Single phase transformer

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- referred quantities
- real transformer
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  - phasor diagram
- rating
- regulation
- efficiency
- no-load current waveform
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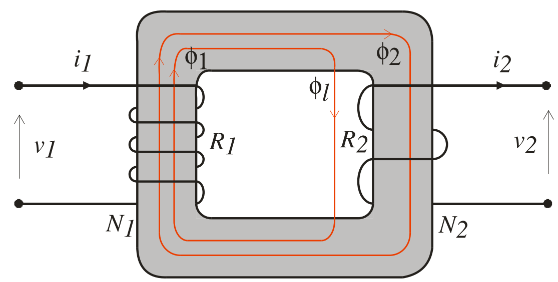
Introduction

⇒ consists of at least 2 windings on a common magnetic circuit

⇒ in practice, arranged so that all or nearly all flux linking 1st coil passes thru' 2nd coil

⇒ achieved by having a well-defined low reluctance magnetic circuit between the 2 coils



Ideal transformer

⇒ ideal transformer applies 4 assumptions:

1) No leakage flux,
   - so $\phi_1 = \phi_2$; or $\phi_l = 0$
2) No reluctance of magnetic circuit
   - $S = 0$; or $\mu_r = \infty$
3) No copper loss ($P_{cu} = 0$)
   - $R_1 = 0$, $R_2 = 0$
4) No iron loss ($P_{Fe} = 0$)
   - $R_p = \infty$

⇒ Faraday's law

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_2}{dt}$$

⇒ with $\phi_1 = \phi_2 = \phi$ for ideal trx

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

⇒ using rms values

$$\frac{V_{1i}}{V_{2i}} = \frac{N_1}{N_2}$$

⇒ subscript *i* denotes 'ideal'

⇒ voltage ratio = turns ratio

⇒ analogous KVL round magnetic circuit

$$F_1 - F_2 = \phi S$$

$$N_1 i_1 - N_2 i_2 = \phi S$$

⇒ for ideal trx, $S = 0$

$$N_1 i_1 = N_2 i_2$$

$$\frac{i_1}{i_2} = \frac{N_2}{N1}$$

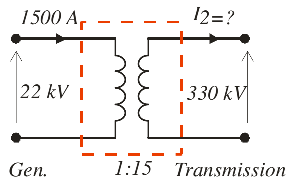
⇒ using rms values

$$\frac{I_{1i}}{I_{2i}} = \frac{N_2}{N1}$$

⇒ hence

$$\frac{V_{1i}}{V_{2i}} = \frac{N_1}{N_2} = \frac{I_{2i}}{I_{1i}}$$

⇒ example:



⇒ on load:

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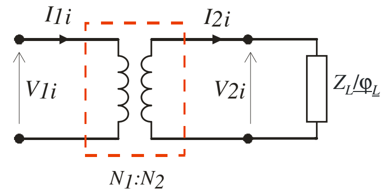
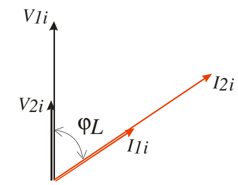


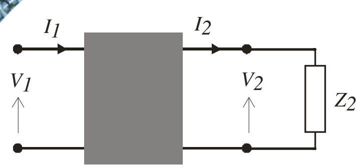


$$I_{2i}=\frac{V_{2i}}{Z_L\angle\varphi_L}\qquad e.g.\ \frac{N_1}{N_2}=2$$

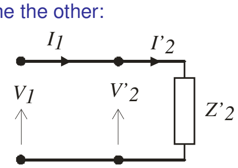
⇒ current & phase both on primary and secondary are determined by the load (not by the supply or power station)

⇒ voltages are determined by the supply or power station

Referred quantities



⇒ referring *V*, *I* and *Z* from one side of trx the the other:



⇒ the dash (‘) is used to indicate a quantity has been referred from one side the other

⇒ what are V'_2, I'_2, and Z'_2 in terms of the original quantities?

$$V'_2=V_1=V_2\frac{N_1}{N_2}\longrightarrow V'_2=V_2\frac{N_1}{N_2}$$

$$I'_2=I_1=I_2\frac{N_2}{N_1}\longrightarrow I'_2=I_2\frac{N_2}{N_1}$$

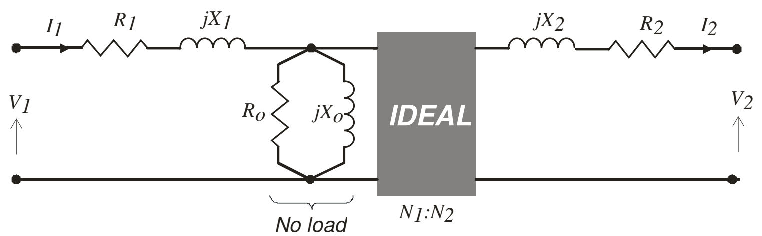
$$Z'_2=\frac{V'_2}{I'_2}=\frac{V_2}{I_2}\frac{N_1/N_2}{N_2/N_1}\longrightarrow Z'_2=Z_2\left(\frac{N_1}{N_2}\right)^2$$

Real transformer

⇒ real (practical) transformer has:

1) leakage flux,
   - $\phi_l \neq 0$
2) finite reluctance of magnetic circuit
   - S = finite ;
3) copper loss ($P_{cu} \neq 0$)
   - $R_1 \neq 0$, $R_2 \neq 0$
4) iron loss ($P_{Fe} \neq 0$)
   - R_p = finite

Equivalent circuit



1) leakage flux is represented by two reactances in the primary and secondary X_1 and X_2; they have small values compared to any other reactances
2) reluctance of mag circuit is not zero, so there is a magnetising flux or mutual flux produced by magnetising current; the effect is represented by a magnetising reactace X_o

$$I_1N_1 - I_2N_2 = \phi_m S$$

ϕ_m is the mutual flux produced by magnetising current

$$\phi_m S = I_{mag} N_1$$

$$I_1N_1 = I_2N_2 + \phi_m S$$

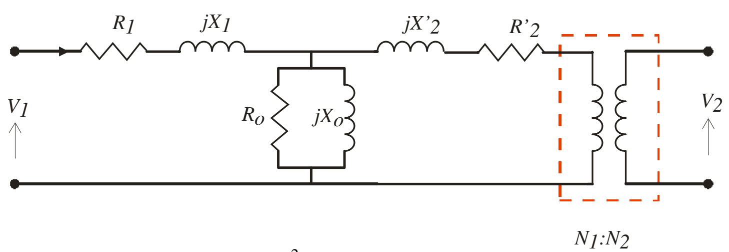
on no-load, I_2 = 0, and

$$I_1 = I_{mag} = \frac{V_1}{jX_o} = \frac{V_1 S}{j\omega N_1^2}$$

3) winding resistance, which incurs P_{cu}, may be represented by the resistances R_1 and R_2,for the primary and secondary, respectively
4) Iron loss, P_{Fe}, is represented by a resistance R_o across the supply terminals

Simplifying equivalent circuit

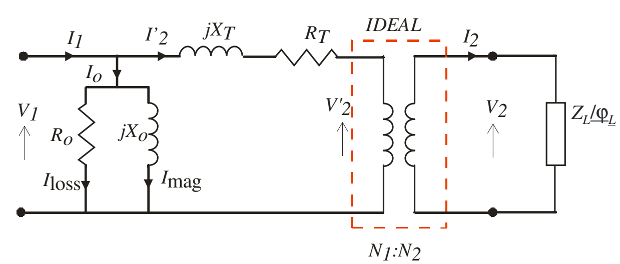
1) Move R_2 and jX_2 from the secondary to primary side



$$R'_2 \text{ as } = R_2\left(\frac{N_1}{N_2}\right)^2$$

$$jX'_2 \text{ as } = X_2\left(\frac{N_1}{N_2}\right)^2$$

2) Change R_o and jX_o across the supply terminals
   - strictly not legitimate, by because series impedances in the circuit are much smaller than parallel components the differences (errors) are insignificant



$$R_T = R_1 + R'_2$$

$$jX_T = j(X_1 + X'_2)$$

⇒ In final equivalent circuit, the designer ensures that

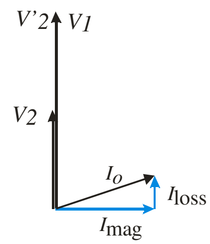
- R_T and X_T (winding resistance and leakage reactance) are small compared to the load impendence Z'_L
- I_o is small, so that R_o and X_o are large (>> Z'_L)
  - Small P_{Fe} gives large R_o
  - Large magnetising inductance L_m or small S gives large X_o

⇒ Example: N_1/N_2 = 2, voltage equation is

$$\overline{V_1} = \overline{V'_2} + R_T \overline{I'}_2 + jX_T \overline{I'}_2$$

Phasor diagrams

On no-load



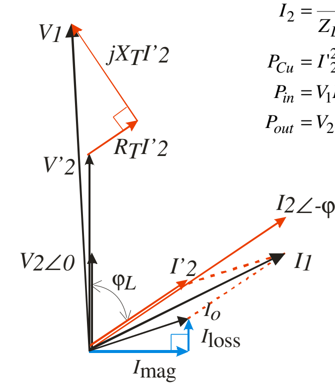
$$I_{Loss} = \frac{V_1}{R_o}$$

$$I_{mag} = \frac{V_1}{X_o}$$

$$I_o^2 = I_{Loss}^2 + I_{mag}^2$$

$$P_{Fe} = \frac{V_1^2}{R_o}$$

On load



$$I_2 = \frac{V_2}{Z_L \angle \varphi_L}$$

$$P_{Cu} = I_2'^2 R_T$$

$$P_{in} = V_1 I_1 \cos \varphi_1$$

$$P_{out} = V_2 I_2 \cos \varphi_L$$

Transformer rating

V_1	V_2	I_1	I_2	VA
$4.44 N_1 A f B_m$				$VA = S_r = V_1 I_1 = V_2 I_2$

⇒ 3 of the 5 quantities are essential to define trx rating, the other 2 can be calculated

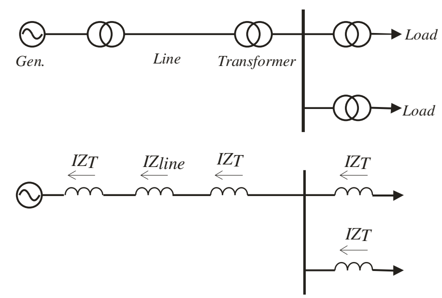
⇒ VA rating for ideal trx is the same on primary as on secondary

⇒ For real trx, the primary and secondary VA are slightly different, but so similar that the there is no distinction between the two

Voltage regulation

⇒ Primary voltage of trx must be such that it delivers rated output power while maintaining rated voltage at secondary

⇒ Due to effect of transformer impendence, secondary voltage may vary from rated value



⇒ Voltage variations are important to consumer because virtually all equipment is designed for some rated voltage

⇒Too high voltage:

- overheating (shortening life of equipment)
- efficiency may suffer

⇒Too Low voltage

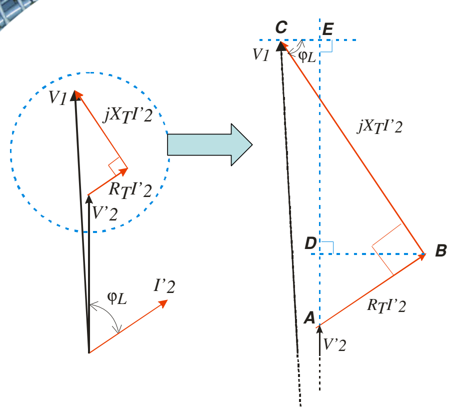
- low power output (light, heat, mechanical power)
- drop in efficiency

⇒ In practice, supply and consumer agree on tolerance of variation e.g. ± 5%

⇒ Regulation definition:

- fractional drop of voltage from no-load to some specified load, while keeping supply voltage constant

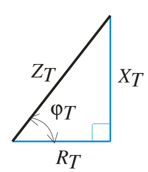
$$\text{Reg} = \left.\frac{|V_{2NL}| - |V_{2L}|}{|V_{2NL}|}\right|_{V_1=\text{const}} = \left.\frac{|V'_{2NL}| - |V'_{2L}|}{|V'_{2NL}|}\right|_{V_1=\text{const}} = \left.\frac{|V_1| - |V'_{2L}|}{|V_1|}\right|_{V_1=\text{const}}$$



$$|V_1| - |V'_{2L}| = OC - OA$$
$$OC \approx OE$$
$$|V_1| - |V'_{2L}| = OE - OA$$
$$= AD + DE$$
$$= I'_2\, R_T \cos\varphi_L + I'_2\, X_T \sin\varphi_L$$

$$\text{Reg} = \frac{|V_1| - |V'_{2L}|}{|V_1|} = \frac{I'_2\, R_T \cos\varphi_L + I'_2\, X_T \sin\varphi_L}{V_1} = \frac{I'_2}{V_1}\left(R_T \cos\varphi_L + I'_2\, X_T \sin\varphi_L\right)$$

⇒ Define Z_T and φ_T , using the impedance triangle (these are constant for a given trx)



$$\text{Reg} = \frac{I'_2}{V_1}\left(Z_T \cos\varphi_T \cos\varphi_L + I'_2\, Z_T \sin\varphi_T \sin\varphi_L\right)$$

$$\text{Reg} = \frac{I'_2\, Z_T}{V_1}\cos(\varphi_T - \varphi_L)$$

⇒ Reg depends on trx parameters (R_T, X_T or Z_T, φ_T) and on load conditions (I_2, φ_L)

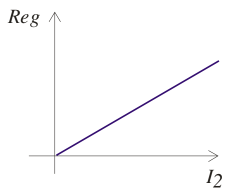
$$\text{Reg} = \frac{I'_2\, Z_T}{V_1}\cos(\varphi_T - \varphi_L)$$

all primary

$$\text{Reg} = \frac{I'_2\, Z_{T\,\text{sec}}}{V_2}\cos(\varphi_{T\,\text{sec}} - \varphi_L)$$

all secondary

Variation of Reg with load current and p.f.

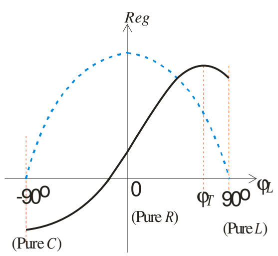


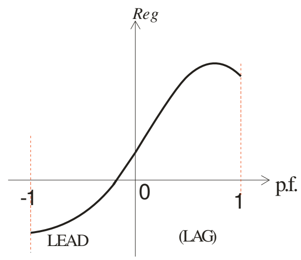
⇒ Max. Reg occurs when $\varphi_L = \varphi_T$ and has value

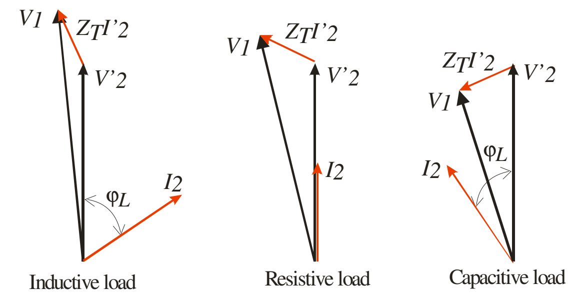
$$\text{Reg} = \frac{I'_2 Z_T}{V_1}$$

⇒ Zero Reg occurs when $\varphi_L - \varphi_T = \pm 90°$

⇒ in terms of p.f., graph is







⇒ eg. $\phi_L = 60° \to$ p.f. = 0.5 (lag)

$\phi_L = -60° \to$ p.f. = 0.5 (lead)

Efficiency

$$\text{power efficiency, } \eta = \frac{Outpt\ \ power,\ \ P_{out}}{Input\ \ power,\ \ P_{in}}$$

$$\eta = \frac{P_{out}}{P_{in}}$$

$$= \frac{P_{out}}{P_{out} + Losses}$$

$$= \frac{V_2 I_2 \cos\varphi_L}{V_2 I_2 \cos\varphi + P_{Fe} + I_2^2 R_{T\sec}}$$

$$\eta = f(I_2, \varphi_L)$$

⇒ define ratio

$$x = \frac{I_2}{I_{2FL}}$$

⇒ x gives the function load current

$$\eta = \frac{V_2 x I_{2FL} \cos\varphi_L}{V_2 x I_2 \cos\varphi_L + P_{Fe} + x^2 I_2^2 R_{T\sec}}$$

$$= \frac{x S_r \cos\varphi_L}{x S_r \cos\varphi_L + P_{Fe} + x^2 P_{CuFL}}$$

Maximum efficiency

1) variable p.f.

$$\eta = \frac{x S_r}{x S_r + \dfrac{P_{Fe} + x^2 P_{CuFL}}{\cos\varphi_L}}$$

⇒ denominator must be minimum, which occurs when $\cos\varphi_L$ is max., ie $\cos\varphi_L = 1$, unity p.f.

2) variable load current, x

$$\eta = \frac{S_r \cos\varphi_L}{S_r \cos\varphi_L + \dfrac{P_{Fe}}{x} + x P_{CuFL}}$$

⇒ denominator must be minimum

$$\frac{d}{dx} = \left(S_r \cos\varphi_L + \frac{P_{Fe}}{x} + x P_{CuFL} \right) = 0$$

$$0 - \frac{P_{Fe}}{x^2} + P_{CuFL} = 0$$

$$x^2 P_{CuFL} = P_{Fe}$$

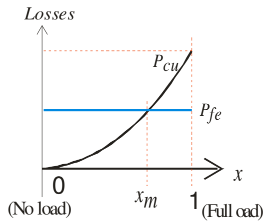
$$P_{Cu} = P_{Fe}$$

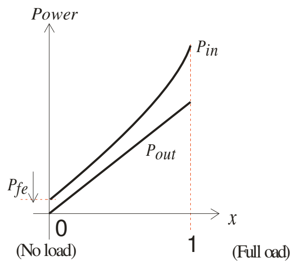
⇒ max η occurs when variable P_{Cu} has the vale of the const P_{Fe}

⇒ max η occurs when

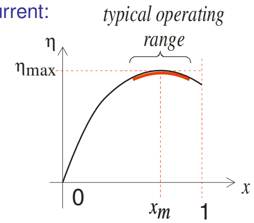
$$x^2 = \frac{P_{Fe}}{P_{CuFL}}$$

$$x = \sqrt{\frac{P_{Fe}}{P_{CuFL}}}$$

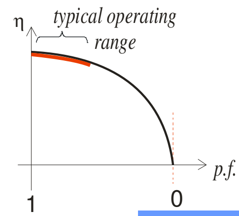




⇒ η with load current:



⇒ η with p.f.:



Waveform of no-load current

$$v = N\frac{d\phi}{dt}$$

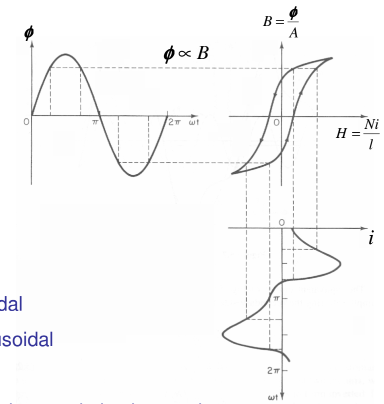
$$\phi = \frac{1}{N}\int v\,dt$$

$$v = V_m \cos\omega t$$

$$\phi = \frac{1}{N}\int V_m \cos\omega t = \frac{V_m}{\omega N}\sin\omega t$$

$$\phi_m = \frac{V_m}{\omega N}$$

$$B_m = \frac{V_m}{\omega NA}$$



⇒ if applied voltage is sinusoidal, flux is sinusoidal

⇒ due to nonlinear *BH* curve, current is nonsinusoidal

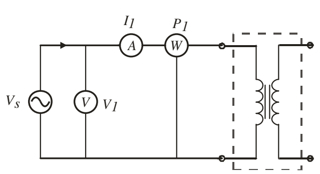
⇒ no-load current is non sinusoidal but periodic

⇒ periodic waveform can be represented as fundamental plus harmonics

⇒ trx no-load current has significant 3rd harmonic

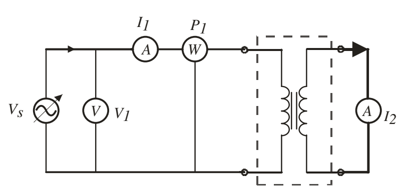
Determination of transformer parameters from tests

Open circuit test



1) Apply V_1 = rated voltage
2) Measure V_1, I_1, P_1

Short circuit test



1) Apply low V_1 gradually until rated current flows
2) Measure V_1, I_1, P_1

$$P_1 = \frac{V_1^2}{R_o} \rightarrow R_o$$

$$I_{Loss} = \frac{V_1}{R_o}$$

$$I_o = \frac{P_1}{V_1}$$

$$I_{mag}^2 + I_{Loss}^2 = I_o^2$$

$$X_o = \frac{V_1}{I_{mag}}$$

⇒ gives R_o and X_o

$$I_1^2 R_T = P_1$$

$$Z_T = \frac{V_1}{I_1}$$

$$Z_T^2 = R_T^2 + X_T^2$$

⇒ gives R_T and X_T