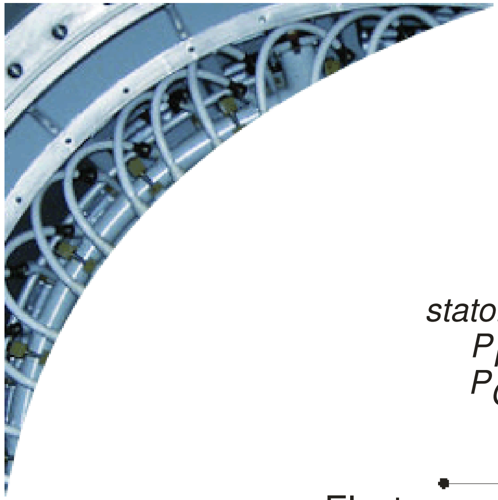
Inject dc current (I_f)
into windings
thru’ slip rings

induction motor

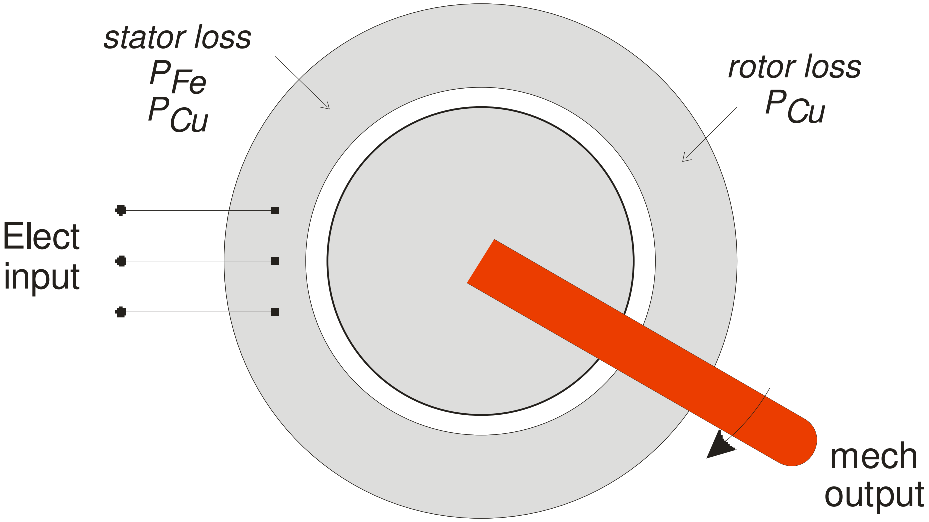
synchronous motor
$n = n_s$

wound-rotor
(with slip rings)

wound-rotor
(with slip rings)



Induction machines

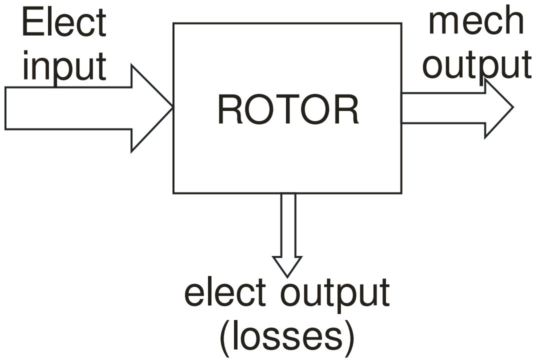


stator loss
P_{Fe}
P_{Cu}

rotor loss
P_{Cu}

Elect input

mech output

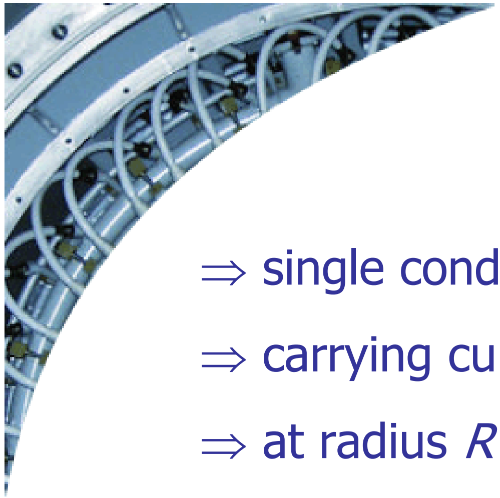


Elect input

ROTOR

mech output

elect output (losses)



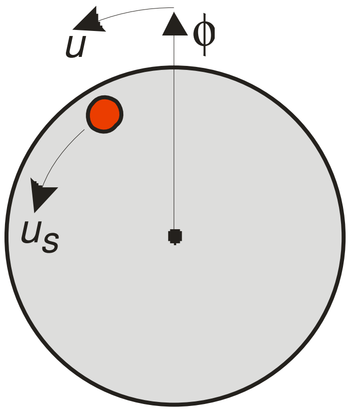
$\Rightarrow$ single conductor of length l

$\Rightarrow$ carrying current i

$\Rightarrow$ at radius R

$\Rightarrow$ rotating magnetic field is moving at speed u_s

$\Rightarrow$ conductor is moving at speed u

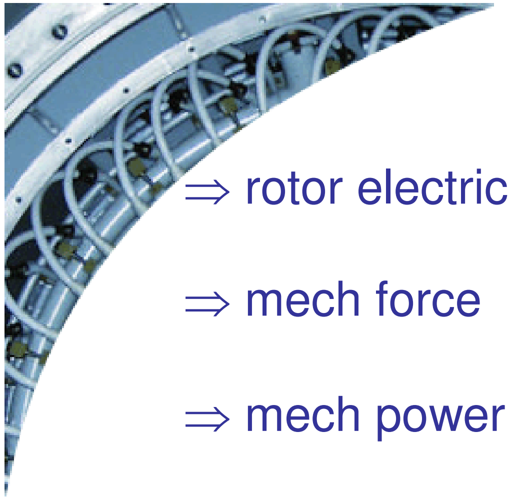


$\Rightarrow$ define (fractional) slip s as $$s = \frac{u_s - u}{u_s}$$

$\Rightarrow$ hence $$s = \frac{2\pi R n_s - 2\pi R n}{2\pi R n_s} = \frac{n_s - n}{n_s}$$

$\Rightarrow$ Faraday’s law gives induced voltage

$$V_{in} = Bl(u_s - u) = Blu_s s$$

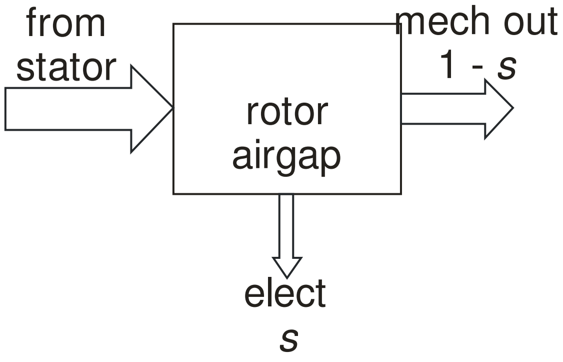


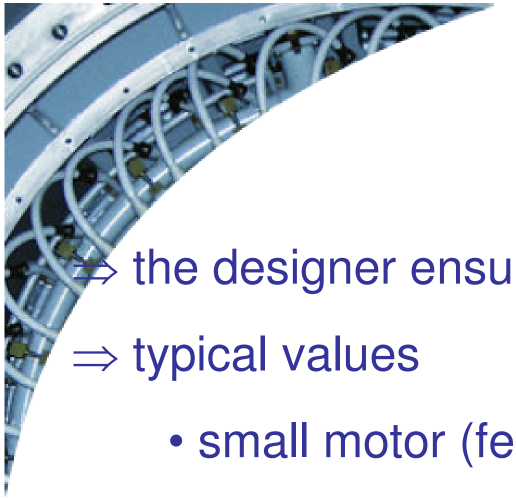
- ⇒ rotor electrical power $V_{in}i = Blu_s si$
- ⇒ mech force Bli
- ⇒ mech power $(Bli)u = Bliu_s(1-s)$
- ⇒ total input power rotor elec power + rotor mech power

$$= Blu_s si + Bliu_s(1-s)$$

$$= Blu_s i$$

- ⇒ thus

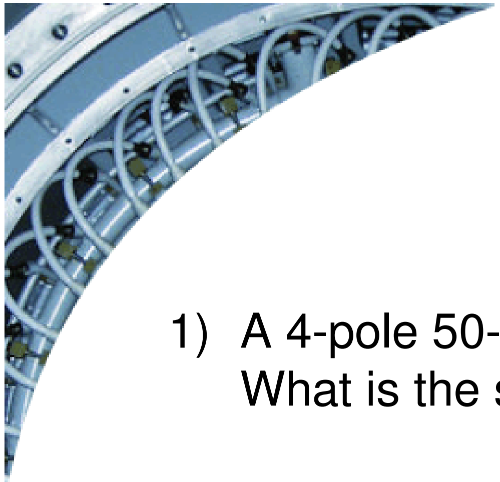




⇒ the designer ensures that *s* is small

⇒ typical values

- small motor (few kW) s = 0.03 (3%)
- large machines s ≤ 0.02



Examples

1) A 4-pole 50-Hz machine is running at $s = 0.025$ and 100 kW enters the rotor. What is the speed n , mechanical output and the rotor copper loss?

2) A 3-phase, 60-Hz, four-pole, 220-V, wound-rotor induction motor has a delta-connected stator winding and a star-connected rotor winding. The rotor has 40% as many turns as the stator. For a rotor speed of 1710 r/min, calculate the

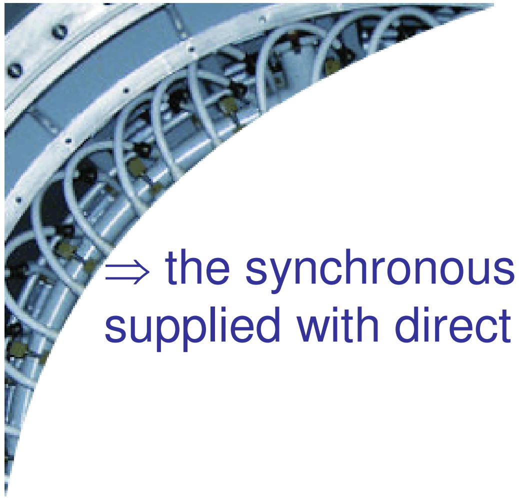
   a) slip

   b) induced phase voltage in the rotor at standstill

   c) induced phase voltage in the rotor at working speed

   d) rotor terminal voltage on open circuit and at standstill

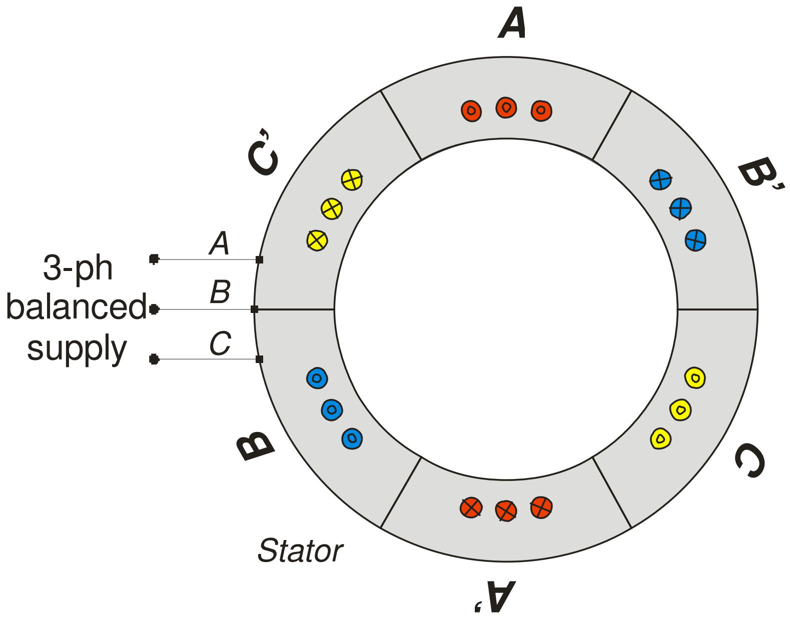
   e) frequency of induced voltage in the rotor



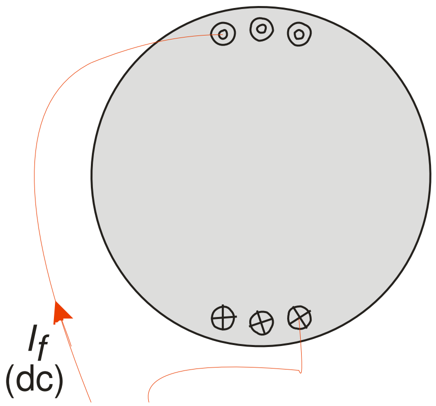
Synchronous machines

⇒ the synchronous machine has a 3-phase winding on the stator while the rotor is supplied with direct current

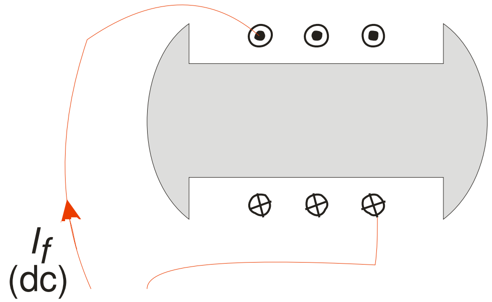
Stator



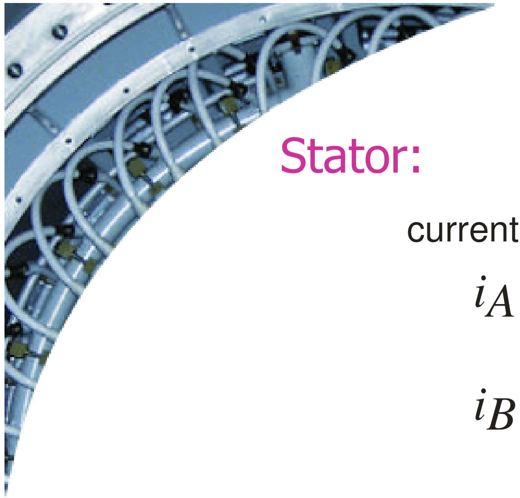
Rotor



cylindrical (round) rotor



salient rotor



Stator:

currents mmf

$$\left.\begin{array}{ccc} i_A & \rightarrow & F_A \\ i_B & \rightarrow & F_B \\ i_C & \rightarrow & F_C \end{array}\right\} F_a \rightarrow \phi$$

⇒ F_a has constant magnitude and rotates a constant speed ω_s

⇒ for a $2p$-pole machine

$$n_s = \frac{\omega_s}{2\pi} = \frac{f}{p}$$

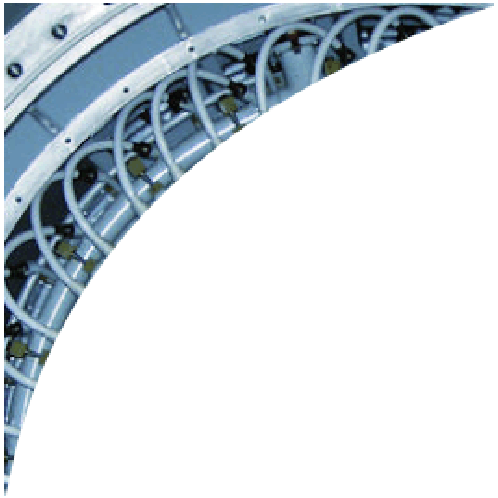
Rotor:

current mmf

$$I_f \rightarrow F_f$$

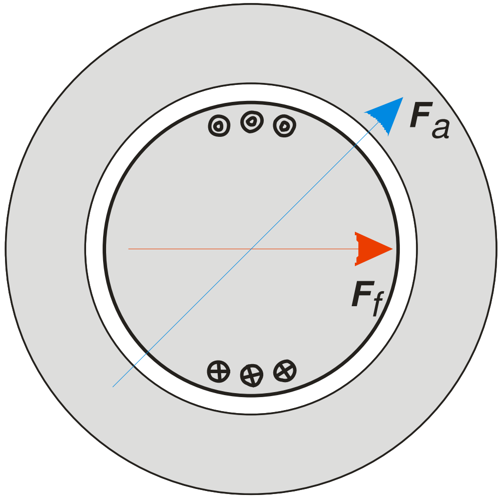
⇒ the direction of the current through the brushes and slip rings to the winding is always in the same direction and so the polarity of on the rotor (N & S) never changes

⇒ F_f is along the axis of the rotor and rotates at ω_r

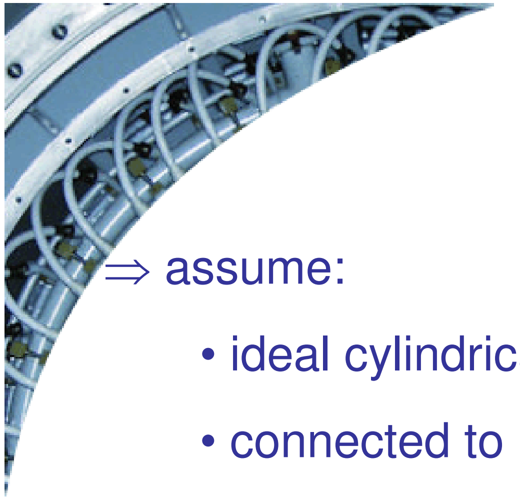


$$\overline{F}_a + \overline{F}_f = \overline{F}_r \rightarrow \phi_r \rightarrow \left\{ \begin{array}{c} e_A \\ \\ e_B \\ \\ e_C \end{array} \right\} E_r$$

⇒ in synchronous machine $\omega_r = \omega_s$



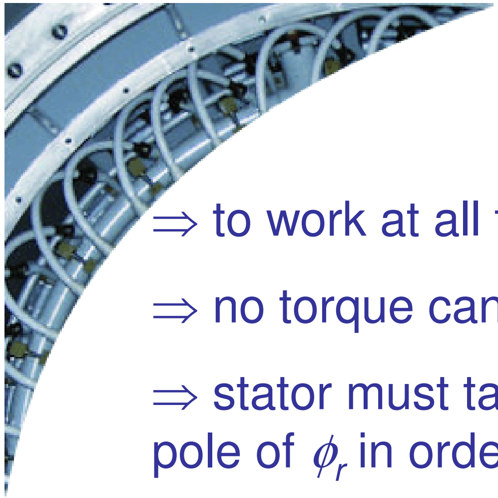
⇒ the machine functions as a motor or as a generator depending on whether the stator fields leads or lags the rotor field



Action of the ideal machine

⇒ assume:

- ideal cylindrical rotor
- connected to ‘infinite’ busbar
- stator windings have
  - negligible resistance
  - negligible leakage reactance
  - uniform air gap
  - high permeability magnetic circuit
  - no saturation
  - balanced load



⇒ to work at all the rotor must rotate at synchronous speed

⇒ no torque can be developed if rotor is unexcited

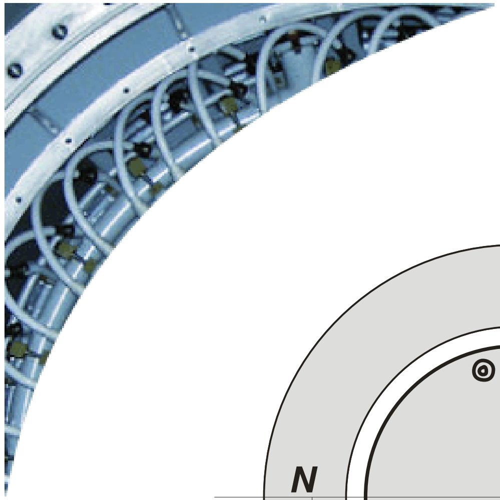
⇒ stator must take in lagging reactive power to magnetise the machine to a gap flux per pole of ϕ_r in order for the stator emf E_r to be induced to balance applied voltage V_a

⇒ if rotor is given a small dc excitation it takes over part of the task of exciting the magnetic circuit, reducing the demand of stator magnetising power (under excitation)

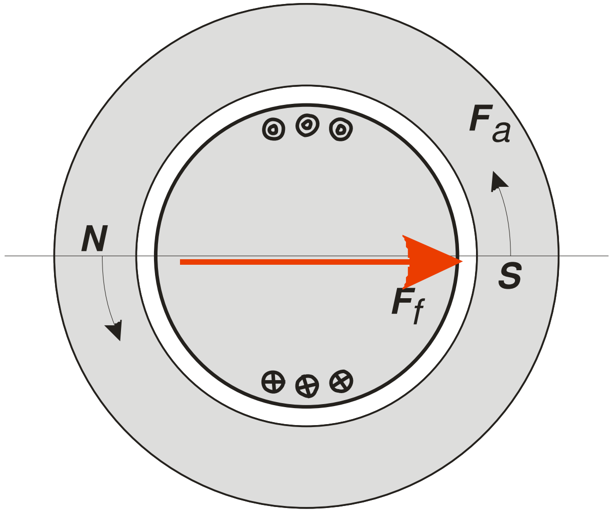
⇒ λ = torque angle

⇒ δ = load angle

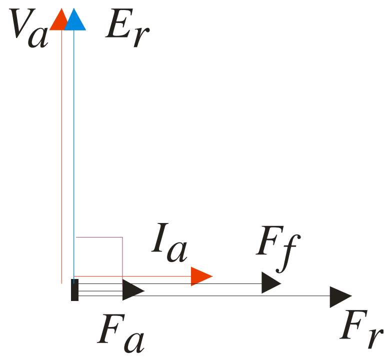
- motor mode: δ is negative
- generator mode: δ is positive



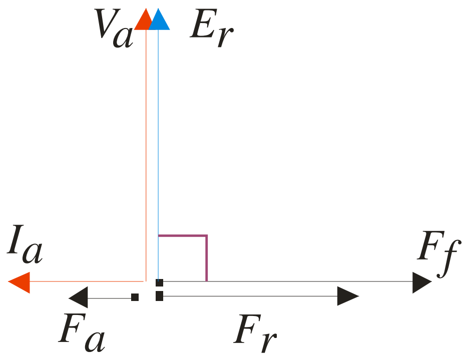
No-load mode



under excited



over excited



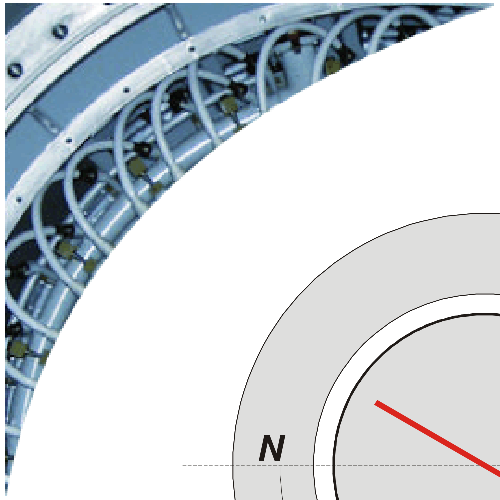
⇒ no torque is developed

- mmf axes are in alignment, torque angle is zero

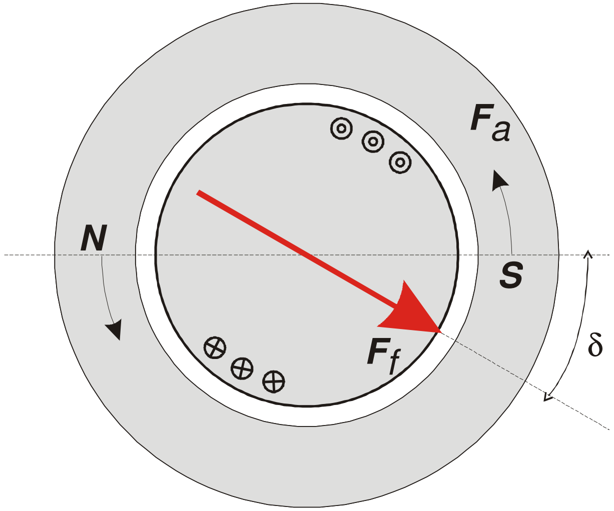
⇒ F_a and F_f combine to give F_r necessary to produce ϕ_r

⇒ if rotor mmf is increased very much into over excitation the stator must produce a demagnetising current so that F_r shall remain unchanged

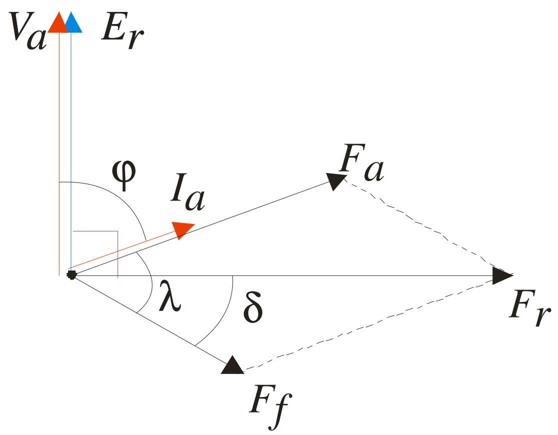
- i.e. stator must produce a leading current



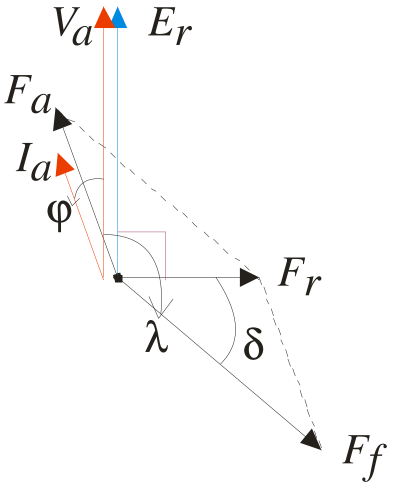
Motor mode



under excited



over excited

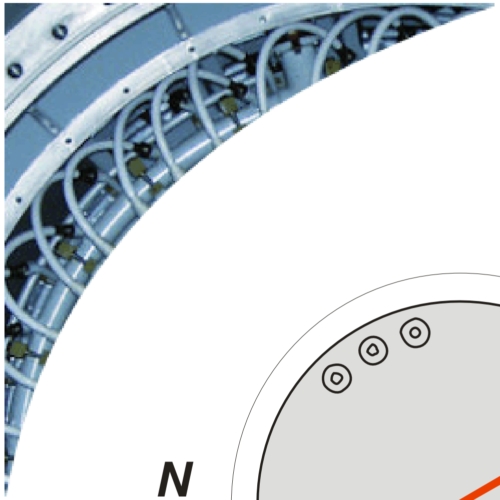


⇒ depending on whether rotor is under or over excited, the stator takes an active current component

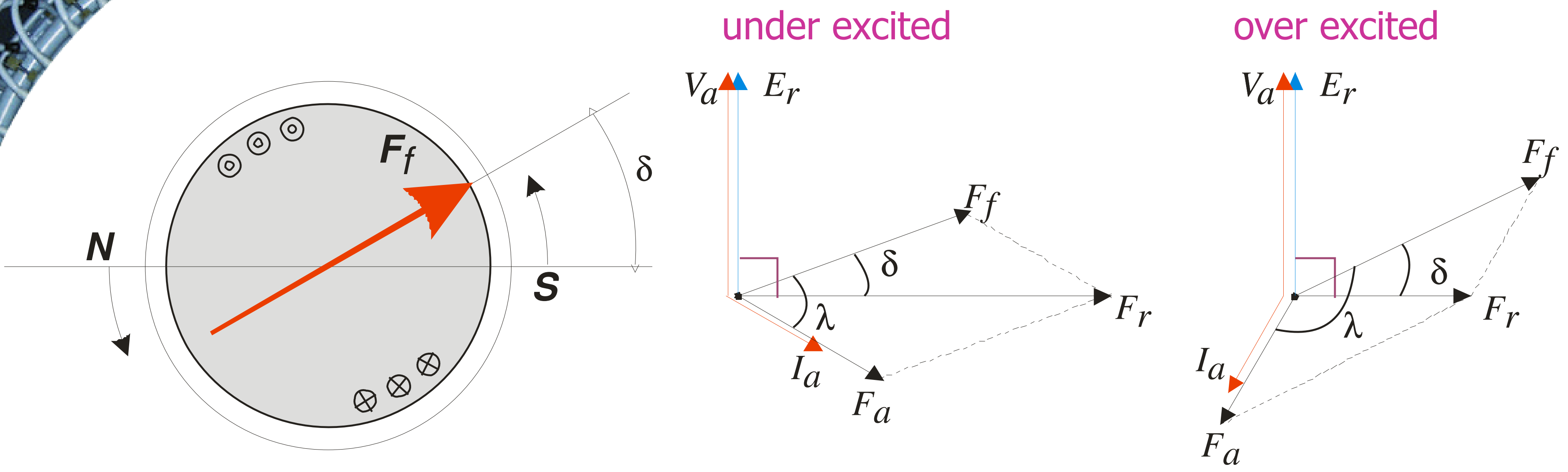
- thus accepting power from the supply and developing a forward torque on the rotor to balance the load torque

⇒ the reactive component of the current, as on no-load, compensates for under or over excitation

$$T \propto \sin\lambda$$



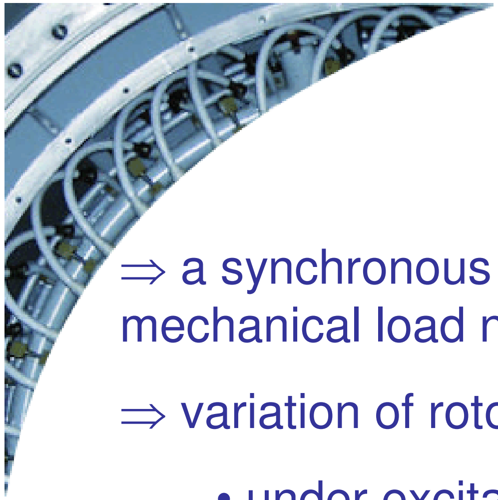
Generator mode



⇒ the active component of stator current reverses

- thus delivering power into the supply and developing a counter torque on the rotor to balance the driving torque

⇒ the reactive component of the current, as on no-load, compensates for under or over excitation



Compensator mode

⇒ a synchronous machine designed to run unloaded and the shaft is connected neither to mechanical load nor prime mover

⇒ variation of rotor excitation causes machine to take purely reactive power

- under excitation → lagging
- over excitation → leading

⇒ applied to control voltage in transmission systems by supplying or consuming reactive power

⇒ a.k.a. synchronous compensator, synchronous capacitor

⇒ in general, the machine has no torque at starting (zero speed) and therefore some other means must be used to bring it to synchronous speed, e.g.

- pony motor
- double-cage arrangement