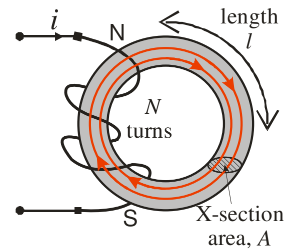
Magnetic circuits

- definition (introduction)
- electric circuit analogy
- composite magnetic circuits
- inductance
  - on dc
  - on ac
- inductor (real)
  - power loss
  - equivalent circuit
  - phasor diagram
- limitations of magnetic circuit model

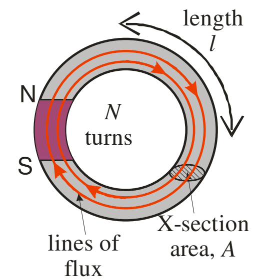
Introduction

⇒ a characteristic of lines of magnetic flux is that each line a closed path

⇒ the complete closed path followed by any group of lines of magnetic flux is referred to as a magnetic circuit



•Coil current as source of mmf, *F*



•PM as source of mmf, *F*

Electric circuit analogy

⇒ in electric circuits: electric current is due to effect of emf

⇒ in magnetic circuits: magnetic flux is due to effect of mmf

⇒ if a current-carrying coil is used as source of mmf, the mmf *F* generated when a current *i* flow in coil on *N* turns is

$$F = Ni$$

⇒ the mmf per unit length of the magnetic circuit is called magnetic field strength, *H*

⇒ if the mean length of the magnetic circuit is *l*, then

$$Hl = Ni$$

$$H = \frac{Ni}{l}$$

⇒ *H* produces *B* wherever it exits:

$$B = \mu H$$

$$\mu = \mu_r \mu_o$$

μ_o = permeability of free space $\quad \mu_o = 4\pi \times 10^{-7}$ H/m

μ_r = relative permeability

⇒ *H* for air, u_r = 1; for other magnetic materials u_r = 500-4000

⇒ for uniform magnetic circuits: X-section area, *A*; length *l*, permeability μ

$$\Lambda = \frac{\phi}{S} = \frac{BA}{Hl} = \mu\frac{A}{l} \qquad S = \frac{1}{\Lambda} = \frac{l}{\mu A}$$

$$\phi = \frac{F}{S} = \frac{Ni}{l/\mu A}$$

Magnetic circuits	Electric circuits
ϕ	I
F	V
Λ	G
$S = \frac{l}{\mu A}$	$R = \frac{l}{\sigma A}$

⇒ in magnetic circuits:

$$F_{\text{source}} = Ni$$

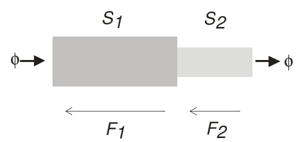
$$F_{\text{used}} = F_{\text{drop}} = \phi S$$

⇒ hence:

$$Ni = \phi S$$

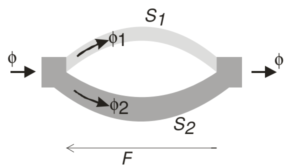
Composite magnetic circuits

I. Series



$$F = F_1 + F_2$$
$$\frac{F}{\phi} = \frac{F_1}{\phi} + \frac{F_2}{\phi}$$
$$S_{tot} = S_1 + S_1$$

II. Parallel

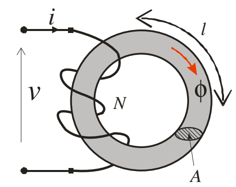


$$\phi = \phi_1 + \phi_2$$
$$\frac{\phi}{F} = \frac{\phi_1}{F} + \frac{\phi_2}{F}$$
$$\frac{1}{S_{tot}} = \frac{1}{S_1} + \frac{1}{S_2}$$

⇒ reluctances in series and parallel combine together in the same way as resistances

Inductance

⇒ any circuit in which a change of current causes change of magnetic flux , and hence a change of voltage, is said to have inductance



⇒ definition (elec. circuit point of view):

$$v = L\frac{di}{dt}$$

⇒ Faraday's law:

$$v = N\frac{d\phi}{dt}$$

⇒ equating:

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$
$$L = N\frac{d\phi}{di}$$

⇒ for uniform case: $L = N\frac{\phi}{i}$

⇒ but $\phi = \frac{F}{S} = \frac{Ni}{S}$

$$L = \frac{N^2}{S}$$

⇒ inductance depends on:

- geometry
- material
- turns

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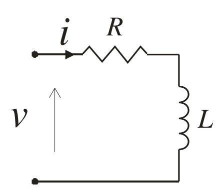
Inductance on dc



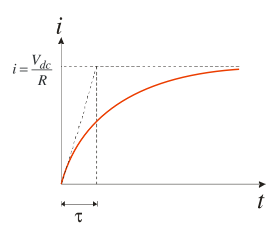
⇒ real coil, with R

$$v = Ri + L\frac{di}{dt}$$

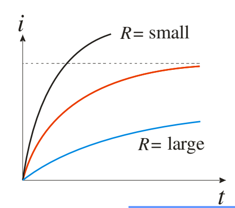
⇒ on dc

$$V_{dc} = Ri + L\frac{di}{dt}$$
$$i = \frac{V_{dc}}{R}\left(1 - e^{-t/\tau}\right)$$

$\tau = L/R$ = time constant



⇒ given L, variable R



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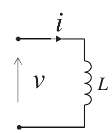
Inductance on ac

⇒ consider pure L



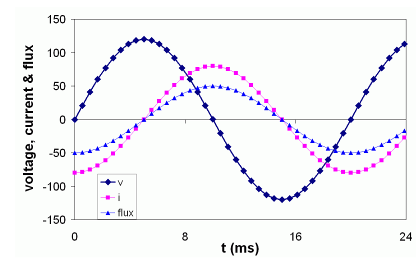
$$v = L\frac{di}{dt}$$

⇒ with sinusoidal supply, v

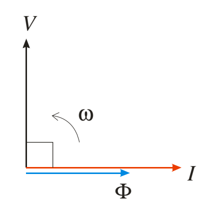
$$v = V_m \sin \omega t$$
$$i = \frac{1}{L}\int v dt = -\frac{V_m}{\omega L}\cos \omega t + \text{const.}$$

⇒ after a long time, const. = 0

$$\phi = \frac{F}{S} = \frac{Ni}{S} = -\frac{NV_m}{S\omega L}\cos \omega t$$



Phasor diagram



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⇒ for flux:

$$\phi = -\frac{V_m}{\omega N}\cos\omega t$$

$$\Phi_m = \frac{V_m}{\omega N}$$

$$V_m = \omega N \Phi_m$$

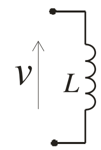
$$\sqrt{2}V = 2\pi f N B_m A$$

$$V = 4.44\, f N B_m A$$

→ *the transformer equation*

rms value (V) *max value* (B_m)

Real inductor



\+ Power Losses:

- Copper loss (*R* of coil), P_{Cu}
- Iron loss, P_{Fe}
  - Eddy current, P_e
  - Hysteresis, P_h

eddy current

⇒ there is induced voltage in the core of the magnetic material caused by the changing magnetic flux (due to the ac current in coil)

⇒ currents circulating in the core due to induced voltage are called eddy currents

hysteresis

⇒ in the iron core the magnetic flux lags the incraes or decrease of the magnetssing force; this creates a hysteresis loop

Iron loss

eddy current loss

⇒ the power loss due to eddy current and the resistance of the iron core

$$P_e = k_e f^2 B_m^2 \qquad [\text{W/m}^3]$$

hysteresis loss

⇒ the energy loss per cycle obtained by product of the area of the hysteresis loop and the volume of the core material

⇒ 3 factor affect the shape & size of hysteresis loop:

- material: narrow-magnetises easily; wide-doe not easily magnetise
- max value of *B*, $B_m \propto V_m$
- initial magnetising state of iron piece

$$P_h = k_h f B_m^n \qquad [\text{W/m}^3]$$

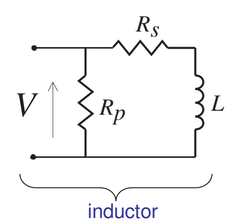
⇒ *n* = Steinmetz index (1.6-2.5)

Inductor equivalent circuit

$P_{Cu} = i^2 R$ ⇒ represent by resistance, R_s = series resistance

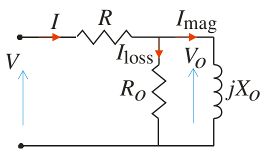
$$P_{Fe} \propto \phi^2 \propto V^2 = kV^2$$

⇒ represent by resistance, R_p such that $P_{Fe} = \frac{V^2}{R_p}$, $k = \frac{1}{R_p}$



⇒ inductance *L* + P_{Cu} + P_{Fe}

Equivalent circuit

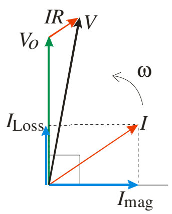


$$j\omega L = jX_o$$

$$I_{Loss} = \frac{V_o}{R_o}$$

$$I_{mag} = \frac{V_o}{jX_o}$$

Phasor diagram



$$\bar{I} = \bar{I}_{Loss} + \bar{I}_{mag}$$

$$\bar{V} = \bar{V}_o + \bar{I}R$$

Limitations of magnetic circuit model

- ⇒ requires homogeneous material
- ⇒ requires isotropic material
- ⇒ neglects non-linear characteristics
- ⇒ neglects saturation
- ⇒ neglects leakage and fringing flux

Examples:

1) The magnetic circuit shown in the figure has a cross-sectional area everywhere of 100 mm^2., and permeability of 10^{-3} H/m. If the coil has 100 turns and carries a steady current of 10 A, calculate the magnetic flux in the various portions A, B and C . Assume that the effective length of each side of the squares is 100 mm, and neglect complications at the corners. What is the self-inductance of the coil?