UNIVERSITY OF ZAMBIA

EE321 LECTURE PART II

SICHILALU SAM

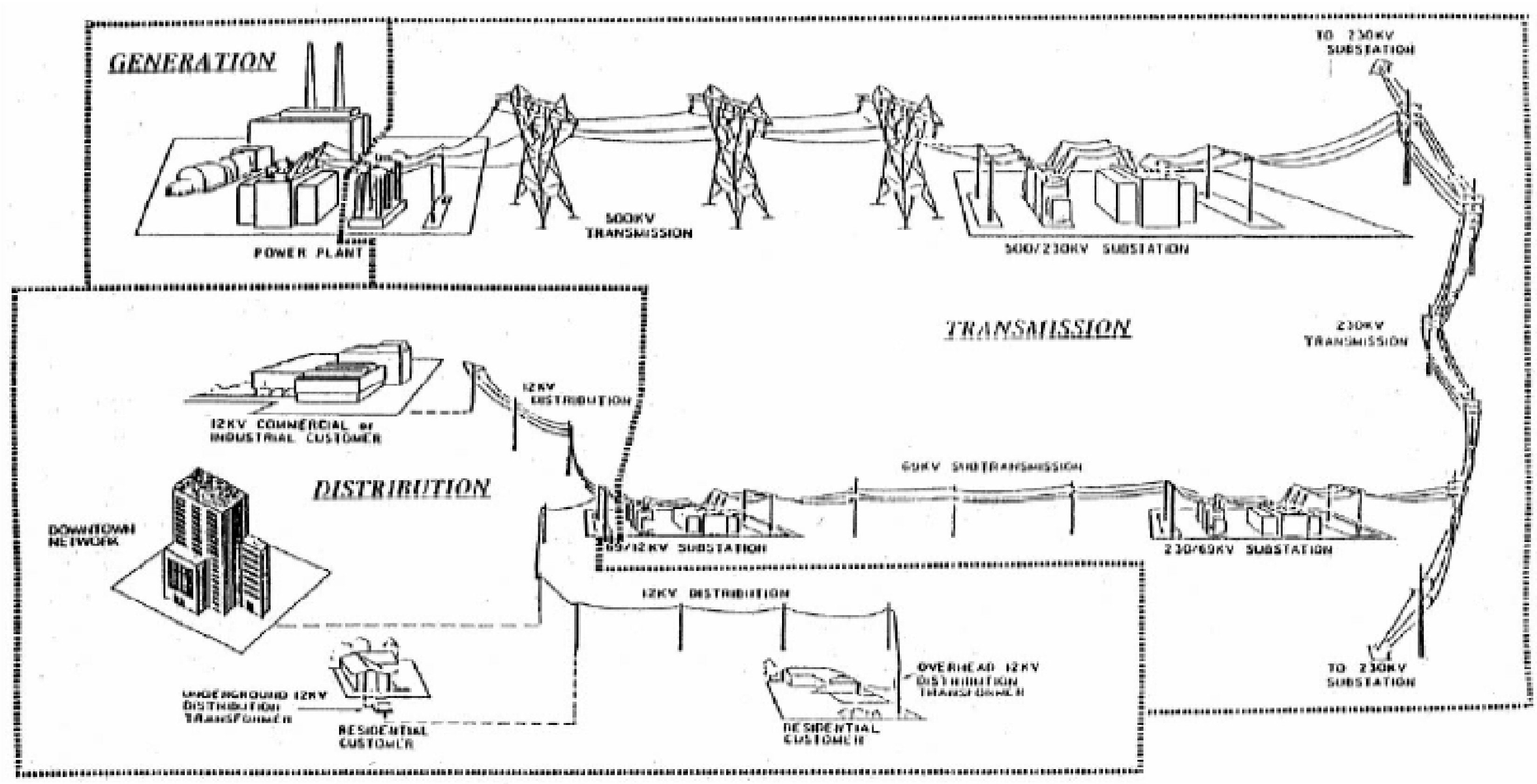
2010

POWER SYSTEMS

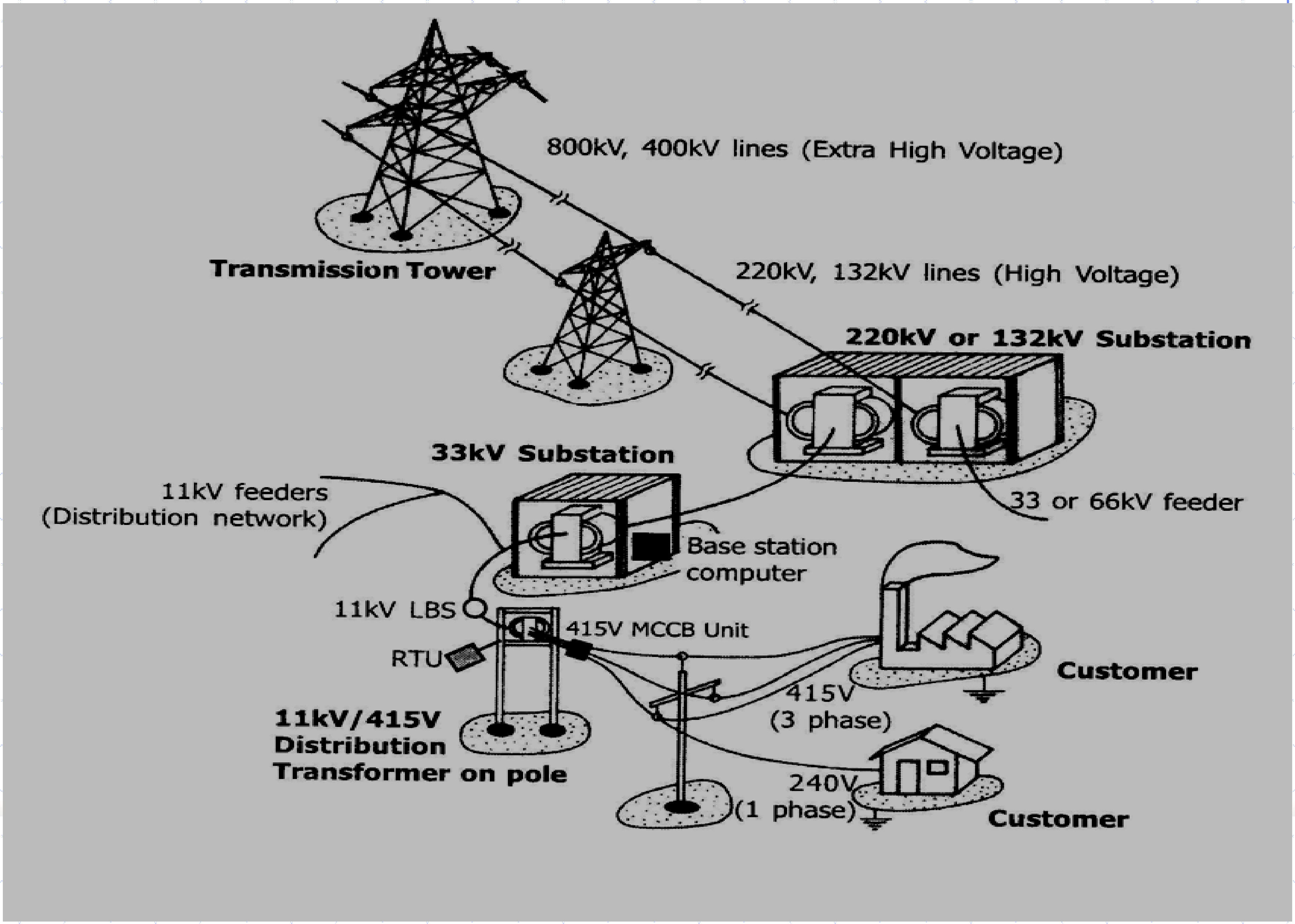
ELECTRIC POWER SYSTEMS

1.Alternator 2. Power transformer 3.Transmission lines

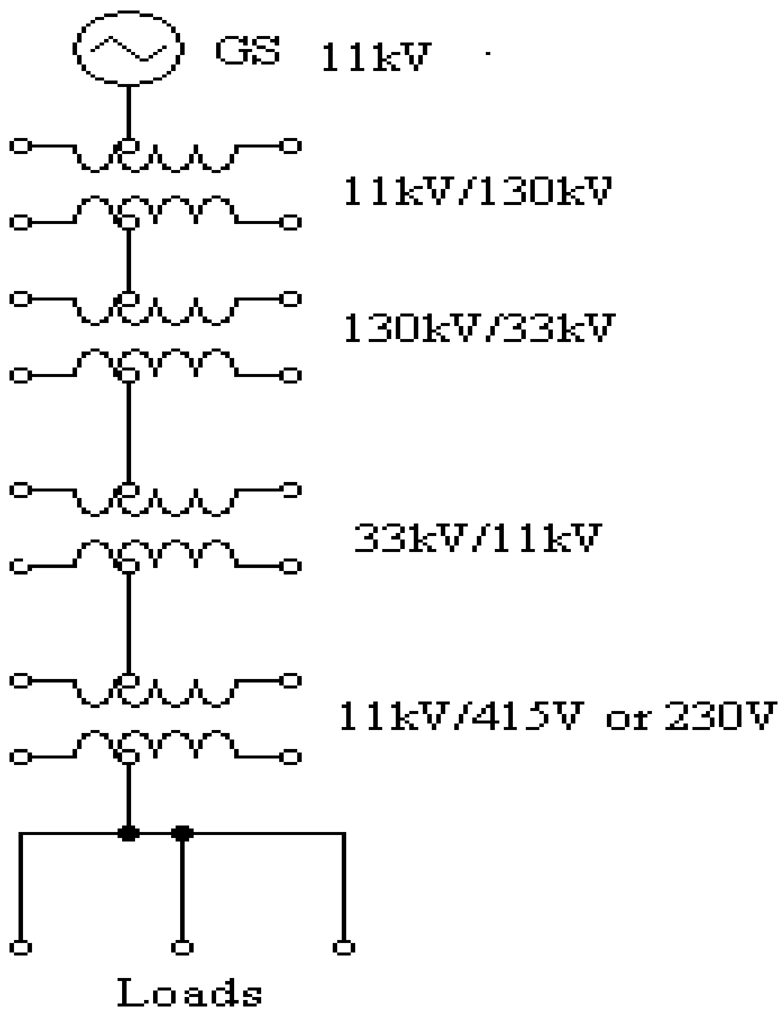
4. Substation transformer 5.Distribution transformer 6.Loads



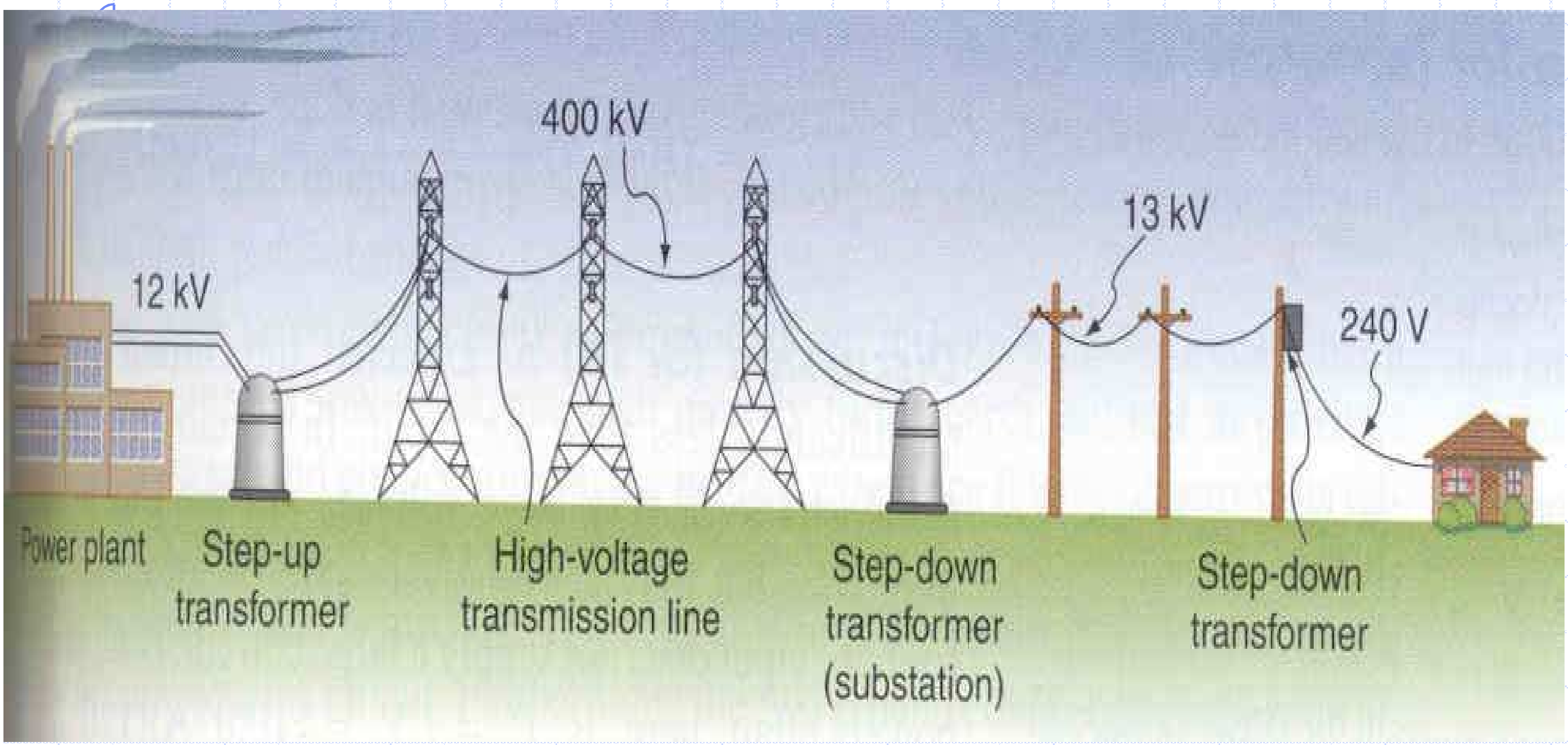
Concept of electric energy transmission.


800kV, 400kV lines (Extra High Voltage)
Transmission Tower
220kV, 132kV lines (High Voltage)
220kV or 132kV Substation
33kV Substation
11kV feeders
(Distribution network)
33 or 66kV feeder
Base station
computer
11kV LBS
415V MCCB Unit
RTU
Customer
415V
(3 phase)
11kV/415V
Distribution
Transformer on pole
240V
(1 phase)
Customer

Power systems and network



Electrical power transmission

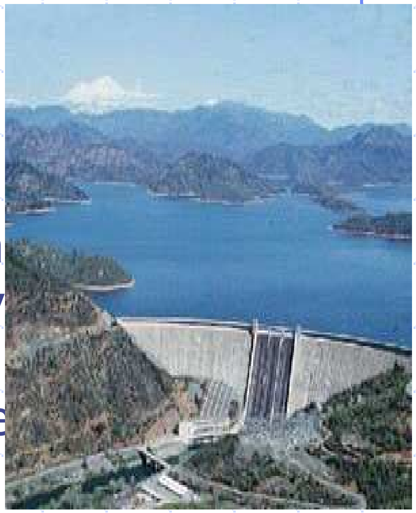


- AC of 50 Hz produced by generator
- Resistance losses are smallest at high voltages and low currents


Electrical Generation

FEATURES OF GENEARATION- SOURCES OF ENERGY, NEEDS AND ELECTRICAL

- Sources : Wood, charcoal, solar, hydropower, nuclear etc…
- Here in zambia, electrical power system is mainly based on hydroelectric power (i.e., from water)
  - The power associated is:
    where ρ: water density= $10^3 kg/m^3$;
    $g = 9.81 m/s^2$;
    Q: water flow rate $[m^3/s]$, and
    H: height [m] .
- Needs/Utilisation: Heating, mechanical power, communication
- Electrical network components: Electricity supply systems hav
power to many types of load.
  - The greater the power supplied, for a given voltage, the
    current.



GENERATION OF THREE PHASE E.M.F

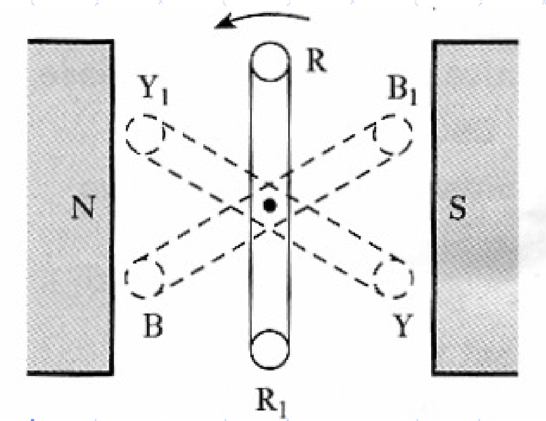


Fig.1: Generation of three-phase e.m.f.s

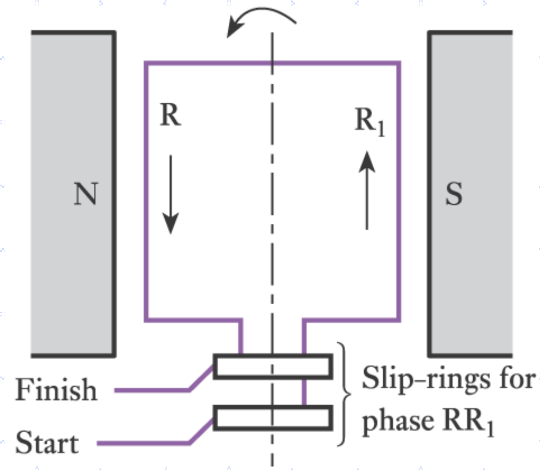
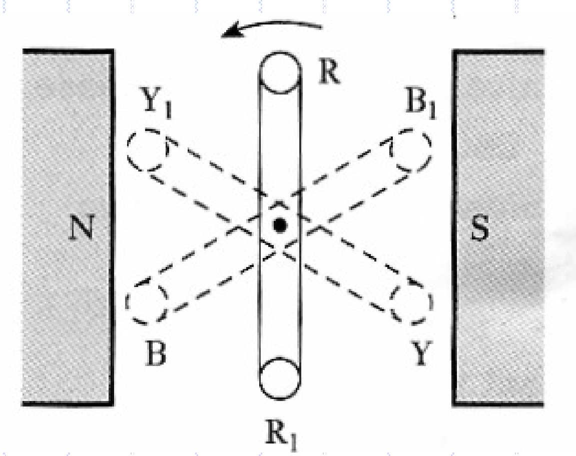
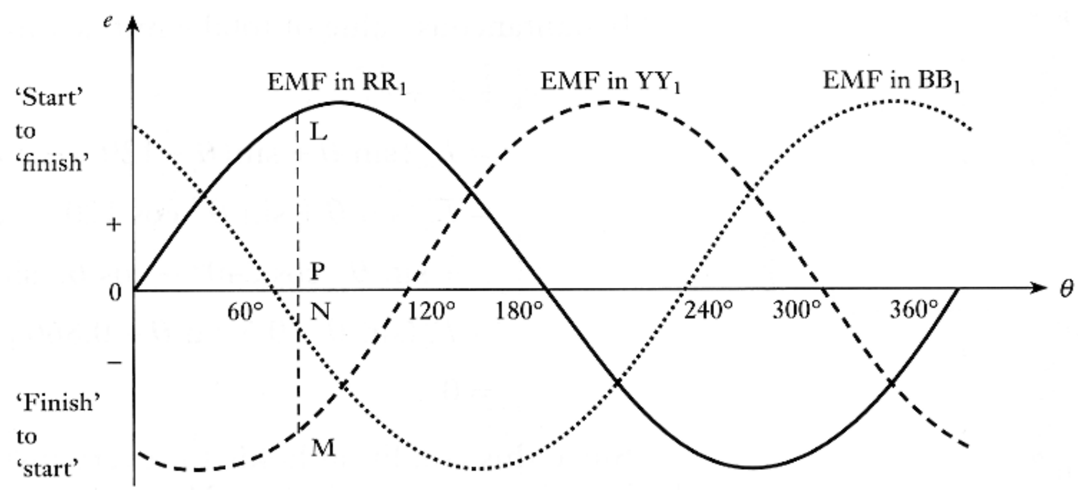


Fig.2: Loop RR_1 at instant of maximum e.m.f.

- In Fig. 4-3.1, RR_1, YY_1 and BB_1 represent three similar loops fixed to one another at angles of 120^0, each loop terminating in a pair of slip-rings carried on the shaft in Fig.4-3.2.
- We shall refer to the slip-rings connected to sides R, Y and B as the 'finishes' of the respective phases and those connected to R_1, Y_1 and B_1 as the 'starts'.

- The letters R, Y and B are abbreviations of 'red', 'yellow' and 'blue', namely the colors used to identify the three phases.
- Also, 'red-yellow-blue' is the sequence that is universally adopted to denote that the e.m.f. in the yellow phase lags that in the red phase by a third of a cycle (120^0), and the e.m.f. in the blue phase lags that in the yellow phase by another third of a cycle.





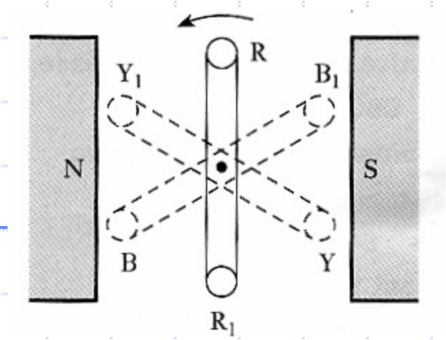


Fig1: Generation of three-phase e.m.f.s

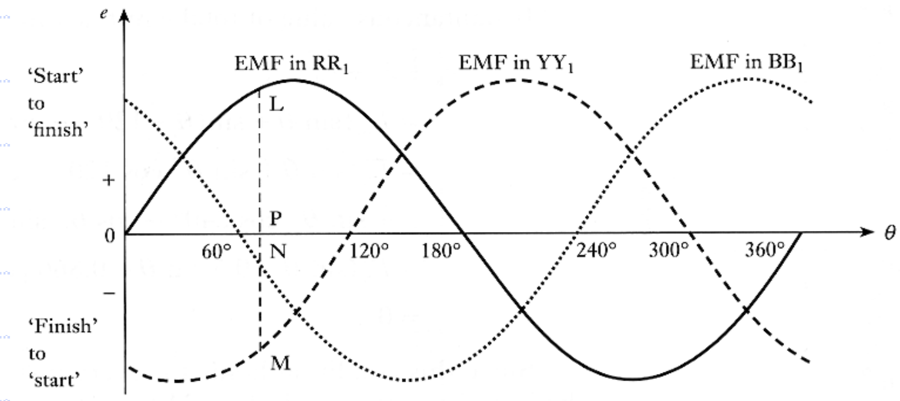


Fig.3: Waveforms of three phase e.m.f.s

- Hence the e.m.f.s generated in loops RR_1, YY_1 and BB_1 are represented by the three equally spaced curves of Fig. 3, the e.m.f.s being assumed positive when their directions round the loops are from 'start' to 'finish' of their respective loops.
- If the instantaneous value of the e.m.f. generated in phase RR_1 is represented by

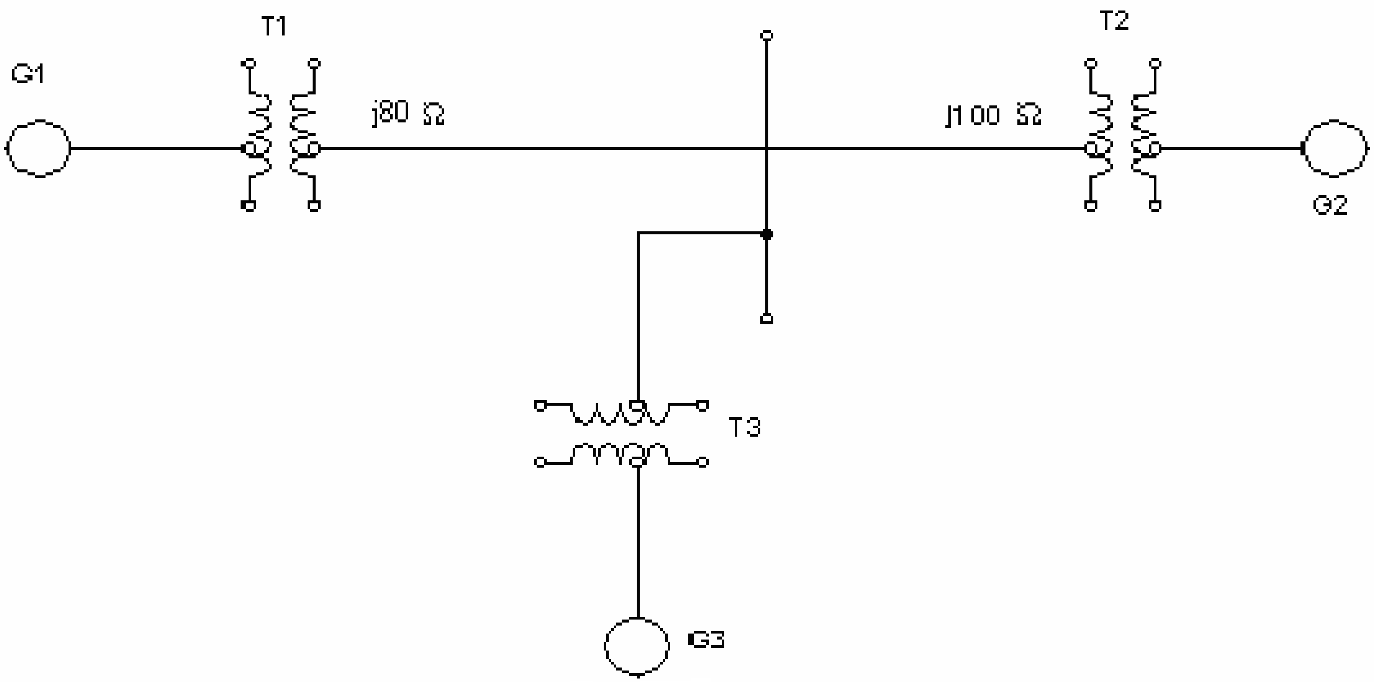
$e_R = E_m \ldots$

then instantaneous e.m.f. in YY_1 is $e_Y = E_m \sin(\ldots)$

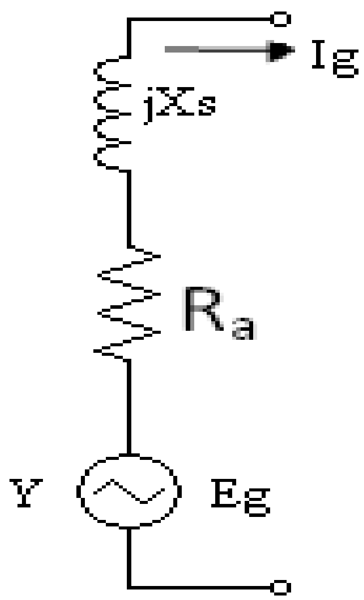
and instantaneous e.m.f. in BB_1 is $e_B = E_m \sin(\ldots)$

SINGLE LINE DIAGRAM

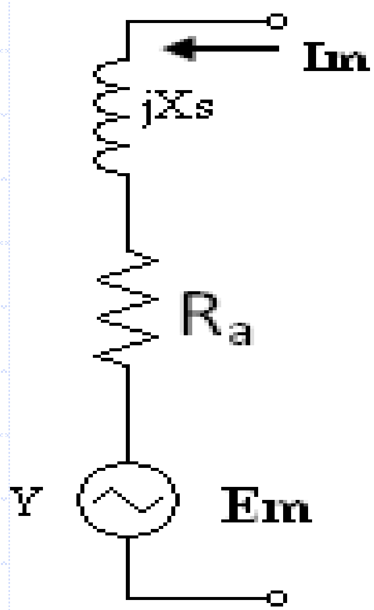
It is a diagrammatic representation of a power system in which the components are represented by their symbols.



MODELLING OF GENERATOR AND SYNCHRONOUS MOTOR

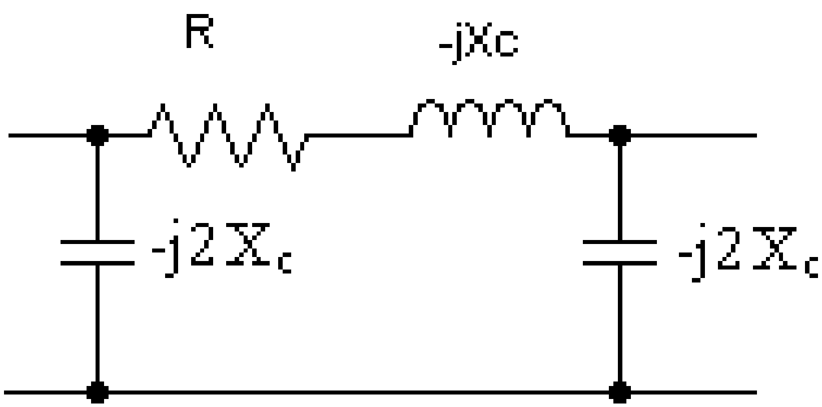


1Φ equivalent circuit of generator

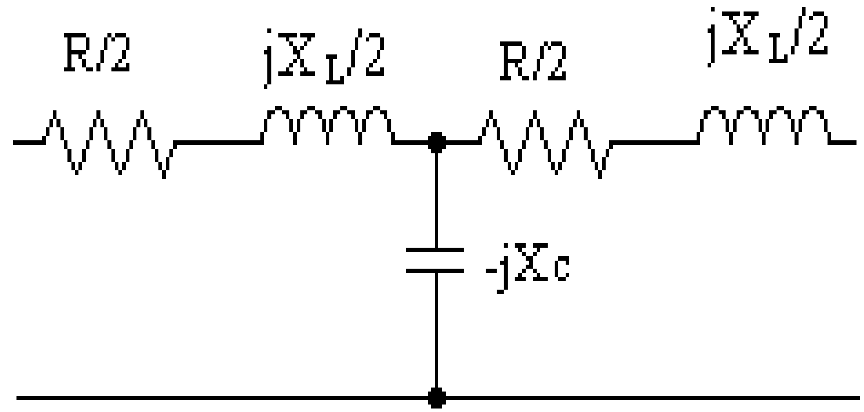


1Φ equivalent circuit of synchronous motor

MODELLING OF TRANSMISSION LINE

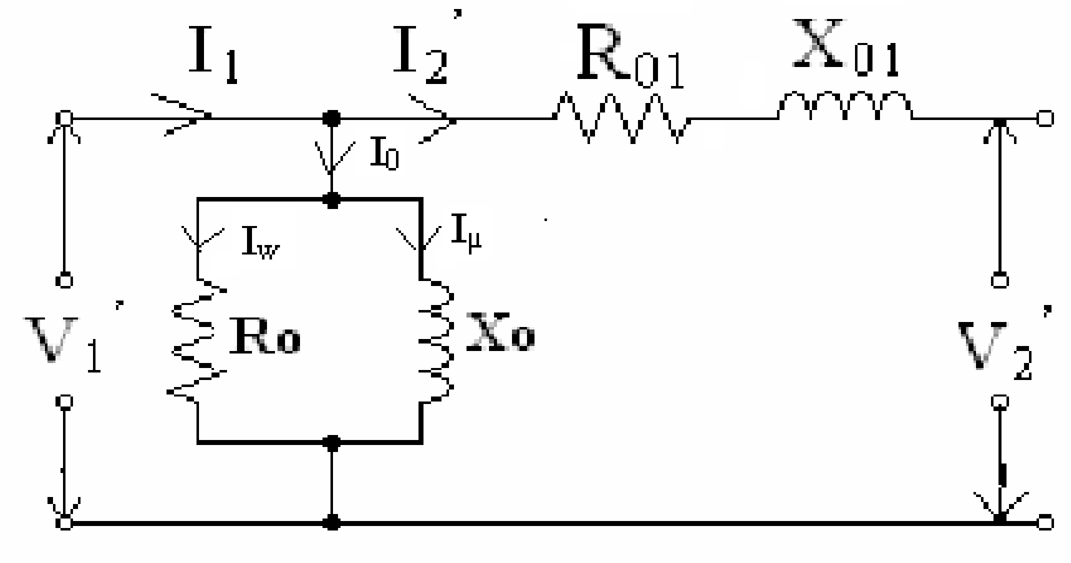


Π type



T type

MODELLING OF TRANSFORMER

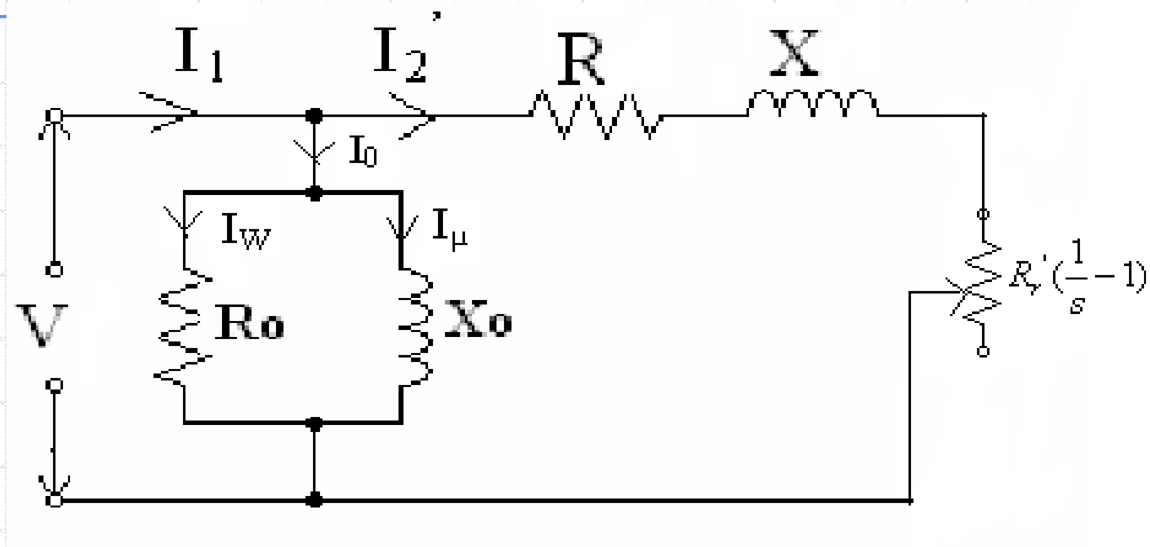


$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$ =Equivalent resistance referred to 1°

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$ =Equivalent reactance referred to 1°

MODELLING OF INDUCTION MOTOR



$R_r'\left(\frac{1}{s}-1\right)$ =Resistance representing load

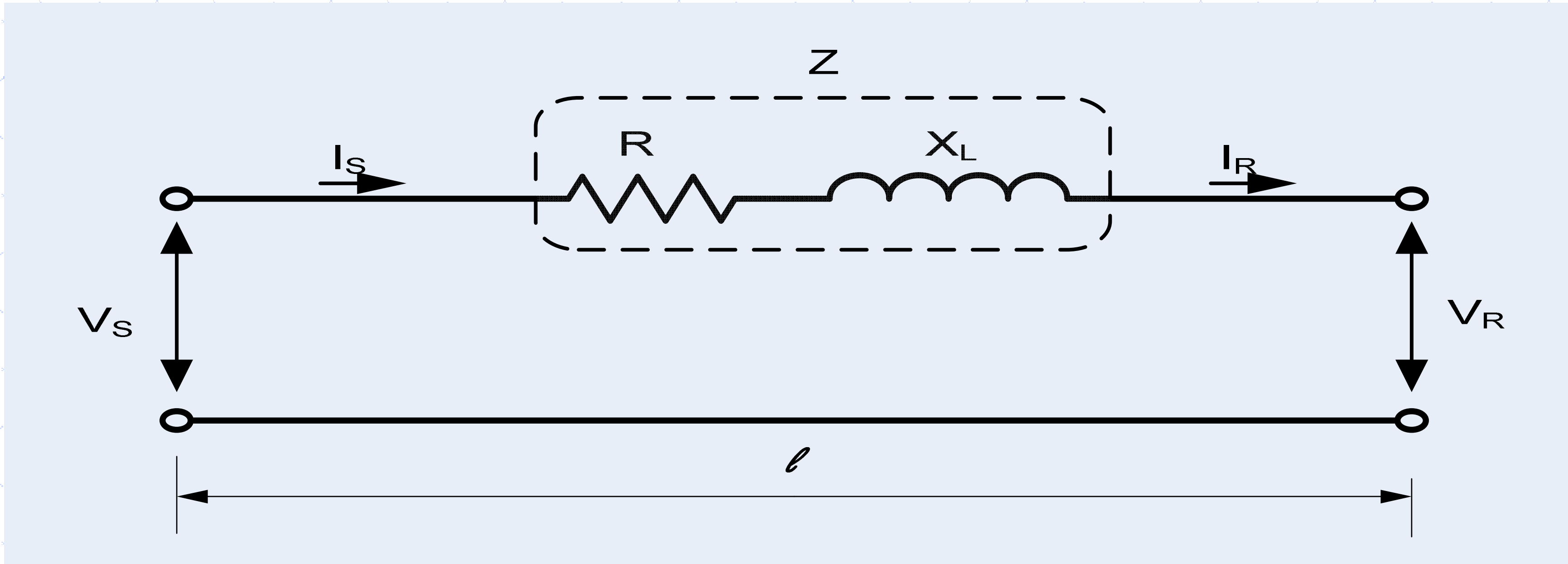
$R = R_s + R_r'$ =Equivalent resistance referred to stator

$X = X_s + X_r'$ =Equivalent reactance referred to stator

Transmission Line Representation

- Short Line Model
  - < 80 km in length
  - Shunt effects are neglected.
- Medium Line Model
  - Range from 80–240 km in length
  - Shunt capacitances are lumped at a few predetermined points along the line.
- Long Line Model
  - >240 km in length.
  - Uniformly distributed parameters.
  - Shunt branch consists of both capacitance and conductance.

Short Line



$$Z = z\ell = (r + j\omega L)\ell$$
$$= R + jX_L$$

$$V_s = I_R Z + V_R$$

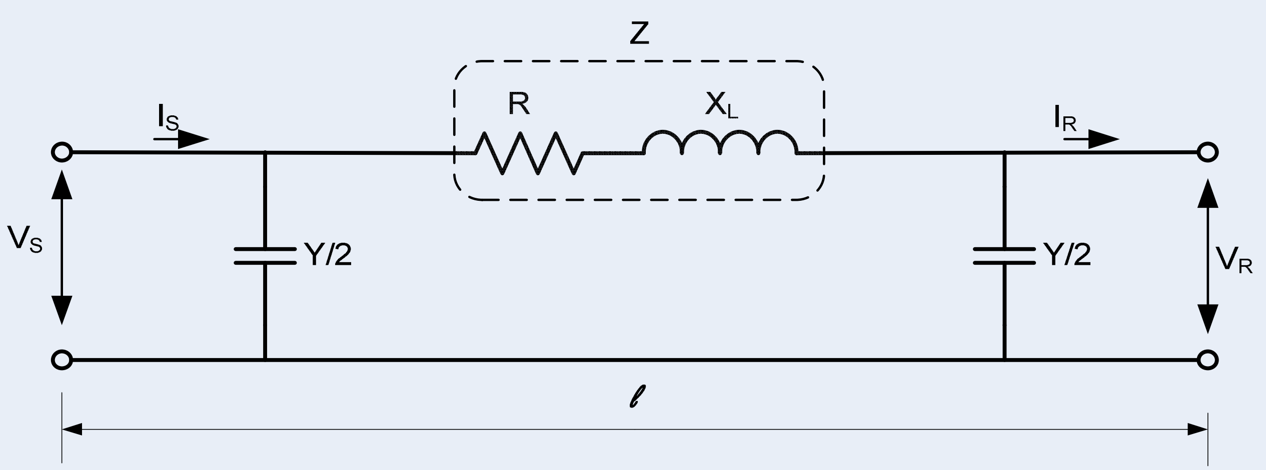
where:

r = per-phase resistance

L = per-phase inductance

ℓ = line length

Medium Line – Nominal π Circuit



Total shunt admittance, Y

$$Y = (g + j\omega C)\ell$$

where :

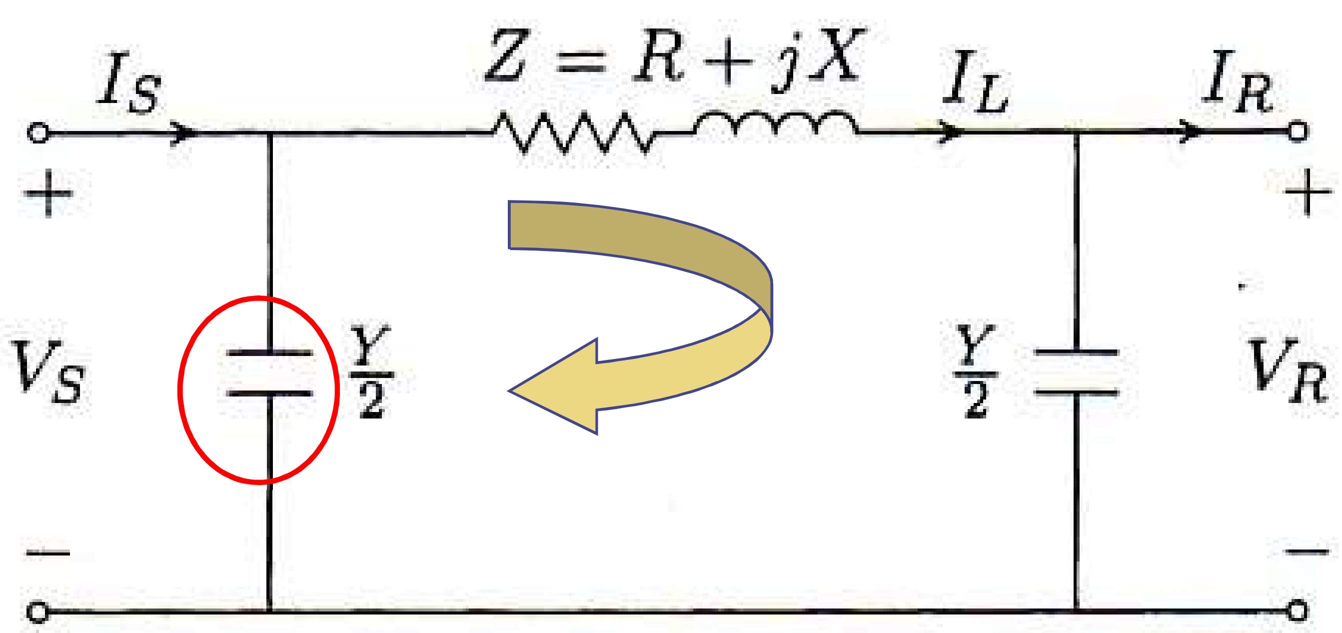
C = line to neutral capacitance per km

g = line conductance per km

ℓ = line length

- Shunt capacitor is considered.
- ½ of shunt capacitor considered to be lumped at each end of the line – π circuit

Medium Line Model



- Using KCL and KVL, the sending–end voltage is:

$$V_S = V_R + ZI_L \qquad ..[1]$$

$$I_L = I_R + \frac{Y}{2}V_R \qquad ..[2]$$

From [1] and [2]

$$V_S = V_R + Z\left(I_R + \frac{Y}{2}V_R\right)$$

$$= \left(1 + \frac{ZY}{2}\right)V_R + ZI_R \qquad ..[3]$$

- Using KCL to obtain equation for sending–end current:

$$I_S = I_L + \frac{Y}{2} V_S \qquad ..[4]$$

Substitute [2] and [3] into [4]

$$I_S = I_R + \frac{V_R Y}{2} + \left[\left(1 + \frac{YZ}{2} \right) V_R + Z I_R \right] \frac{Y}{2}$$

$$= Y \left(1 + \frac{YZ}{4} \right) V_R + \left(1 + \frac{YZ}{2} \right) I_R \qquad ..[5]$$

Complex Power

Remember!

$$\left|V_{line}\right| = \sqrt{3}\left|V_{phase}\right|$$

- Sending end power

$$S_{S(3\phi)} = 3V_{S(phase)} I^{*}_{S(phase)}$$

$$or$$

$$S_{S(3\phi)} = \sqrt{3} V_{S(line)} I^{*}_{S(line)}$$

- Receiving end power

$$S_{R(3\phi)} = 3V_{R(phase)} I^{*}_{R(phase)}$$

$$or$$

$$S_{R(3\phi)} = \sqrt{3} V_{R(line)} I^{*}_{R(line)}$$

Transmission Line Efficiency

- Total Full–Load Line Losses

$$S_{L(3\phi)} = S_{S(3\phi)} - S_{R(3\phi)}$$

- Transmission Line Efficiency

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} \qquad \%\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} \times 100$$

- Note that only Real Power are taken into account!

Example

- A 220-kV, three-phase transmission line is 40 km long. The resistance per phase is 0.15 Ω/km and the inductance per phase is 1.5915 mH/km. The shunt capacitance is negligible.

Use the line model to find the voltage and power at the sending end and efficiency when the line is supplying a three-phase load of

a) 381 MVA at 0.8 pf lagging at 220 kV

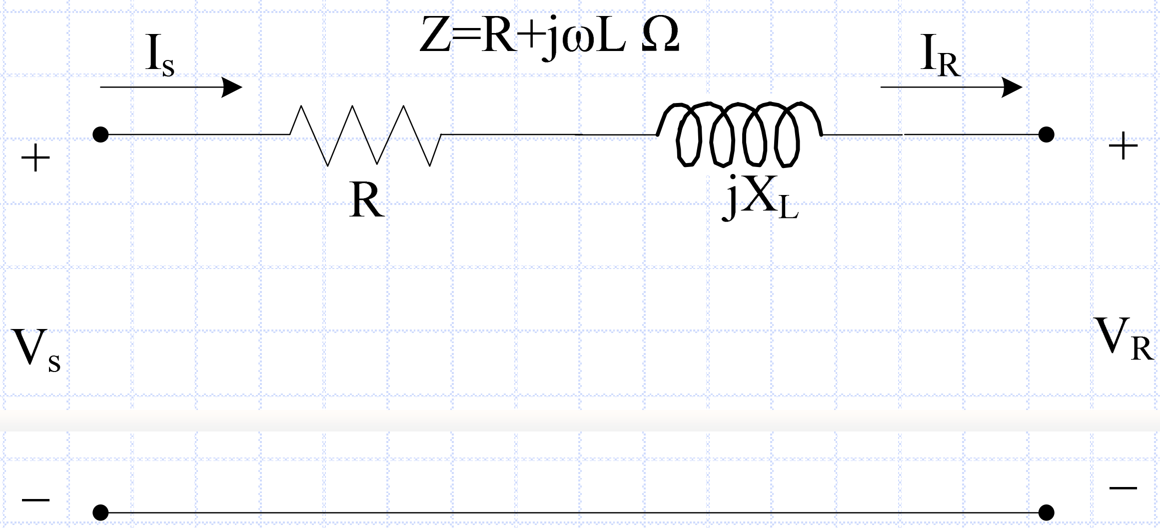
Solution

- Given

  R = 0.15 Ω/km , L = 1.5915 mH/km

  S =381 MVA with pf 0.8 lag

  $V_{R(line)}$=220 kV



Find sending end voltage, $V_S = V_R + ZI_R$

Therefore, find V_R, Z, and I_R

$$V_{R(phase)} = \frac{V_{R(Line)}}{\sqrt{3}}$$

$$= \frac{220\angle 0^o\, kV}{\sqrt{3}}$$

$$= 127\angle 0^o\, kV$$

The series impedance per phase;

$$Z_{40km} = (r + j\omega L)l$$

$$= (0.15 + j(2\pi)(50)(1.5915m))40$$

$$= 6 + j20\Omega$$

$$S = 381 \text{ MVA}, \quad \theta = \cos^{-1} 0.8 = 36.87^o$$

Thus ,

$$S_R = 381 \angle 36.87^o \, MVA = 304.8 MW + j228.6 M \text{ var}$$

$$S_R = 3V_{R(Phase)} I_R^*$$

$$I_R^* = \frac{S_R}{3V_{R(Phase)}}$$

$$I_R = \frac{S_R^*}{3V_{R(Phase)}^*} = \frac{381 \angle -36.87^o \, MVA}{3(127 \angle 0^o \, kV)}$$

$$= 1000 \angle -36.87^o \, A$$

Therefore,

$$\mathrm{V}_{\mathrm{S(Phase)}} = \mathrm{V}_{\mathrm{R(Phase)}} + ZI_R$$

$$= 127\angle 0^o\, kV + (6 + j20\Omega)(1000\angle -36.87^o$$

$$= 144.3\angle 4.93^o\, kV$$

$$\left|V_{S(Line)}\right| = \sqrt{3}\left|V_{S(Phase)}\right|$$

$$= \sqrt{3}\left|144.3\right|$$

$$= 250\mathrm{V}$$

Find Sending - end Power, $S_S = 3V_{S(Line)} I_S$

$$I_S = I_R = 1000 \angle -36.87^o A$$

$$S_S = 3V_{R(Phase)} I_R^*$$

$$= 3 (144.33 \angle 4.93^o V)(1000 \angle 36.87^o A)$$

$$= 322.8 MW + j288.6 M \text{ var}$$

$$= 433 \angle 41.8^o MVA$$

Effiency, η

$$\%\eta = \frac{P_R}{P_S} \times 100$$

$$= \frac{304.8}{322.8} \times 100$$

$$= 94.4\%$$

DELTA CONNECTION OF THREE-PHASE WINDINGS.

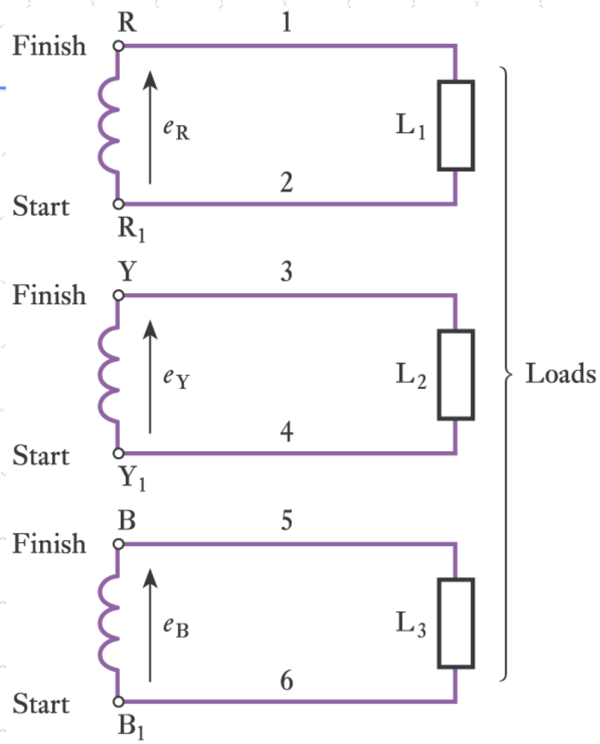


Fig.4-4.1: Three-phase windings with six lines conductors

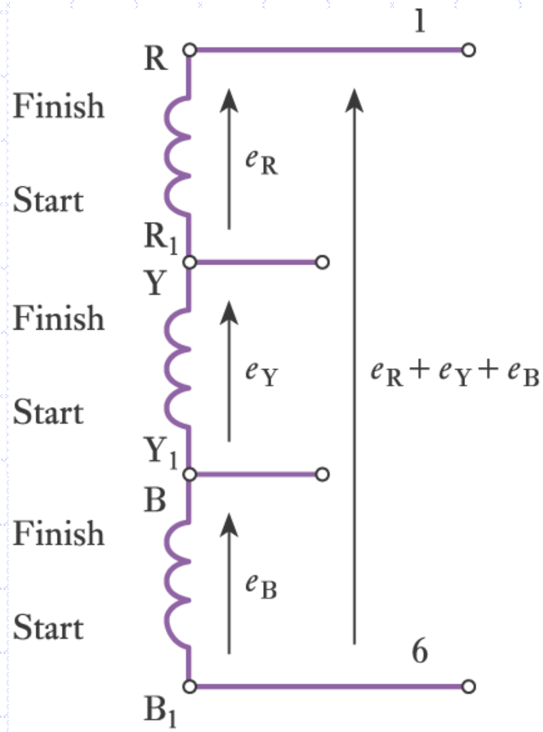
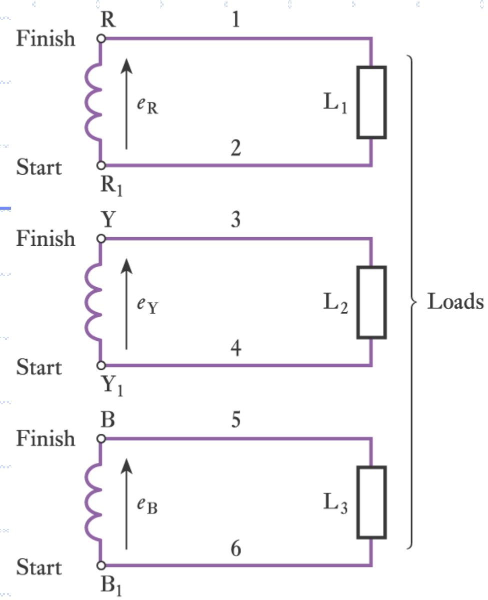


Fig.4-4.2: Resultant e.m.f. in a delta-connected winding

- The three phases can, for convenience, be represented as in Fig. 4-4.4 where the phases are shown isolated from one another; L_1, L_2, and L_3 represent loads connected across the respective phases.

Fig.4-4.3



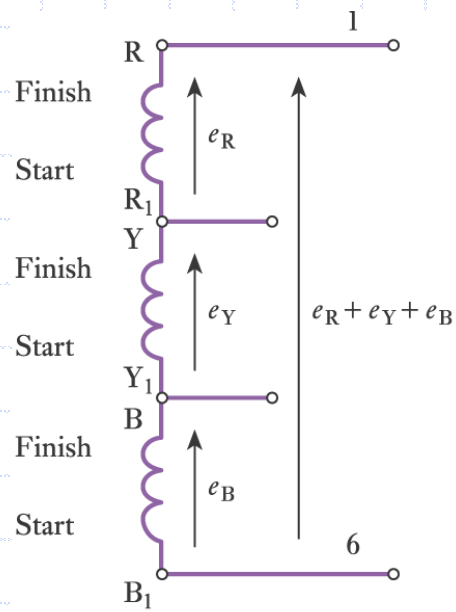


Fig.4-4.4

- Since the e.m.f.s are assumed acting from 'start' to 'finish', they can be represented by the arrows e_R, e_Y and e_B in Fig. 4-4.3.
- This arrangement being cumbersome and expensive, let us consider how it may be simplified.
  - For instance, let us join R_1 and Y together as in Fig. 4-4.4, thereby enabling conductors 2 and 3 of Fig. 4-4.4 to be replaced by a single conductor.
  - Similarly, let us join Y_1 and B together so that conductors 4 and 5 may be replaced by another single conductor.

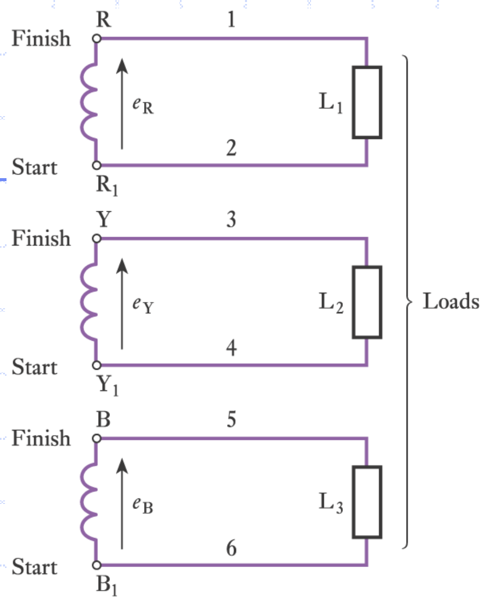


Fig.4-4.3

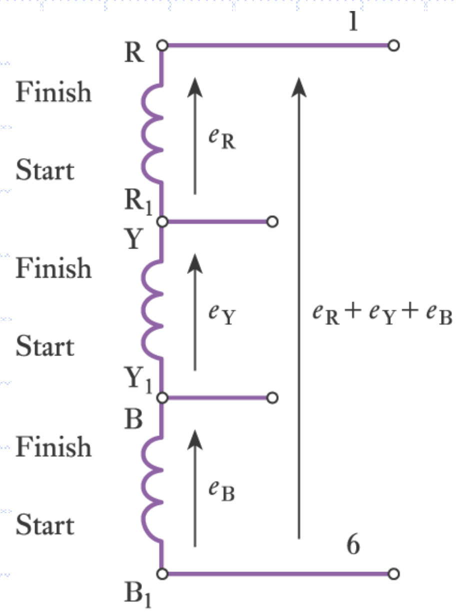


Fig.4-4.4

- If we join 'start' B_1 to 'finish' R, there will be three e.m.f.s chasing each other around the loop and these would produce a circulating current in that loop.

- However, we can next show that the resultant e.m.f. between these two points is zero and that there is therefore no circulating current when these points are connected together.

- Instantaneous value of total e.m.f. acting from B_1 to R is:

$$e_R + e_Y + e_B$$
$$= E_m\{\sin\theta + \sin(\theta - 120°) + \sin(\theta - 240°)\}$$
$$= E_m(\sin\theta + \sin\theta \cdot \cos 120° - \cos\theta \cdot \sin 120° + \sin\theta \cdot \cos 240° - \cos\theta \cdot \sin 240°)$$
$$= E_m(\sin\theta - 0.5\sin\theta - 0.866\cos\theta - 0.5\sin\theta + 0.866\cos\theta)$$
$$= 0$$

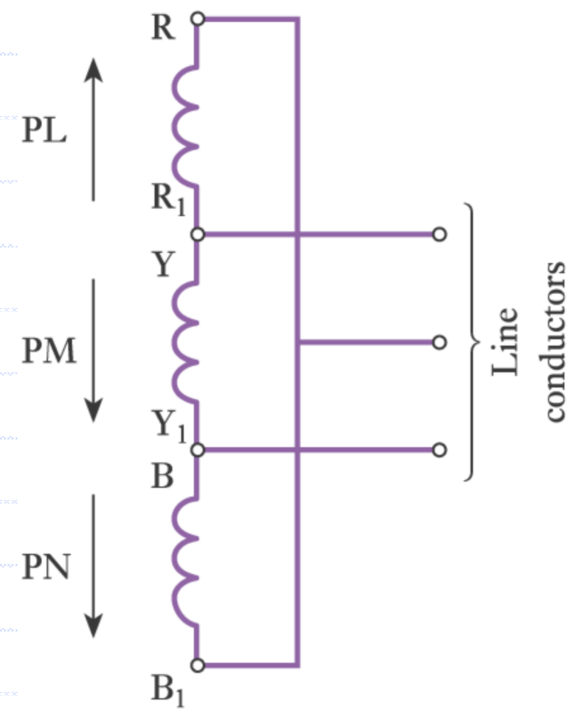


Fig.4-4.5

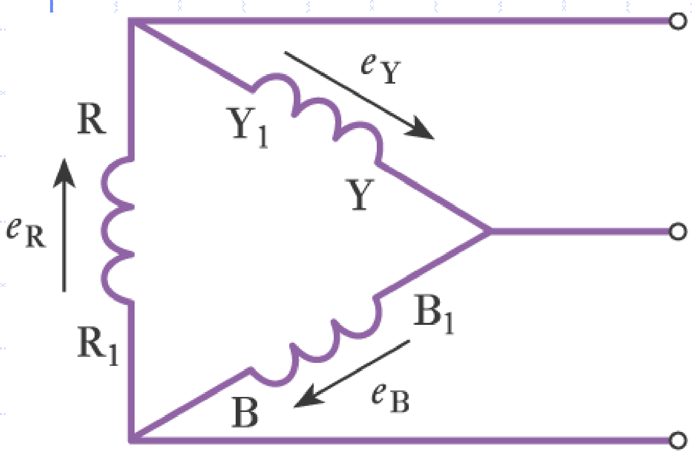


Fig.4-4.6: Conventional representation of **a delta** or **mesh-connected** winding.

- **Since this condition holds for every instant, it follows that R and B_1 can be joined together, as in Fig.4-4.5, without any circulating current being set up around the circuit.**

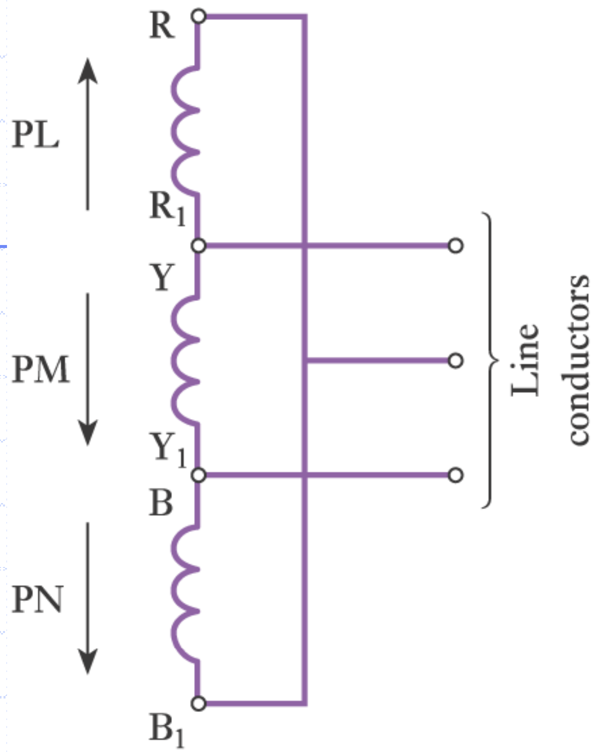


Fig.4-4.5

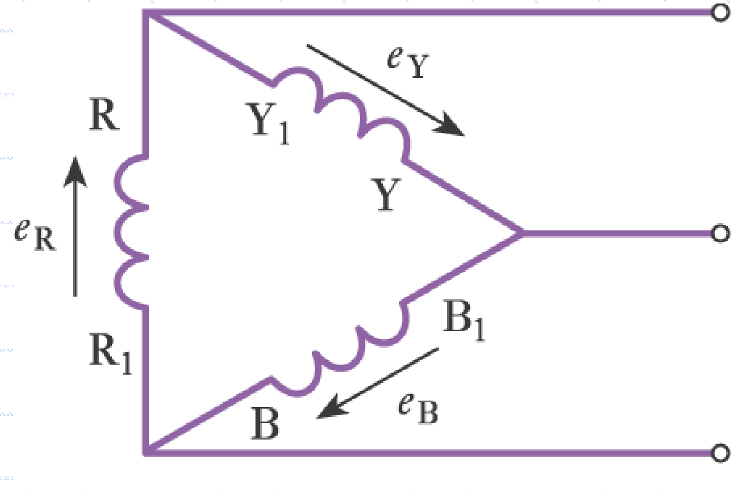


Fig.4-4.6

- By visual inspection, the algebraic sum of the e.m.f.s round the closed circuit formed by the three windings is zero at any instant.

- It should be noted that the directions of the arrows in Fig. 4-4.6 represent the directions of the e.m.f. at a *particular instant*, whereas arrows placed alongside symbole, as in Fig.4-4.5, represent the *positive* directions of the e.m.f.s.

STAR CONNECTION OF THREE-PHASE WINDINGS.

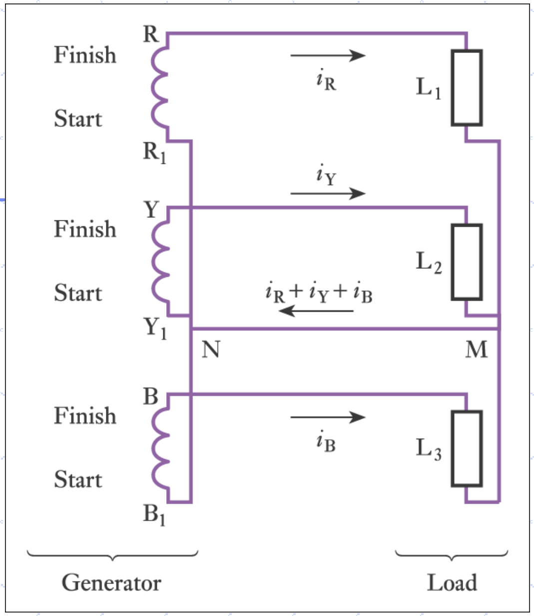


Fig.4-5.1: Star connection of three-phase winding

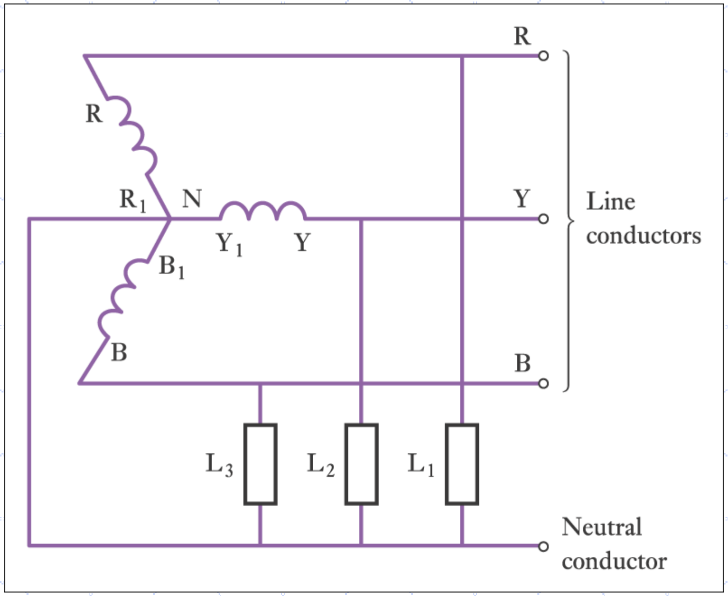


Fig.4-5.2: Four-wire star-connected system

- **Let us go back to Fig. 4-3.4 and join together the three 'starts', R_1, Y_1 and B_1 at N, as in Fig. 4-5.1, so that the three conductors 2, 4 and 6 of Fig.4-4.3 can be replaced by the single conductor NM of Fig.4-5.1.**

- **Since the generated e.m.f. has been assumed positive when acting from 'start' to 'finish, the current in each phase must also be regarded as positive when flowing in that direction, as represented by the arrows in Fig. 4-5.1.**

Fig.4-5.1

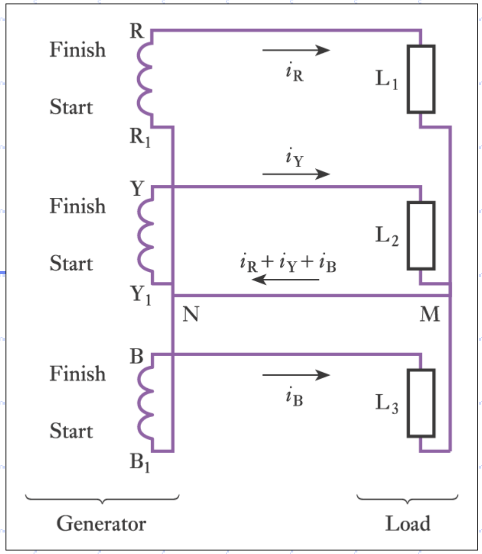
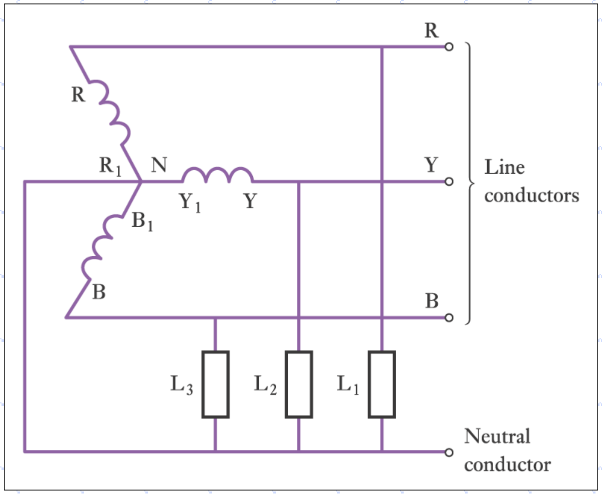


Fig.4-5.2



- If i_R, i_Y and i_B are the instantaneous values of the currents in the three phases, the instantaneous value of the current in the common wire MN is ($i_R + i_Y + i_B$), having its positive direction from M to N.

- This arrangement is referred to as a *four-wire star-connected* system and is more conveniently represented as in Fig.4-5.2, and junction N is referred to as the *star* or *neutral point.*

- Three-phase motors arc connected to the line conductors R, Y and B, whereas lamps, heaters, etc. are usually connected between the line and neutral conductors, as indicated by L_1, L_2 and L_3, total load being distributed as equally as possible between the three lines.

- If these three loads are exactly alike, **the phase currents have the same peak value, 'm and differ in phase by 120^0.**
- Hence if the instantaneous value of the current in **load L_1** is represented by:

$$i_1 = I_m \sin \theta$$

instantaneous current in L_2 is

$$i_2 = I_m \sin(\theta - 120°)$$

and instantaneous current in L_3 is

$$i_3 = I_m \sin(\theta - 240°)$$

- Hence instantaneous value of **the resultant current in neutral conductor MN** is:

$$\begin{aligned} & i_1 + i_2 + i_3 \\ & \quad = I_m\{\sin \theta + \sin(\theta - 120°) + \sin(\theta - 240°)\} \\ & \quad = I_m \times 0 = 0 \end{aligned}$$

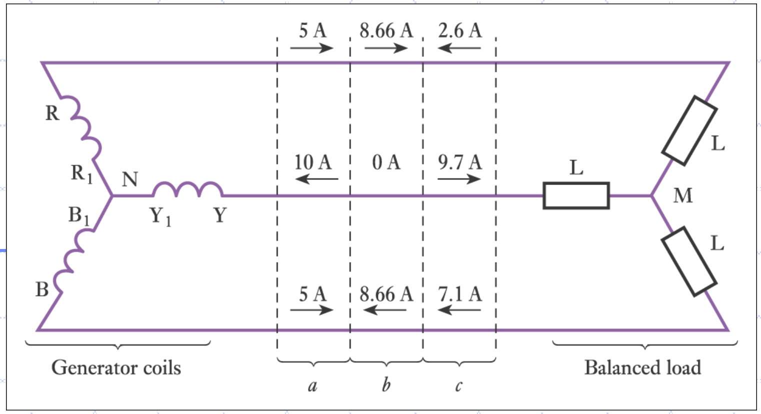


Fig.4-5.3: Three-wire star- connected system with balanced load

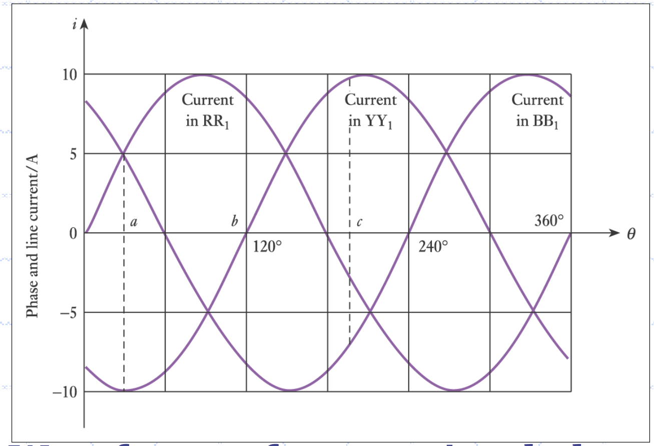


Fig.4-5.4: Waveforms of current in a balanced 3-φ system

i.e. with a balanced load the resultant current in the neutral conductor is zero at *every* instant; hence this conductor can be dispensed with, thereby giving us the *three-wire star-connected* system shown in Fig.4-5.3

- **When we are considering the distribution of current in a three-wire, three-phase system it is helpful to bear in mind:**
  - **That arrows such as those of Fig.4-5.1, placed alongside *symbols*, indicate the direction of the current when it is assumed to be *positive* and not the direction at a particular instant.**
  - **That the current flowing outwards in one or two conductors is equal to that flowing back in the remaining conductor or conductors (see Fig.4-5.4)**

VOLATGES AND CURRENTS IN STAR CONNECTED SYSTEM.

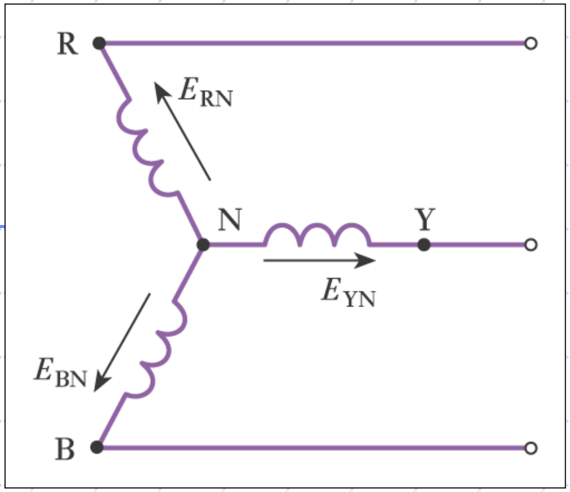


Fig.4-6.1: Star-connected generator

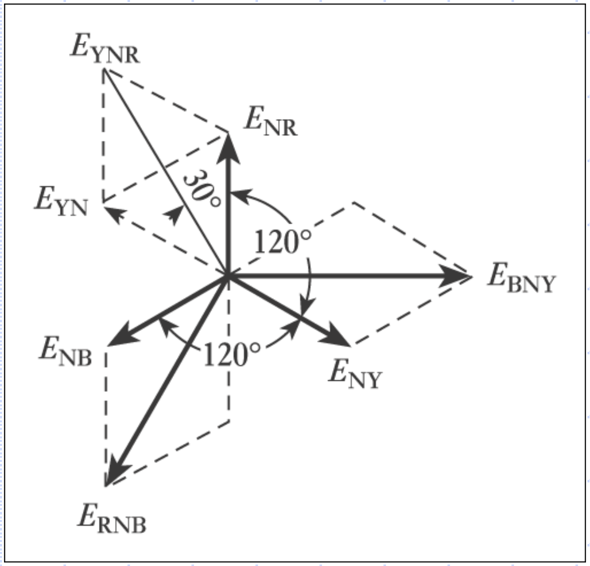


Fig.4-6.2: Phasor diagram

- **Let us again assume the e.m.f. in each phase to be positive when acting from the neutral point outwards, so that the r.m.s. values of the e.m.f.s generated in the three phases can be represented by E_{NR}, E_{NY} and E_{NB} in Figs.4-6.1 and 4-6.2.**

- **When the relationships between line and phase quantities are being derived for either the star- or the delta—connected system, it is essential to relate the phasor diagram to a circuit diagram and to indicate on each phase the direction in which the voltage or current is assumed to be positive. A phasor diagram by itself is meaningless.**

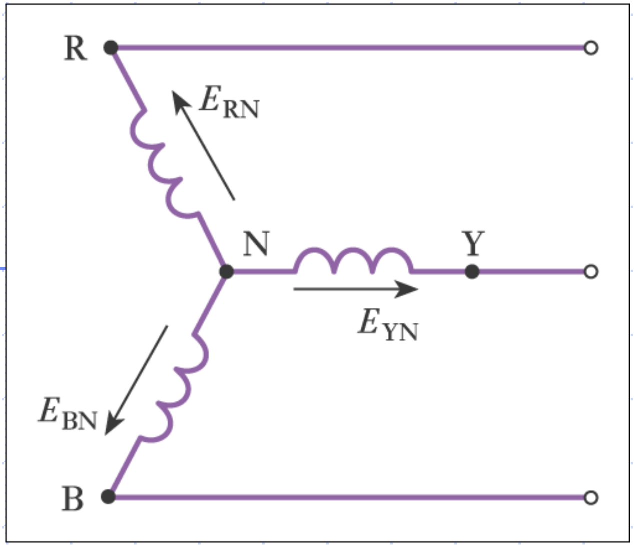
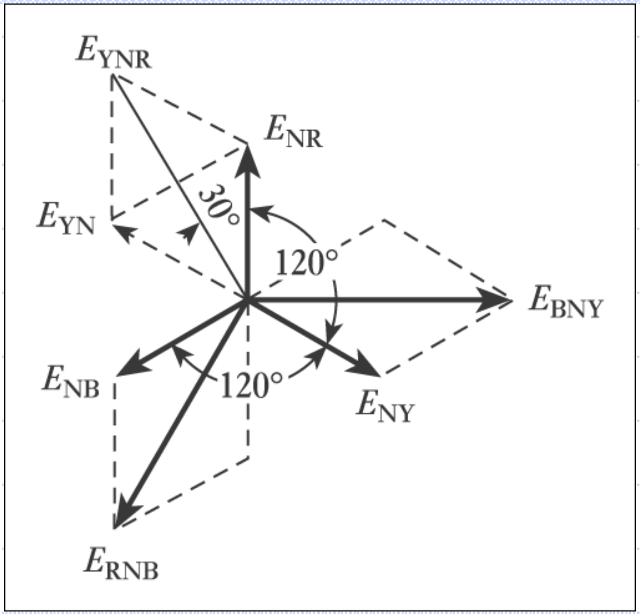


Fig.4-6.1.

Fig.4-6.2



- Here E_{RNB} is obtained by subtracting E_{NR} from E_{NB}, and E_{BNY} is obtained by subtracting E_{NB} from E_{NY}, as shown in Fig.4-6.2.

- From the symmetry of this diagram it is evident that the line voltages are equal and are spaced 120^0 apart.

- Further, since the sides of all the parallelograms are of equal length, the diagonals bisect one another at right angles. Also, they bisect the angles of their respective parallelograms; and, since the angle between E_{NR} and E_{YN} is 60°

$$\therefore \quad E_{YNR} = 2E_{NR}\cos 30° = \sqrt{3}E_{NR}$$

i.e. Line voltage = 1.73 × star (or phase) voltage

Fig.4-6.1.

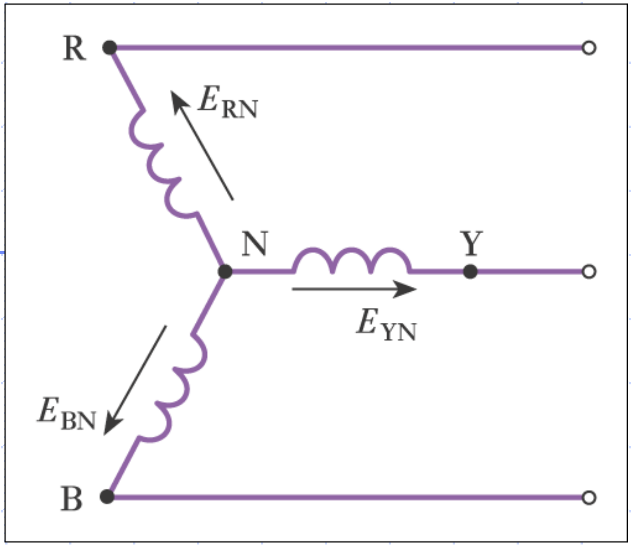
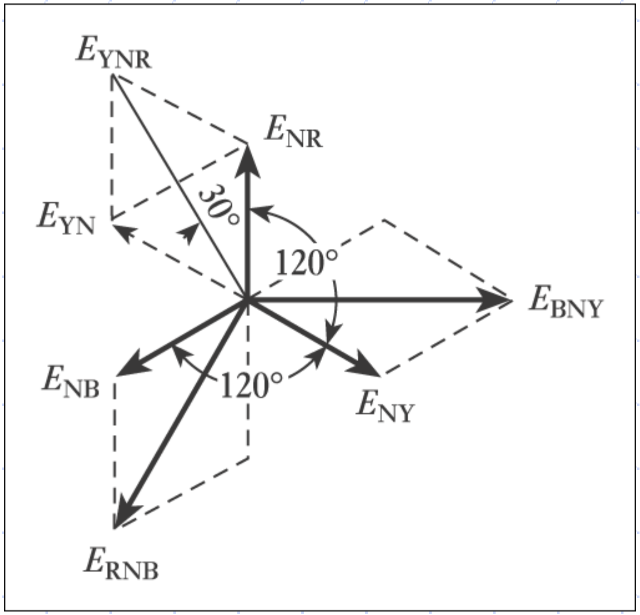


Fig.4-6.2



From Fig.4-6.1 it is obvious that in a star-connected system the current in a line conductor is the same as that in the phase to which that line conductor is connected.

Hence, in general, if

V_L = p.d. between any two line conductors

= line voltage

and

V_P = p.d. between a line conductor and the neutral point

= star voltage (or voltage to neutral)

and if I_L and I_p are **line and phase currents** respectively, **then for a star-connected system**

[illegible]

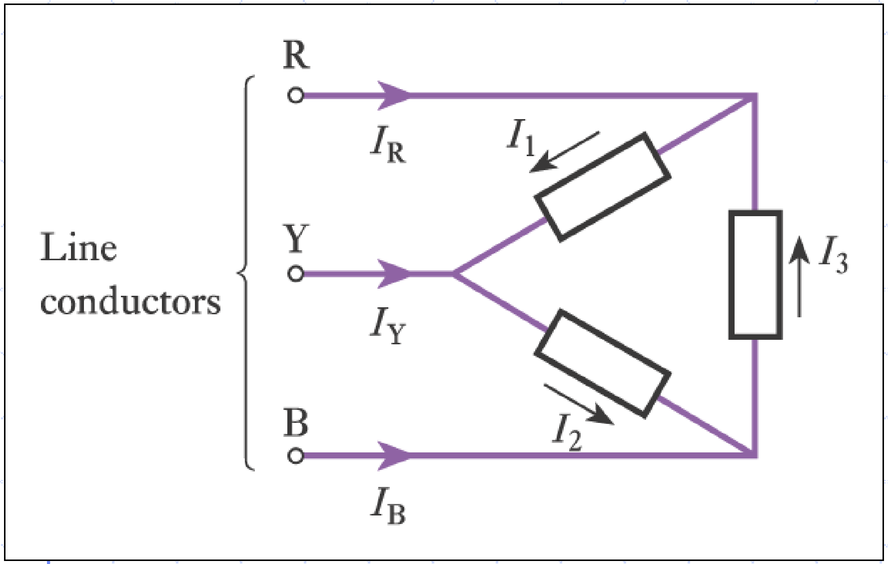
[illegible]

- In practice, it is the **voltage between two line conductors or between a line conductor and the neutral point** that is measured.

- Owing to the internal impedance drop in the windings, **this p.d. is different from the corresponding e.m.f. generated in the winding**, except when **the generator is on open circuit**; hence, **in general, it is preferable to work with the potential difference, *V*, rather than with the e.m.f., *E*.**

- The **voltage given for a three-phase system** is **always the line voltage** unless it is stated otherwise.

VOLATGES AND CURRENTS IN DELTA COONECTED SYSTEM.



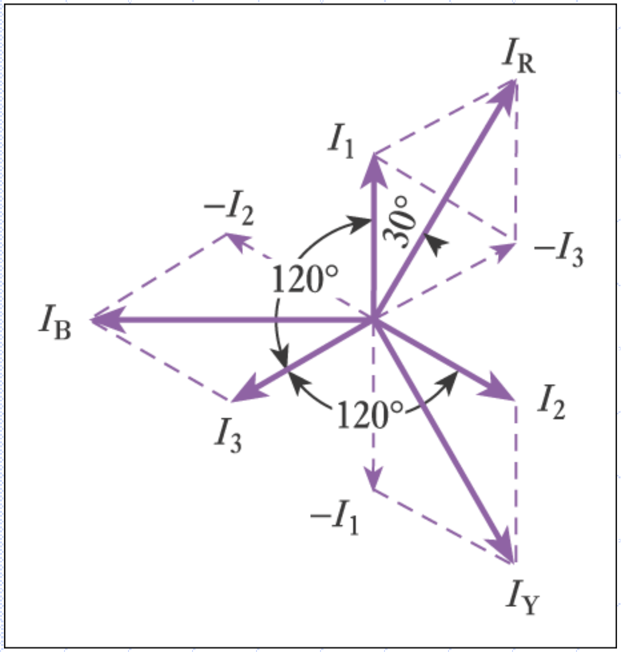
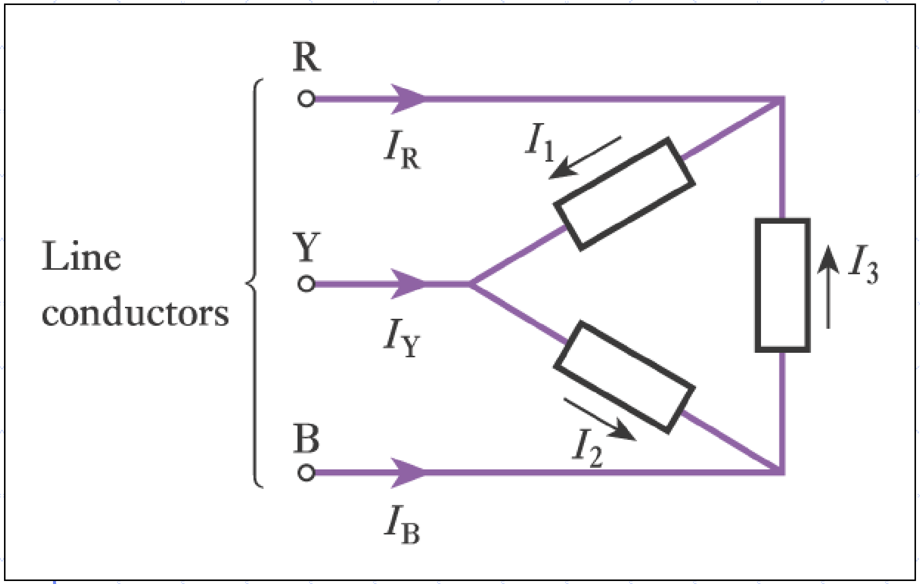
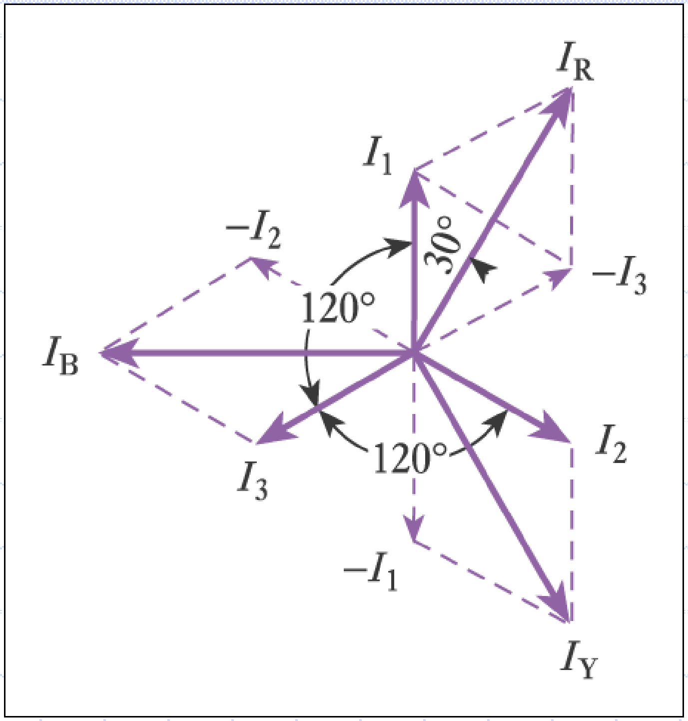


Fig.4-7.1: Delta-connected system with balanced load

- **In Fig. 4-7.1, the load is assumed to be balanced, hence these currents are equal in magnitude and differ in phase by 120°, as shown in Fig.4-7.2.**

- **From Fig.4-7.1 it will he seen that I_1, when positive, flows away from line conductor R, whereas I_3, when positive, flows towards it.**





- Consequently, I_R is obtained by **subtracting I_3 from I_1**, as in **Fig.4-15.**

- Similarly, I_Y **is the phasor difference of** I_2 and I_1, and I_B **is the phasor difference of I_3 and I_2.**

- From **Fig.4-15,** it is **evident that the line currents are equal in magnitude and differ in phase by 120°.**

- **Also**

$$I_R = 2I_1 \cos 30° = \sqrt{3}I_1$$

Hence for a delta-connected system with a balanced load

Line current = 1.73 × phase current

i.e.

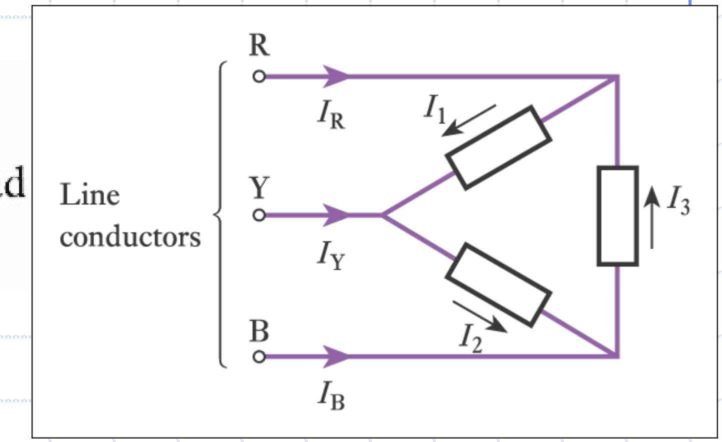


Fig.4-7.1

- **From Fig.4.1.4 it can be seen that in a delta-connected system, the line and the phase voltages are the same, i.e.**

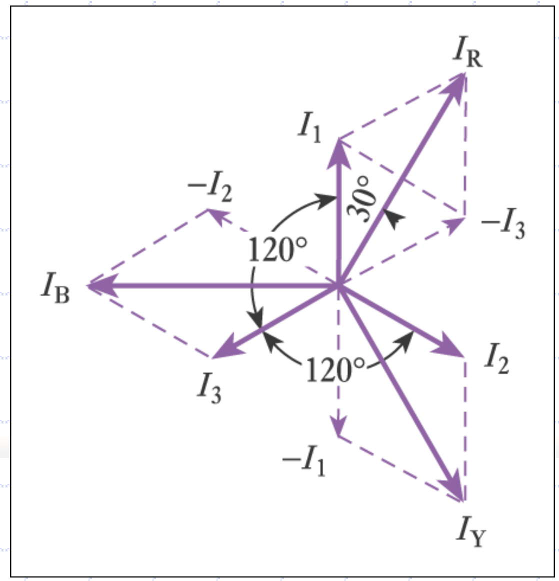


Fig.4-7.2

POWER IN A THREE-PHASE SYSTEM WITH A BALANCED LOAD.

- **If I_p is the r.m.s. value of the current in each phase and V the r.m.s. value of the p.d. across each phase,**

$$\text{Active power per phase} = I_P V_P \times \text{power factor}$$

and

$$\text{Total active power} = 3 I_P V_P \times \text{power factor}$$

p P

If I_L and V_L are the r.m.s. values of the line current and voltage respectively, then for a *star-connected system*,

$$V_P = \frac{V_L}{1.73} \quad \text{and} \quad I_P = I_L$$

- **Substituting for I_p and V_p, we get**

$$\text{Total active power in watts} = 1.73 I_L V_L \times \text{power factor}$$

For a *delta-connected system*

$$V_P = V_L \quad \text{and} \quad I_P = \frac{I_L}{1.73}$$

POWER IN A 3-ϕ SYSTEM WITH A BALANCED LOAD-Cntd.

Again, substituting for I_P and V_P

$$\text{Total active power in watts} = 1.73 I_L V_L \times \text{power factor}$$

Hence it follows that, for any balanced load,

$$\text{Active power in watts} = 1.73 \times \text{line current} \times \text{line voltage} \times \text{power factor} = 1.73 I_L V_L \times \text{power factor}$$

$$P = \sqrt{3}\, I_L V_L \cos\phi$$

MEASUREMENT OF ACTIVE POWER IN A THREE-PHASE THREE WIRES SYSTEM.

STAR-CONNECTED BALANCED LOAD,WITH NEUTRAL POINT ACCESSIBLE

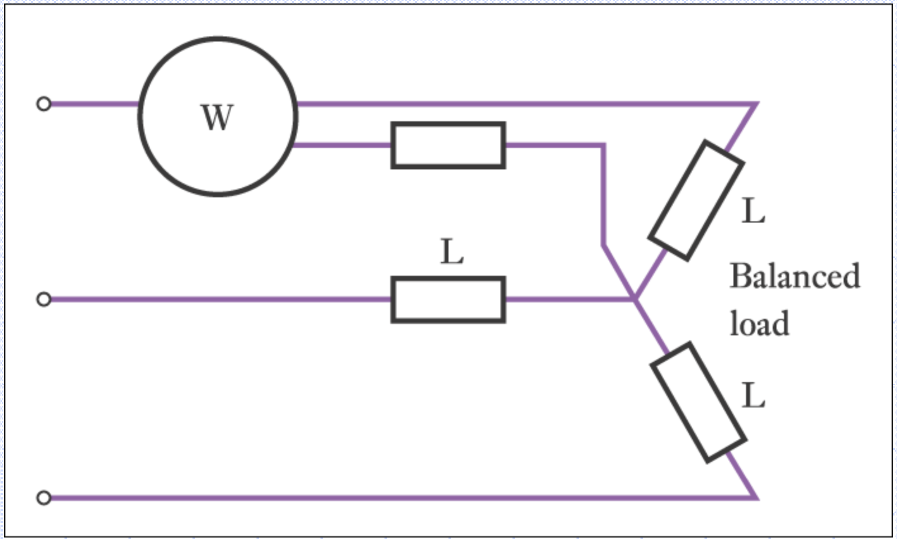


Fig.4-9.1:Measurement of active power in a star-connected balanced load

- If a wattmeter W is connectcd with its current coil in one line and the voltage circuit between that line and the neutral point, as shown in Fig.4-9.1, the reading on the wattmeter gives the power per phase:

$$\therefore \quad \text{Total active power} = 3 \times \text{wattmeter reading}$$

4.9.2 BALANCED OR UNBALANCED LOAD, STAR-OR DELTA-CONNECTED, THE TWO-WATTMETER METHOD.

- Suppose the three loads L_1, L_2 and L_3 are connected in star, as in Fig.4-9.2.
- The current coils of the two wattmeters are connected in any two lines, say the 'red' and 'blue' lines, and the voltage circuits are connected between these lines and the third line.
- Suppose v_{RN}, v_{YN} and v_{BN} are the instantancous values of the p.d.s across the loads, these p.d.s being assumed positive when the respective line conductors are positive in relation to the neutral point.

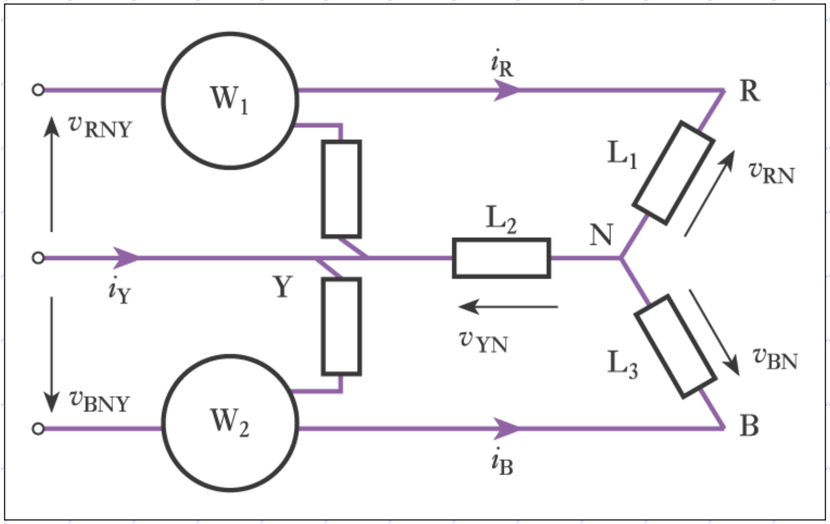


Fig.4-9.2: Measurement of power by two wattmeters

- Also, suppose, I_R, I_Y and I_B are the corresponding instantaneous values of the line (and phase) currents.
- Therefore instantancous power in load L1 = $i_R v_{RN}$, instantaneous power in load in $L_2 = i_Y v_{YN}$, and instantancous power in load $L_3 = i_B v_{BN}$.

- Therefore, total instantaneous power is

  power = $i_R v_{RN} + i_Y v_{YN} + i_B v_{BN}$

- From Fig.4-9.2 it is seen that instantaneous current through current coil of W_1 is i_R and instantaneous p.d. across voltage circuit of W_1 is: $v_{RN} - v_{YN}$. Therefore, instantaneous power measured by W_1 is:
  $W_1 = i_R(v_{RN} - v_{YN})$

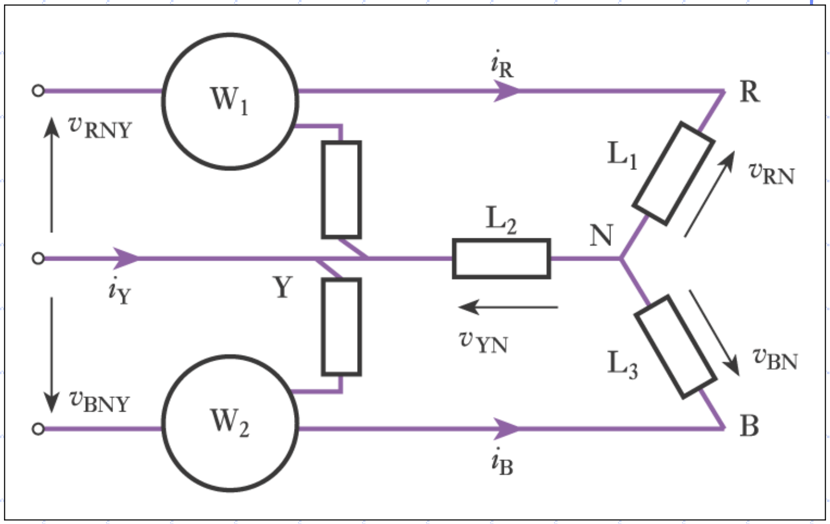


Fig.4-9.2

- Similarly, instantaneous current through current coil of W_2 is i_B and instantaneous p.d. across voltage circuit of W_2 is: $v_{BN} - v_{YN}$.

- Note that this pd. is not $v_{YN} - v_{BN}$. This is due to the fact that a wattmeter reads positively when the currents in the current and voltage coils are *both* flowing from the junction of these coils or *both* towards that junction; and since the positive direction of the current in the current coil of W_2 has already been taken as that of the arrowhead alongside i_B in Fig. 4-9.2 it follows that the current in the voltage circuit of W_2 is positive when flowing from the 'blue' to the 'yellow' line.

- The instantaneous measured by

$$W_2 = i_B(v_{BN} - v_{YN})$$

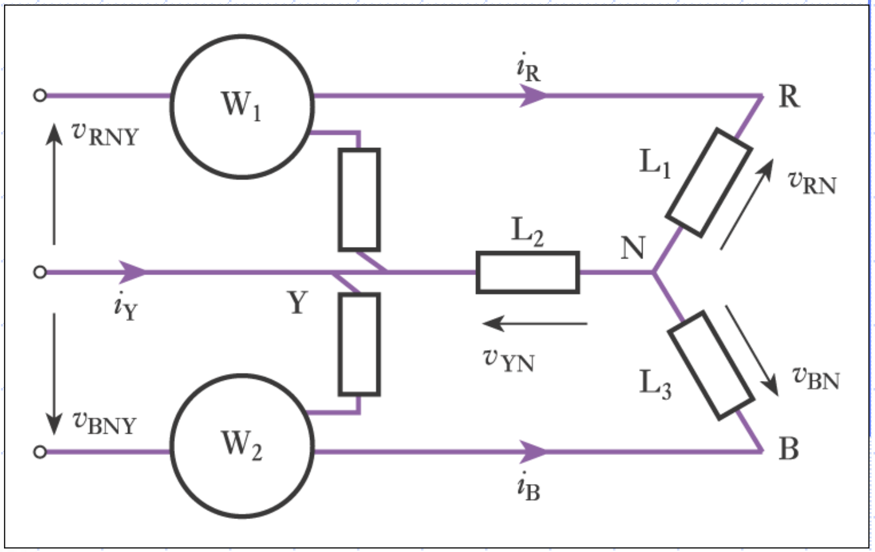


Fig.4-9.2

- Hence the sum of the instantaneous powers of W_1 and W_2 is

$$i_R(v_{RN} - v_{YN}) + i_B(v_{BN} - v_{YN})$$
$$= i_R v_{RN} + i_B v_{BN} - (i_R + i_B)v_{YN}$$

- From KCL's law, the algebraic sum of the instantancous currents at N is zero, i.e.

$$i_R + i_Y + i_B = 0$$
$$\therefore \quad i_R + i_B = -i_Y$$

so that sum of instantancous powers measured bv W_1 and W_2 is

$$i_R v_{RN} + i_B v_{BN} + i_Y v_{YN}$$
$$= \text{total instantaneous power}$$

POWER FACTOR MEASUREMENT BY MEANS OF THE TWO-WATTMETERS

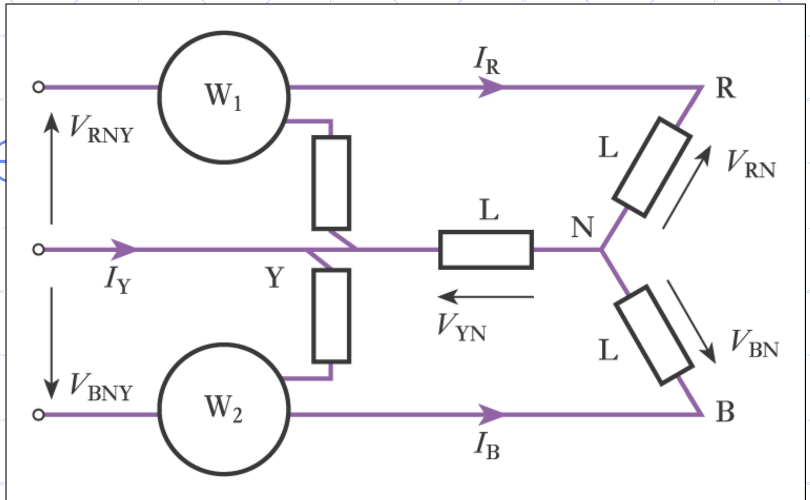


Fig.4-9.3: Measurement of active power and power factor by two wattmeters

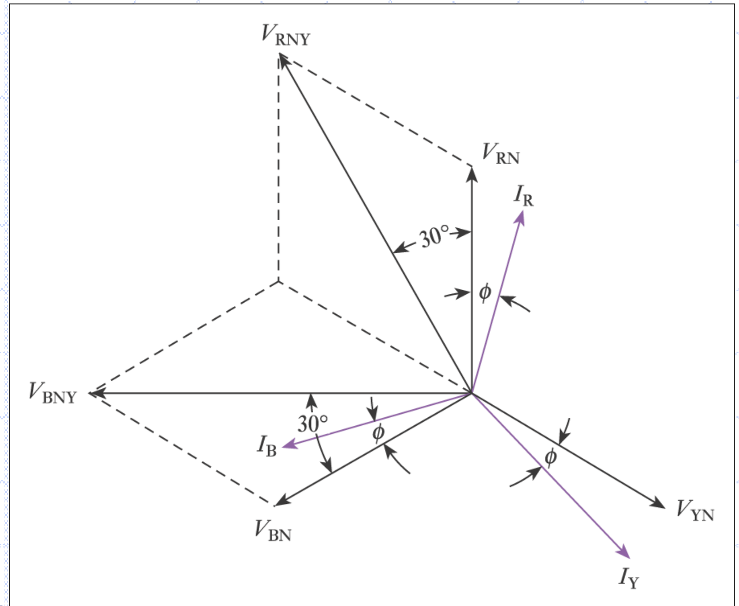


Fig.4-9.4: Phasor diagram

- Suppose L in Fig.4-9.3 to represent **three similar loads connected in star**, and suppose V_{RN} and V_{BN} to be the **r.m.s. values of the phase voltages** and I_R, I_Y and I_B be the **r.m.s. values of the currents.**

- Since these voltages and currents are assumed sinusoidal, they can be **represented by phasors**, as in **Fig. 4-9.3**, the **currents being assumed to lag the corresponding phase voltages by an angle** Φ.

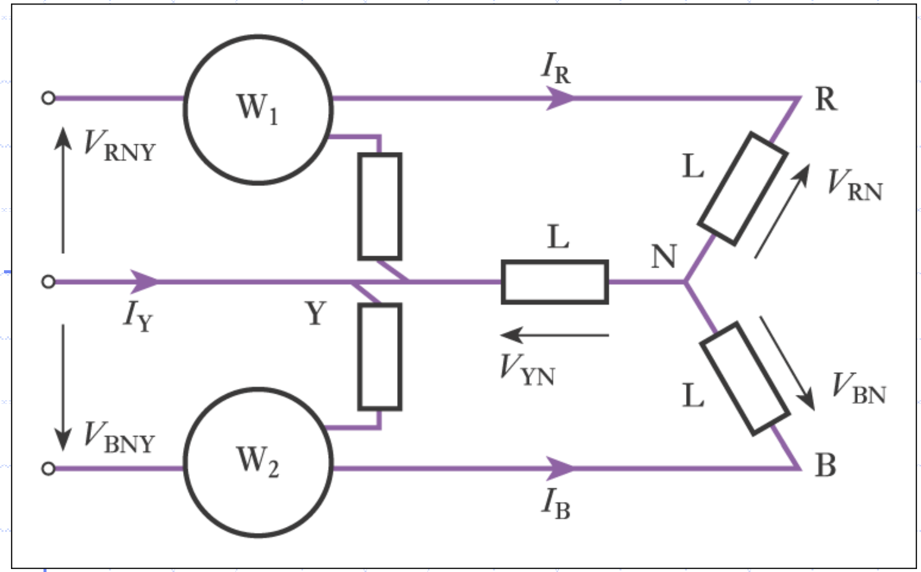


Fig.4-9.2

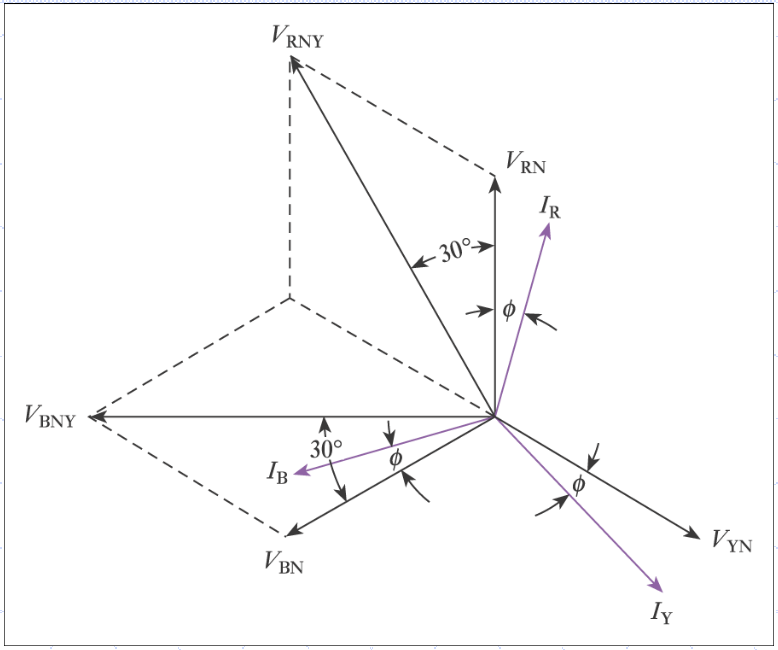


Fig.4-9.3

- **Current through current coil of W_1 is I_R. Potential difference across voltage circuit of W_1 is**

Phasor difference of V_{RN} and $V_{YN} = V_{RNY}$

Phase difference between I_R and $V_{RNY} = 30° + \phi$. Therefore reading on W_1 is

$$P_1 = I_R V_{RNY} \cos(30° + \phi)$$

- Current through current coil of $W_2 = I_B$. Potential difference across voltage circuit of W_2 is

  Phasor difference of V_{BN} and $V_{YN} = V_{BNY}$

- Phase difference between I_B and $V_{BNY} = 30° - \phi$. Therefore reading on W_2 is

$$P_2 = I_B V_{BNY} \cos(30° - \phi)$$

Since the load is balanced,

$$I_R = I_Y = I_B = \text{(say) } I_L\text{, numerically}$$

and

$$V_{RNY} = V_{BNY} = \text{(say) } V_L\text{, numerically}$$

Hence

$$P_1 = I_L V_L \cos(30° + \phi)$$

and

$$P_2 = I_L V_L \cos(30° - \phi)$$

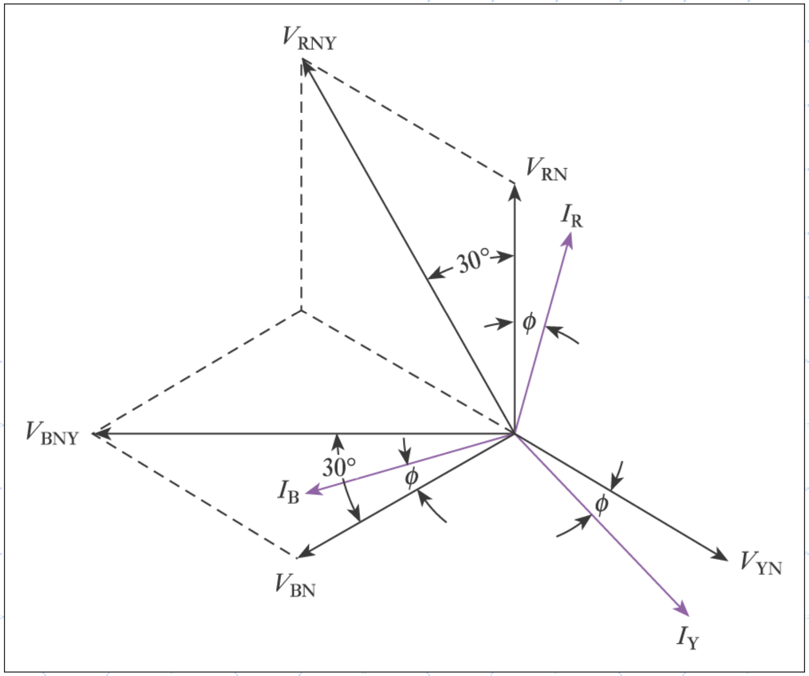


Fig.4-9.3.

$$P_1 + P_2 = I_L V_L\{\cos(30° + \phi) + \cos(30° - \phi)\}$$

$$P_1 + P_2 = I_L V_L(\cos 30° \cdot \cos\phi - \sin 30° \cdot \sin\phi$$
$$+ \cos 30° \cdot \cos\phi + \sin 30° \cdot \sin\phi)$$

$$P_1 + P_2 = 1.73 I_L V_L \cos\phi$$

- This is an **alternative method of proving that the sum of the two wattmeter readings gives the total active power, but it should be noted that this proof assurred a balanced load and sinusoidal voltages and currents.**
- **By division of P_1/P_2, gives:**

$$\frac{P_1}{P_2} = \frac{\cos(30° + \phi)}{\cos(30° - \phi)} = (\text{say})\, y$$

$$y = \frac{(\sqrt{3}/2)\cos\phi - (1/2)\sin\phi}{(\sqrt{3}/2)\cos\phi + (1/2)\sin\phi}$$

so that

$$\sqrt{3} y \cos\phi + y \sin\phi = \sqrt{3}\cos\phi - \sin\phi$$

from which

$$\sqrt{3}(1 - y)\cos\phi = (1 + y)\sin\phi$$

$$\therefore \qquad 3\left(\frac{1-y}{1+y}\right)^2 \cos^2\phi = \sin^2\phi = 1 - \cos^2\phi$$

$$\left\{1 + 3\left(\frac{1-y}{1+y}\right)^2\right\}\cos^2\phi = 1$$

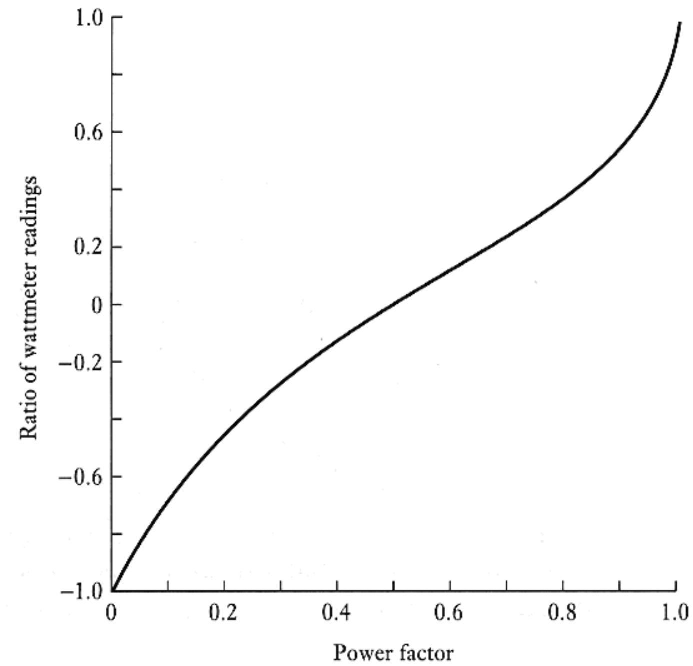


Fig.4-9.4: Reationship between power factor and ratio of wattmeter readings

- **A more convenient method is to draw a graph of the power factor for various ratios of P_1/P_2 and in order that these ratios may lie between +1 and -1, it is always the practice to take P_1 as the smaller of the two readings.**
- **By adopting this practice, it is possible to derive reasonably accurate values of the power factor from the graph.**

- When the power factor of the load **is 0.5 lagging**, **Φ is 60**; and the reading on **$W_1 = I_L V_L \cos 90° = 0$.**

- When the power factor is **less than 0.5 lagging, Φ is greater than 60°** and **(30^0 +Φ) is therefore greater than 90°.** Hence the reading on **W_1 is negative.**

- To measure **this active power it is necessary to reverse the connections to either the currcnt or the voltage coil, but the reading thus obtaincd must be taken as negative when the total active power** and **the ratio of the wattmeter readings are being calculated.**

- An alternative **method of deriving the power factor is as follows:**

$$P_2 - P_1 = I_L V_L \sin \phi$$

and

$$\tan \phi = \frac{\sin \phi}{\cos \phi} = 1.73 \left(\frac{P_2 - P_1}{P_2 + P_1} \right)$$

ASSIGNMENT ON POWER MEASUREMENT

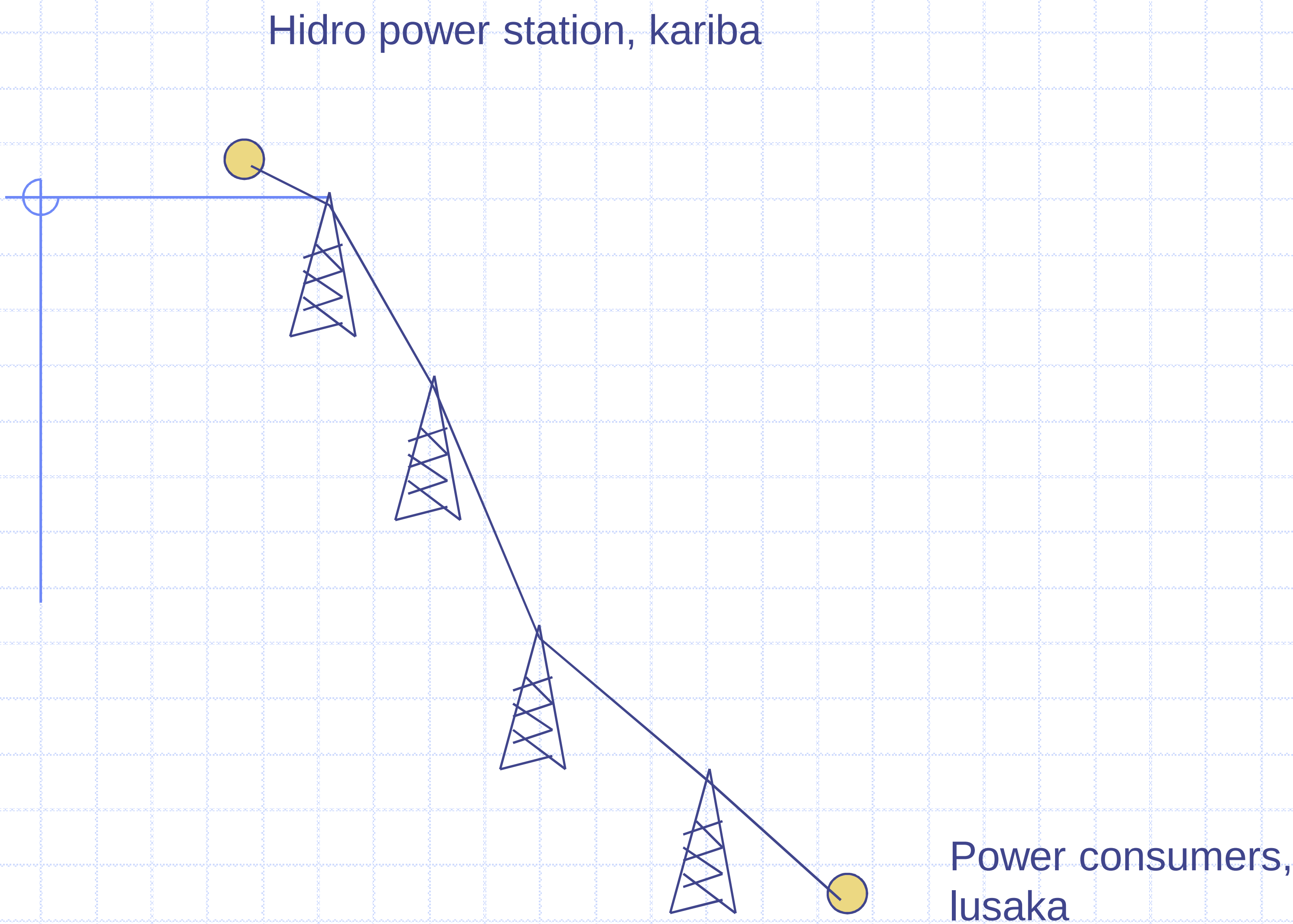
Explain in details the three wattmeter power measurement method, Its advantage and disadivantages.

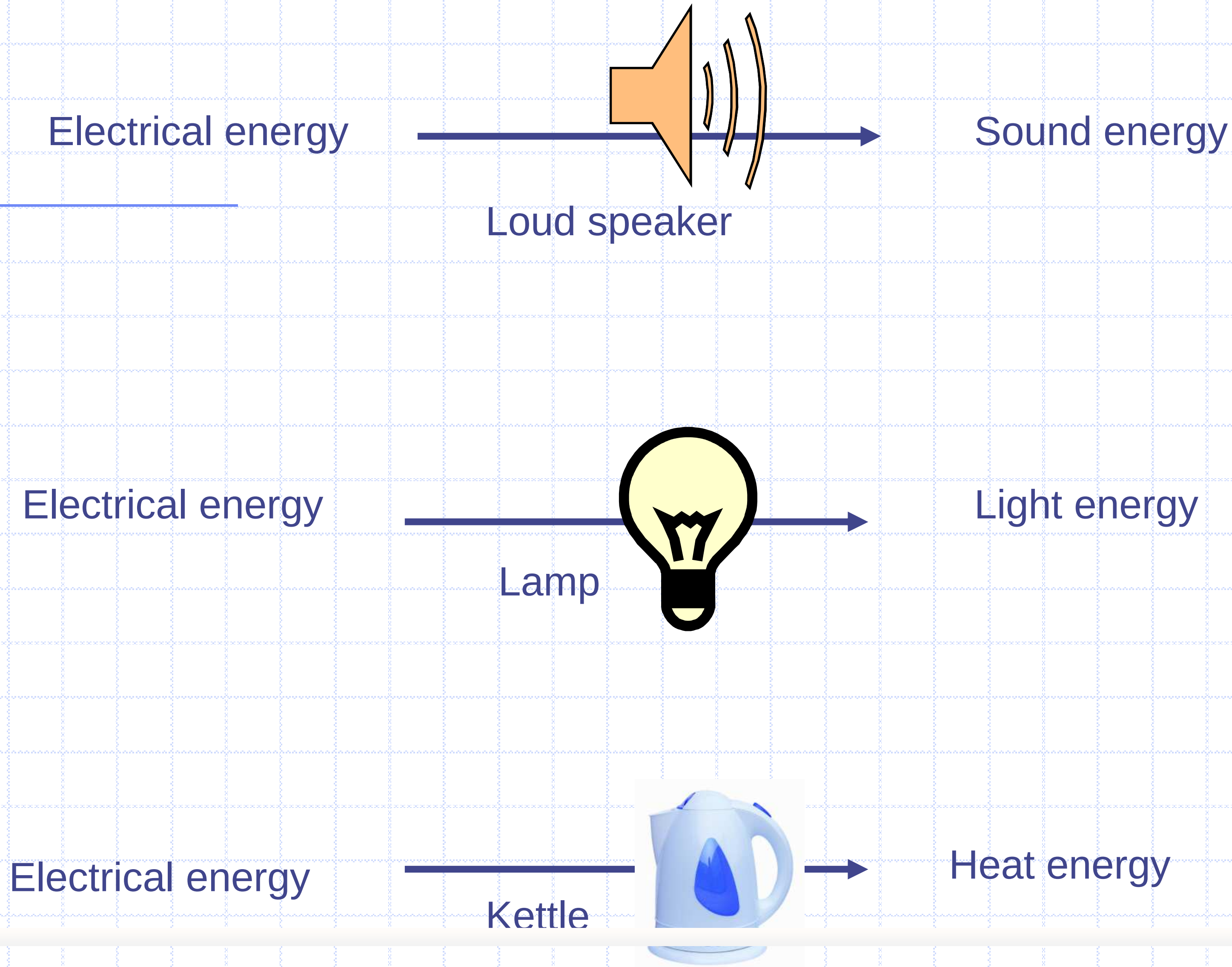
ELECTROMECHANICAL ENERGY CONVERSION

Electrical energy is the most popular form of energy, because:

1. it can be transmitted easily for long distance, at high efficiency and reasonable cost.

2. It can be converted easily to other forms of energy such as sound, light, heat or mechanical energy.


Hidro power station, kariba
Power consumers,
lusaka


Electrical energy
Loud speaker
Sound energy
Electrical energy
Lamp
Light energy
Electrical energy
Kettle
Heat energy

Electromechanical energy conversion device:

- converts electrical energy into mechanical energy or converts mechanical energy into electrical energy.

There are various electromechanical conversion devices may categorized as under:

a. Small motion

- telephone receivers, loud speakers, microphones

b. Limited mechanical motion

- electromagnets, relays, moving-iron instruments, moving-coil instruments, actuators

c. Continuous energy conversion

- motors, generators

Principle of Energy Conversion

According to the *principle of conservation of energy*, energy can neither be created nor destroyed, it can merely be converted from one form into another.

The total energy in a system is therefore constant.

Energy conversion in electromechanical system

In an energy conversion device, out of the total input energy, some energy is converted into the required form, some energy is stored and the rest is dissipated.

It is possible to write an equation describing energy conversion in electromechanical system:

Electrical energy from source = Mechanical energy to load + Increase of field energy + Energy converted to heat (losses) —— 3.1

$$\text{Electrical energy from source} = \text{Mechanical energy to load} + \text{Increase of field energy} + \text{Energy converted to heat (losses)} \quad \text{—— } 3.1$$

The last term on the right-hand side of Eq. 3.1 (the losses) may be divided into three parts:

$$\text{Energy converted to heat (losses)} = \text{Resistance losses} + \text{Friction and windage losses} + \text{Field losses} \quad \text{—— } 3.2$$

Then substitution from Eq. 3.2 in Eq. 3.1 yields

Electrical energy from source minus resistance losses = Mechanical energy to load plus friction and windage losses + Increase of magnetic coupling field energy plus core losses — 3.3

Now consider an electromechanical system (actuator) illustrated in Fig. 3.1.

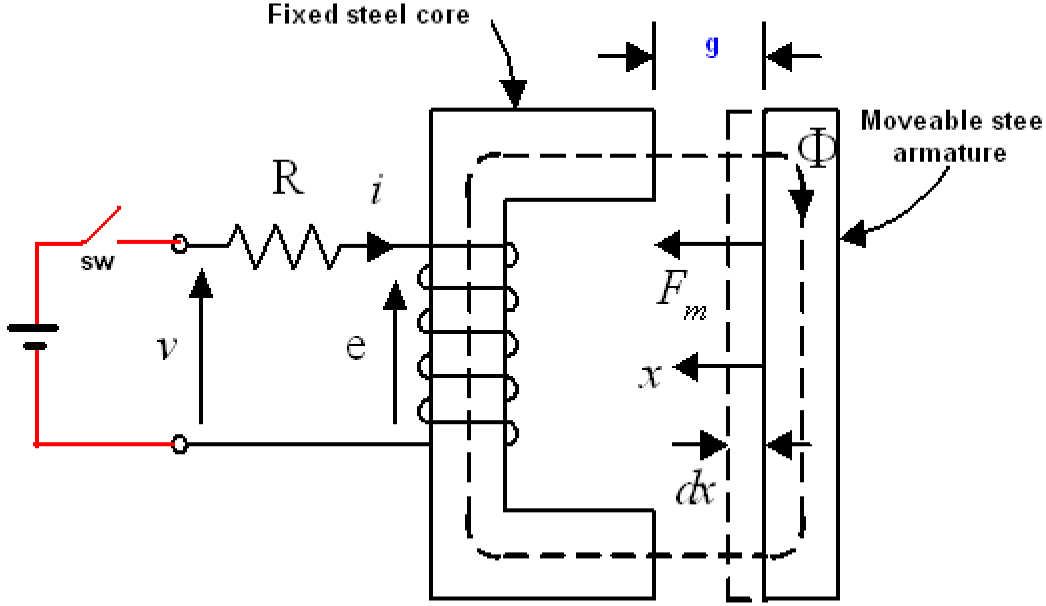


fig 3.1

At any instant, the emf e induced in the coil by the change in the flux linkage λ is

$$e = \frac{d\lambda}{dt} \quad \text{volt} \qquad (3.4)$$

Consider now a differential time interval dt, during which the current in the coil is changing and the armature is moving.

Therefore, the differential energy transferred in time *dt* from the electric source to the coupling field is given by the energy output of the source minus the resistance loss:

$$dW_e = vidt - Ri^2dt$$

$$= (v - Ri)idt$$

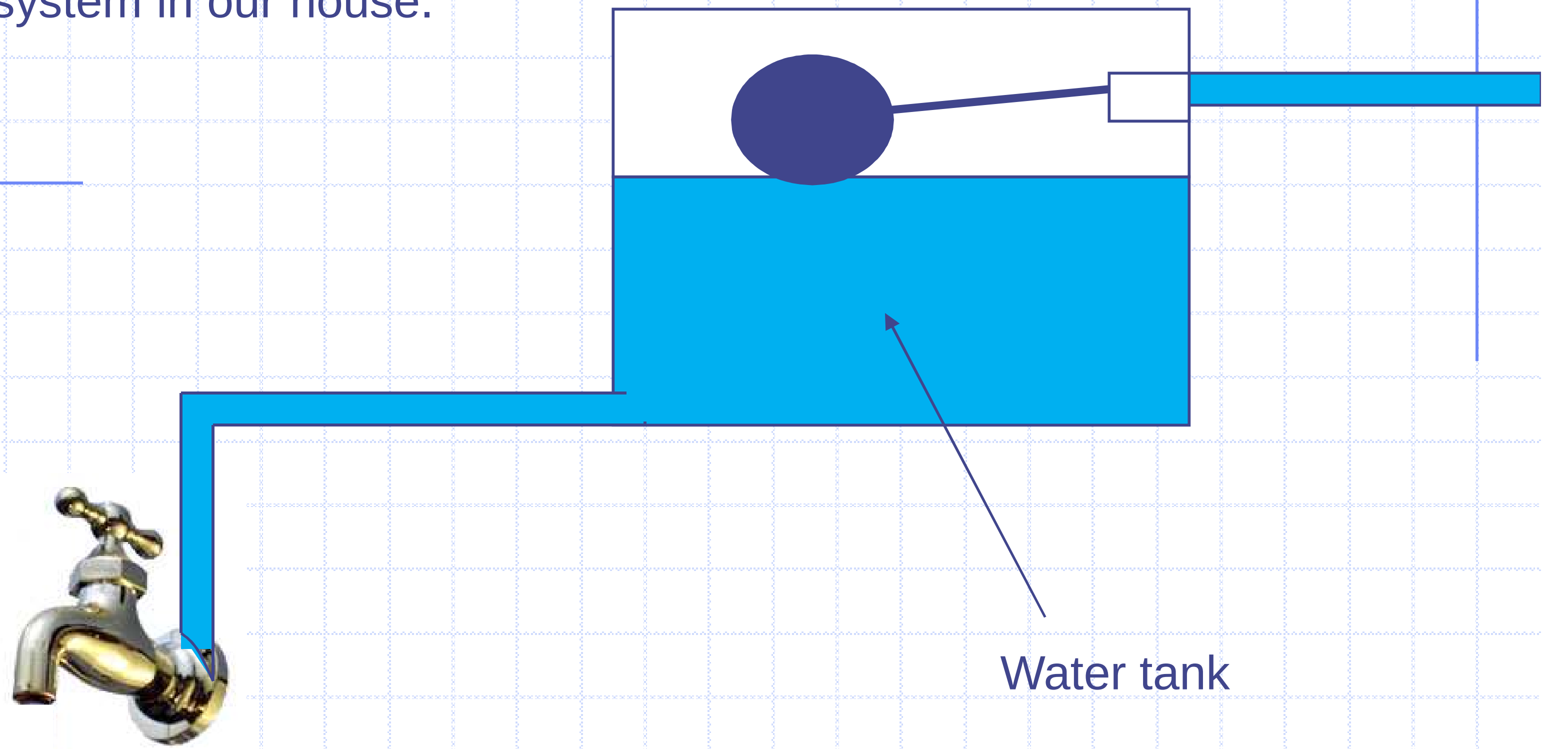
$$= eidt$$

$$\Rightarrow \quad dW_e = eidt \quad \text{Joule} \qquad (3.5)$$

The coupling field forms an energy storage to which energy supplied by the electric system. At the same time, energy is released from the coupling field to the mechanical system.

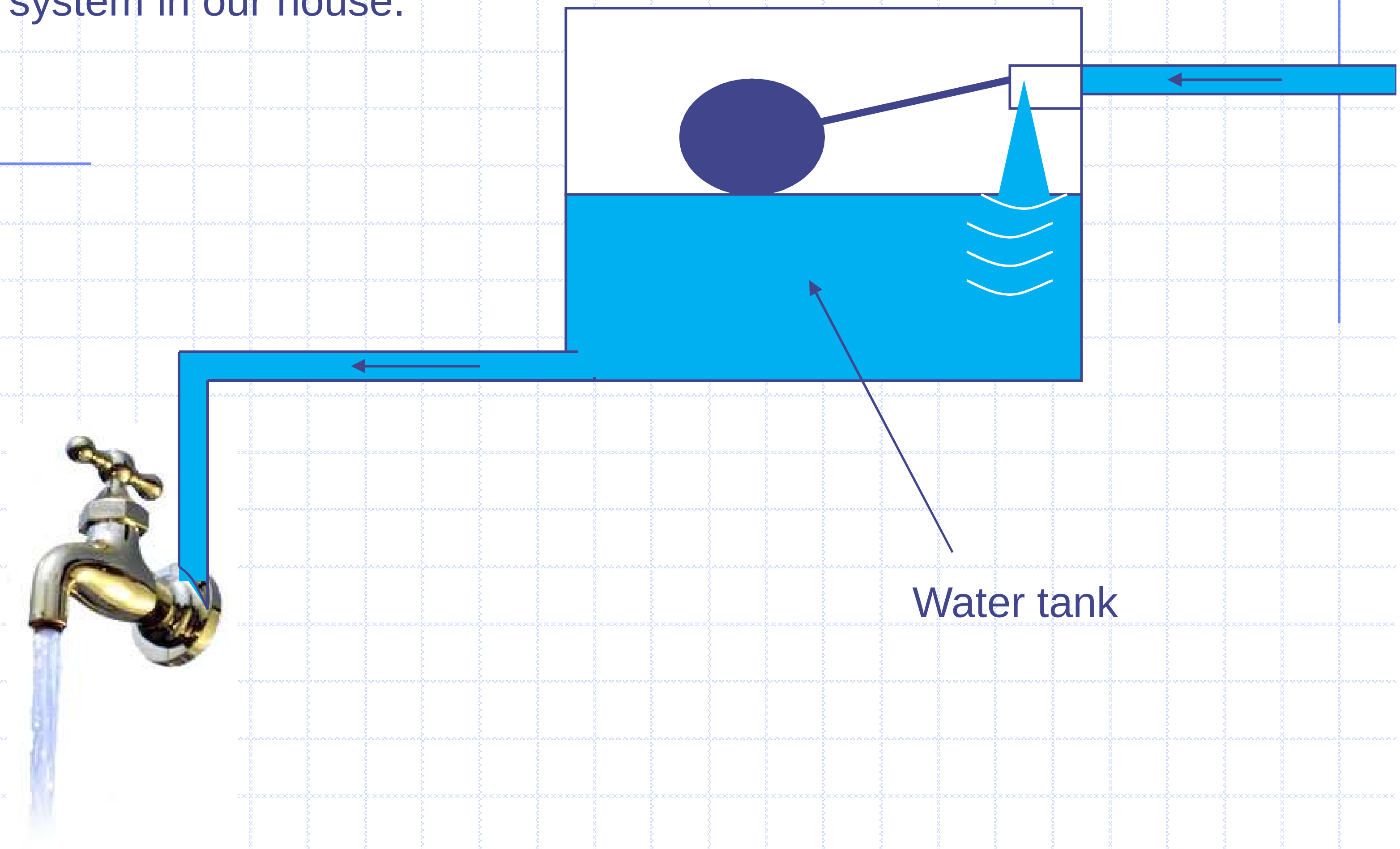
The rate of release energy is not necessarily equal at any instant to the rate of supply of energy to the field, so that the amount of energy stored in the coupling field may vary.

It’s like a pipe system in our house.



The water out from the tap will make water flow into the storage tank from the supply.

It's like a pipe system in our house.



The water out from the tap will make water flow into the storage tank from the supply.

In time *dt*, let dW_f be the energy supplied to the field and either stored or dissipated. Let dW_m be the energy converted to mechanical form, useful or as loss, in the same time, *dt*.

Then, by the *principle of conservation of energy*, the following equation may be written for the field:

$$dW_e = dW_m + dW_f \qquad (3.6)$$

Field Energy

To obtain an expression for for dW_f of Eq. 3.6 in terms of the system variables, it is first necessary to find an expression for the energy stored in the magnetic field for any position of the armature. The armature will therefore be clamped at some value of air-gap length *g* so that no mechanical output can be produced.

$$dW_m = 0 \qquad (3.6)$$

If switch SW in Fig. 3.1 is now closed, the current will rise to a value *V/R*, and the flux will be established in the magnetic system. Let the relationship between coil flux linkage λ and the current *i* for the chosen air-gap length be that shown in Fig. 3.2

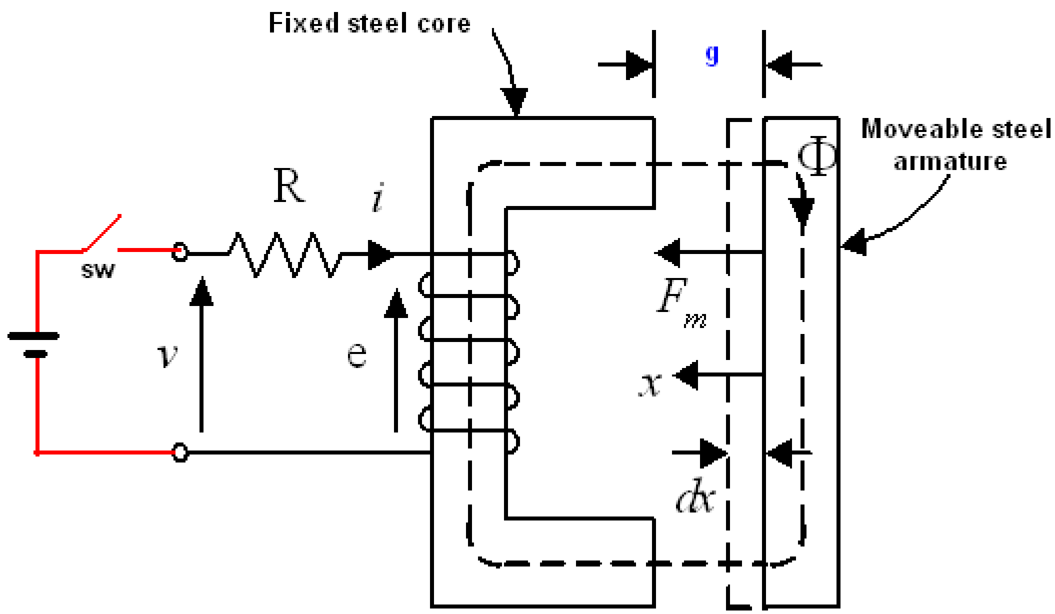


fig 3.1

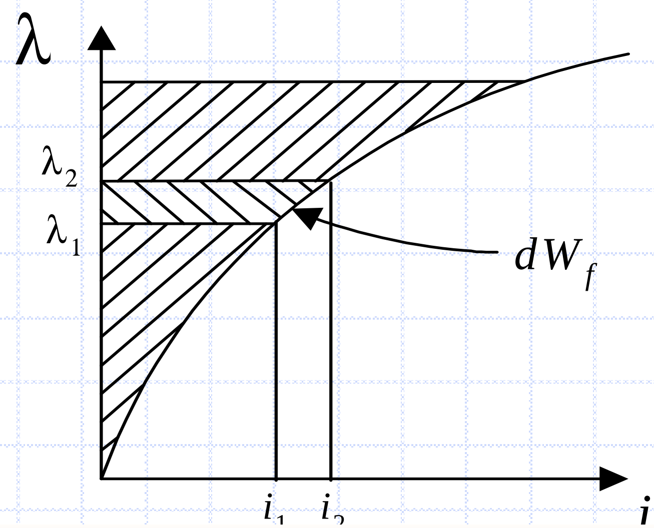
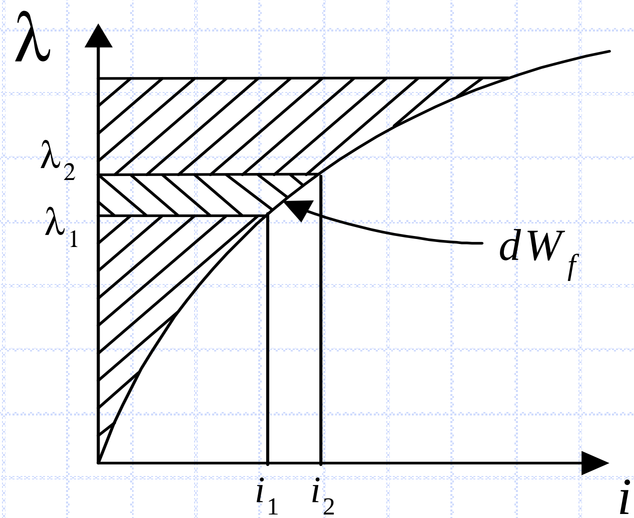


Fig. 3.2

Field Energy (continue…..)

Since core loss is being neglected, this will be a single-valued curve passing through the origin. In the absence of any mechanical output energy, all of the electric input energy must be stored in the magnetic field:



$$dW_e = dW_f \qquad (3.8)$$

Substitution from Eqs. 3.4 and 3.8 in Eq. 3.5 yields

$$dW_f = dW_e = i.edt = id\lambda \quad \text{J} \qquad (3.9)$$

If now v is changed, resulting in a change in current from i_1 to i_2, there will be a corresponding change in flux linkage from λ_1 to λ_2 .

The increase in energy stored during the transition between these two states is

$$dW_f = \int_{\lambda_1}^{\lambda_2} i d\lambda \quad \text{J} \tag{3.10}$$

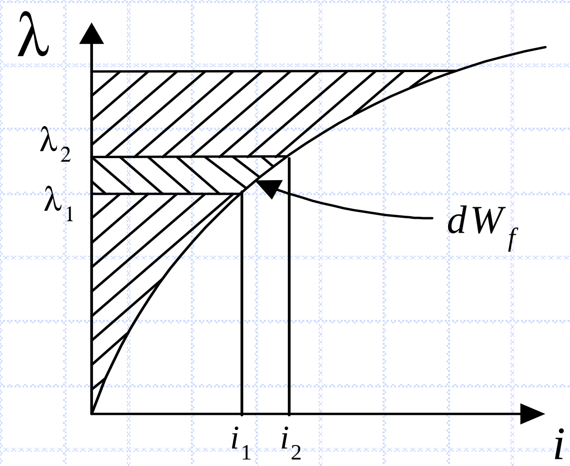


Fig 3.2.

The area is shown in Fig 3.2. When the flux linkage is increased from zero to λ, the total energy stored in the field is

$$W_f = \int_{0}^{\lambda} i d\lambda \quad \text{J} \tag{3.11}$$

Field Energy (continue…..)

This integral represents the area between the λ–i characteristic and the λ–axis, the entire shaded area of Fig. 3.2.

If it is assumed that there is no leakage flux, so that all flux Φ in the magnetic system links all N turns of the coil, then

$$\lambda = N\Phi \quad \text{Wb} \longrightarrow 3.12$$

From Eqs. 3.9 and 3.12,

$$dW_f = id\lambda = Nid\Phi = \mathscr{F}d\Phi \quad \text{J} \qquad (3.13)$$

where $\mathscr{F} = Ni \quad \text{A} \qquad (3.14)$

$\mathscr{F}$ is mmf (magneto-motive force)

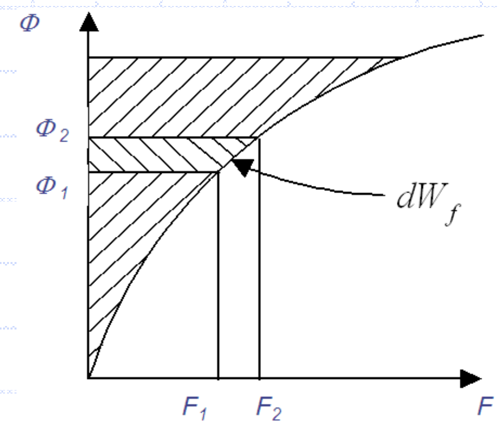


Fig. 3.3

The characteristic of Fig. 3.3 can also represent the relationship between Φ and $\mathscr{F}$.

Field Energy (continue…..)

If the reluctance of the air gap forms a large part of the total reluctance of the magnetic system, then that of the steel may be neglected and the λ–i characteristic becomes the straight line through the origin shown in Fig. 3.3. For this system,

$$\lambda = Li \quad \text{Wb} \qquad (3.15)$$

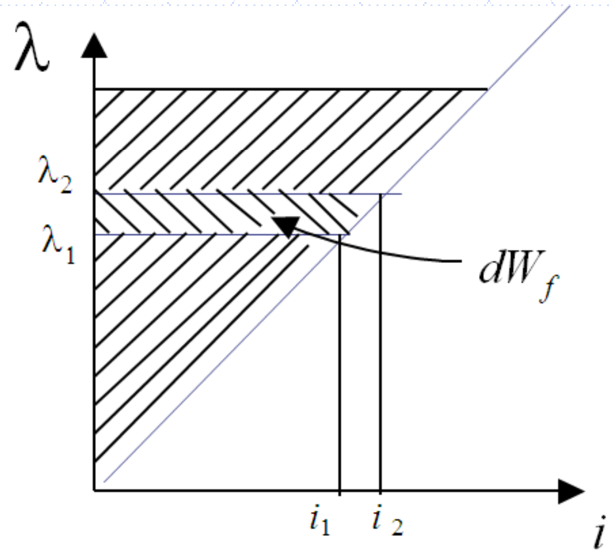


Fig. 3.3

Where L is the inductance of the coil. Substitution in Eq. 3.11 gives the energy W_f in several useful forms:

$$W_f = \int_0^{\lambda} \frac{\lambda}{L} d\lambda = \frac{\lambda^2}{2L} = \frac{Li^2}{2} = \frac{i\lambda}{2} \quad \text{J} \qquad (3.16)$$

Field Energy (continue…..)

If the reluctance of the magnetic system (that is, of the air gap) as seen from the coil is S, then $\mathscr{F} = S\,\Phi$, and from Eq. 3.13,

$$W_f = \int_0^{\phi} \mathscr{F}\, d\lambda = \frac{S\phi^2}{2} = \frac{\mathscr{F}^2}{2S} \quad \text{J} \qquad (3.17)$$

If A is the cross-section area of the core and $l = 2g$ is the total length of air gap in a flux path, then from Eq. 3.16,

$$W_f = \frac{i\lambda}{2} = \frac{F\phi}{2} = \frac{1}{2} HBlA \quad \text{J} \qquad (3.18)$$

Where B is the flux density in the air gaps. Since *B/H*=μ_0 and *lA* is the total gap volume, it follows from Eq. 3.18 that the energy density in the air gaps is

$$w_f = \frac{W_f}{lA} = \frac{1}{2}BH = \frac{1}{2}\mu_0 H^2 = \frac{1}{2}\frac{B^2}{\mu_0} \quad \text{J/m}^3 \qquad (3.19)$$

Equations 3.16, 3.17 and 3.19 represent three different ways of expressing the field energy.

Example 3.1 The core and armature dimensions of the actuator of Fig. 3.1 are shown in Fig. 3.4. Both parts are made of mild steel, whose magnetization curve is given in Fig. 3.5. Given l_a = 160 mm, l_b = 80 mm. The coil has 2000 turns. Leakage flux and fringing may be neglected. The armature is fixed, so that the length of the air gas, l_u= 9 mm, and a direct current is passed through the coil, producing a flux density of 0.8 T in the air gap.

a) Determine the required coil current.

b) Determine the energy stored in the air gap.

c) Determine the energy stored in the steel.

d) Determine the total field energy.

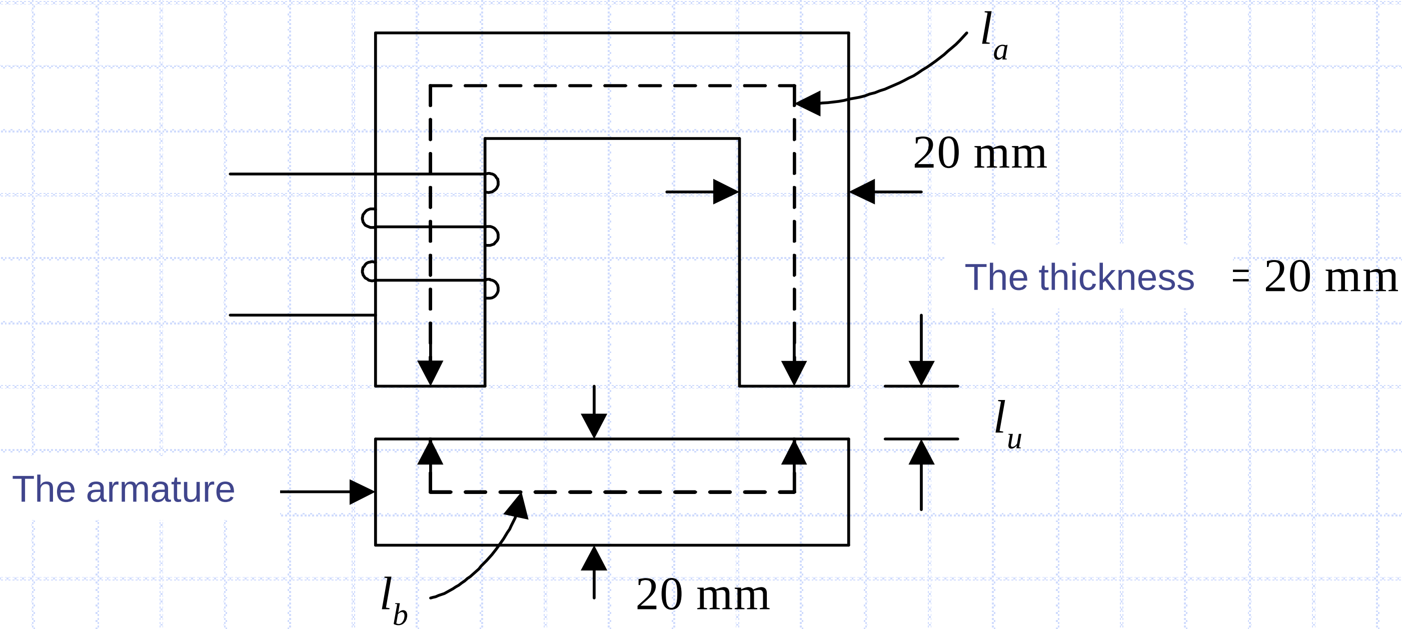


Fig. 3.4

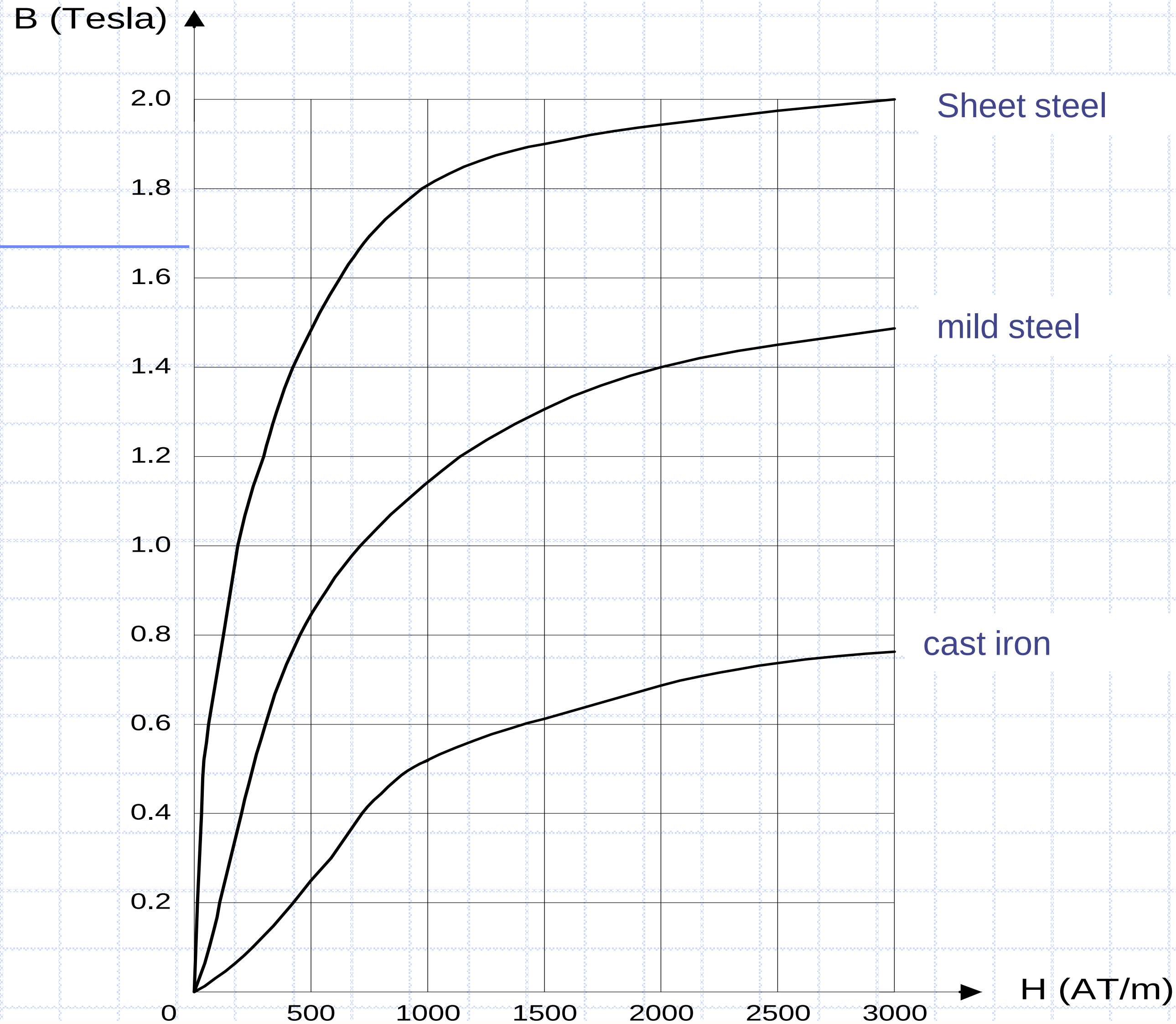

B (Tesla)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
500
1000
1500
2000
2500
3000
H (AT/m)
Sheet steel
mild steel
cast iron

Fig. 3.5

Solution

(a) Area, $A = (20 \times 10^{-3})(20 \times 10^{-3}) = 4 \times 10^{-4}\ m^2$.

$Ni = H_t l_t + H_u l_u$

$l_t = 160 + 80 = 240\ mm = 240 \times 10^{-3}\ m$
$l_u = 2 \times 9\ mm = 18\ mm = 18 \times 10^{-3}\ m$

Given $B_u = 0.8\ T$
$B_u = B_t = 0.8\ T$

From Fig. 3.6, magnetic field intensity in the steel is,
$H_t = 450\ A/m$

For the air gaps

$$H_u = \frac{B_u}{\mu_o} = \frac{0.8}{4\pi \times 10^{-7}} = 636.62 \times 10^3 \quad \text{A/m}$$

$$i = \frac{(450)(240 \times 10^{-3}) + (636.62 \times 10^3)(18 \times 10^{-3})}{2000}$$

$$= \frac{11567.16}{2000} = 5.78 \quad \text{A}$$

(b) Energy density in the air gaps is

$$w_{fu} = \frac{B^2}{2(4\pi \times 10^{-7})} = 254.65 \times 10^3 \quad \text{J/m}^3$$

Volume of air gaps = length of air gaps × area of air gaps

$= 0.018 \times 0.02 \times 0.02$

$= 7.2 \times 10^{-6}\ m^3$

Energy stored in the air gaps,

W_{fu} = the volume of air gaps × w_{fu}

$= (7.2 \times 10^{-6}) \times 254.65 \times 10^3$

$= 1.834$ Joule.

(c) Energy density in the steel,

$$w_{ft} = \int_0^{0.8} H dB$$

Energy density in the steel is given by the area enclosed between the characteristic and the *B* axis in Fig. 3.6 up to value of 0.8 T.

$$w_{ft} \cong \tfrac{1}{2} \times 0.8 \times 450 = 180 \text{ J/m}^3 \text{ (straight-line approximation)}$$

Volume of steel= length of steel × area of steel

$$= (240 \times 10^{-3}) \times (0.02 \times 0.02)$$

$$= 9.6 \times 10^{-5} \text{ m}^3$$

∴ Energy stored in the steel,

$$W_{ft} = 9.6 \times 10^{-5} \times 180 = 0.01728 \text{ Joule}$$

(d) Total field energy,

$$W_f = W_{ft} + W_{fu}$$

$$= 0.01728 + 1.834$$

$$= 1.851 \text{ Joule.}$$

The proportion of field energy stored in the steel is, therefore, seen to be negligibly.

Coenergy

Coenergy, W_f' is the area enclosed between the λ-i characteristic and the i axis of Fig.3.2.

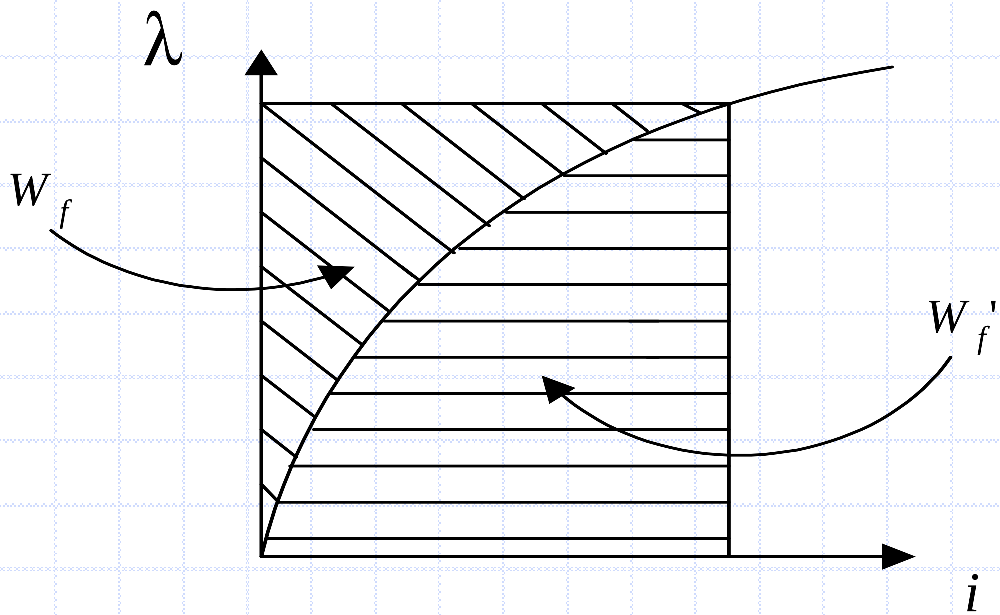


Fig. 3.6 Field energy and coenergy

For linear λ-i characteristic, $W_f' = W_f$.
For nonlinear λ-i characteristic, $W_f' > W_f$.

Mechanical Energy in a Linear System

It will be assumed that the armature of the actuator in Fig. 3.1 may move from position x_1 to position x_2, as a result, the length of air gaps is reduced. The λ–i characteristics for the two extreme positions of the armature may be assumed to be the two straight lines (linear).

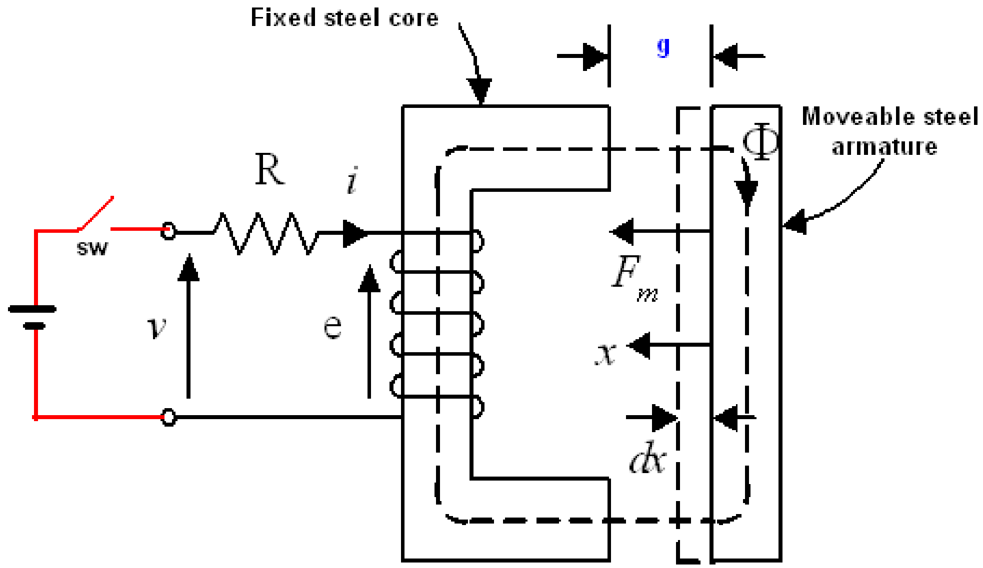
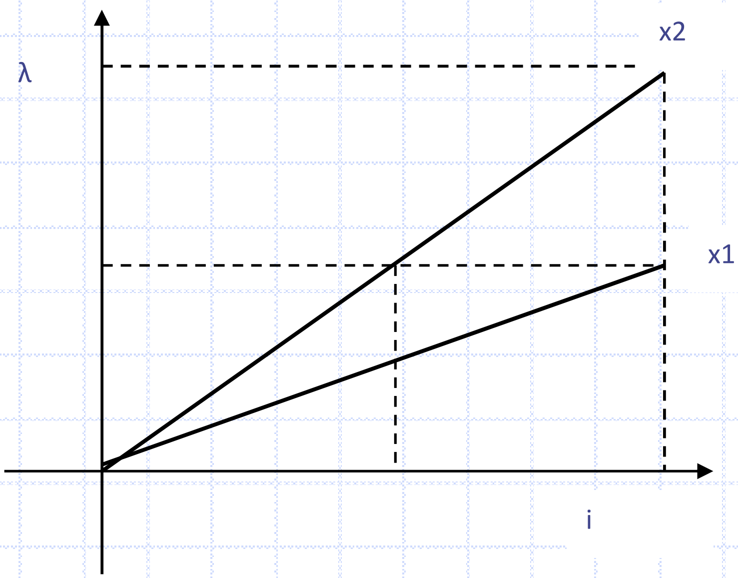


fig 3.1



Consider a very slow armature displacement. It may assumed that it takes place at essentially constant current as illustrated in Fig. 3.7 (as $d\lambda/dt$ is negligible). The operational point has changed from ***a*** to ***b***.

At the moment of armature movement,

$$\Delta W_e = \int eidt = \int_{\lambda_1}^{\lambda_2} id\lambda = i_o(\lambda_2 - \lambda_1) \qquad (3.20)$$

The change of field energy,

$$\Delta W_f = \tfrac{1}{2} i_o \lambda_2 - \tfrac{1}{2} i_o \lambda_1 = \tfrac{1}{2} i_o(\lambda_2 - \lambda_1) \qquad (3.21)$$

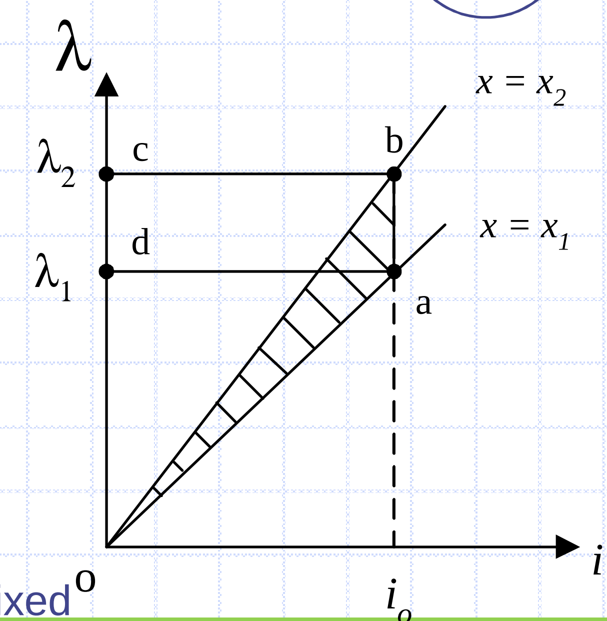


Fig. 3.7 Current is fixed

From Eq. (3.6),

$$\Delta W_e = \Delta W_m + \Delta W_f$$

$$\Delta W_m = \Delta W_e - \Delta W_f$$

$$\therefore \quad = i_o(\lambda_2 - \lambda_1) - \tfrac{1}{2} i_o(\lambda_2 - \lambda_1)$$

$$= \tfrac{1}{2} i_o(\lambda_2 - \lambda_1)$$

$= \Delta W_f$

$= \Delta W_f'$ = the change of coenergy

For small change of x or dx,

$$\because \quad dW_m = dW_f'$$

$$\Rightarrow \quad F_m dx = dW_f' \qquad (3.21)$$

where

$$dW_m = F_m dx$$

F_m = mechanical force on moving part (armature)

Eq. 3.21 can be written as,

$$F_m = \frac{\partial W_f'}{\partial x}(i, x)\Big|_{i = \text{constant}} \quad \text{N} \qquad (3.22)$$

Eq. 3.22 is partial differential since W_f is function of more than one variable.

Consider now a very rapid differential armature displacement *dx*. It may be assumed that it takes place at essentially constant flux linkage λ_o, as illustrated in Fig. 3.8. At the instant, the current is changed from i_1 to i_2 , where $i_1 > i_2$.

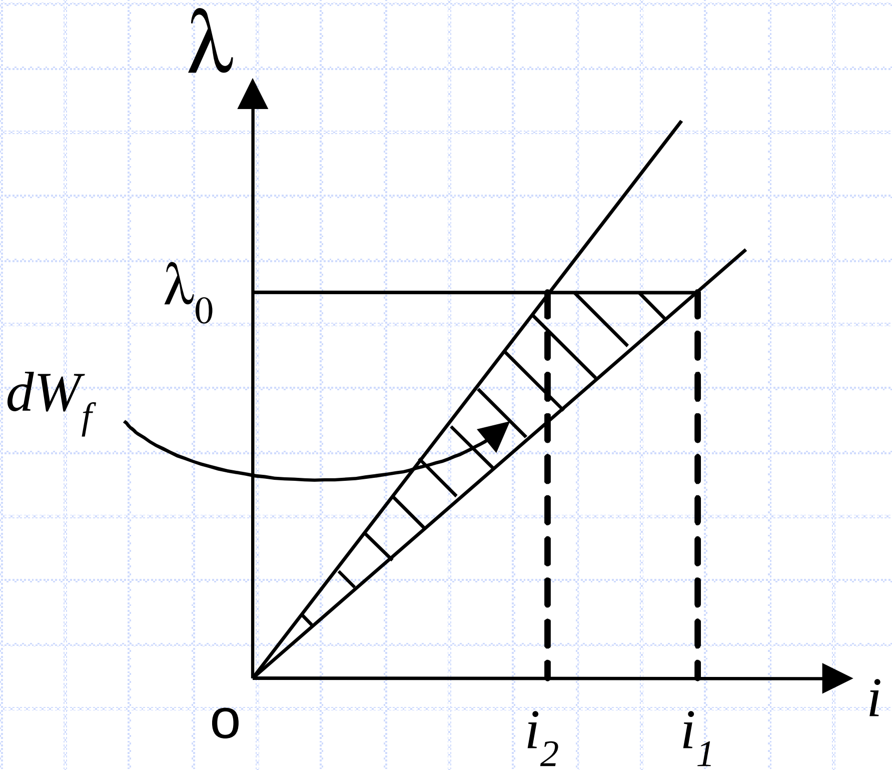


Fig. 3.8

Mechanical Energy in a Linear System

$$dW_e = dW_m + dW_f$$

$$\therefore -F_m dx = dW_f \qquad (3.25)$$

$$\Rightarrow \quad -F_m dx = \tfrac{1}{2}\lambda_o (i_2 - i_1) \qquad (3.26)$$

= the change of field energy

Eq. 3.26 can be written as,

$$F_m = -\frac{\partial W_f}{\partial x}(\lambda, x)\Big|_{\lambda = \text{constant}} \qquad (3.27)$$

Since the electrical input energy is zero, the mechanical output energy has been supplied entirely by the coupling field.

For a linear electromagnetic system,

$$\lambda = L(x)\, i \qquad (3.28)$$

where
$L(x)$ = the inductance of the coil which dependent on length of the air gaps.

From Eqs. 3.11 and 3.28,

$$W_f = \int_0^{\lambda} i\, d\lambda = \int_0^{\lambda} \frac{\lambda}{L(x)}\, d\lambda = \frac{\lambda^2}{2L(x)} \qquad (3.29)$$

$$= \frac{L(x)^2 i^2}{2L(x)} = \tfrac{1}{2} L(x) i^2 \qquad (3.30)$$

$$W_f = W_f' = \tfrac{1}{2} L(x) i^2 \qquad (3.31)$$

From Eqs. 3.22 and 3.31,

$$F_m = \frac{\partial W_f'}{\partial x}(i, x) \Big|_{i = \text{constant}}$$

$$= \frac{\partial}{\partial x}\left(\tfrac{1}{2} L(x) i^2\right) \Big|_{i = \text{constant}}$$

$$= \frac{1}{2} i^2 \frac{dL(x)}{dx} \qquad (3.32)$$

$$\Rightarrow \qquad F_m = \frac{1}{2} i^2 \frac{dL(x)}{dx} \qquad (3.33)$$

From Fig. 3.1 (for linear system),

$$Ni = H_u 2g$$

$$= \frac{B_u}{\mu_o} 2g \qquad \text{(3.34)}$$

From Eq. 3.18

W_f = volume of air gaps $\times \frac{B_u^{\,2}}{2\mu_o}$

$$= A_u 2g \times \frac{B_u^{\,2}}{2\mu_o} \qquad \text{(3.35)}$$

where A_u = cross section area of air gap

From Fig. 3.1, it is seen that a positive displacement *dx* will correspond to a reduction *dg* in the air gap length. Thus,

$$dx = -\,dg \quad \text{m} \qquad (3.36)$$

From Eqs. 3.27, 3.35 and 3.36 yield,

$$F_m = \frac{\partial}{\partial g}\left(A_u 2g \times \frac{B_u{}^2}{2\mu_o} \right)$$

$$\Rightarrow \qquad F_m = 2A_u \frac{B_u{}^2}{2\mu_o} \qquad (3.37)$$

where

$2A_u$ = The total cross-section area of air gaps

∴ The force per unit area of air gaps, f_m is

$$f_m = \frac{B_u^{\;2}}{2\mu_o} \quad \text{N/m}^2 \qquad (3.38)$$

Example 3.2

An electromagnet system is shown in Fig. 3.9.

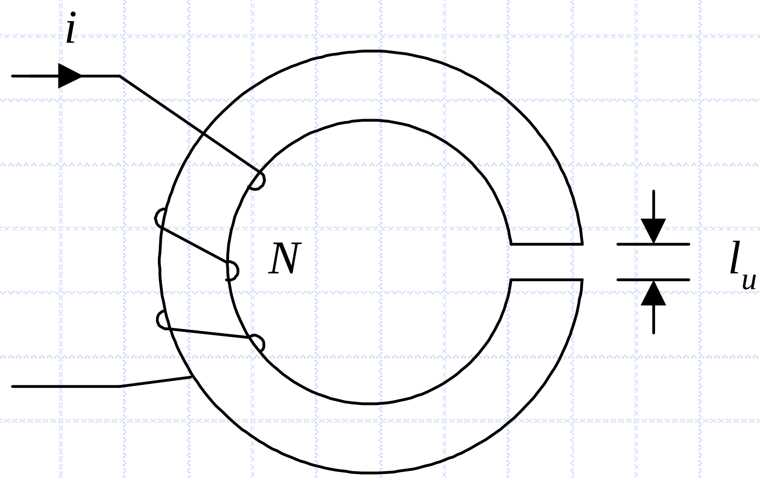


Fig. 3.9: linear system

Given that N = 600, i = 3 A, cross section area of air gap is 5 cm^2 and air gap length is 1.5 mm. By neglecting core reluctance, leakage flux and fringing effects, find:

(a) Force between the electromagnetic surfaces.

(b) Energy stored in the air gap.

Solution

(a) The total cross-section area of air gap = A_u, Eq. 3.37 becomes,

$$F_m = A_u \frac{B_u^2}{2\mu_o} \qquad (3.39)$$

For linear system,

$$Ni = H_u l_u = \frac{B_u l_u}{\mu_o}$$

$$\Rightarrow \quad B_u = \frac{\mu_o Ni}{l_u} \qquad (3.40)$$

Substitution from Eq. 3.40 in Eq. 3.39 yields

$$\therefore \quad F_m = \frac{A_u \mu_o N^2 i^2}{2 l_u^{\,2}}$$

$$= \frac{(5\times10^{-4})(4\pi\times10^{-7})(600)^2(3)^2}{2(1.5\times10^{-3})^2}$$

= 452.39 N

(b) Since the system is linear, the entire field energy is stored in the air gap,

$$W_f = \text{volume of air gap} \times \frac{B_u^{\,2}}{2\mu_o}$$

$$= l_u \times A_u \times \frac{B_u^{\,2}}{2\mu_o}$$

$= l_u \times F_m$

$= (1.5 \times 10\text{-}3) \times 452.39$ Nm

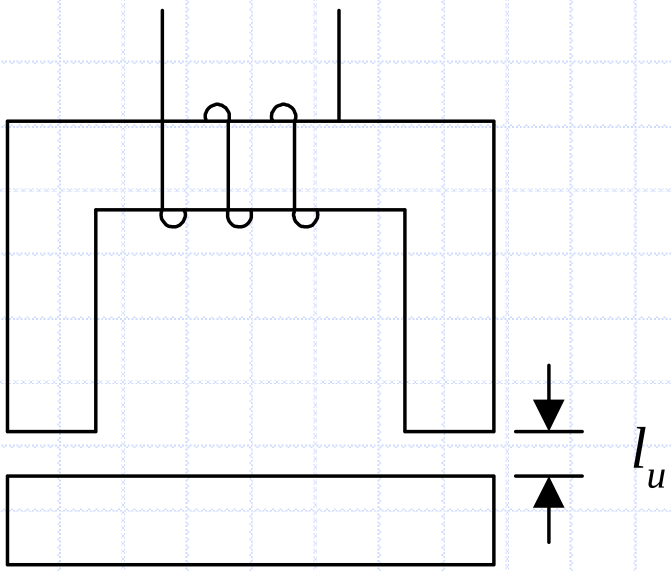
$= 0.6789$ Nm

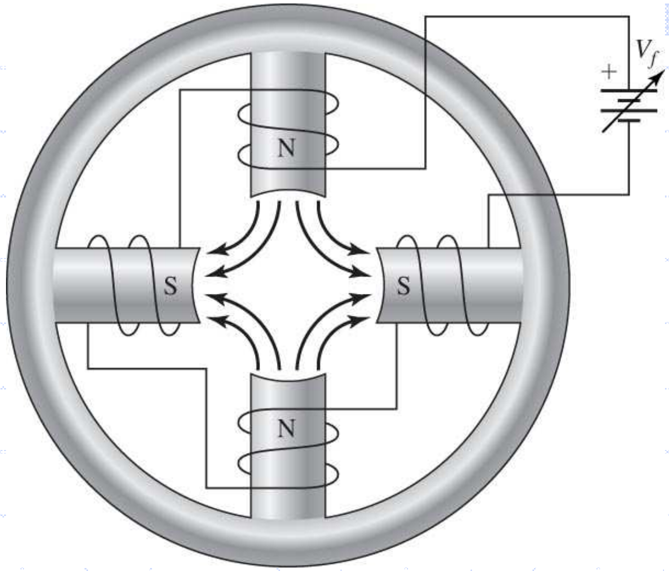
= <u>0.6789 Joule</u>

ASSIGNMENT ON ELECTRO-MECH ENERGY CONVERSION

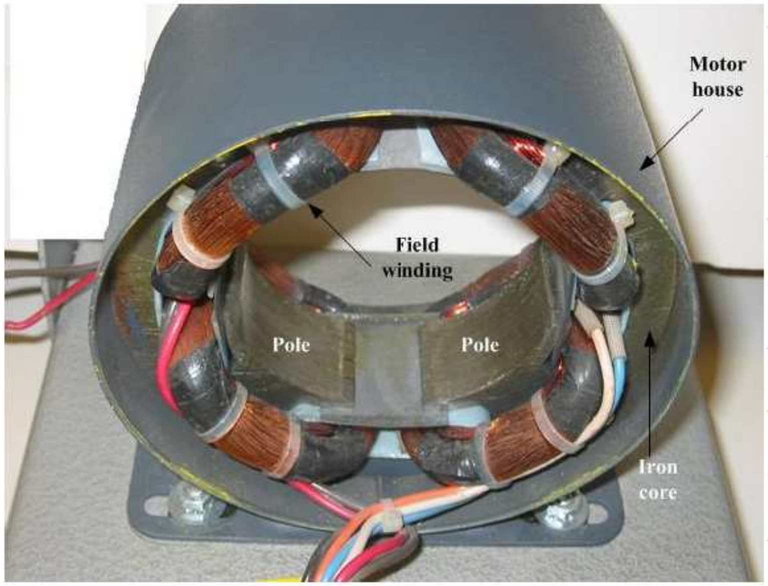
Electromagnet system in Fig. 3.10 has cross-section area 25 cm^2. The coil has 350 turns and 5 ohm resistance. Magnetic core reluctance, fringing effects and leakage flux can be neglected. If the length of air gap is 4 mm and a 110 V DC supply is connected to the coil, find

(a) Stored field energy
(b) Lifting force





Direct-Current Machine

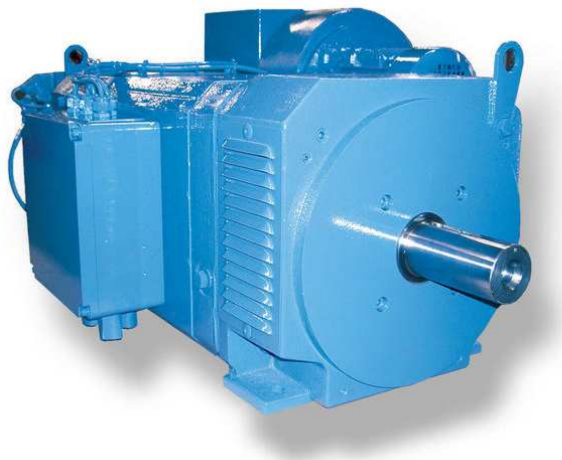


Electric Machine

- Electric machines can be used as motors and generators
- Electric motor and generators are rotating energy-transfer electromechanical motion devices
- Electric motors convert electrical energy to mechanical energy
- Generators convert mechanical energy to electrical energy

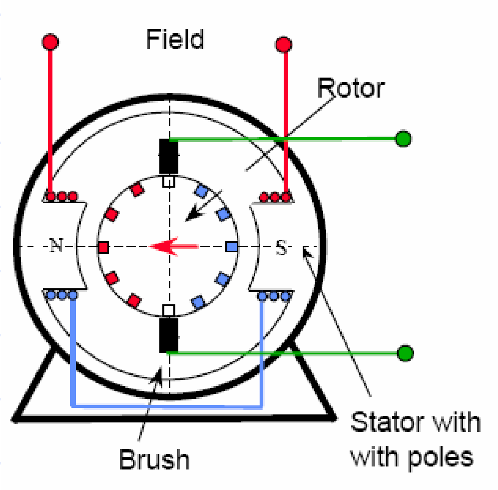
Electric Machine

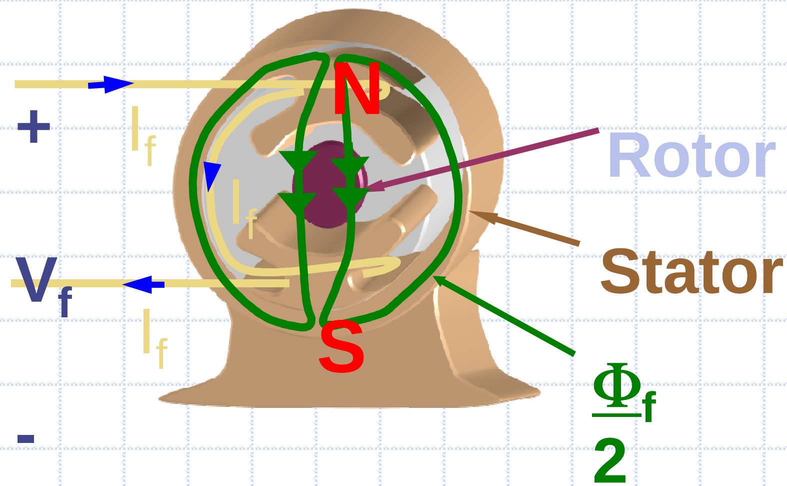
- Electric machines can be divided into 2 types:
  - AC machines
  - DC machines
- Several types DC machines
  - Separately excited
  - Shunt connected
  - Series connected
  - Compound connected
  - Permanent magnet



Electric Machine

- All Electric machines have:
  - Stationary members (stator)
  - rotating members (rotor)
  - Air gap which is separating stator and rotor

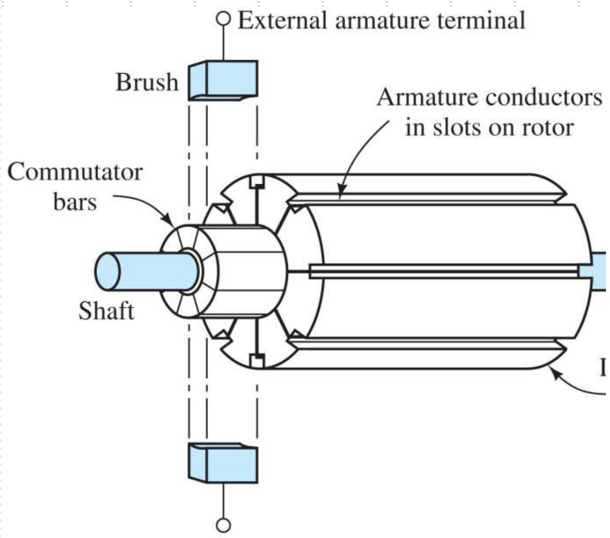


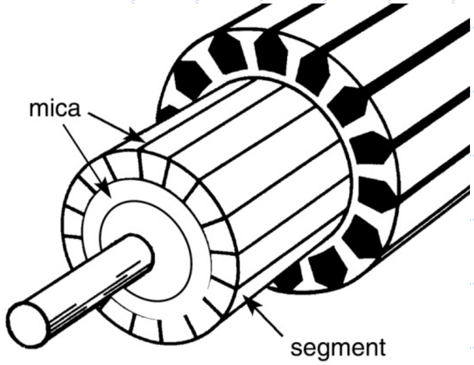


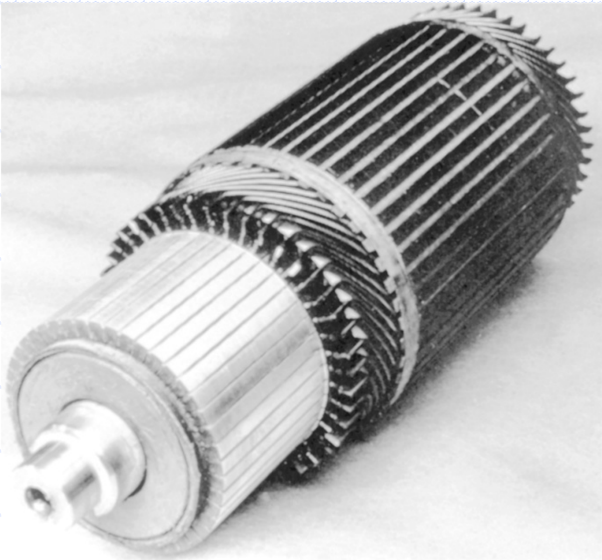
- The rotor and stator are coupled magnetically

Electric Machine

- The armature winding is placed in the rotor slot and connected to rotating commutator which rectifies the induced voltage
- The brushes which are connected to the armature winding, ride on commutator

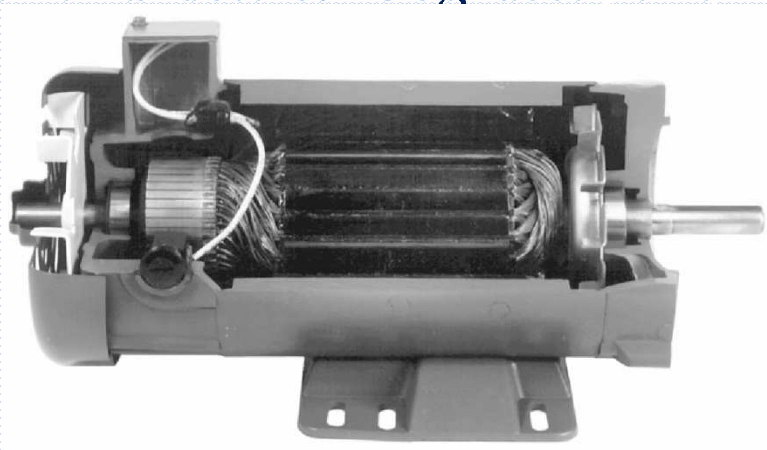


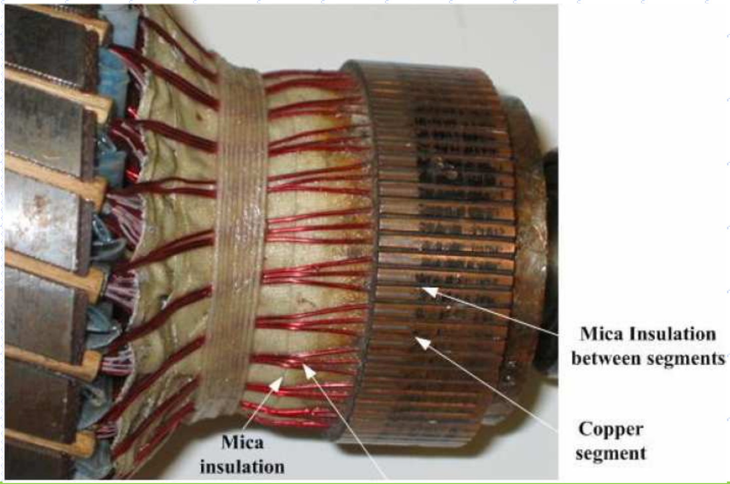




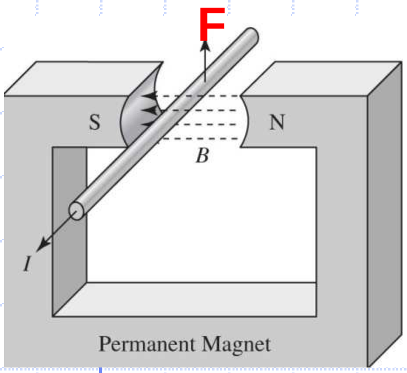
Electric Machine

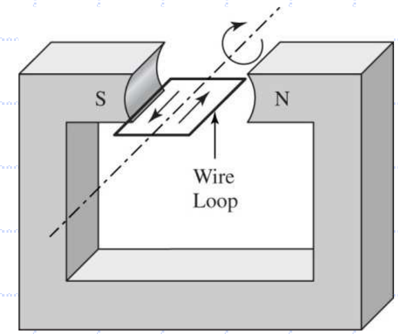
- The armature winding consists of identical coils carried in slots that are uniformly distributed around the periphery of the rotor
- Conventional DC machines are excited by direct current, in particular if a voltage-fed converter is used a dc voltage is supplied to the stationary field winding
- Hence the excitation magnetic field is produced by the field coils
- Due to the commutator, armature and field windings produce stationary magnetomotive forces that are displaced by 90 electrical degrees

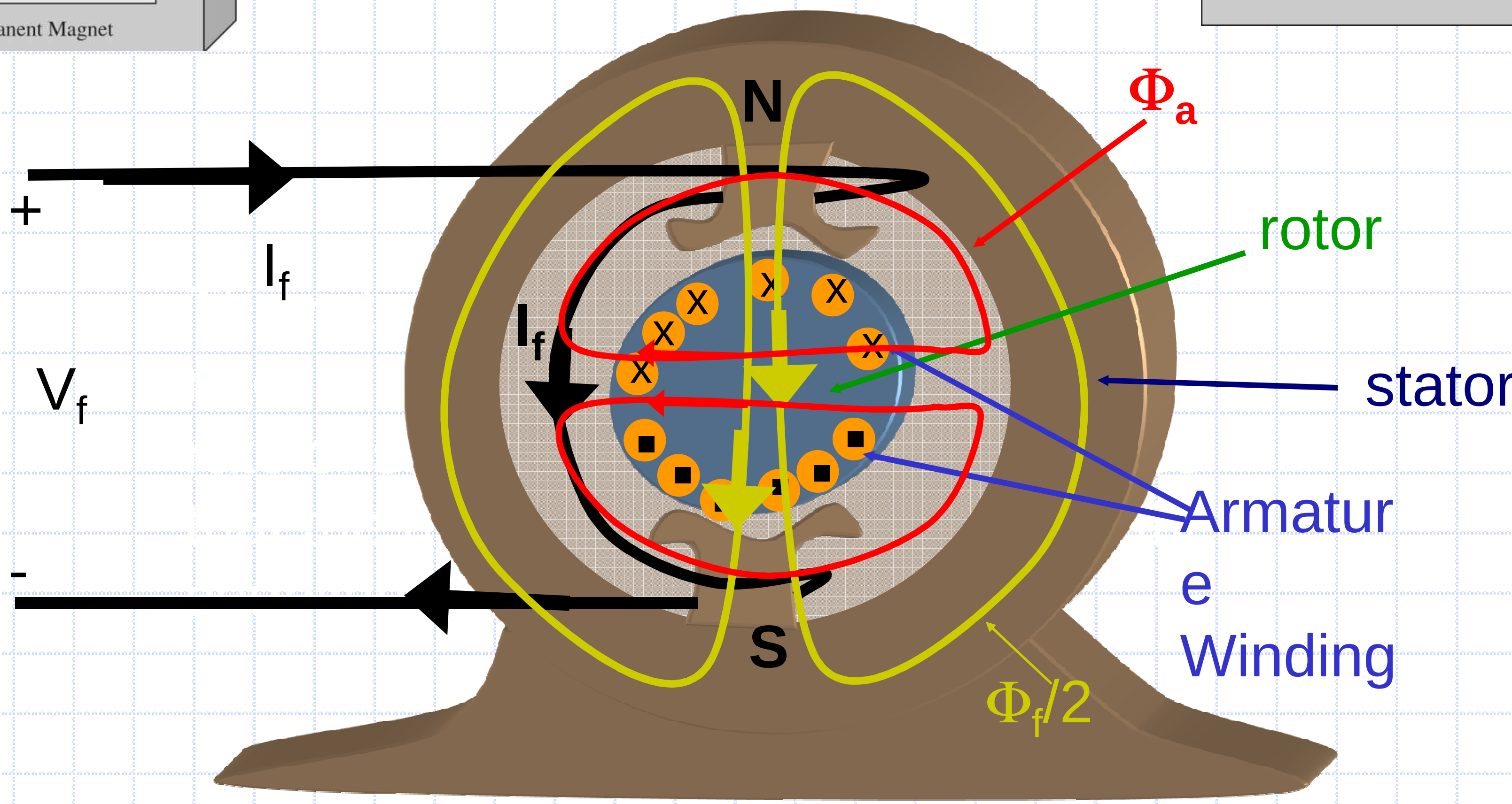




Magnetic Flux in DC machines







DC Machines

- The current is induced in the **Rotor Winding** (**i.e.** the **Armature Winding**) since it is placed in the field (***Flux Lines***) of the Field Winding.

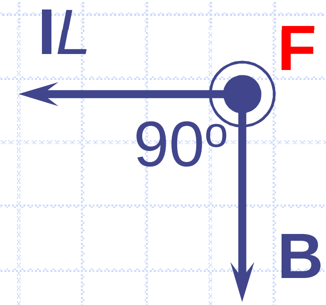
Orthogonality of Magnetic Fields in DC Machines

- mmf produced by the armature and mmf produced by the field winding are orthogonal.

$$\mathbf{F} = \mathbf{I}L \times \mathbf{B} = ILB\sin\left(90^o\right)$$

$\mathbf{B}$ → Magnetic field due to field winding

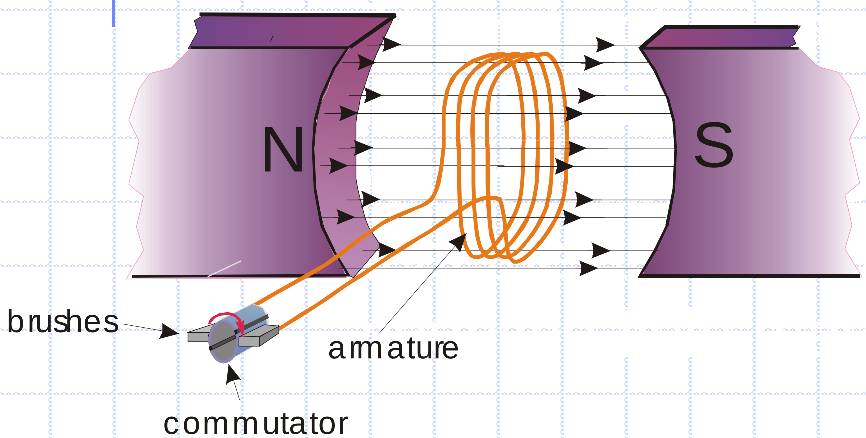
$\mathbf{I}L$ → Magnetic field due to armature winding

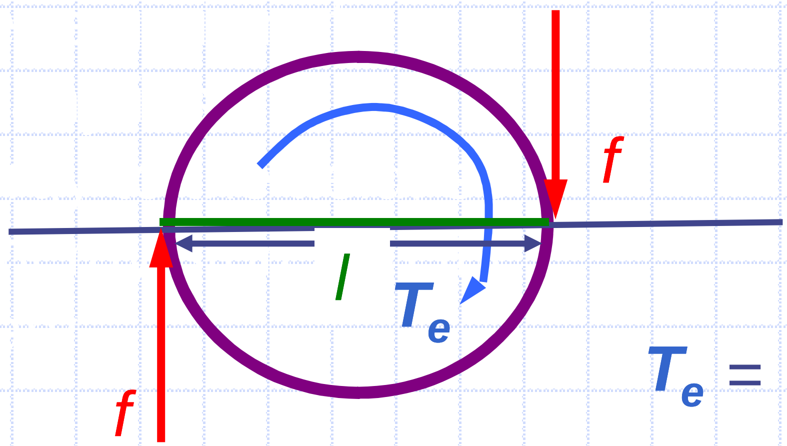


DC Machines

- The force acting on the rotor, is expressed as

$$f = \underbrace{I\,L}_{\substack{\text{Due to}\\\text{the Armature}}} \times \underbrace{B}_{\substack{\text{Due to}\\\text{the Field}}}$$

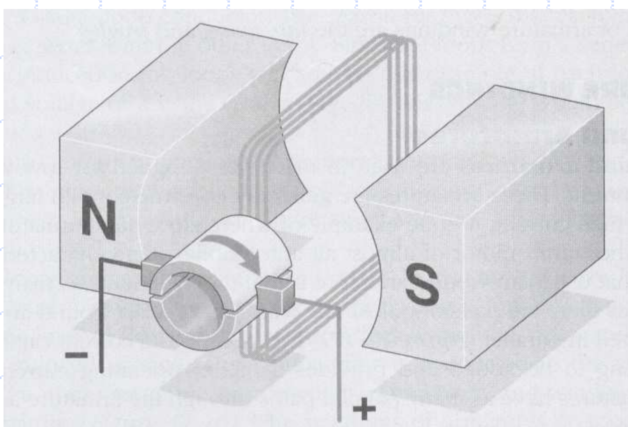


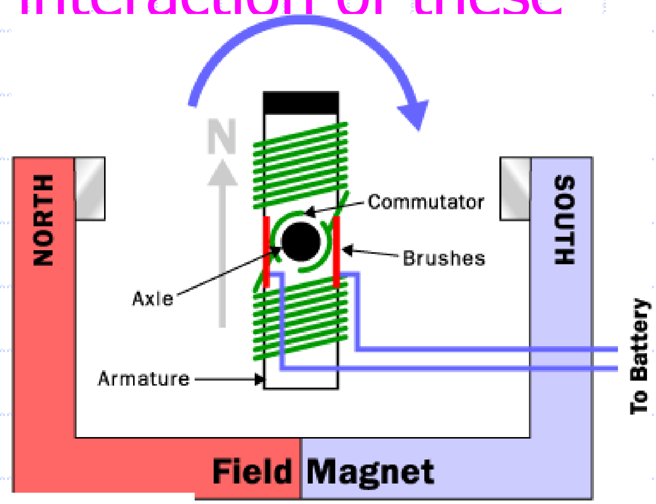


$$T_e = f \times l$$

DC Machines

- The Field winding is placed on the stator and the current (voltage) is induced in the rotor winding which is referred also as the armature winding.
- In DC Machines, the ***mmf*** produced by the field winding and the ***mmf*** produced by the armature winding are at right-angle with respect to each other.
- The torque is produced from the interaction of these two fields.





Types of DC Machines