THE UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

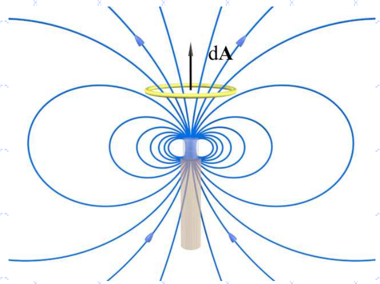
ELECTRICAL AND ELECTRONIC DEPARTMENT

EE321 ELECTROMECHANICS & MACHINES

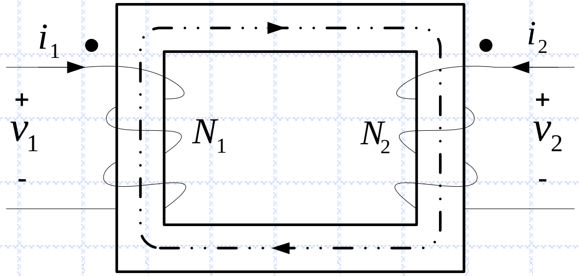
SICHILALU SAM (MSc)

2010

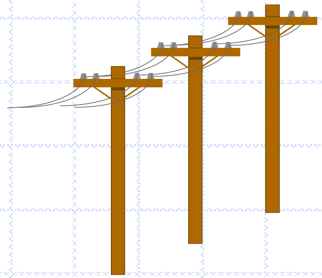
LEARNING OUTCOMES



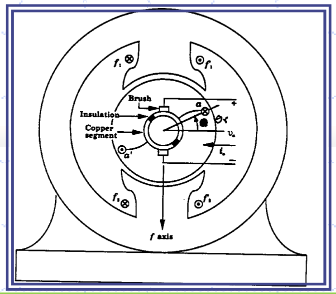
UNDERSTANDING OF MAGNETISM
AND ITS APPLILCATIONS



TRANSFORMERS AND THEIR PRINCIPLE
OF OPERATIONS



UNDERSTANDING OF ELECTRICAL
POWER SYSTEMS AND GENERATION



PRINCIPLE OF OPERATIONS OF ELECTRICAL
MACHINES

Units and Dimensions

Objectives

- Know the difference between units and dimensions
- Understand the SI, systems of units
- Know the SI prefixes from nano- to giga-
- Understand and apply the concept of dimensional homogeneity

Units & Dimensions

- A **system of units** is described by:
  - a set of **fundamental dimensions** from which all other dimensions may be *derived,* and
  - a set of **base units**.

Dimensions

Dimension	Symbol
Length	[L]
Mass	[M]
time	[T]
force	[F]
electric current	[A]
absolute temperature	[θ]
luminous intensity	[I]

Base Units

Fundamental Dimension	Base Unit
time	second (s)
electric current	ampere (A)
absolute temperature	kelvin (K)
luminous intensity	candela (cd)
amount of substance	mole (mol)

The International System of Units (SI)

Fundamental Dimension	Base Unit
length [*L*]	meter (m)
mass [*M*]	kilogram (kg)
time [*T*]	second (s)
electric current [*A*]	ampere (A)
absolute temperature [θ]	kelvin (K)
luminous intensity [*I*]	candela (cd)
amount of substance [*n*]	mole (mol)

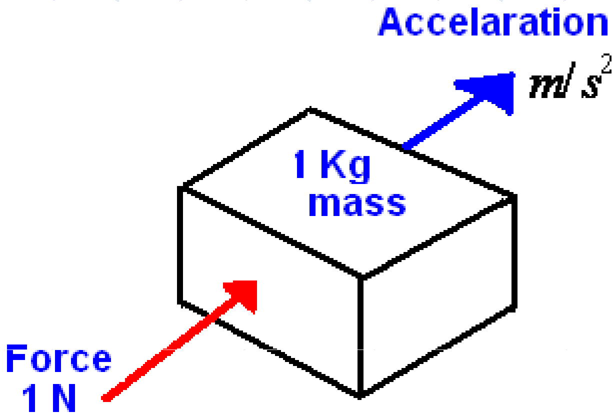
SI Prefixes

Prefix	Decimal Multiplier	Symbol
nano	10^{-9}	n
micro	10^{-6}	µ
milli	10^{-3}	m
centi	10^{-2}	c
deci	10^{-1}	d
deka	10^{+1}	da
hecto	10^{+2}	h
kilo	10^{+3}	k
mega	10^{+6}	M
giga	10^{+9}	G

Supplementary SI Dimensions

Supplementary Dimension	Base Unit
plane angle	radian (rad)
solid angle	steradian (sr)

SI System of Units
Force = (mass) (acceleration)



F=ma

W=mg

$$\text{Force} = ma = \frac{kg \cdot m}{s^2} = \textit{Newton} = N$$

SI System of Units: Work/Energy

$$\text{Work/ Energy} = \text{Force X Distance}$$

$$= \text{N·m}$$

$$= \frac{kg.m^2}{s^2}$$

$$= \textit{Joule}$$

$$= \text{J}$$

SI System of Units: Power

Power = Work / Time

$$= \frac{N \cdot m}{s} = \frac{Joule}{s} = \frac{J}{s}$$

$$= \frac{kg \cdot m^2}{s^3}$$

= *Watt*

= W

SI System of Units: Stress/Pressure

Pressure = Force / Area

$$= \frac{N}{m^2} = \frac{kg \cdot m / s^2}{m^2}$$

$$= \frac{kg}{m \cdot s^2}$$

$= \mathit{Pascal}$

= Pa

Consistency of Units

- Principle of consistency of units:
  - units on the left side of an equation must be the same as those on the right side of an equation
  - dimensional homogeneity

Vector & Scalar Quantities

Characteristics of a Scalar Quantity

- Only has magnitude
- Requires 2 things:
  1. A value
  2. Appropriate units

Ex. Mass: 5kg

Temp: 21° C

Speed: 65 mph

Characteristics of a Vector Quantity

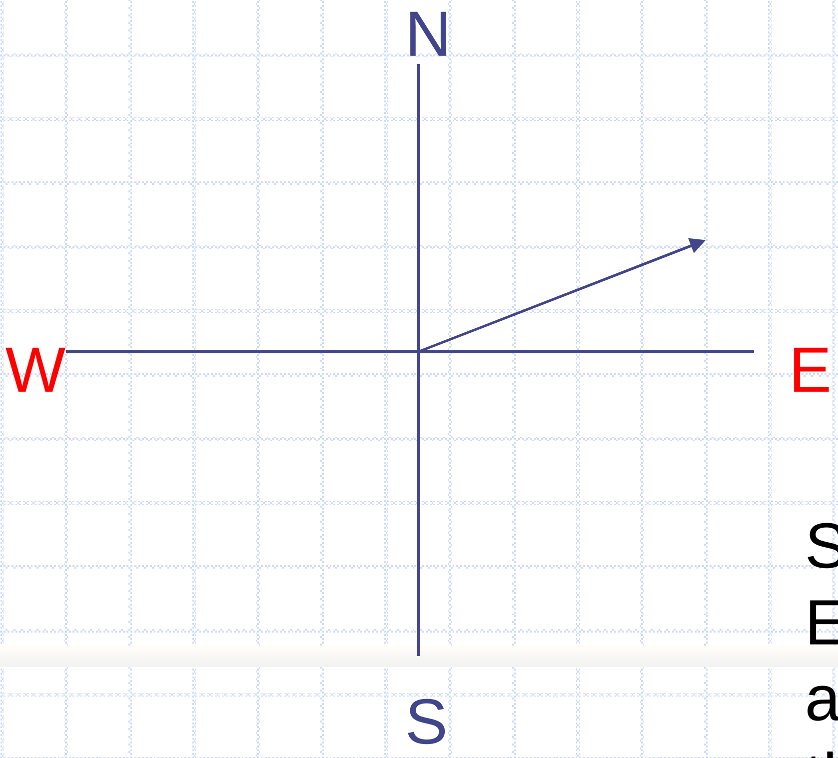
- Has magnitude & direction
- Requires 3 things:
  1. A value
  2. Appropriate units
  3. A direction!

Ex. Acceleration: 9.8 m/s^2 down

Velocity: 25 mph West

Understanding Vector Directions

To accurately draw a given vector, start at the **second** direction and move the given degrees to the **first** direction.

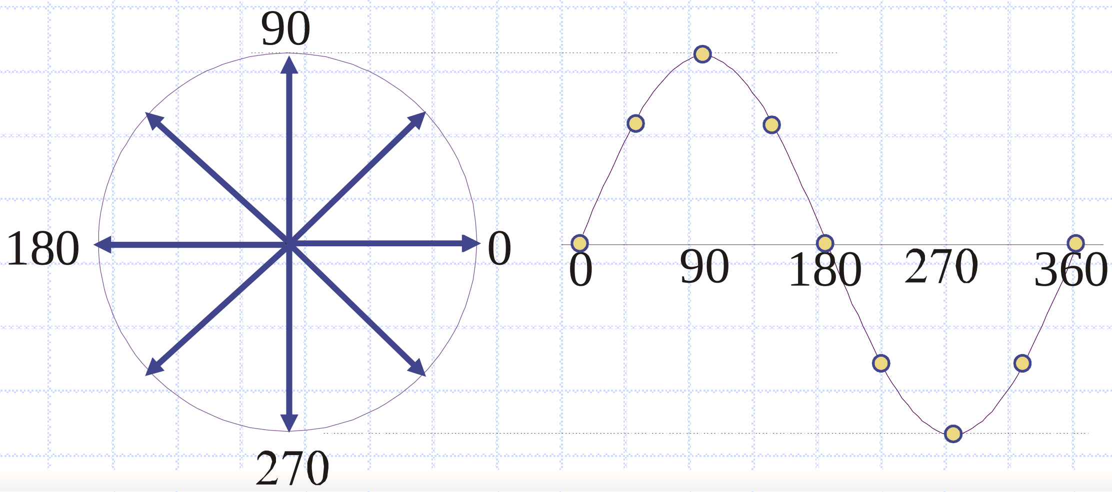


30° N of E

Start on the East origin and turn 30° to the North

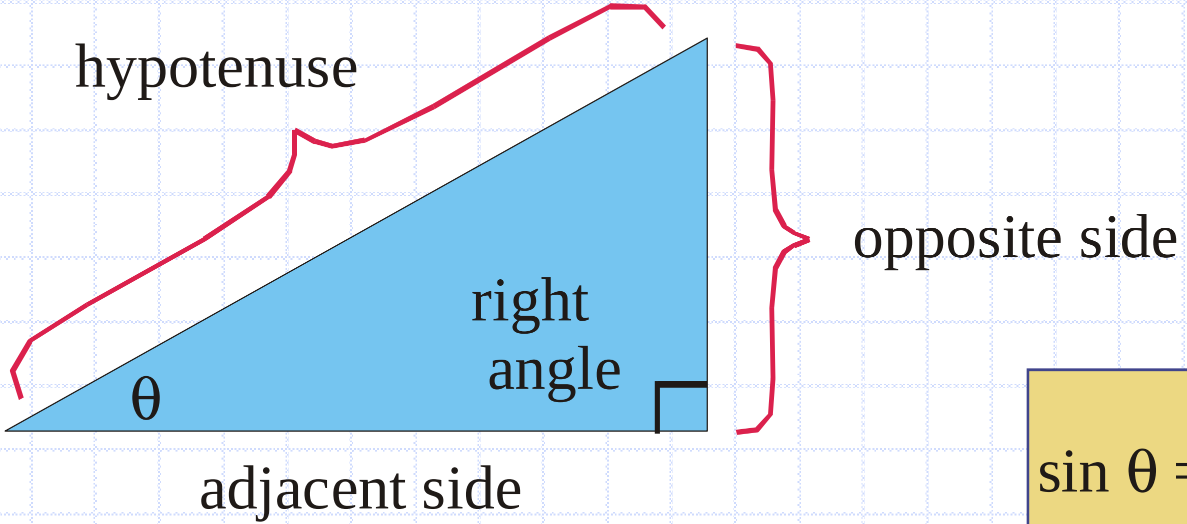
Phasors

The sine wave can be represented as the projection of a vector rotating at a constant rate. This rotating vector is called a **phasor**.



Phasors

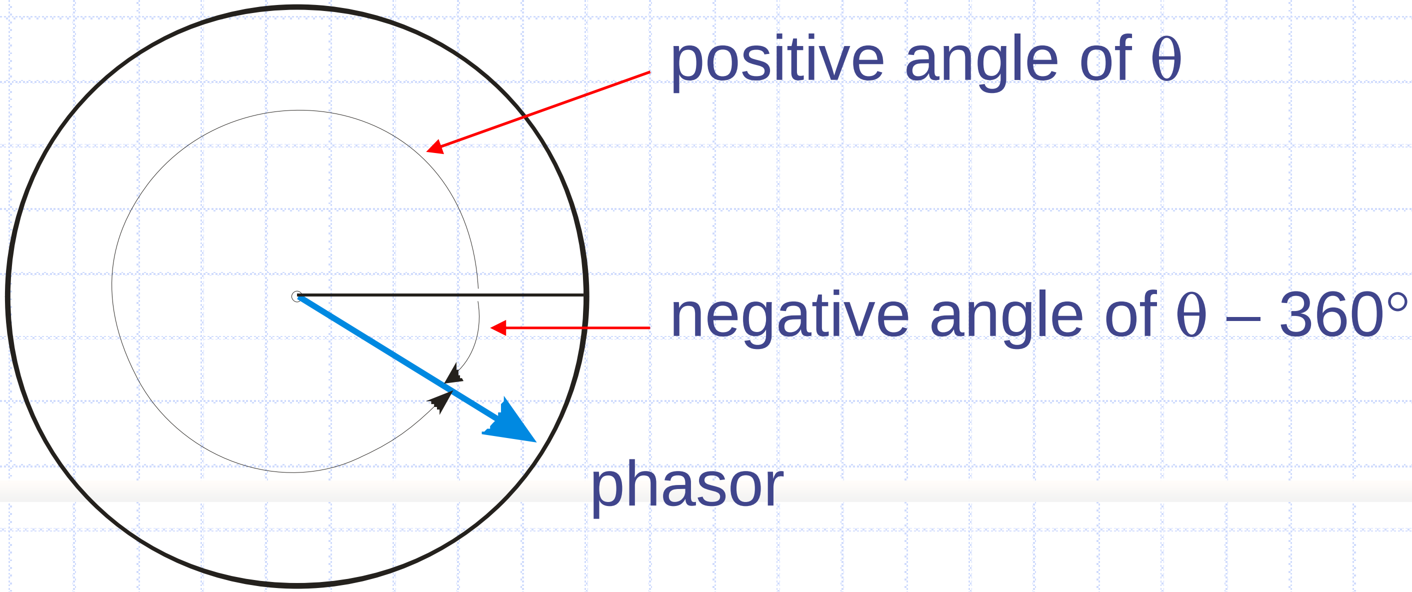
Phasors allow ac calculations to use basic trigonometry. The sine function in trigonometry is the ratio of the opposite side of a right triangle to the adjacent side.



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Phasors

The position of a phasor at any instant can be expressed as a positive angle, measured counterclockwise from 0° or as a negative angle equal to $\theta - 360°$.



Angular velocity of a phasor

When a phasor rotates through 360° or 2π radians, one complete cycle is traced out.
The velocity of rotation is called the **angular velocity** (ω).

$$\omega = 2\pi f$$

(Note that this angular velocity is expressed in radians per second.)

The instantaneous voltage at any point in time is given by

$$v = V_p \sin 2\pi f$$

ELECTROSTATICS

ELECTROSTATICS

Electrostatic is the science of electricity at rest. An atom is electrically neutral, number positive charge of protons are equal to negative charge of electrons. Any imbalance of charges results into a body being positively or negatively charged.

The total deficiency or excess of electrons in a body is known as its Charge

Absolute and Relative permittivity of a medium

All medium posse two certain property of permittivity.

(i) Absolute permittivity ε (ii) Relative permittivity ε_r

For measuring of relative permittivity , vacuum/free space is made reference medium. It has an **Absolute permittivity** $\varepsilon_o = 8.854x10^{-12} F/m$ and **Relative permittivity** $\varepsilon_r = 1$

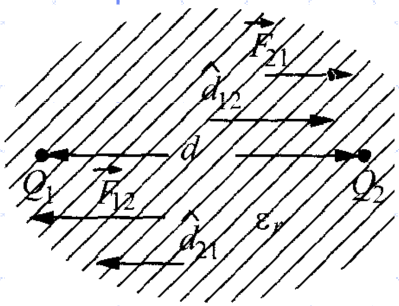
For any other medium if its relative permittivity compare to vacuum is ε_r then its absolute permittivity is $\varepsilon = \varepsilon_o \varepsilon_r F/m$

LAWS OF ELECTROSTATICS

First law→ like charges of electricity repel each other, whereas unlike charges attract each other .

Second Law→ the force exerted between two point charges
(i) is directly proportional to the product of their strengths
(ii) is inversely proportional to the square of the distance between them

This law is known as Coulomb's Law $F \propto \frac{Q_1 Q_2}{d^2}$ or $F = k\frac{Q_1 Q_2}{d^2}$



In vector form, where d is the unit vector (unit length in direction of distance d) $\hat{d} = \vec{d}/d$

$$\vec{F} = k\frac{Q_1 Q_2}{d^2}\hat{d} \quad = \quad \vec{F} = k\frac{Q_1 Q_2}{d^2}\vec{d}$$

Where is $\vec{d}$ vector notation of $\hat{d}$, which is scalar notation

where **d = r** interchangeable

$$\vec{F}_{21} = k\frac{Q_1 Q_2}{d_{12}^{\ 2}}\hat{d}_{12} \quad = \quad \vec{F}_{21} = k\frac{Q_1 Q_2}{d_{12}^{\ 2}}\vec{d}_{12}$$

$\vec{F}_{21}$ Is the force on Q_2 due to Q_1 and $\vec{d}_{12}$ is the unit vector in direction from Q_1 to Q_2 .The opposite is true for $\vec{F}_{12}$

in **SI** the constant $k = \frac{1}{4\pi\varepsilon} = \frac{1}{4\pi\varepsilon_o\varepsilon_r} = \frac{1}{4\pi x 8.854 \cdot 10^{12}} = 8.9878x10^9 = 9x10^9$

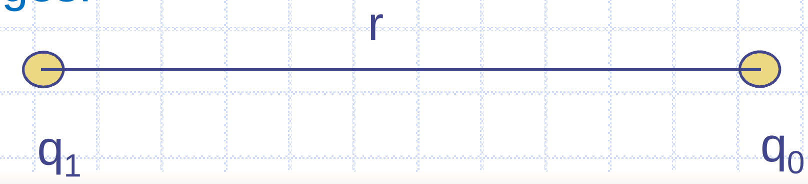
The final equation $F = 9x10^9\frac{Q_1 Q_2}{\varepsilon_r d^2}$ in medium $F = 9x10^9\frac{Q_1 Q_2}{d^2}$ in space/vacuum

The Electric Field

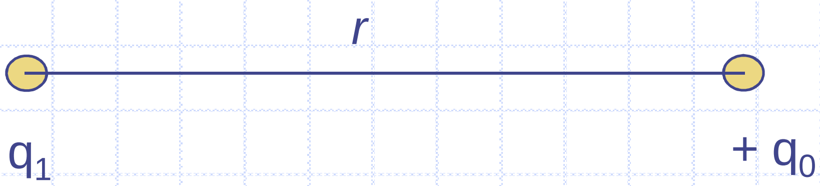
Definition of the electric field. Whenever charges are present and if I bring up another charge, it will feel a net Coulomb force from all the others.

Electric field therefore, is field which is equal to the force per unit positive charge. ***$E=F/q_0$.*** The direction of the electric field is along ***r*** and points in the direction a positive test charge would move.

This idea was proposed by Michael Faraday in the 1830's. The idea of the field replaces the charges as defining the situation. Consider two point charges:



where ***r*** = ***d***



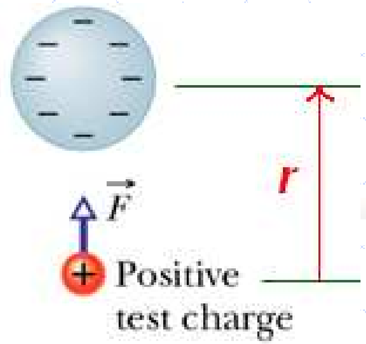
The Coulomb force is $F = 9x10^9 \frac{Q_1 Q_0}{r^2}$

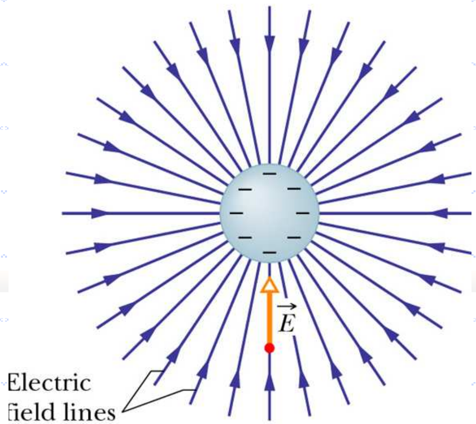
The force per unit charge is $E = \frac{F}{Q_o}$ and then the electric field at r is $E = 9x10^9 \frac{Q_1 \cancel{Q_0}}{r^2 \cancel{Q_o}}$ due to the point charge q_1 .

The units are Newton/Coulomb. The electric field has direction and is a vector. How do we find the direction? The direction is the direction a unit positive test charge would move.

If q_1 were positive

Point negative charge



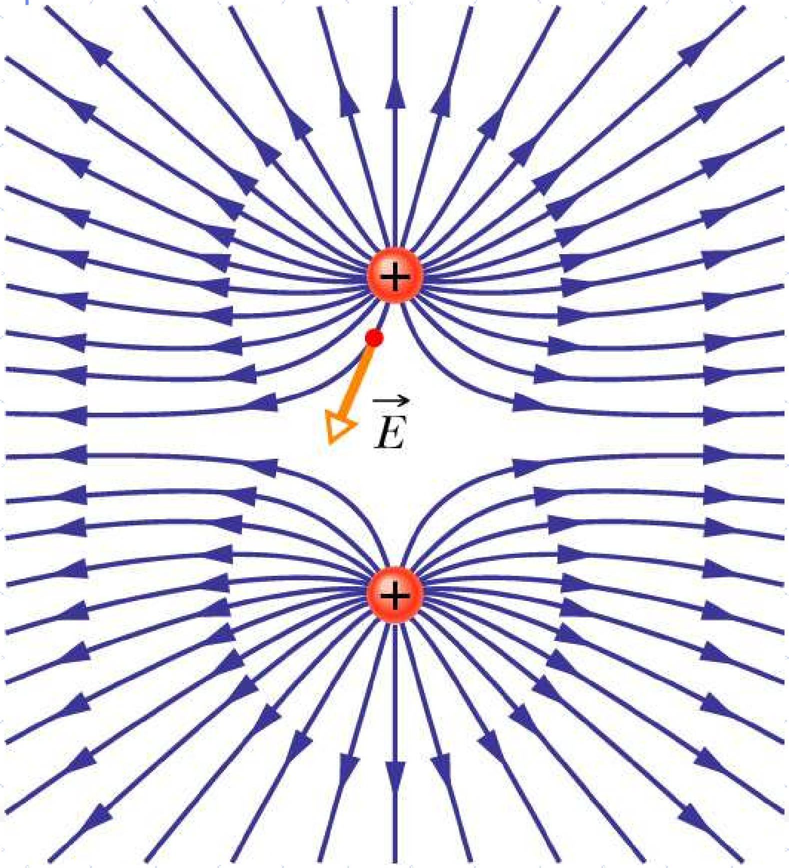


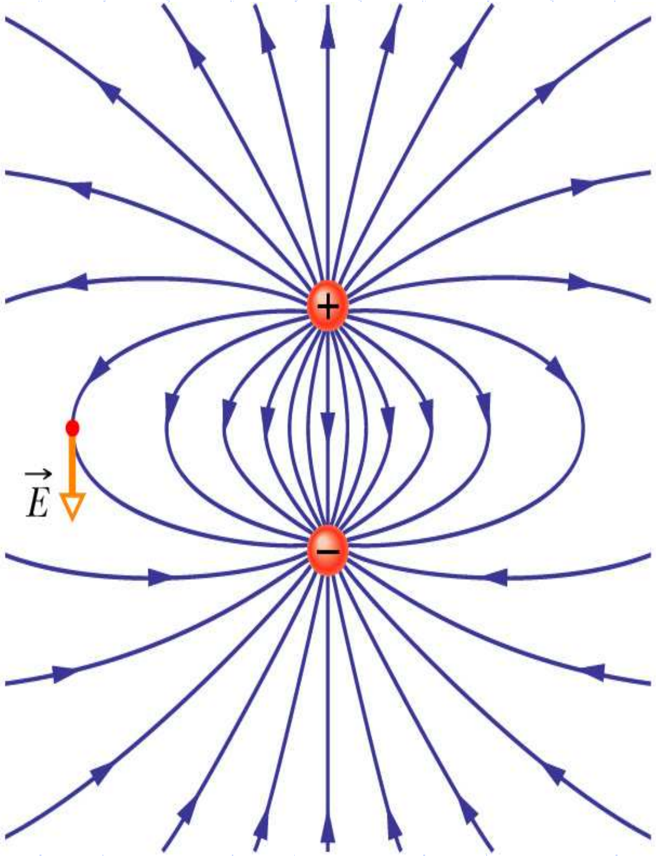
$E = 9x10^9 \frac{Q_1 \cancel{Q_0}}{r^2 \cancel{Q_o}}$

Electric Field Lines

Like charges (++)

Opposite charges (+ -)





This is called an electric dipole.

Methods of evaluating electric fields

- Direct evaluation from Coulombs Law or brute force method

If we know where the charges are, we can find E from $E = \sum k \frac{Q_i}{r_i^2}$

This is a vector equation and can be complex and messy to evaluate and we may have to resort to a computer. The principle of superposition guarantees the result.

- Instead of summing the charge we can imagine a continuous distribution and integrate it. This distribution may be over a volume, a surface or just a line.

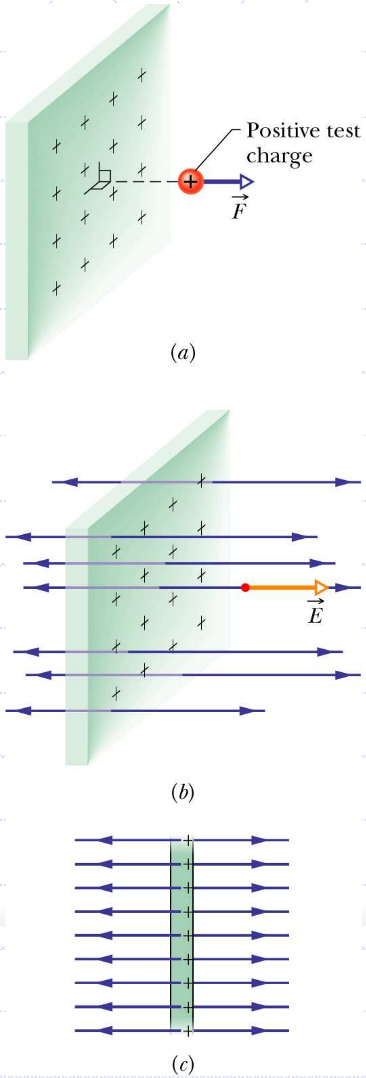
  - $E = \int dE = \int k\, Q_i / r_i^2 \, dq$ where ***r*** is a unit vector directed from charge *dq* to the field point.
  - $dq = \rho dV$, or $dq = \delta dA$, or $dq = \lambda dl$

Example of field lines for a uniform distribution of positive charge on one side of a very large non conducting sheet

This is called a uniform electric field.

How would the electric field change if both sides were charged?

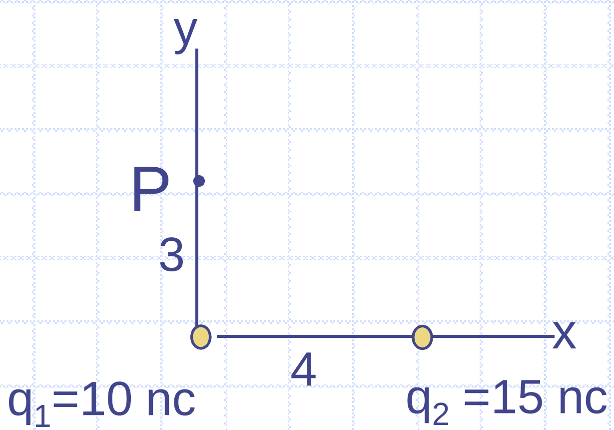
How would things change if the sheet were conducting?



Example of finding electric field from two charges

Example1

Given that we have q1=+10 nC at the origin, q2 = +15 nC at x=4 m as shown below . Find the electric field E at y=3 m and x=0? point P

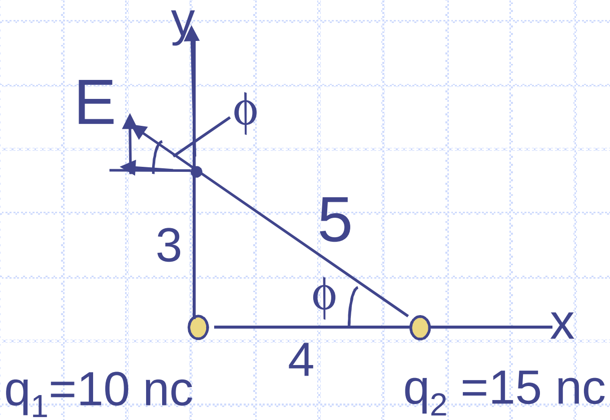


Use principle of superposition

Find x and y components of electric field due to both charges and add them up

Example continued

Recall $E = kq/r^2$
and $k = 8.99 \times 10^9$ N.m²/C²



Field due to q_1

$E = 10^{10}$ N.m²/C² 10 X10^{-9} C/(3m)² = 11 N/C
in the y direction.

E_y= 11 N/C

E_x= 0

Field due to q_2

$E = 10^{10}$ N.m²/C² 15 X10^{-9} C/(5m)² = 6 N/C

at some angle ϕ Resolve into x and y components

$E_y = E \sin \phi = 6 * 3/5 = 18/5 = 3.6$ N/C

$E_x = E \cos \phi = 6 * (-4)/5 = -24/5 = -4.8$ N/C

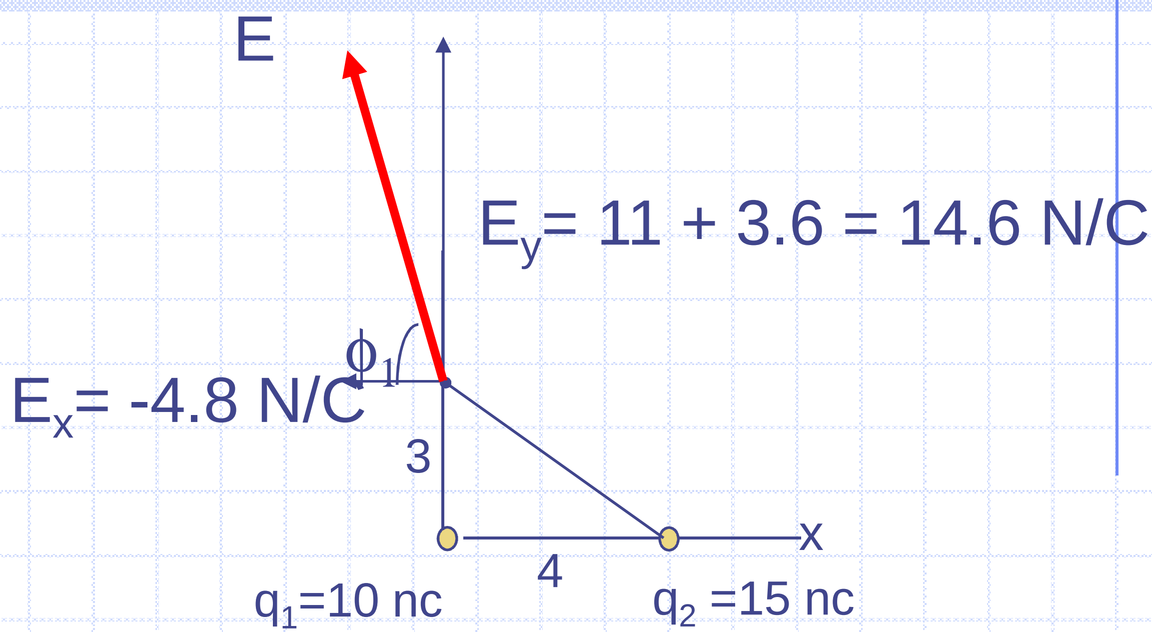
Now add all components

E_y= 11 + 3.6 = 14.6 N/C
E_x = -4.8 N/C

Magnitude

$$E = \sqrt{E_x^2 + E_y^2}$$

Example continued



Magnitude of electric field

$$E = \sqrt{E_x^2 + E_y^2}$$

$$E = \sqrt{(14.6)^2 + (-4.8)^2} = 15.4\, N/C$$

Using unit vector notation we can also write the electric field vector as:

$$\vec{E} = -4.8\,\hat{i} + 14.6\,\hat{j}$$

ϕ_1 = atan E_y/E_x= atan (14.6/-4.8)= 72.8 deg

MAGNETOSTATIC

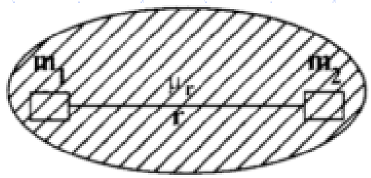
MAGNETOSTATIC
(Terms and Definitions)

Laws of magnetism

Coulomb was the first to determine experimentally the quantitative expression for the magnetic force between two isolated point poles. Magnetic poles always exits in pairs and impossible is isolate single pole at current level of scientific knowledge.

The force between two magnetic poles placed in medium is;

(i) is directly proportional to their pole strengths
(ii) is inversely proportional to the square of the distance between them and
(iii) is inversely proportional to the absolute permeability of the surrounding medium.



$$\vec{F} = k\frac{Q_1Q_2}{d^2}\hat{d}$$

$$F \propto \frac{m_1m_2}{\mu r^2} \quad \text{or} \quad F = k\frac{m_1m_2}{\mu r^2} \quad \text{or} \quad \vec{F} = \frac{k\, m_1m_2}{\mu r^2}\hat{r} \quad \text{in vector from}$$

where $\hat{r}$ is a unit vector to indicate direction of r.

or

$$\vec{F} = k\frac{m_1 m_2}{r^3}\vec{r}$$ where $\vec{F}$ and $\vec{r}$ are vectors

In the S.I. system of units, the value of the constant k is = $1/4\pi$.

$$F = \frac{m_1 m_2}{4\pi\mu r^2}\text{ N} \quad \text{or} \quad F = \frac{m_1 m_2}{4\pi\mu_0\mu_r r^2}\text{ N}$$ – in a medium

In vector form

$$\vec{F} = \frac{m_1 m_2}{4\pi\mu r^3}\vec{r} = \frac{m_1 m_2}{4\pi\mu_0 r^2}\text{ N}$$

If the above equation is

$m_1 = m_2 = m$ (say) ; $r = 1$ metre ; $F = \dfrac{1}{4\pi\mu_0}$ N

then

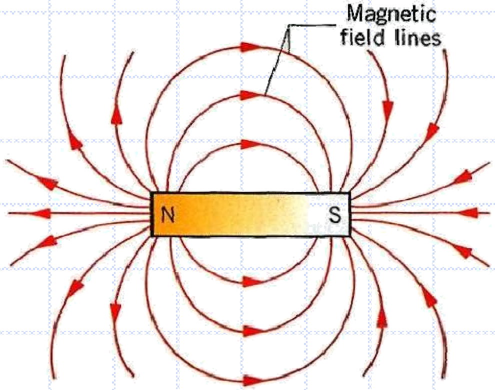
$m^2 = 1$ or $m = \pm 1$ weber*

Hence, a unit magnetic pole may be defined as *that pole which when placed in vacuum at a distance of one metre* from a similar and equal pole repels it with a force of $1/4\pi\,\mu_0$ *newtons*.

1. Magnetic Flux φ (WEBER)

a. Flux Lines have the following characteristics:

i. Direction
ii. Continuous Loops
iii. Flux lines repel each other
iv. Flux lines arrange themselves to be as short as possible



$$\varphi = F / R$$

φ = Magnetic Flux in Webers (Wb)
F = Magnetomotive Force (MMF) in Ampere Turns
R = Reluctance

2. Flux Density: B (Wb/m2) (Tesla)

B = φ/A

B = Magnetic flux density

φ = flux, in webers

A = area, in square meters

Magnetic potential : M (J/Wb)

The magnetic potential at any point within a magnetic field is measured by the work done in shifting a *N*-pole of one weber from infinity to that point against the force of the magnetic field. It is given by

$$M = \frac{m}{4\pi\mu_0 r} \text{ J/Wb}$$

It is a scalar quantity.

3. Permeability: μ (measured in weber/ampere-turn-meter)

Definition: The ease with which flux lines may be established within a material. It is represented by the Greek letter μ or noted as (Webers/Amp-meter).

The amount of flux density (B) that will occur for a given magnetic field intensity (H).

$$\mu = B/H$$

μ = Permeability of a material
B = Flux Density
H = Magnetic Field Intensity

4. Relative Permeability: $\mu_{relative}$

Definition: the ratio of its absolute permeability (μ) to the permeability of a vacumn (μ_0)

$$\mu_{relative} = \mu/\mu_0$$

$\mu_0 = 4\pi x10\text{-}7$ ***Wb/A-m***. *Permeability of a vacuum*

5. **Magnetomotive Force: $\mathscr{F}$ (MMF)**

Definition: The amount of force produced by means of current carrying conductive coil.

$$\mathscr{F} = N*I \quad = \text{Ampere Turns}$$

$\mathscr{F}$ = Magnetomotive force, in ampere-turns

N = Number of turns of a conductor

I = Current through the N turns

6. **Magnetic Field Intensity: *H*** (measured in Ampere turns per meter)

$$H = \mathscr{F}/l = NI/l$$

H = Magnetic Field Intensity

$\mathscr{F}$ = Magnetomotive Force (Ampere turns)

l = average length of path (in meters)

It would be helpful to remember that following terms are sometimes interchangeably used with field intensity : Magnetising force, strength of field, magnetic intensity and intensity of magnetic field.

7. Reluctance $\mathscr{R}$ (synonymous with Resistance in an electric circuit)

Definition: The opposition to the establishment of a magnetic field in a material

$$\mathscr{R} = l/\mu A$$

$\mathscr{R}$ = Reluctance

l = average length of path (in meters)

μ = Permeability of material

A = Area in m^2

8.Intensity of Magnetization (I)

It may be defined as the induced pole strength developed per unit area of the bar.

Let m = pole strength induced in the bar in Wb

A = face or pole area of the bar in m^2

Then $I = m/A$ Wb/m^2

Hence, it is seen that intensity of magnetisation of a substance may be defined as *the flux density produced in it due to its own induced magnetism.*

If l is the magnetic length of the bar, then the product $(m \times l)$ is known as its magnetic moment M.

$$\therefore \quad I = \frac{m}{A} = \frac{m \times l}{A \times l} = \frac{M}{V} = \text{magnetic moment/volume}$$

9. Susceptibiliy (K)

Susceptibility is defined as *the ratio of intensity of magnetisation I to the magnetising force H*.

$\therefore \quad K = I/H$ henry/metre.

Relation between B, H, I and K

flux density B in a material is given by

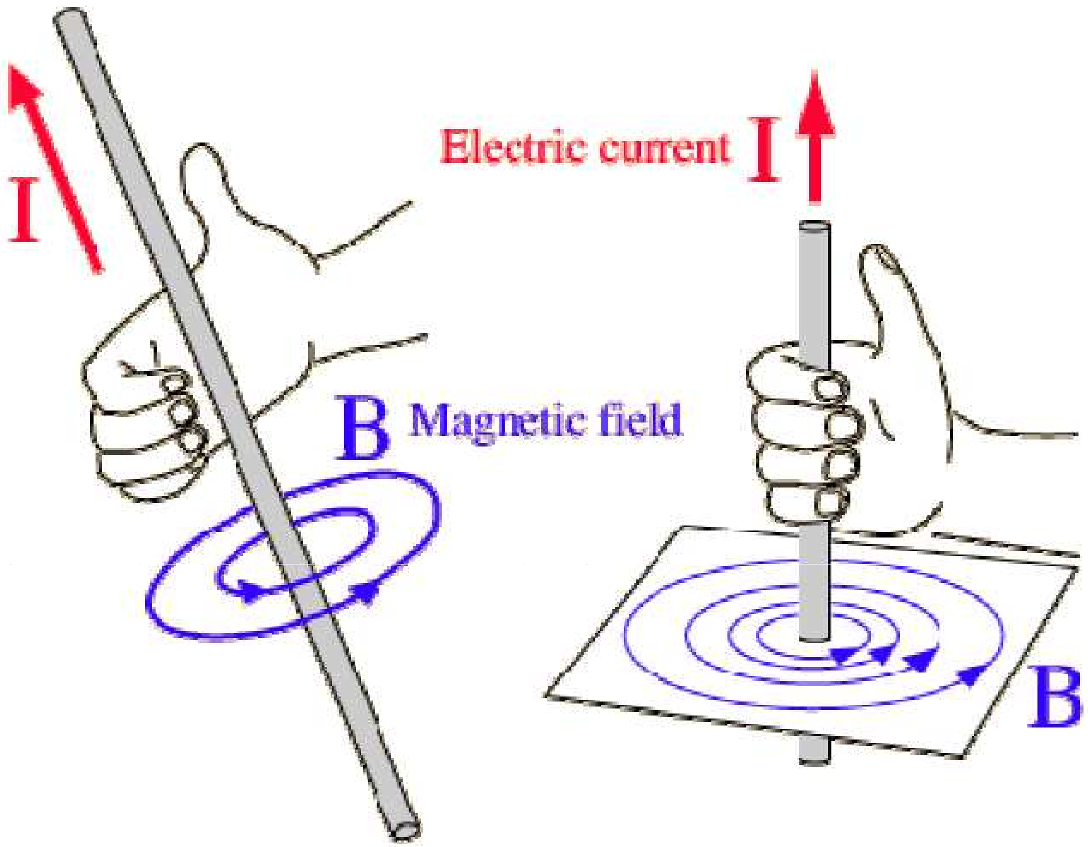
$$B = B_0 + m/A = B_0 + I \qquad \therefore \quad B = \mu_0 H + I$$

Now absolute permeability is $\mu = \frac{B}{H} = \frac{\mu_0 H + I}{H} = \mu_0 + \frac{I}{H} \qquad \therefore \quad \mu = \mu_0 + K$

Also $\mu = \mu_0 \mu_r \quad \therefore \quad \mu_0 \mu_r = \mu_0 + K$ or $\mu_r = 1 + K/\mu_0$

Quantity	Symbol	SI Units
Magnetic Pole Strength	p	A m
Permeability	μ_o, μ	H/m
Relative permeability	μ_r	unitless
Magnetic Flux Density	B	Wb/m^2=Tesla
Magnetic Intensity	H	A/m
Magnetic Polarization	J	A/m
Magnetic Moment	M	A/m^2

Current's Relationship to magnetism

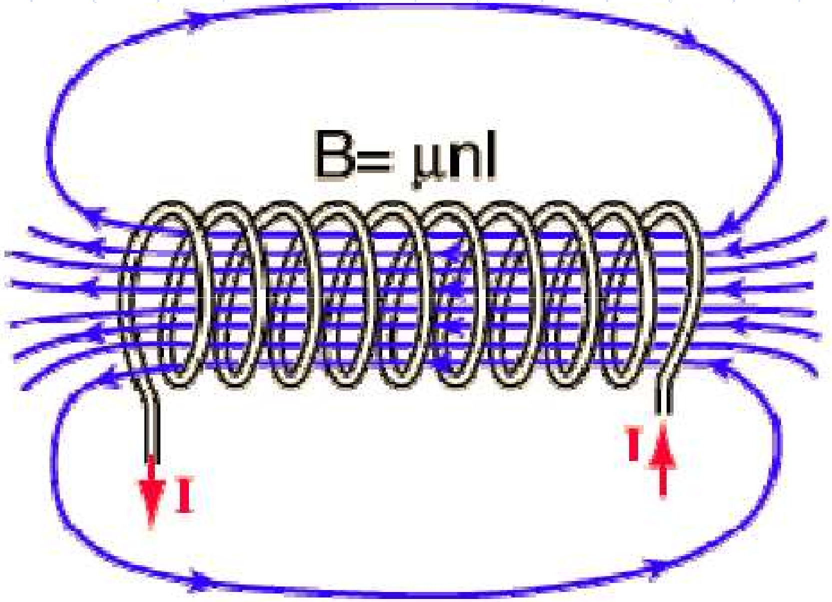


Right Hand Rule (illustration)

As current flows through a conductor the direction of the magnetic field can be determined by use of the **Right Hand Rule.** Wrap your right hand around the conductor as shown with your thumb pointing in the direction of electron flow. Your fingers will illustrate the direction of the magnetic field**(B).**

Magnetism in a solenoid

The Magnetic field concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.



Flux density of a coil- can be increased by increasing the current

Inductors

$L = N^2 \mu A/l$ Henrys's

N = number of turns

µ = Core permeability

A = Cross-sectional area of core, in square meters

l = average length of core, in meters

Example

The magnetic susceptibility of oxygen gas at 20°C is 167×10^{-11} H/m. Calculate its absolute and relative permeabilities.

Solution. $\mu_r = 1 + \frac{K}{\mu_0} = 1 + \frac{167 \times 10^{-11}}{4\pi \times 10^{-7}} = \mathbf{1.00133}$

Now, absolute permeability $\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1.00133 = \mathbf{12.59 \times 10^{-7}}$ ***H*/m**

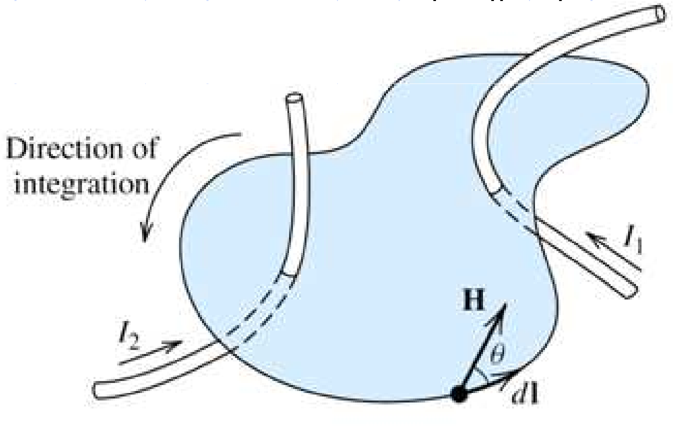
Ampere's Law

Ampére's Law

The line integral of the magnetic field intensity around a closed path is equal to the sum of the currents flowing through the area enclosed by the path.

$$\oint \vec{H} . \vec{dl} = \sum i$$

$$\vec{H} . \vec{dl} = \left|\vec{H}\right|\left|\vec{dl}\right| \cos \theta$$



$\oint \mathbf{H} \bullet d\mathbf{l} = I_1 + I_2$

Ampère's law states that the line integral of magnetic field intensity around a closed path is equal to the sum of the currents flowing through the surface bounded by the path.

Example of Ampére's Law

Find the magnetic field along a circular path around an infinitely long Conductor carrying 'I' ampere of current.

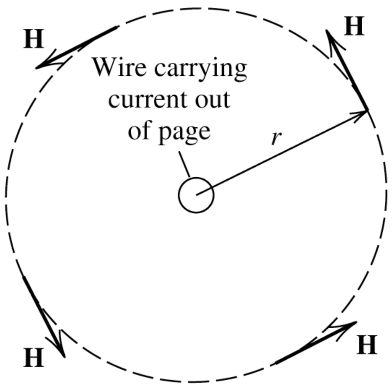
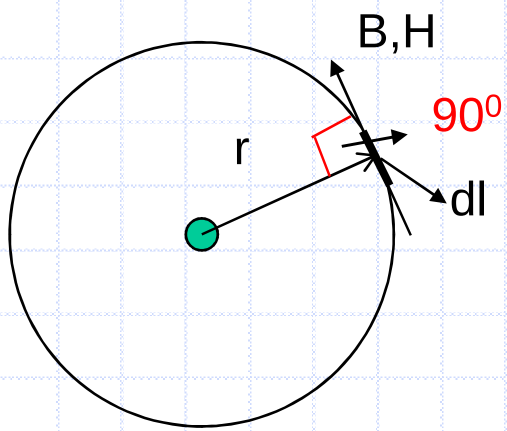


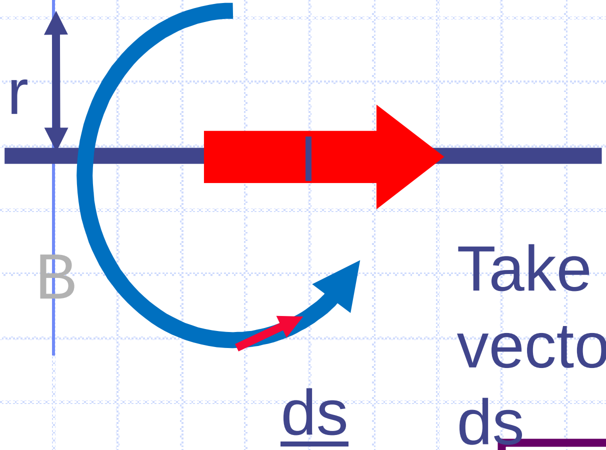
Figure 15.7 The magnetic field around a long straight wire carrying a current can be determined with Ampère's law aided by considerations of symmetry.



Since both $\vec{dl}$ and $\vec{H}$ are perpendicular to radius 'r' at any point 'A' on the circular path, the angle θ is zero between them at all points. Also since all the points on the circular path are equidistant from the current carrying conductor $\vec{H}$ is constant at all points on the circle

$$\oint \vec{H}.\vec{dl} = \left|\vec{H}\right| \oint \vec{dl} = \left|\vec{H}\right| 2\pi r = I \quad \text{or} \quad \left|\vec{H}\right| = \frac{I}{2\pi r}$$

Magnetic Field from a long wire surface dot integral



Using Biot-Savart Law

$$|B| = \frac{\mu_0 I}{2\pi r}$$

Take a short vector on a circle, ds

$$\mathbf{B} \bullet \mathbf{ds} = |\mathbf{B}| \bullet |\mathbf{ds}| \cos\theta$$

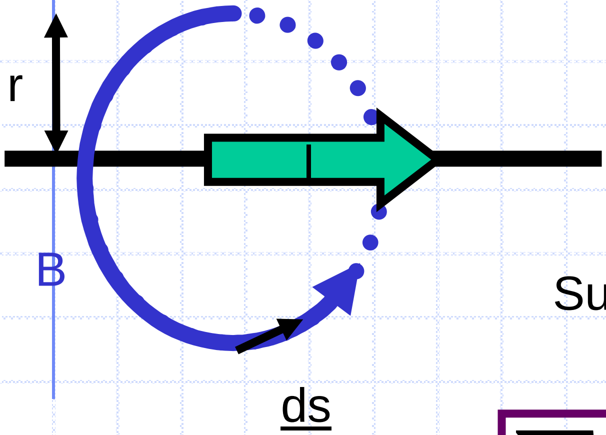
$$\theta = 0 \Rightarrow \cos\theta = 1$$

$$\mathbf{B} \bullet \mathbf{ds} = |\mathbf{B}||\mathbf{ds}|$$

Thus the dot product of B & the short vector ds is:

$$\mathbf{B} \bullet \mathbf{ds} = \frac{\mu_0 I}{2\pi r} ds$$

Sum **B.ds** around a circular path



$$\mathbf{B}\bullet\mathbf{ds} = \frac{\mu_0 I}{2\pi r} ds$$

Sum this around the whole ring

$$\sum \mathbf{B}\bullet\mathbf{ds} = \sum \frac{\mu_0 I}{2\pi r} ds = \frac{\mu_0 I}{2\pi r} \sum ds$$

Circumference of circle

$$\sum ds = 2\pi r$$

$$\Rightarrow \sum \mathbf{B}\bullet\mathbf{ds} = \frac{\mu_0 I}{2\pi r} 2\pi r = \mu_0 I$$

Sum **B.ds** around any path

$$\sum_{\text{circ.}} \mathbf{B} \bullet \mathbf{ds} = \mu_0 I$$

N.B. this does not depend on r

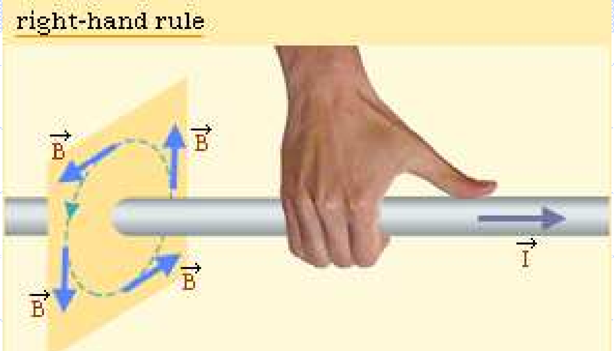
In fact it does not depend on path

$$\sum_{\text{path}} \mathbf{B} \bullet \mathbf{ds} = \mu_0 I$$

Ampere's Law: $\sum_{\text{path}} \mathbf{B} \bullet \mathbf{ds}$ on any closed loop $= \mu_0 I$

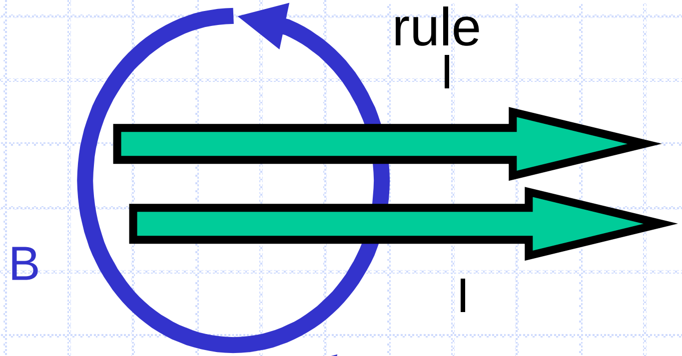
where **I** is the current flowing through the loop

Ampere's Law

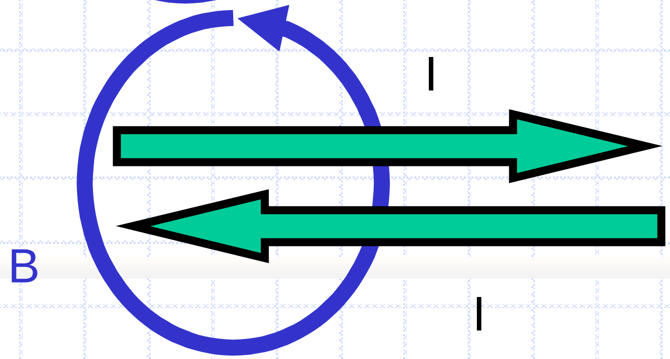


Sign comes from direction of loop, current & right hand rule

$$\sum_{\text{path}} \mathbf{B} \bullet \mathbf{ds} = \mu_0 I$$

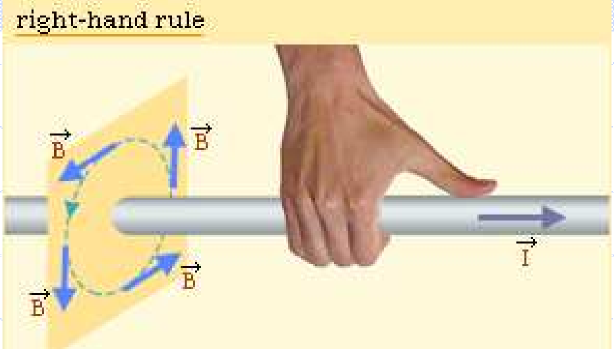


$$\sum_{\text{path}} \mathbf{B} \bullet \mathbf{ds} = 2\mu_0 I$$



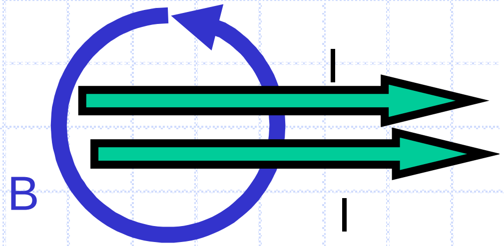
$$\sum_{\text{path}} \mathbf{B} \bullet \mathbf{ds} = 0$$

Ampere's Law

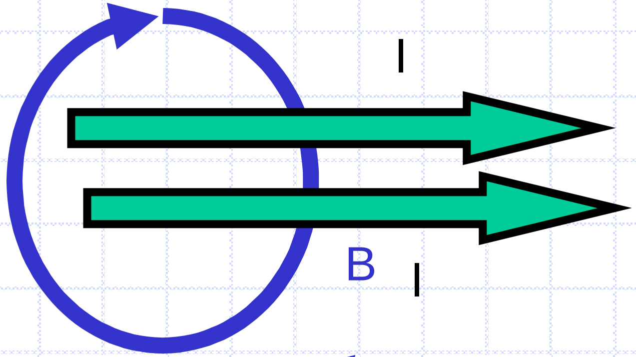


Sign comes from direction of loop, current & right hand rule

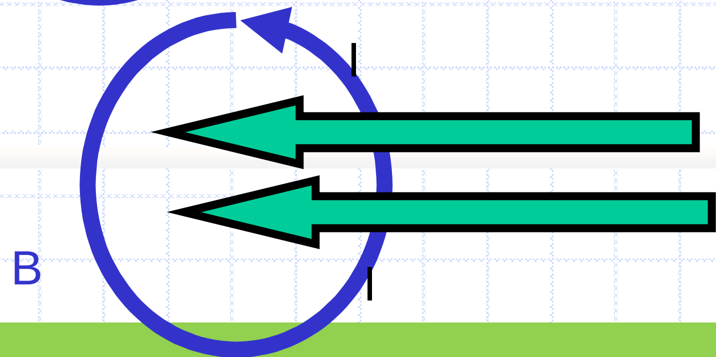
$$\sum_{path} \mathbf{B} \bullet \mathbf{ds} = 2\mu_0 I$$



$$\sum_{path} \mathbf{B} \bullet \mathbf{ds} = -2\mu_0 I$$



$$\sum_{path} \mathbf{B} \bullet \mathbf{ds} = -2\mu_0 I$$



Ampere's Law

$$\sum_{\text{path}} \mathbf{B} \bullet \mathbf{ds} = \mu_0 I$$

$$= \oint \mathbf{B} \bullet \mathbf{ds}$$

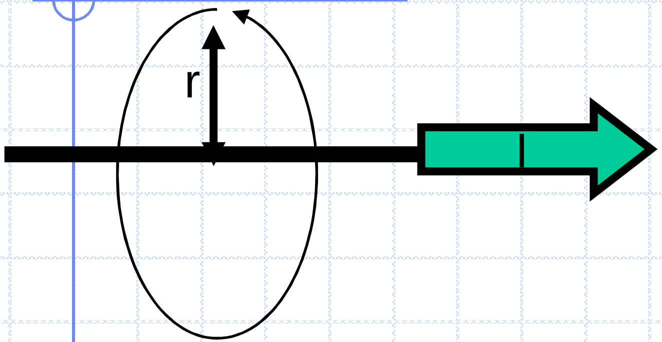
- No Different Physics from Biot-Savart Law
- Useful in cases where there is a high degree of symmetry
- C.f. Coulomb's Law and Gauss's Law in electrostatics

Examples of using Ampere’s Law

- Long-straight wire
- Insider a long straight wire

Magnetic Field from a long wire

Tangential component



Take a circle of radius r as the Ampere Loop

$$\sum_{\text{Circle}} \mathbf{B} \bullet \mathbf{ds} = \mu_0 I$$

Tangential component

$$\mathbf{B} \bullet \mathbf{ds} = |\mathbf{B}||\mathbf{ds}|$$

By symmetry at constant r

$$|B| = \text{constant}$$

L.H.S.

$$\sum_{\text{Circle}} \mathbf{B} \bullet \mathbf{ds} = |B| \sum_{\text{Circle}} |\mathbf{ds}| = 2\pi r B$$

L.H.S. = R.H.S

$$2\pi r \mathrm{B} = \mu_0 I$$

or

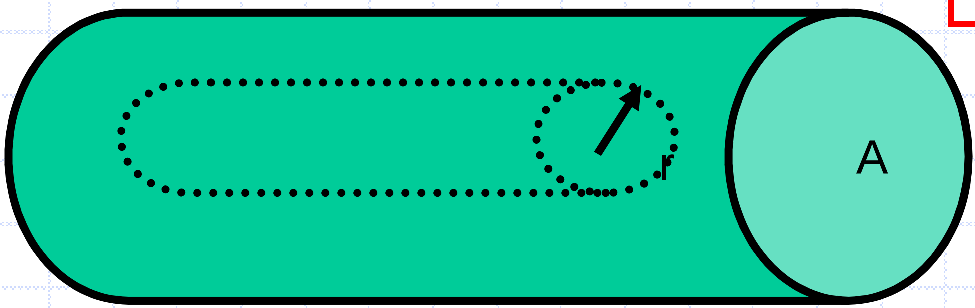
$$B = \frac{\mu_0 I}{2\pi r}$$

Inside a wire current carrying I_0

We Take our Ampere loop to be a circle of radius *r*

Assuming that the current density is uniform then the current flowing through the loop is

$$I = \frac{a}{A} I_0 = \frac{\pi r^2}{\pi R^2} I_0 = \frac{r^2}{R^2} I_0$$



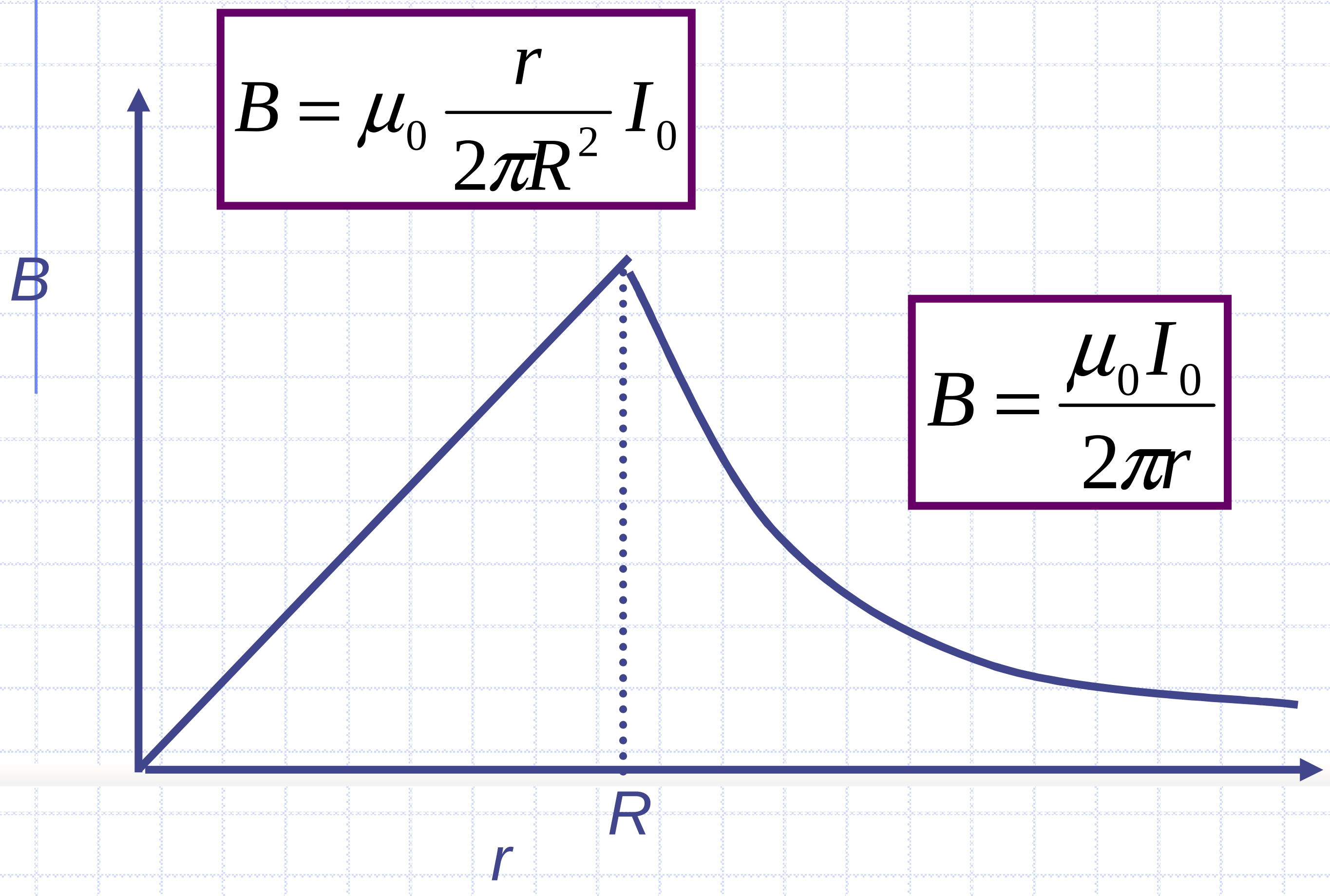
Now same as before

$$\sum_{\text{Circle}} \mathbf{B} \bullet \mathbf{ds} = 2\pi r \mathrm{B} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \mu_0 \frac{r}{2\pi R^2} I_0$$

Field from a long wire



Example on ampere laws

Calculate the magnetizing force and flux density at a distance of 5cm from a long straight circular conductor carrying a current of 250A and placed in air. Draw a curve showing the variation of B from the conductor surface outwards if its diameter is 2mm.

solution

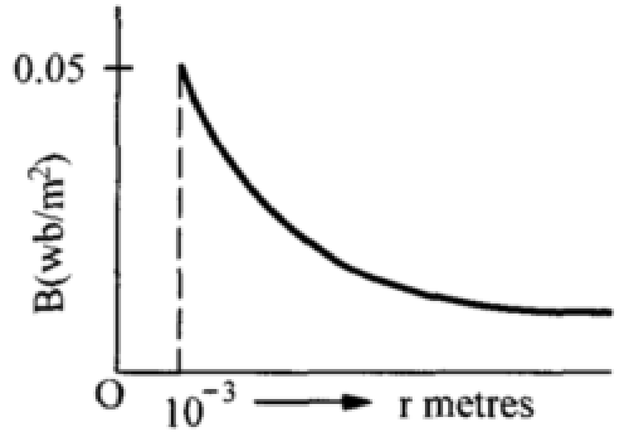
$$H = \frac{I}{2\pi r} = \frac{250}{2\pi \times 0.05} = \mathbf{795.6\ AT/m}$$

$$B = \mu_0 H = 4\pi \times 10^{-7} \times 795.6 = \mathbf{10^{-3}\ Wb/m^2}$$

In general, $B = \dfrac{\mu_0 I}{2\pi r}$

Now, at the conductor surface, $r = 1\ \text{mm} = 10^{-3}\ \text{m}$

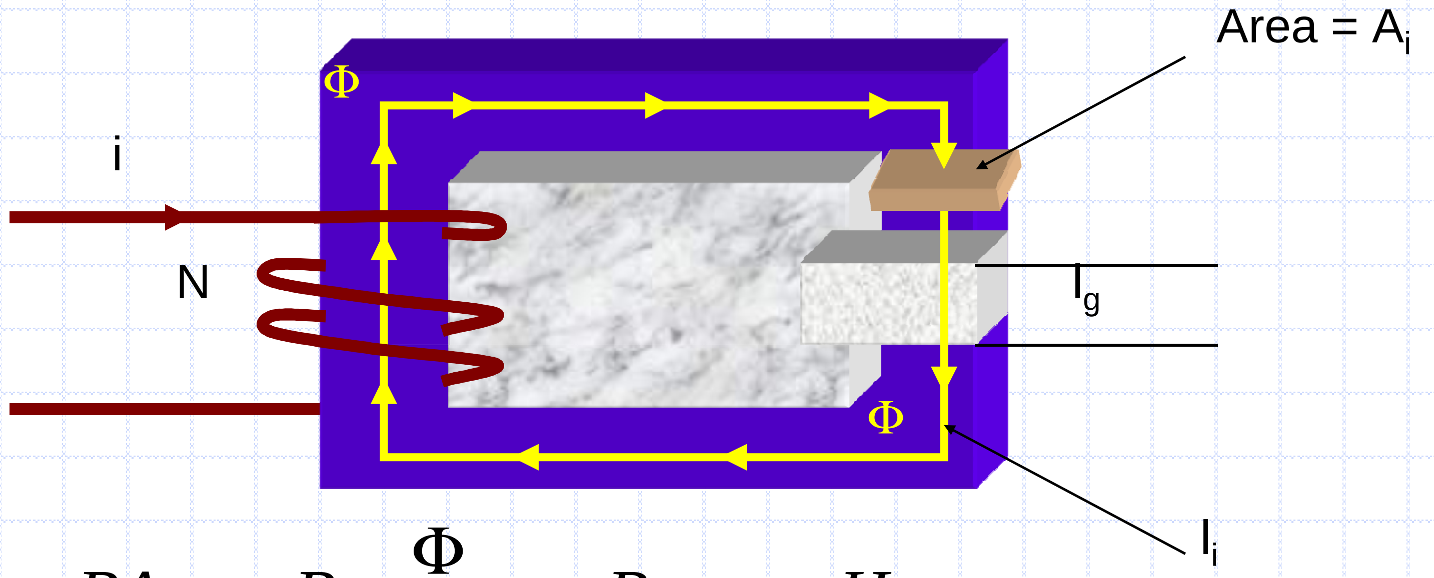
$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 250}{2\pi \times 10^{-3}} = \mathbf{0.05\ Wb/m^2}$$



Magnetic Circuits

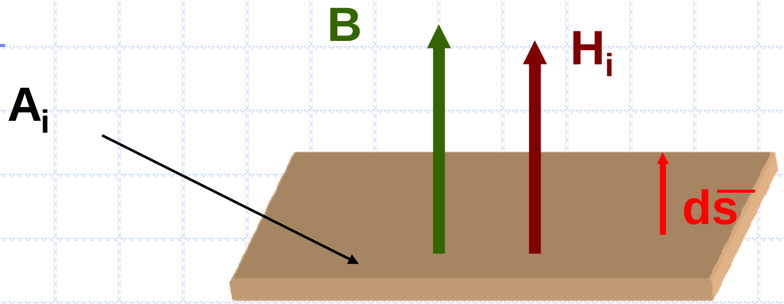
Magnetic circuit - **is defined as the route or path which is followed by magnetic flux. The Laws of magnetic circuit are quite similar to those of electric circuits**

- Consider the following coil



$$\Phi = BA \qquad B = \frac{\Phi}{A} \qquad B = \mu \quad H$$

- Consider a cross section of the magnetic circuit depicted before.



- The flux density "B" is related to the field intensity "H " by

$$B = \mu\ H \quad \left[\frac{Weber}{m^2}\right]$$

- Recall that Ampere's Law states that the line integral of the field intensity "**H**" about a closed path is equal to the net current enclosed within this closed path of integration, that is,

$$\oint H \cdot dl = Ni_{enclosed}$$

$$\int_a^b H_i dl + \int_b^a H_g dl = Ni$$

$$H_i l_i + H_g l_g = Ni$$

- If an uniform flux density is assumed, the following relationships can be obtained,

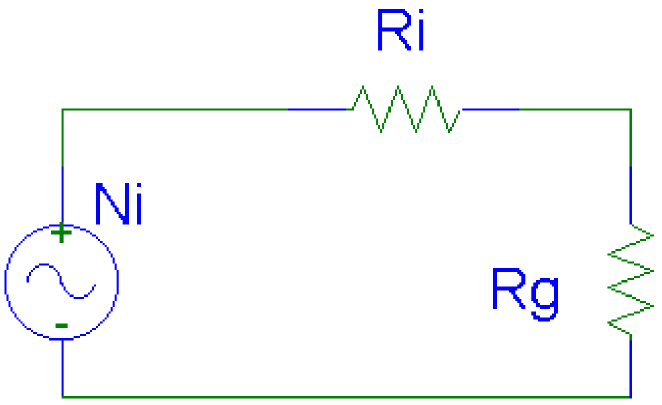
$$\Phi_i = B_i \cdot A_i \qquad \Phi_g = B_g \cdot A_g \qquad A_g = A_i \qquad \Phi_g = \Phi_i = \Phi$$

- Substituting for H and B, we will have the following expression

$$\frac{B_i}{\mu_i} l_i + \frac{B_g}{\mu_g} l_g = Ni$$

$$\frac{l_i}{\mu_i \quad A_i}\Phi + \frac{l_g}{\mu_g \quad A_g}\Phi = Ni$$

$$\mu_i = \mu_{ri}\mu_o \qquad \mu_{ri} = 500 \rightarrow 4000$$

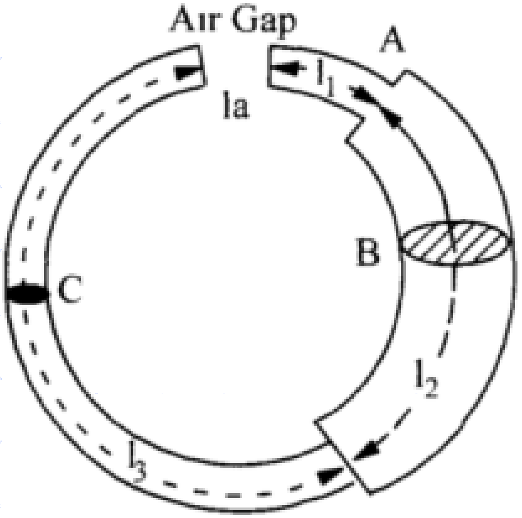


$$\Re_i = \frac{l_i}{\mu_i A_i} \qquad \Re_g = \frac{l_g}{\mu_g A_g}$$

$$(\Re_i + \Re_g)\Phi = Ni \qquad \Phi = \frac{Ni}{(\Re_i + \Re_g)}$$

Composite Series Magnetic Circuit

The composite circuits contains different reluctances therefore
The total reluctance is equal to sum of individuals



$$\text{total reluctance} = \sum \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2} + \frac{l_3}{\mu_0 \mu_{r_3} A_3} + \frac{l_a}{\mu_0 A_g}$$

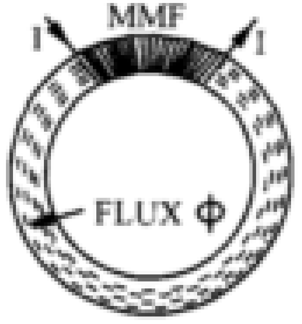
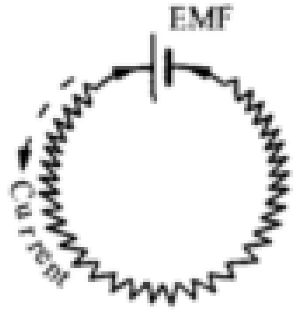
$$\text{flux } \Phi = \frac{\text{m.m.f.}}{\dfrac{l}{\mu_0 \mu_r A}}$$

Calculation procedure for composite magnetic path

$H = NI/l$ AT/m or $NI = H \times l$

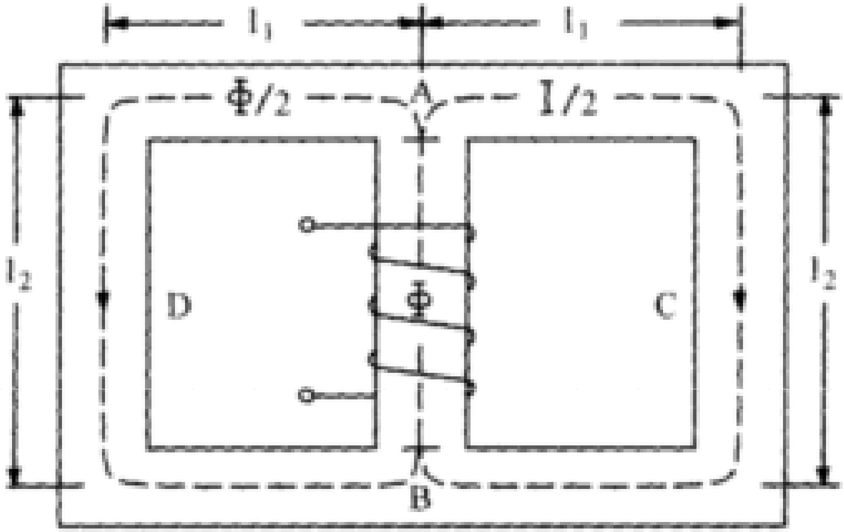
(*i*) Find H for each portion of the composite circuit. For air, $H = B/\mu_0$, otherwise $H = B/\mu_0\mu_r$.
(*ii*) Find ampere-turns for each path separately by using the relation AT = $H \times l$.
(*iii*) Add up these ampere-turns to get the total ampere-turns for the entire circuit.

Analogy Between Magnetic and Electric Circuits

Magnetic Circuit	Electric Circuit
	
1. Flux = $\frac{\text{m.m.f.}}{\text{reluctance}}$	Current = $\frac{\text{e.m.f.}}{\text{resistance}}$
2. M.M.F. (ampere-turns)	E.M.F. (volts)
3. Flux Φ (webers)	Current I (amperes)
4. Flux density B (Wb/m^2)	Current density (A/m^2)
5. Reluctance $S = \frac{l}{\mu A}\left(= \frac{l}{\mu_0 \mu_r A}\right)$	resistance $R = \rho \frac{l}{A} = \frac{l}{\rho A}$
6. Permeance (= 1/reluctance)	Conductance (= 1/resistance)
7. Relucitivity	Resistivity
8. Permeability (= 1/reluctivity)	Conductivity (= 1/resistivity)
9. Total m.m.f. = $\Phi S_1 + \Phi S_2 + \Phi S_3 + \ldots$	9. Total e.m.f. = $IR_1 + IR_2 + IR_3 + \ldots$

Parallel Magnetic Circuit

The circuit has two magnetic paths ACB and ADB acted upon same MMF. Each path has average length **2(L1 +L2)**



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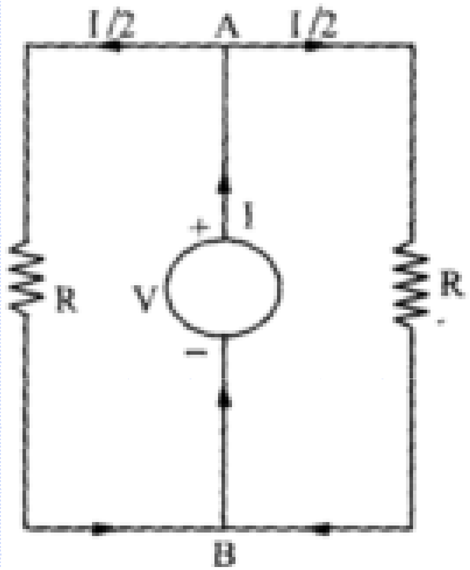


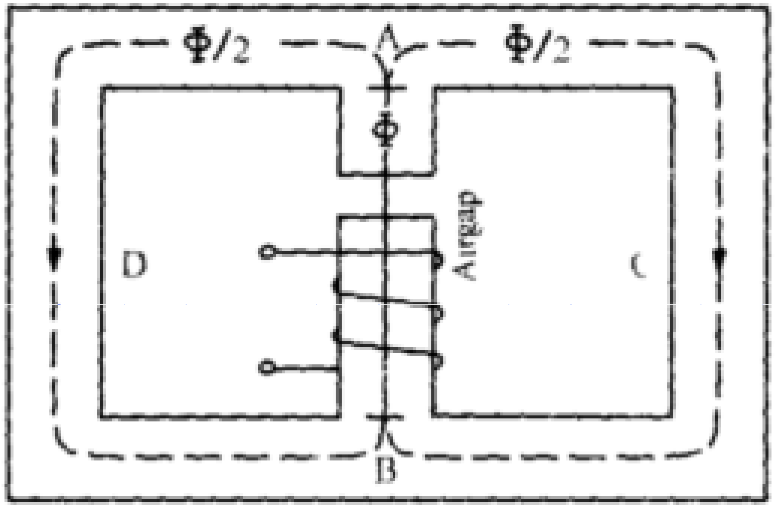
The flux produced by the coil wound on the central core is divided equally at point *A* between the two outer parallel paths. The reluctance offered by the two parallel paths is = half the reluctance of each path.

equivalent electrical circuit where resistance offered to the voltage source is $= R \| R = R/2$

The reluctance offered by central core AB is negligible in magnetic circuit

Series- Parallel Magnetic Circuit

The circuit has two magnetic paths ACB and ADB, connected across the Common magnetic path with air gap. The reluctance of central core Will be equal only to air gap as central core is negligible.



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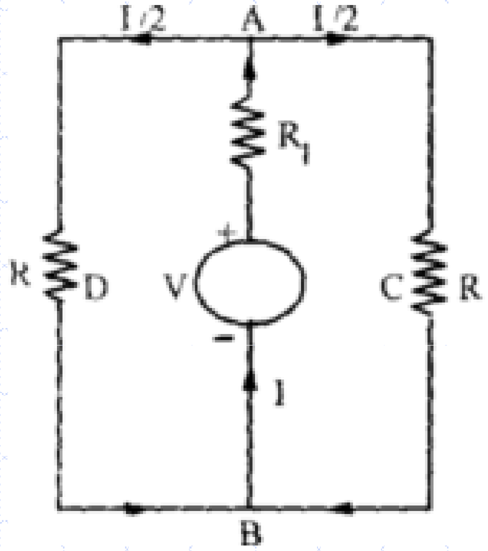


Hence the MMF required for this Circuit would be the sum of

(i) That required for air-gap and

(ii) That required for either of two paths *(not both)*

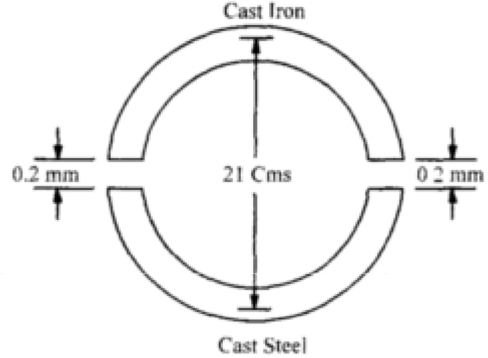
The equivalent electrical circuit total resistance offered $= R_1 + R \| R = R_1 + R/2.$

Example on magnetic circuit

A ring has a diameter of 21 cm and a cross-sectional area of 10 cm^2. The ring is made up of semicircular sections of cast iron and cast steel, with each joint having a reluctance equal to an air-gap of 0.2 mm. Find the ampere-turns required to produce a flux of 8×10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively.

Neglect fringing and leakage effects.

Solution



i. Finding the flux density $B = \Phi/A$

$$\Phi = 8 \times 10^{-4} \text{ Wb} ;\ A = 10 \text{ cm}^2 = 10^{-3} \text{ m}^2 ;\ B = 8 \times 10^{-4}/10^{-3} = 0.8 \text{ Wb/m}^2$$

ii. Finding the Magnetic force F required in Air-gap

Air gap

$$H = B/\mu_0 = 0.8/4\pi \times 10^{-7} = 6.366 \times 10^5 \text{ AT/m}$$

$$\text{Total air-gap length} = 2 \times 0.2 = 0.4 \text{ mm} = 4 \times 10^{-4} \text{ m}$$

$$\therefore \quad \text{AT required} = H \times l = 6.366 \times 10^5 \times 4 \times 10^{-4} = 255$$

Example Cont….

iii. Finding the Magnetic force F required for Caste steel path

$$H = B/\mu_0\, \mu_r = 0.8/4\pi \times 10^{-7} \times 800 = 796 \text{ AT/m}$$

$$\text{path} = \pi\, D/2 = 21\ \pi/2 = 33 \text{ cm} = 0.33 \text{ m}$$

$$\text{AT required} = H \times l = 796 \times 0.33 = 263$$

iv. Finding the Magnetic force F required for Caste Iron path

$$H = 0.8/\pi \times 10^{-7} \times 166 = 3{,}835 \text{ AT/m} \;;\; \text{path} = 0.33 \text{ m}$$

$$\text{AT required} = 3{,}835 \times 0.33 = 1265$$

ANSWER

$$\text{Total AT required} = 255 + 263 + 1265 = \mathbf{1783.}$$

ASSIGNMENT 1 MAGNETIC CIRCUIT

(I)

The length of the magnetic circuit of a relay is 25 cm and the cross-sectional area is 6.25 cm^2. The length of the air-gap in the operated position of the relay is 0.2 mm. Calculate the magnetomotive force required to produce a flux of 1.25 mWb in the air gap. The relative permeability of magnetic material at this flux density is 200. Calculate also the reluctance of the magnetic circuit when the relay is in the unoperated position, the air-gap then being 8 mm long (assume μ_r, remains constant).

(II)

A steel- ring of 25cm mean diameter and of circular section 3cm in diameter has a air-gap of 1.5mm length. It is wound uniformly with 700 turns of wire carrying a current of 2 amp. Neglect leakage and assume that iron path takes 35% of total magneto motive force. Calculate:

(i) Magneto motive force
(ii) Flux density
(iii) Magnetic flux
(iv) Relative permeability

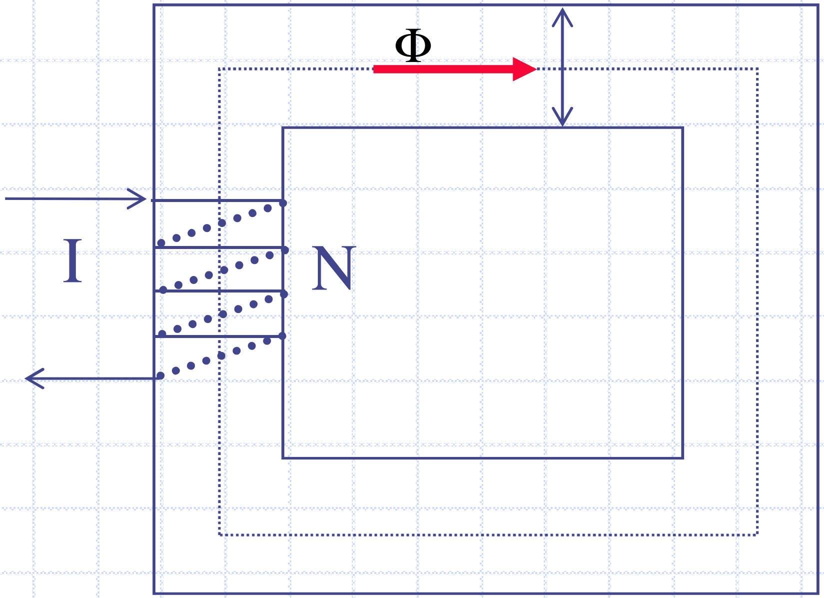
Inductance(L)

Definition: Flux Linkage(λ) per unit of current(**I**) in a magnetic circuit

$$L = \frac{\lambda}{I} = \frac{N\Phi}{I}$$

$$\Phi = \frac{NI}{\Re}$$

$$\therefore L = \frac{N^2}{\Re}$$



Thus inductance depends on the geometry of construction

Faraday's law of Electromagnetic Induction

The EMF (Electromotive Force) induced in a magnetic circuit is Equal to the rate of change of flux linked with the circuit

$$e = \frac{d\lambda}{dt} = \frac{d(N\Phi)}{dt} = N\frac{d\Phi}{dt}$$

$$\because Li = N\Phi$$

$$\therefore e = \frac{dLi}{dt} = L\frac{di}{dt}$$

Lenz's Law

The polarity of the induced voltage is given by Lenz's law

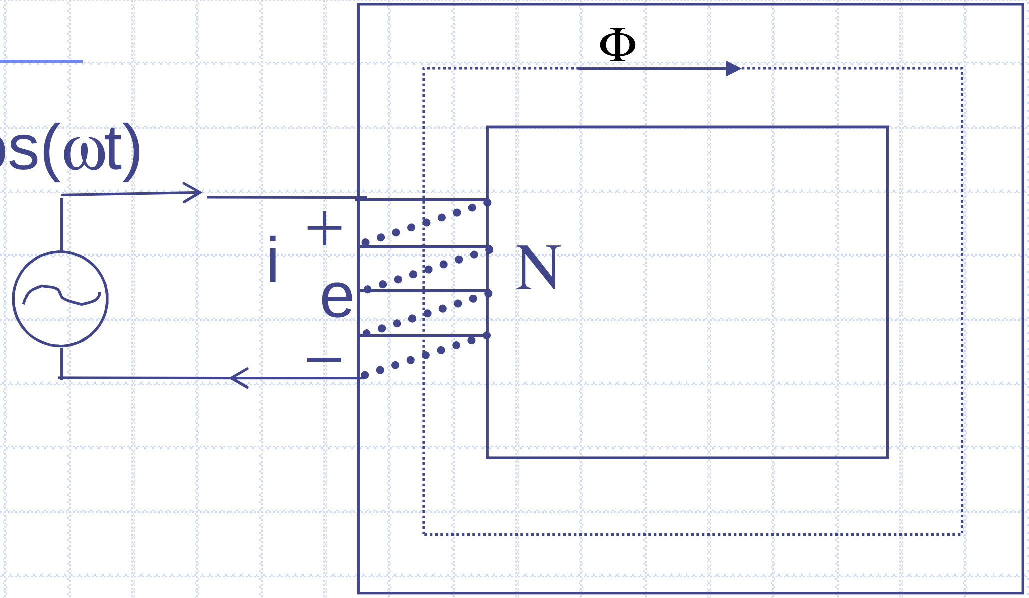
The polarity of the induced voltage will be such as to oppose the very cause to which it is due

Thus *sometimes* we write

$$\therefore e = -\frac{dLi}{dt} = -L\frac{di}{dt}$$

Applicable use to Transformers

$V = V_m \, Cos(\omega t)$



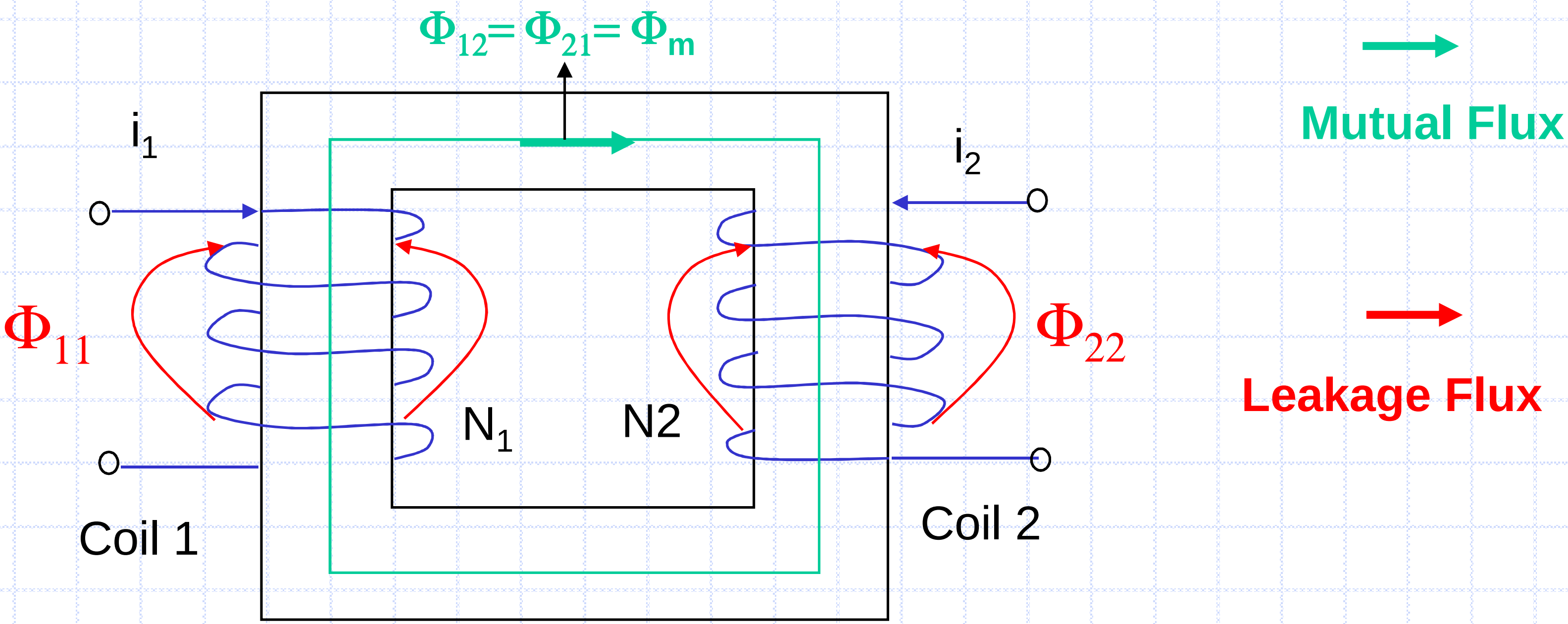
$\Phi = \Phi_m \, Sin(\omega t)$

Ideally

$$e = -N\frac{d\Phi}{dt} = -N\Phi_m \omega Cos(\omega t) = -E_m Cos(\omega t) = -V_m Cos(\omega t)$$

$$i = \frac{N\Phi}{L} = \frac{N\Phi_m Sin(\omega t)}{L} = I_m Sin(\omega t)$$

Self Inductance, Mutual inductance and Leakage Flux



Self ,Leakage and Mutual Flux

Φ_{11} is the *leakage flux* of coil 1. This flux does not link coil 2 and links only coil 1.

Similarly Φ_{22} is the *leakage flux* of coil 2. This flux does not link coil 1 and links only coil 2.

$\Phi_{12} = \Phi_{21} = \Phi_m$ is the *mutual flux* that links both coil 1 and 2

Then *Self flux of coil 1* is $\Phi_1 = \Phi_{11} + \Phi_{12} = \Phi_{11} + \Phi_m$

Then *Self flux of coil 2* $\Phi_2 = \Phi_{22} + \Phi_{21} = \Phi_{22} + \Phi_m$

Self Inductance

Definition: **Total flux linked by a coil per unit of its own current**

Self flux linking coil 1 is $\lambda_{11} = N_1\Phi_1 = N_1(\Phi_{11} + \Phi_{12})$

Self flux linking coil 2 is $\lambda_{22} = N_2\Phi_2 = N_2(\Phi_{22} + \Phi_{21})$

L_1=Self Inductance of coil 1 = $\dfrac{\lambda_{11}}{i_1}$

L_2=Self Inductance of coil 2 = $\dfrac{\lambda_{22}}{i_2}$

A coil always links all the flux it produces

Mutual Inductance

Definition: **Portion of flux produced by one coil (say 2) that links the other coil (say 1) per unit of current in the flux producing coil (coil 2).**

M_{12}=Mutual Inductance of coil 1 due to current in coil 2 = $\frac{N_1 \Phi_m}{i_2}$

M_{21}=Mutual Inductance of coil 2 due to current in coil 1 = $\frac{N_2 \Phi_m}{i_1}$

Normally **$M_{12} = M_{21} = M$**

Relationship between Mutual and Self Inductance

$$L_1 = \frac{N_1\Phi_1}{i_1} \; ; \quad L_2 = \frac{N_2\Phi_2}{i_2}$$

Let $k_1 = \frac{\Phi_m}{\Phi_1}$ and $k_2 = \frac{\Phi_m}{\Phi_2}$

Then $$L_1L_2 = \frac{N_1\Phi_1}{i_1}\frac{N_2\Phi_2}{i_2} = \frac{N_1\Phi_m}{i_1k_1}\frac{N_2\Phi_m}{i_2k_2} = \frac{1}{k_1k_2}\frac{N_2\Phi_m}{i_1}\frac{N_1\Phi_m}{i_2}$$

$$L_1L_2 = \frac{M^2}{k_1k_2}$$

or $M = k\sqrt{L_1L_2}$ Where $k = \sqrt{k_1k_2}$ is the coefficient of coupling

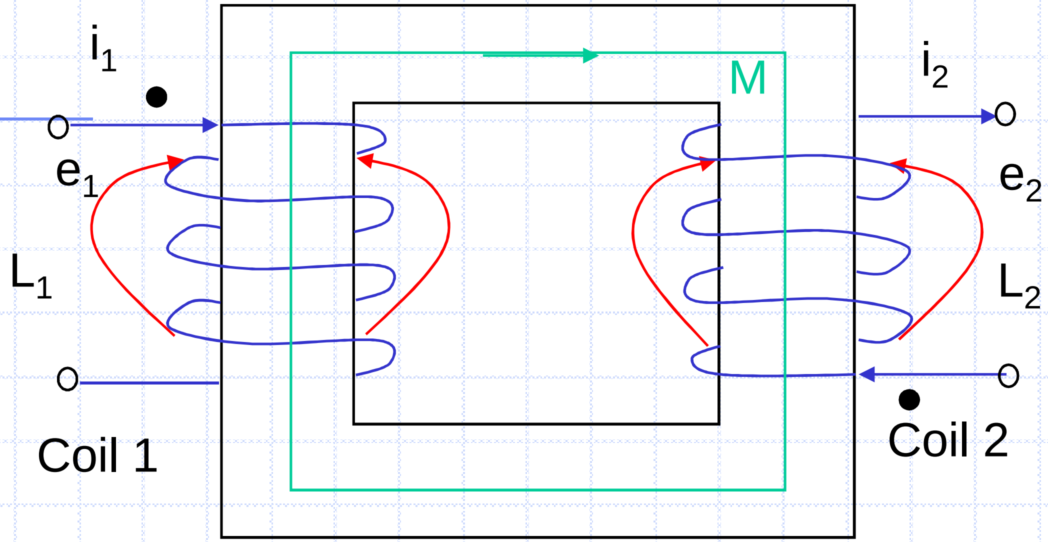
Normally k < 1 (meaning leakage flux cannot be avoided in practice)

Dot Convention

The dots are placed in such a manner that the currents entering (or leaving) both the dotted terminals will produce adding magnetic flux. In this case the mutual flux linkages will add to the self flux linkages.(Case I)

Conversely, if current enters through one dotted terminal and leaves through the other, they produce opposing flux. In this case mutual flux linkages subtract from self linkages.(Case II)

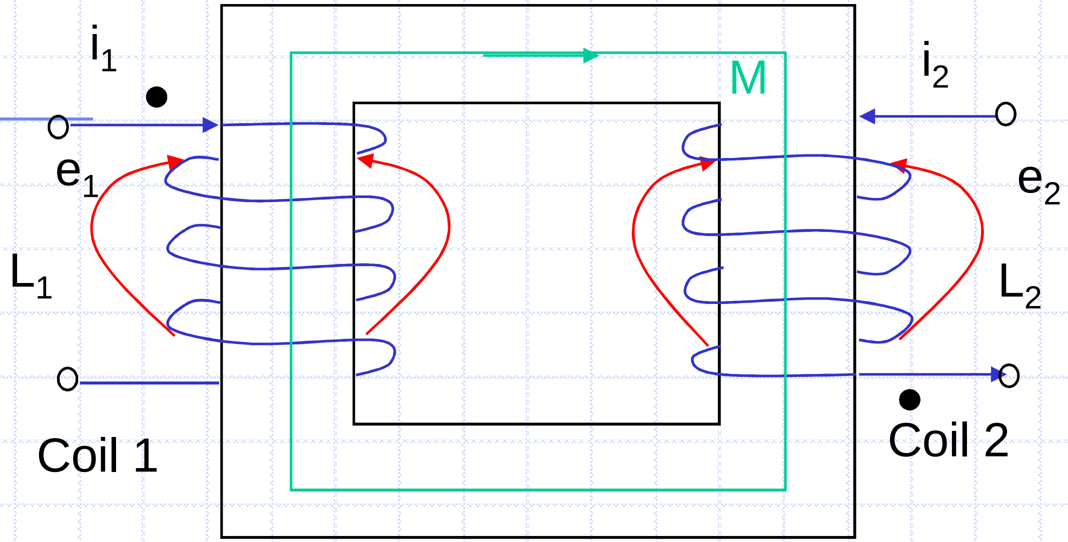
Case I



$$e_1 = \frac{d\lambda_1}{dt} = \frac{d(L_1 i_1 + M i_2)}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$e_2 = \frac{d\lambda_2}{dt} = \frac{d(L_2 i_2 + M i_1)}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Case II



$$e_1 = \frac{d\lambda_1}{dt} = \frac{d(L_1 i_1 - M i_2)}{dt} = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$e_2 = \frac{d\lambda_2}{dt} = \frac{d(L_2 i_2 - M i_1)}{dt} = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Iron Losses in Magnetic Circuit

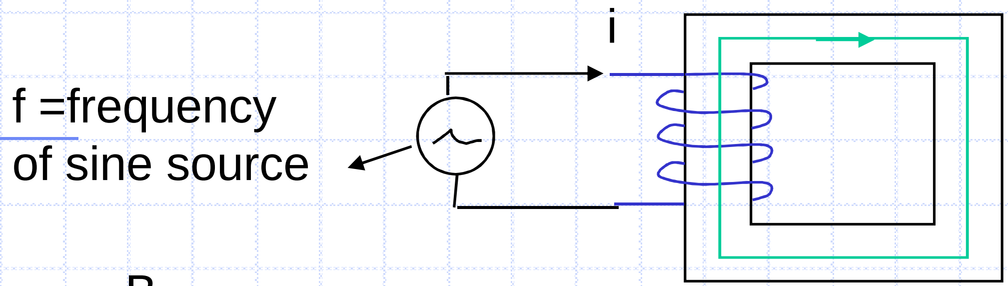
There are two types of iron losses

a) Hysteresis losses

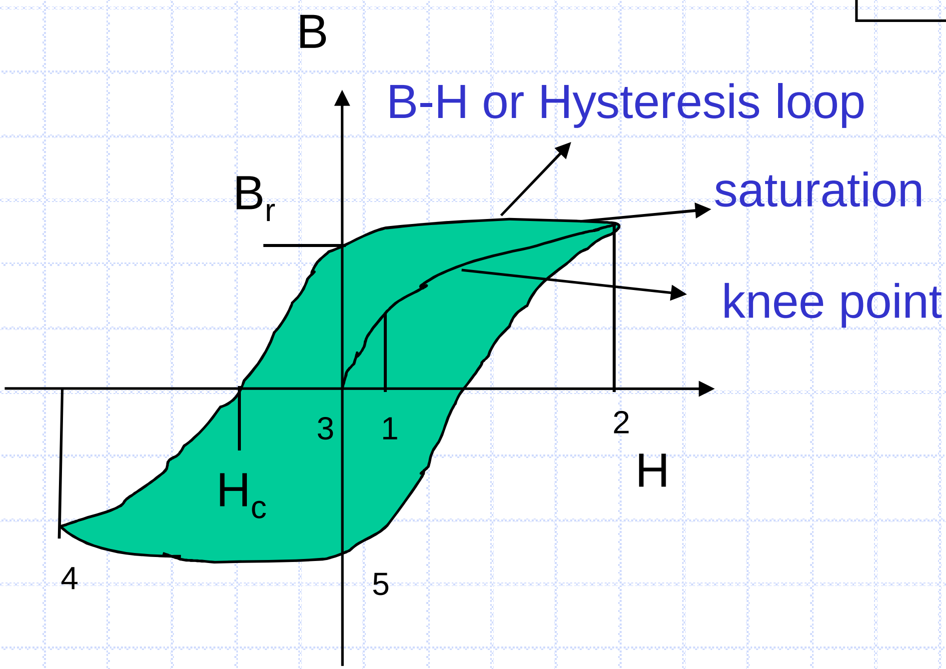
b) Eddy Current Losses

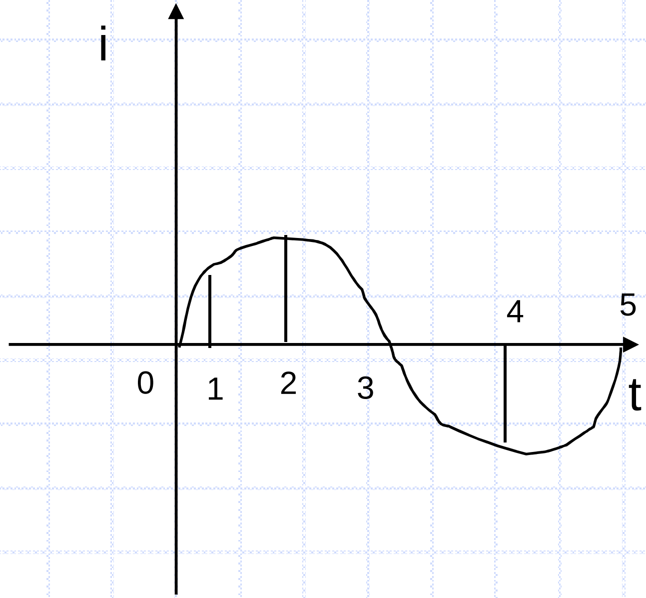
Total iron loss is the sum of these two losses

Hysteresis losses



$$f = \frac{1}{T}$$





B_r = Retentive flux density (due to property of retentivity)

H_c= Coercive field intensity (due to property of coercivity)

Hysteresis losses

•In each of the current cycle the energy lost in the core is proportional to the area of the B-H loop

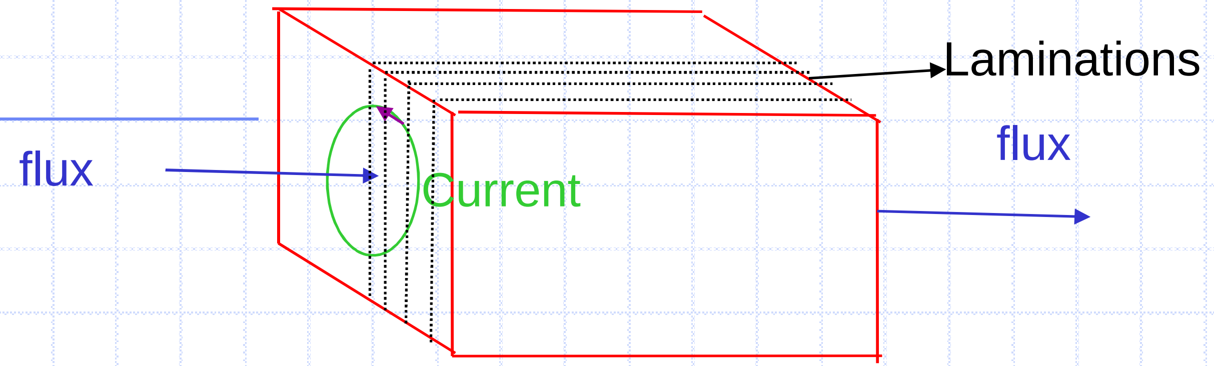
•Energy lost/cycle = $V_{core} \oint BdH$

• P_h = Hysteresis loss = $f V_{core} \oint BdH = k_h B^n_{max} f$

k_h = Constant

B_{max} = Peak flux density

Eddy current loss



Because of time variation of flux flowing through the magnetic material as shown, current is induced in the magnetic material, following Faraday's law. This current is called eddy current.
The direction of the current is determined by Lenz's law. This current can be reduced by using laminated (thin sheet) iron structure, with Insulation between the laminations.

- P_e = Eddy current loss $= k_e B^2_{max} f^2$

k_h = Constant , B_{max}= Peak flux density

Example- Inductance

Two identical coils X and Y of 1000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. If a current of 5A flowing In X produces a flux of 0.5mWb in it, find the mutual inductance between X and Y

Solution

$$M = \frac{N_2 \Phi_1}{I_1} \text{ H}$$

Flux produced in $X = 0.5 \text{ mWb} = 0.5 \times 10^{-3}$ Wb

Flux linked with $Y = 0.5 \times 10^{-3} \times 0.8 = 0.4 \times 10^{-3}$ Wb

$$M = \frac{1000 \times 0.4 \times 10^{-3}}{5} = 0.08 \text{ H}$$

Quiz

1. A unit of flux density that is the same as a Wb/m^2 is the

   a. ampere-turn

   b. ampere-turn/weber

   c. ampere-turn/meter

   d. tesla

Quiz

2. If one magnetic circuit has a larger flux than a second magnetic circuit, then the first circuit has

   a. a higher flux density

   b. the same flux density

   c. a lower flux density

   d. answer depends on the particular circuit.

Quiz

3. The *cause* of magnetic flux is

a. magnetomotive force

b. induced voltage

c. induced current

d. hysteresis

Quiz

4. The measurement unit for permeability is

   a. weber/ampere-turn

   b. ampere-turn/weber

   c. weber/ampere-turn-meter

   d. dimensionless

Quiz

5. The measurement unit for *relative* permeability is

   a. weber/ampere-turn

   b. ampere-turn/weber

   c. weber/ampere-turn meter

   d. dimensionless

Quiz

6. The property of a magnetic material to behave as if it had a memory is called

a. remembrance

b. hysteresis

c. reluctance

d. permittivity

Quiz

7. Ohm's law for a magnetic circuit is

a. $F_m = NI$

b. $B = \mu H$

c. $\varphi = \dfrac{F_m}{\mathcal{R}}$

d. $\mathcal{R} = \dfrac{l}{\mu A}$

Quiz

8. The control voltage for a relay is applied to the

    a. normally-open contacts

    b. normally-closed contacts

    c. coil

    d. armature

Quiz

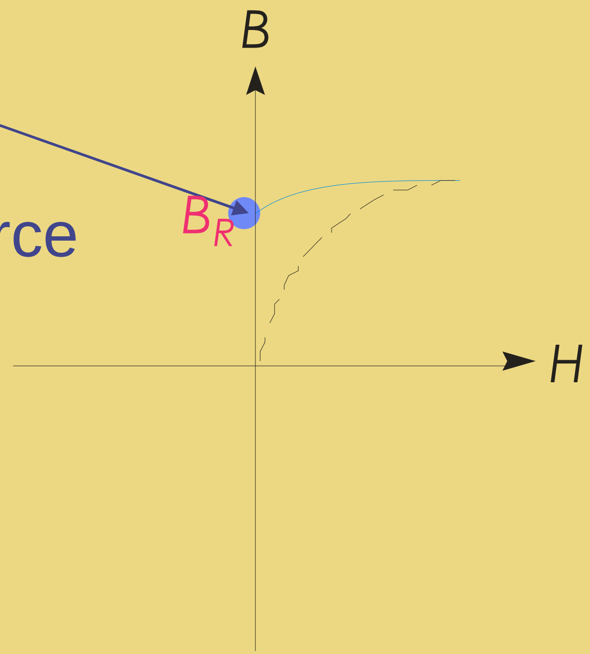
9. A partial hysteresis curve is shown. At the point indicated, magnetic flux

   a. is zero

   b. exists with no magnetizing force

   c. is maximum

   d. is proportional to the current



Quiz

10. When the current through a coil changes, the induced voltage across the coil will

a. oppose the change in the current that caused it

b. add to the change in the current that caused it

c. be zero

d. be equal to the source voltage

Quiz solutions next…

Quiz

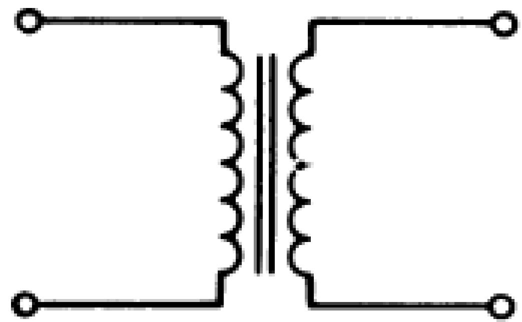
Answers:

1. d
2. d
3. a
4. c
5. d
6. b
7. c
8. c
9. b
10. a

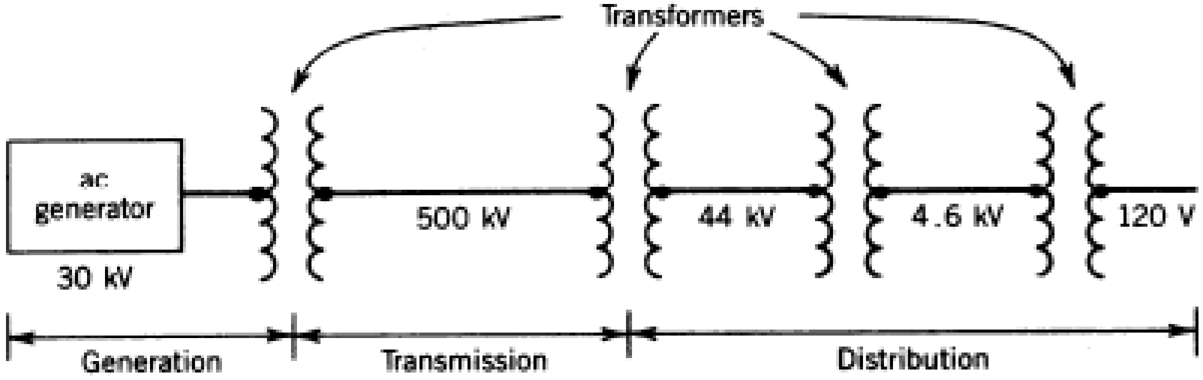
TRANSFORMERS

Transformers

- Change voltage level
- Isolation of circuits
- Match the impedance

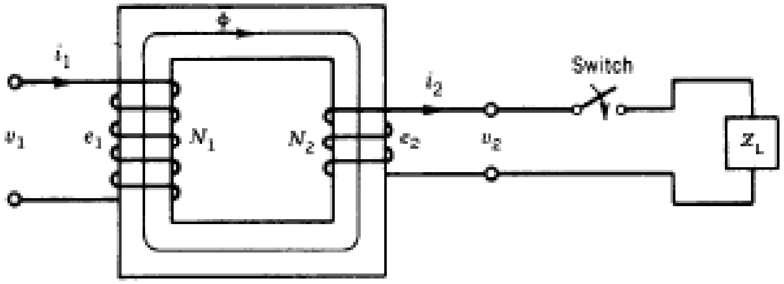


- power transmission



Ideal Transformer

- no losses
- no leakage



- voltages

$$v_1 = e_1 = N_1 \frac{d\Phi}{dt}$$

$$v_2 = e_2 = N_2 \frac{d\Phi}{dt}$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a$$ turns ratio

- power

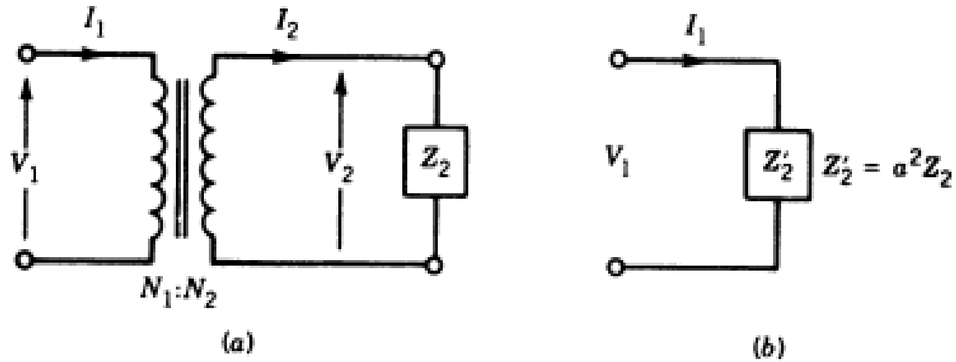
$$v_1 i_1 = v_2 i_2$$

- currents

$$N_1 i_1 - N_2 i_2 = 0$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

Impedance transfer

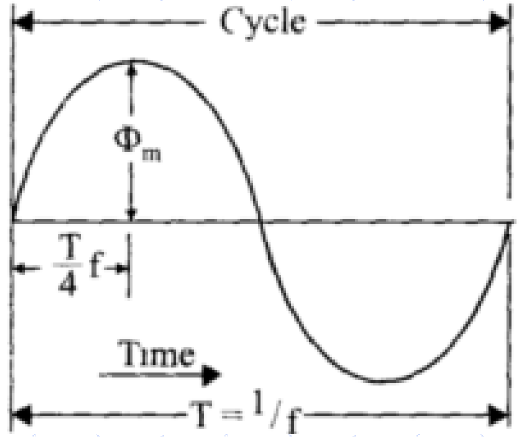


$$Z_2 = \frac{V_2}{I_2}$$

$$Z_1 = \frac{V_1}{I_1} = \frac{aV_2}{I_2/a} = a^2 \frac{V_2}{I_2} = a^2 Z_2$$

E.M. F and Maximum Flux Equations

Let N_1 = No. of turns in primary
N_2 = No. of turns in secondary
Φ_m = maximum flux in core in webers
$= B_m \times A$
f = frequency of a.c. input in Hz



The maximum flux happens at 1/4f of a second

$$\therefore \quad \text{average rate of change of flux} = \frac{\Phi_m}{1/4f} = 4 f \Phi_m \text{ Wb/s or volt}$$

Now, rate of change of flux per turn means induced e.m.f. in volts.

$$\text{average e.m.f./turn} = 4 f \Phi_m \text{ volt}$$

If flux Φ varies *sinusoidally*, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor.

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = 1.11$$

$$\text{r.m.s. value of e.m.f./turn} = 1.11 \times 4 f \Phi_m = 4.44 f \Phi_m \text{ volt}$$

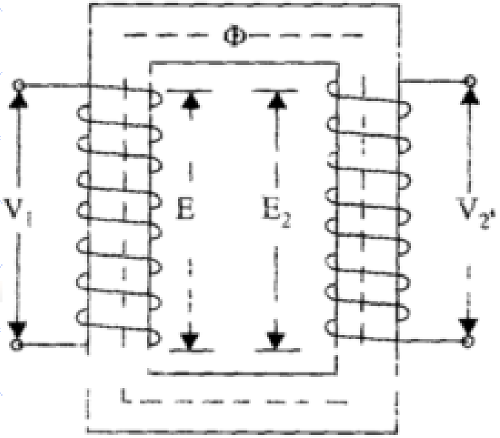
Therefore the r.ms value of emf primary winding

$$E_1 = 4.44 f N_1 \Phi_m = 4.44 f N_1 B_m A$$

Therefore the r.ms value of emf secondary winding

$$E_2 = 4.44 f N_2 \Phi_m = 4.44 f N_2 B_m A$$

In an ideal transformer on no-load, $V_1 = E_1$ and $E_2 = V_2$



Example on ideal transformer

The maximum flux density in the core of a 250/3000-volts, 50-Hz single-phase transformer is 1.2 Wb/m². If the emf per turn is 8 volt, determine
(i) primary and secondary turns (ii) area of the core.

Solution

(i)

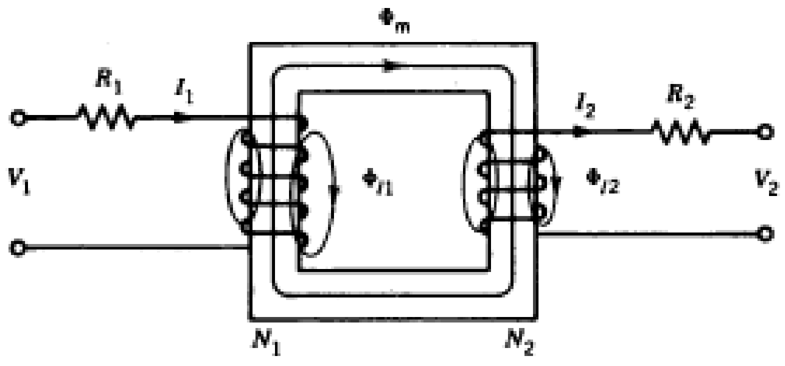
$$E_1 = N_1 \times \text{emf induced/turn}$$
$$N_1 = 250/8 = 32; N_2 = 3000/8 = 375$$

(ii)

$$E_2 = -4.44 f N_2 B_m A$$
$$3000 = 4.44 \times 50 \times 375 \times 1.2 \times A; \mathbf{A = 0.03m2.}$$

Practical transformer

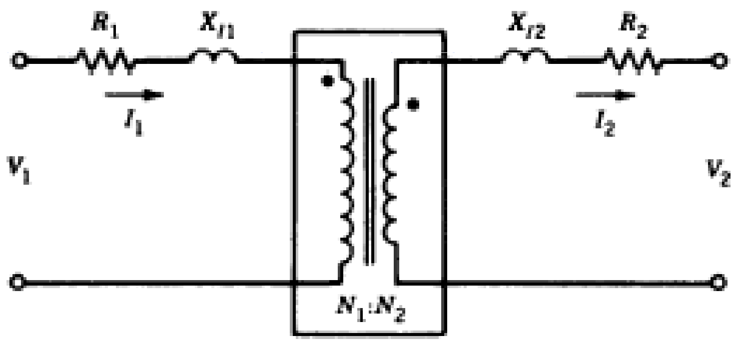
- Winding resistance
- Flux leakage
- Finite permeability
- Core losses



(a)

model

- physical reasoning
- mathematic model of coupled circuits



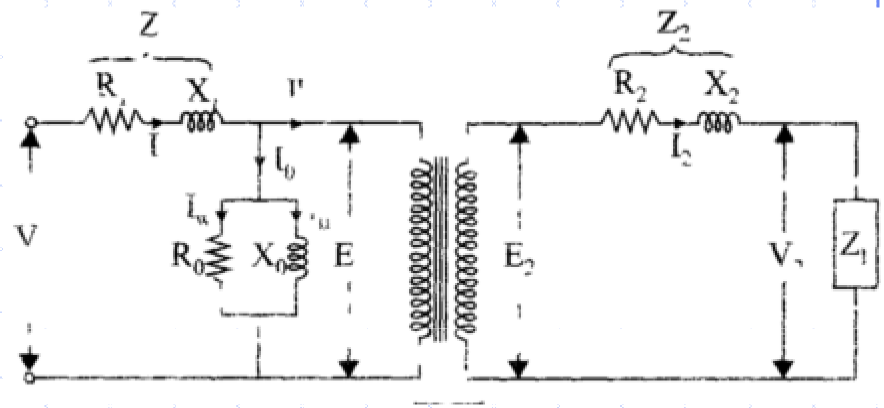
Tightly coupled transformer

(b)

- Winding resistance in series with leakage inductance
- Magnetizing inductance in parallel with core resistance

Equivalent Circuit of practical transformer

The no-load current I_0 is simulated by pure inductance X_0 taking the magnetising component I_μ and a non-inductive resistance R_0 taking the working component I_w connected in parallel across the primary circuit.



The value of E_1 is obtained by subtracting vectorially $I_1 Z_1$ from V_1. The value of $X_0 = E_1/I_0$ and of $R_0 = E_1/I_w$. It is clear that E_1 and E_2 are related to each other by expression $E_2/E_1 = N_2/N_1 = K$

The primary equivalent of the secondary induced voltage is $E_2' = E_2/K = E_1$

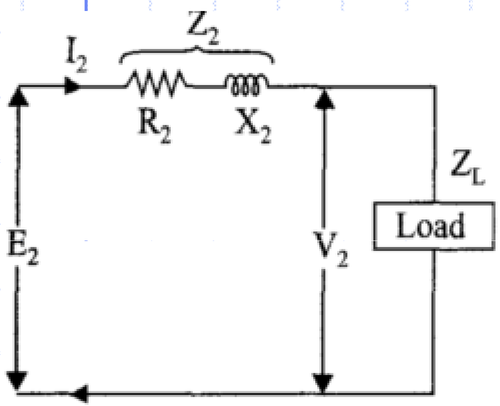
Similarly, primary equivalent of secondary terminal or output voltage is $V_2' = V_2/K$

Primary equivalent of the secondary current is $I_2' = KI_2$

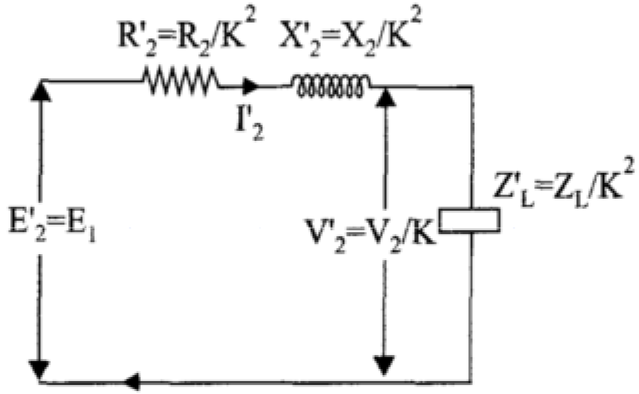
For transferring secondary impedance to primary K^2 is used.

$$R_2' = R_2/K^2, X_2' = X_2/K^2, Z_2' = Z_2/K^2$$

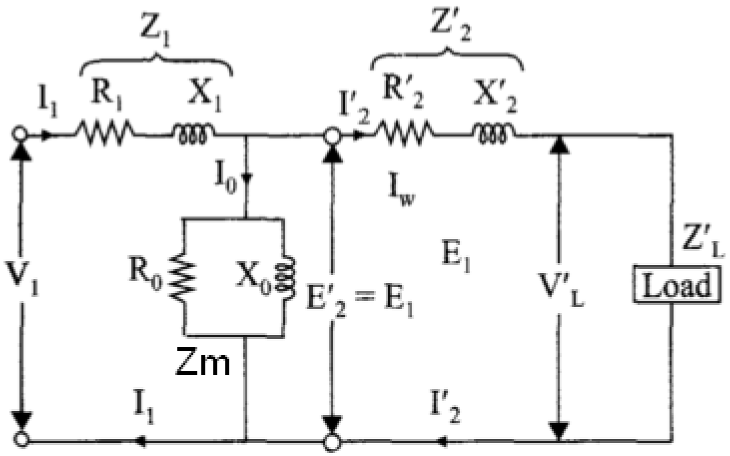
The secondary circuit

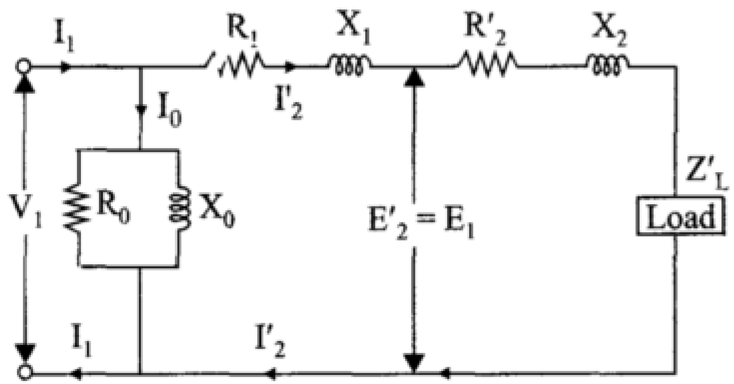


its equivalent primary values



The final primary referred equivalent circuit





total impedance between the input terminal is

It should be noted that in this case $X_0 = V_1/I_\mu$.

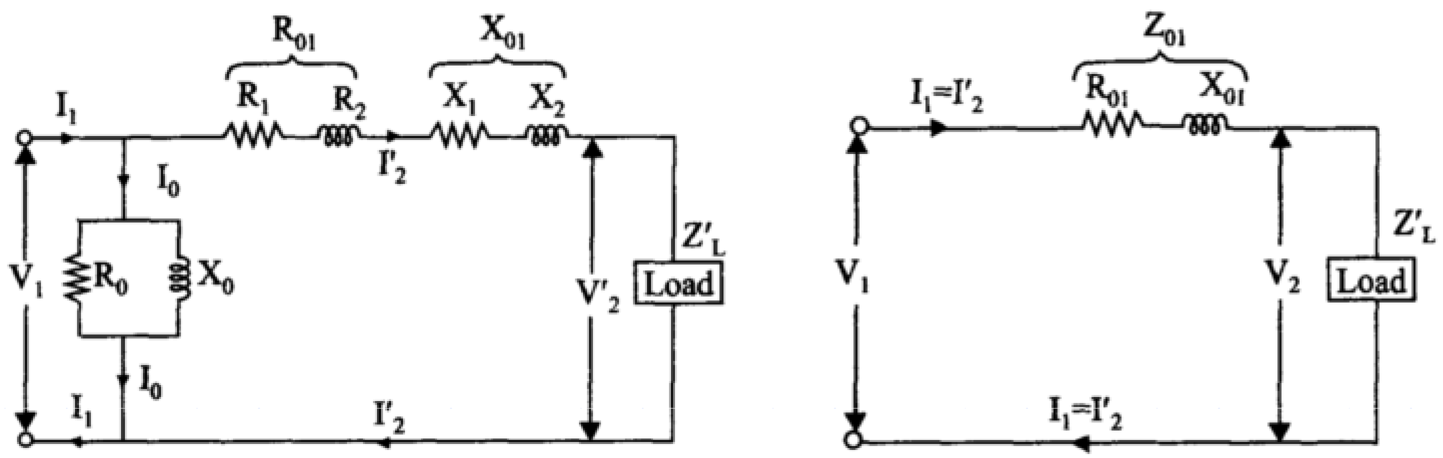
total impedance between the input terminal is $\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_m \,||\, (\mathbf{Z}_2' + \mathbf{Z}_L') = \left(\mathbf{Z}_1 + \dfrac{\mathbf{Z}_m\,(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')} \right)$

where $\mathbf{Z}_2' = R_2' + jX_2'$

$\mathbf{Z}_m$ = impedance of the exciting circuit.

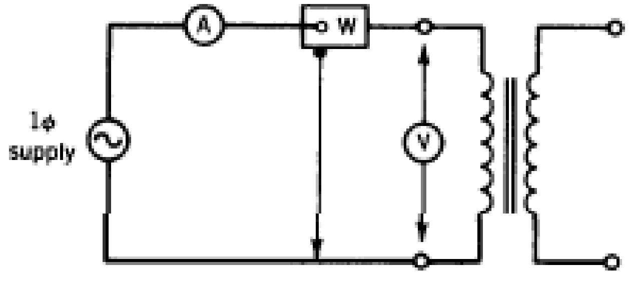
Therefore the total primary voltage $\mathbf{V}_1 = \mathbf{I}_1 \left[\mathbf{Z}_1 + \dfrac{\mathbf{Z}_m\,(\mathbf{Z}_2' + \mathbf{Z}_L')}{\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L')} \right]$

Further simplification may be achieved by omitting I_0 :

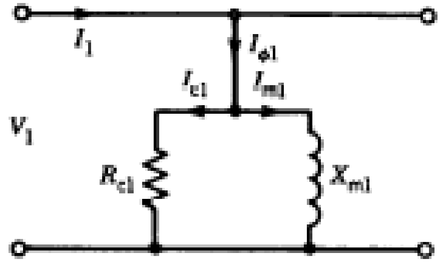


Determination of Equivalent Circuit Parameters

- **No-load test** (rated voltage on one side whereas the other side is open)

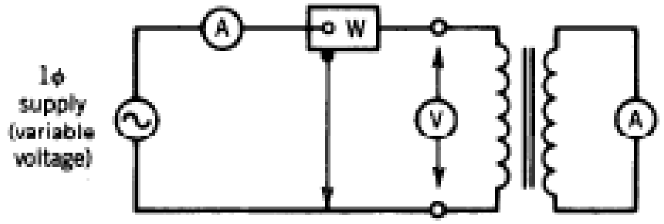


(a)

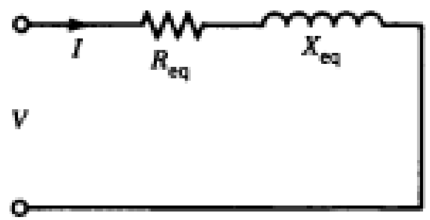


(b)

- **Short-Circuit test** (rated current on one side whereas the other side is short-circuited)



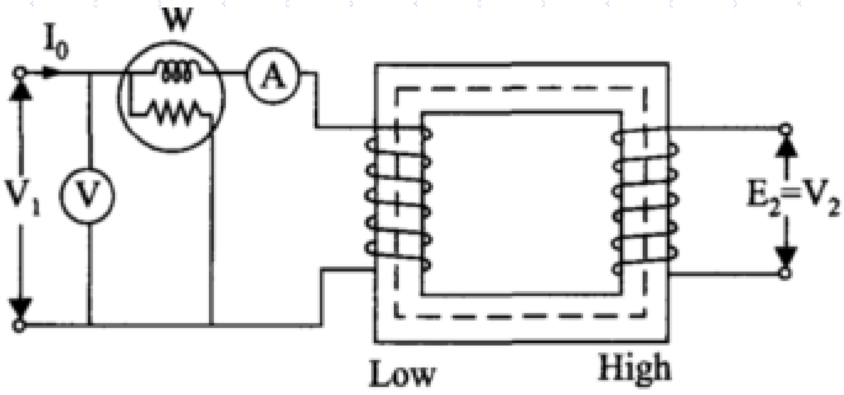
(a)



(b)

- Nameplate
  S kVA, V1/V2 volts

Open Circuit Test (no Load Test)



primary no-load current I_0

If W is the wattmeter reading

$$W = V_1 I_0 \cos\phi_0 \quad \therefore \quad \cos\phi_0 = W/V_1 I_0$$

$$I_\mu = I_0 \sin\phi_0, \quad I_w = I_0 \cos\phi_0 \quad \therefore \quad X_0 = V_1/I_\mu \text{ and } R_0 = V_1/I_w$$

Admittance Y_o $\quad I_0 = V_1 Y_0$ or $Y_0 = I_0/V_1$

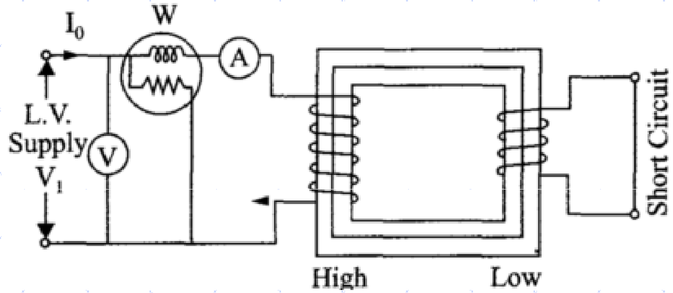
Conductance G_o $\quad W = V_1^2 G_0$ or $G_0 = W/V_1^2$

The exciting susceptance $B_0 = \sqrt{(Y_0^2 - G_0^2)}$

Short Circuit Test (ImpedanceTest)

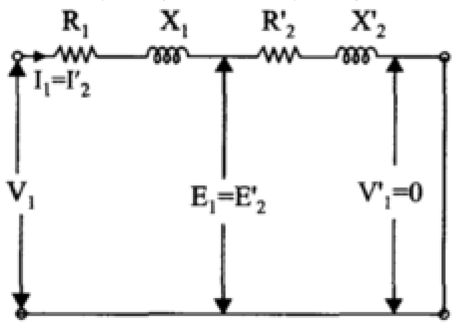
(*i*) Equivalent impedance ($\mathbf{Z}_{01}$ or $\mathbf{Z}_{02}$), leakage reactance (X_{01} or X_{02}) and total resistance (R_{01} or R_{02}) of the transformer as referred to the winding

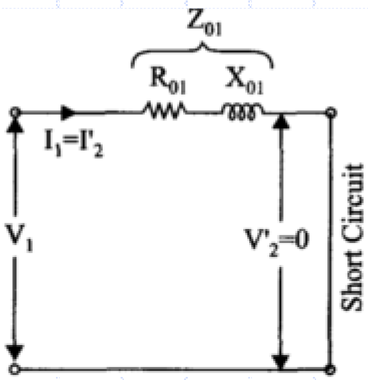
(*ii*) Cu loss at full load (and at any desired load). This loss is used in calculating the efficiency of the transformer.



(*iii*) Knowing Z_{01} or Z_{02}, the total voltage drop in the transformer as referred to primary or secondary can be calculated and hence regulation of the transformer determined.

A low voltage (usually 5 to 10% of normal primary voltage) at correct frequency





$Z_{01} = V_{SC}/I_1$

Also $\quad W = I_1^2 R_{01}$

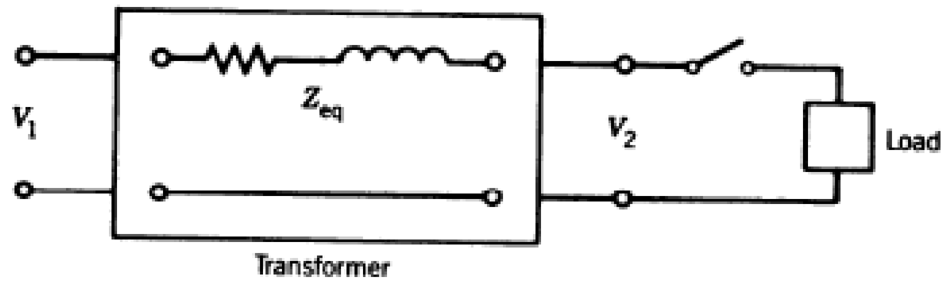
$\therefore \quad R_{01} = W/I_1$

$\therefore \quad X_{01} = \sqrt{\left(Z_{01}^2 - R_{01}^2\right)}$

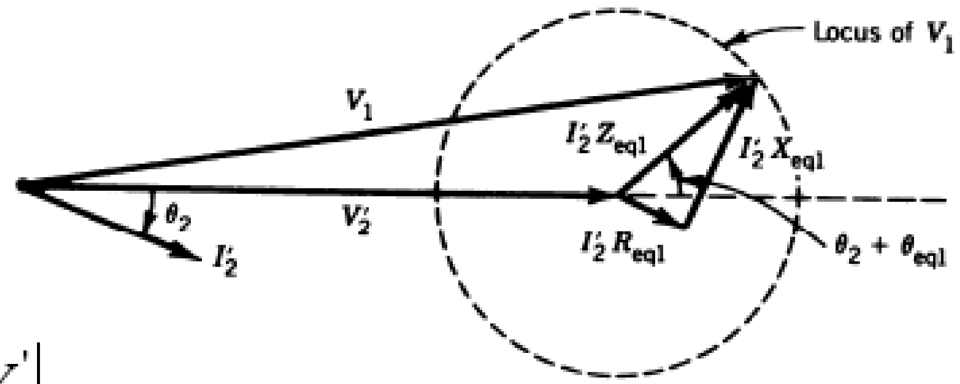
If R_1 can be measured, then knowing R_{01}, we can find $R_2' = R_{01} - R_1$.

Voltage regulation

- No load $V_2 = V_1/a$
- Loaded $V_2 = V_1/a \pm \Delta V_2$



(a)



(b) Phasor diagram

$$\text{Voltage regulation} = \frac{|V_2|_{NL} - |V_2'|_L}{|V_2'|_L}$$

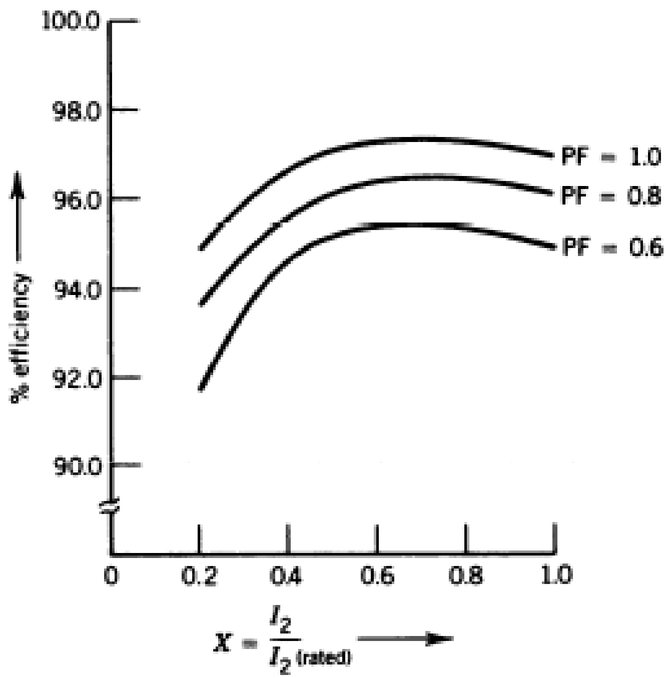
- Maximum voltage regulation occur if $\theta_L = -\theta_{eq1}$

Efficiency - All-day efficiency

$$\eta = \frac{P_{out}}{P_{out} + P_c + P_{cu}}$$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + I_2^2 R_{eq2}}$$

Max efficiency occurs for:

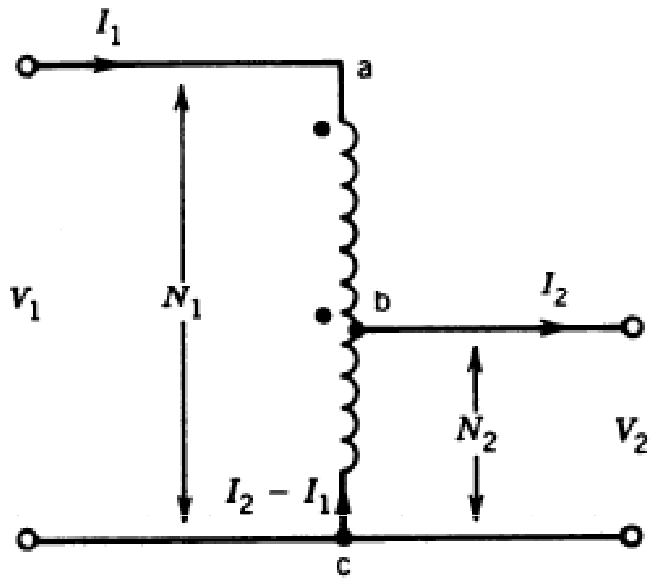


- fixed V_2 and θ_2 — $Pc = I_2^2 R_{eq2}$
- fixed V_2 and I_2 — $\cos\theta_2 = 1$

- All-day efficiency:

$$\eta_{AD} = \frac{enrgy\ output\ over\ 24\ hours}{enrgy\ input\ over\ 24\ hours}$$

Autotransformer



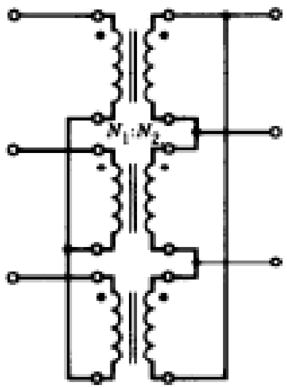
- Same operation as two windings transformer
- Physical connection from primary to secondary
- Sliding connection allows for variable voltage
- Higher kVA delivery than two windings connection

Three-phase transformer

- Three similar single-phase transformers connected to form a three-phase transformer

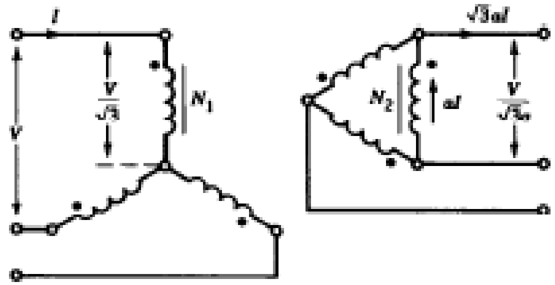
- Four possible connection:

  Y-Δ Δ-Y Δ-Δ Y-Y



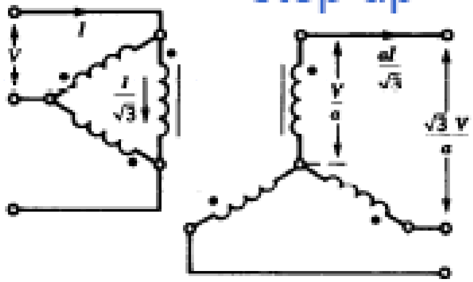
(a) Y - Δ

- step down



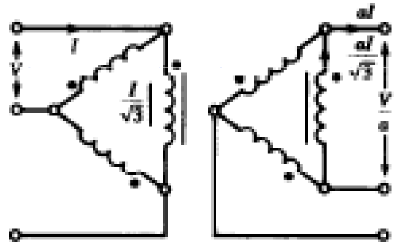
(b) Y – Δ

- step up



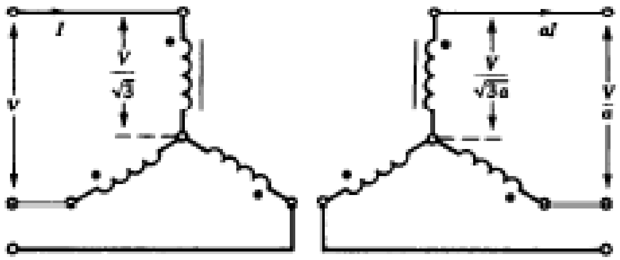
(c) Δ - Y

- one can be removed



(d) Δ – Δ

- seldom used

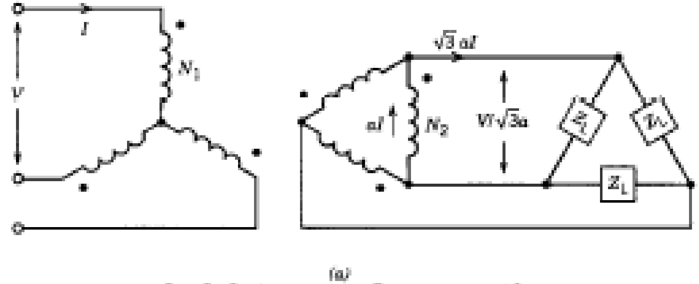


(e) Y - Y

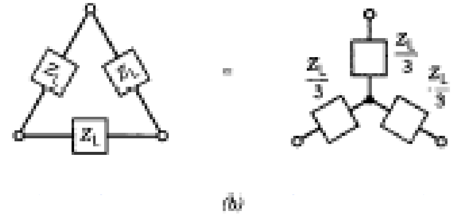
Three-phase transformer - single-phase equivalent circuit

- Validity conditions:

  Identical transformers
  balanced source and load

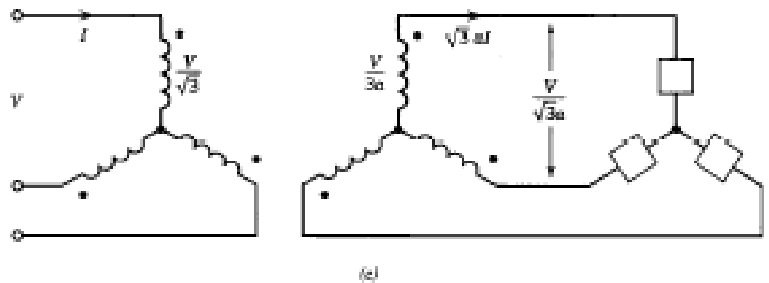


Δ-Y transformation

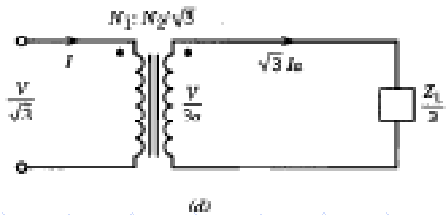


- Only one phase variables are used, the other phases are similar.

- equivalent Y-representation

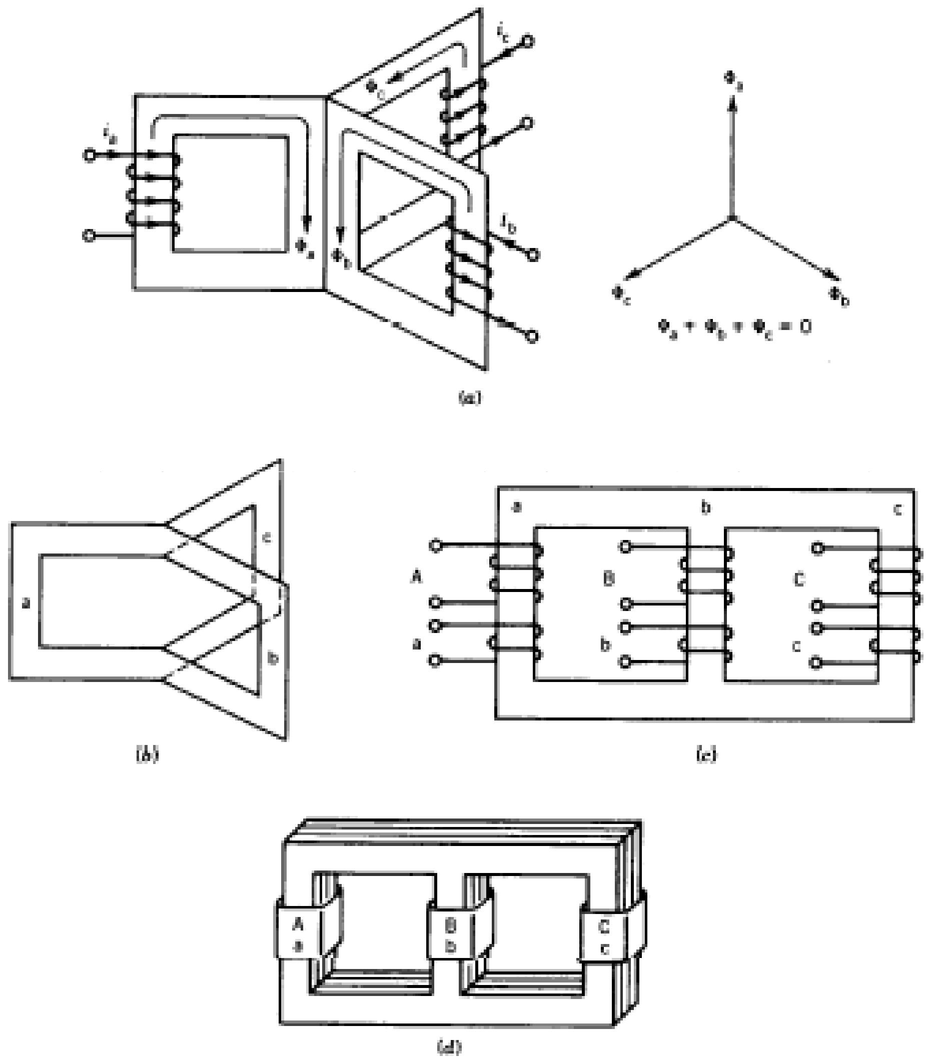


- line-to-neutral = phase voltage



Three-phase transformer unit

- balanced three-phase voltage
- balanced three-phase flux
- return leg can be removed
- In-plan construction easy to manufacture
- Same operation as transformer bank

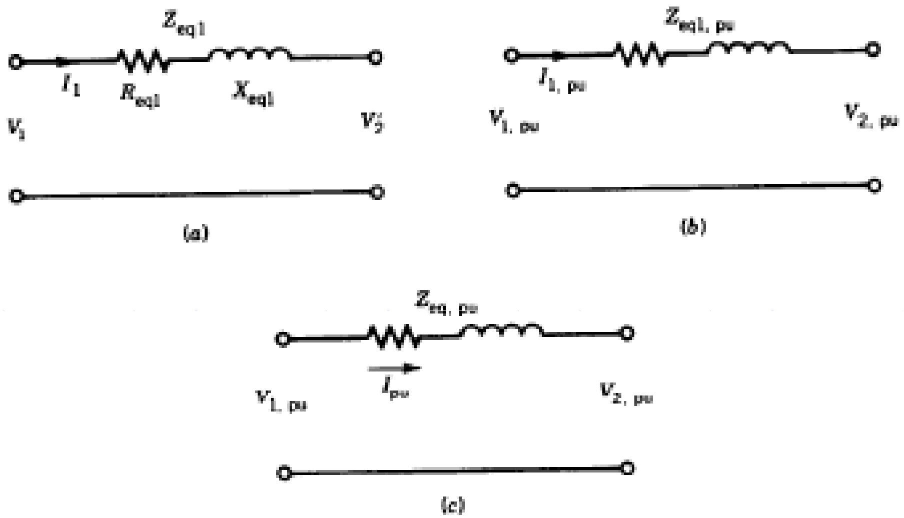


Per-unit system

- base (reference) value of the quantity P_{base}, V_{base}

$$I_{\text{base}} = \frac{P_{\text{base}}}{V_{\text{base}}}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}}$$



$$\text{pu-quantity} = \frac{\text{actual q}}{\text{base q}}$$

- pu voltage equation and full load copper losses

$$V_{1,\text{pu}} = I_{1,\text{pu}} Z_{\text{eq1,pu}} + V_{2,\text{pu}}$$ Independent of the side

$$P_{\text{Cu,FL}} = R_{\text{eq1,pu}}$$

ASSIGNMENT 2 ON TRANSFORMERS

The parametres of a 2300/230 V, 50-Hz transfomer are given below :

$R_1 = 0.286\ \Omega$ $R_2' = 0.319\ \Omega$ $R_0 = 250\ \Omega$

$X_1 = 0.73\ \Omega$ $X_2' = 0.73\ \Omega$ $X_0 = 1250\ \Omega$

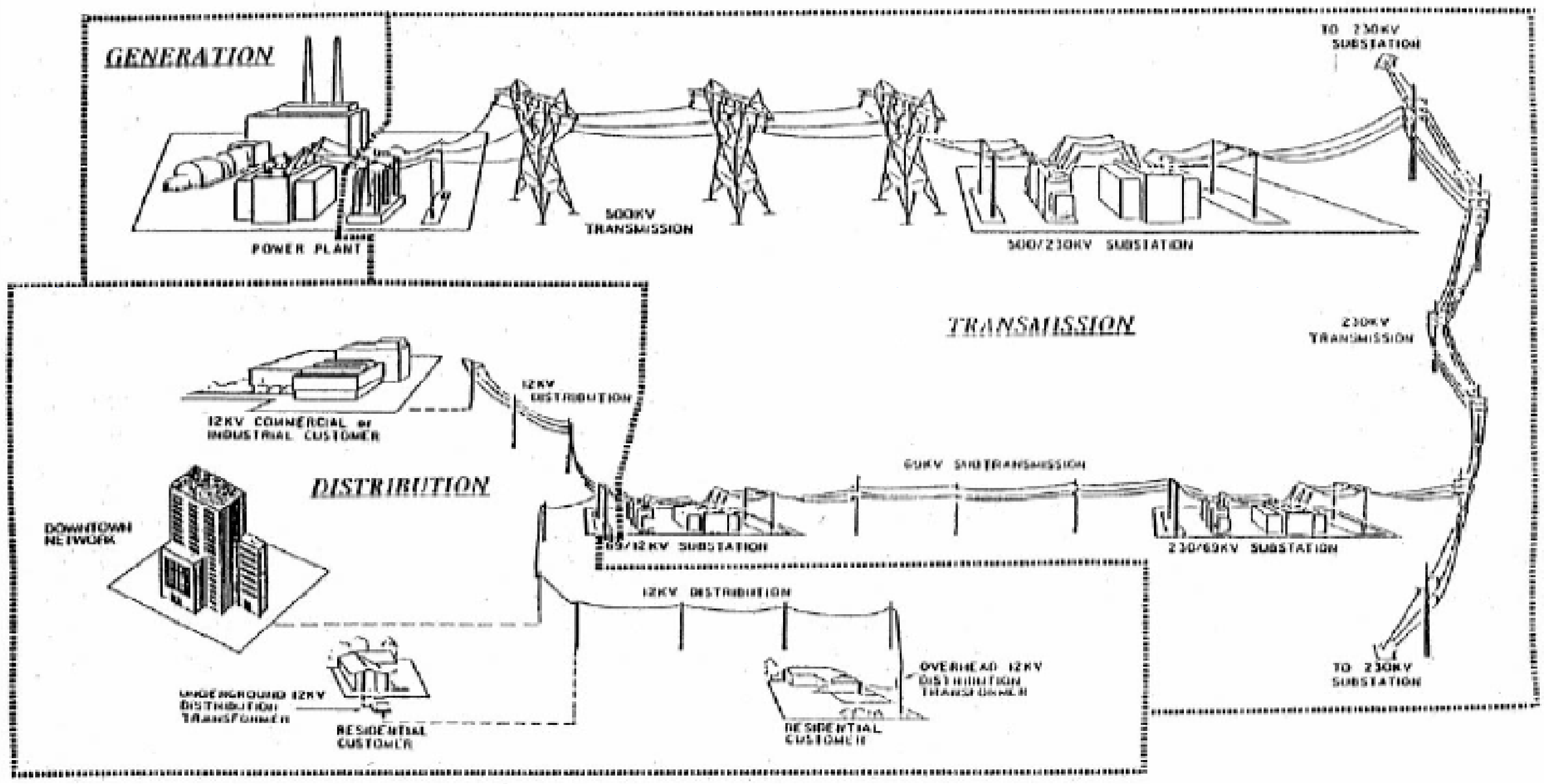
The secondary load impedance $Z_L = 0.387 + j\ 0.29$. *Solve the exact equivalent circuit with normal voltage across the primary.*

POWER SYSTEMS

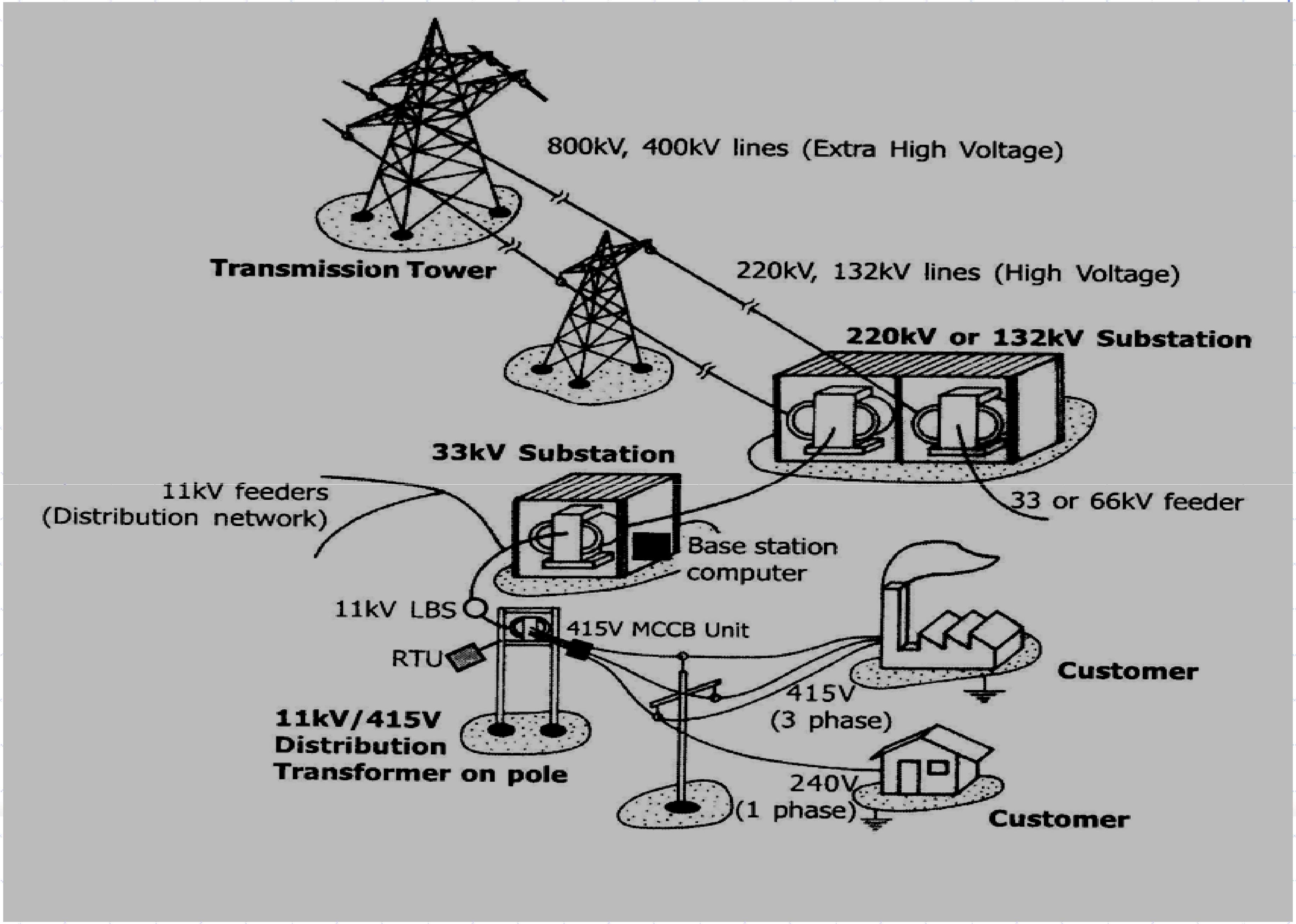
ELECTRIC POWER SYSTEMS

1.Alternator 2. Power transformer 3.Transmission lines

4. Substation transformer 5.Distribution transformer 6.Loads

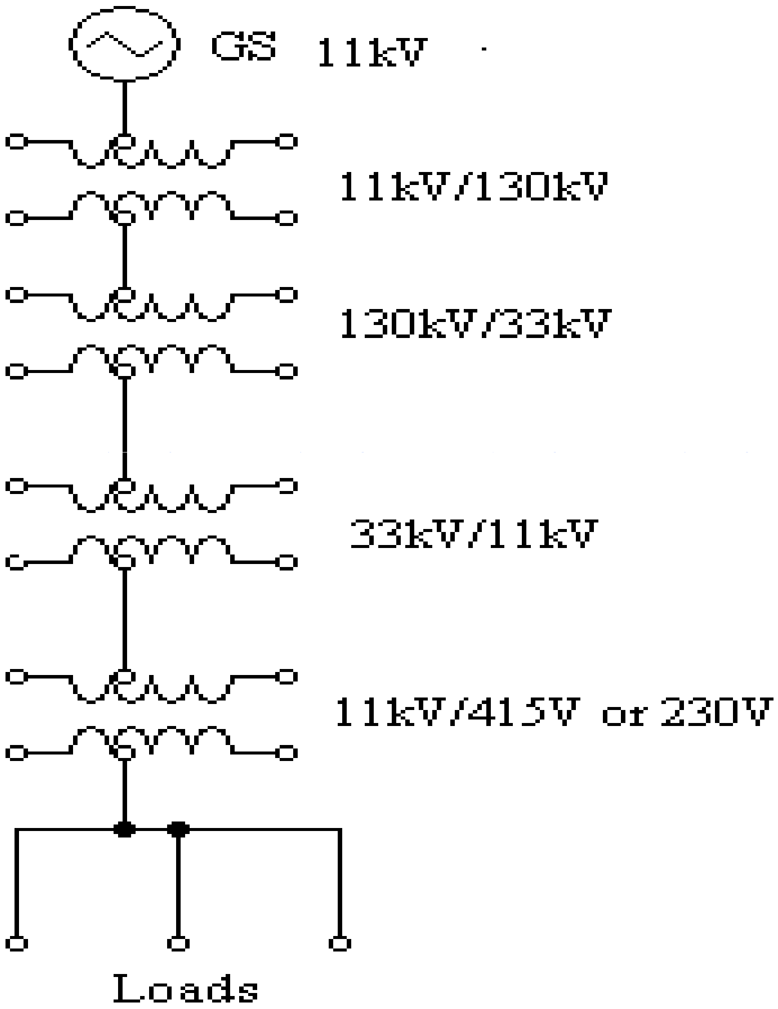


Concept of electric energy transmission.

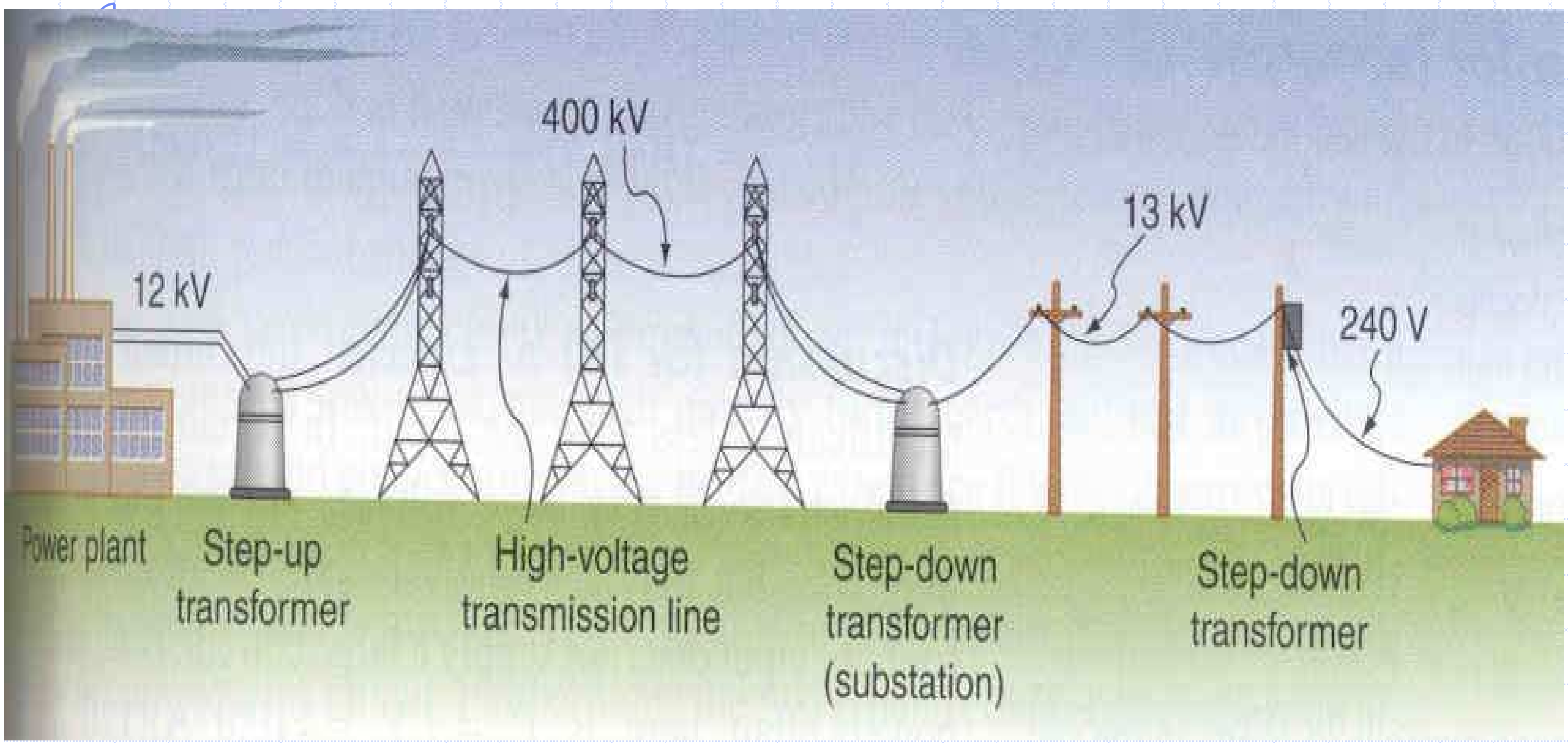


800kV, 400kV lines (Extra High Voltage)
Transmission Tower
220kV, 132kV lines (High Voltage)
220kV or 132kV Substation
33kV Substation
11kV feeders
(Distribution network)
33 or 66kV feeder
Base station
computer
11kV LBS
415V MCCB Unit
RTU
Customer
415V
(3 phase)
11kV/415V
Distribution
Transformer on pole
240V
(1 phase)
Customer

Power systems and network



Electrical power transmission



- AC of 50 Hz produced by generator
- Resistance losses are smallest at high voltages and low currents


Electrical Generation

FEATURES OF GENEARATION- SOURCES OF ENERGY, NEEDS AND ELECTRICAL

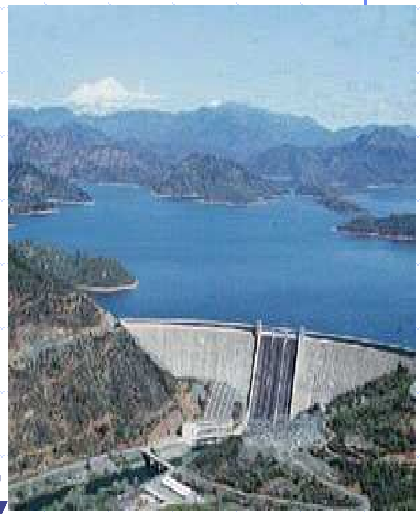
- **Sources :** Wood, charcoal, solar, hydropower, nuclear etc...
- Here in zambia, electrical power system is mainly based on hydroelectric power (i.e., from water)
  - The power associated is:

    where ρ: water density= $10^3 kg/m^3$;

    $g = 9.81 m/s^2$;

    Q: water flow rate $[m^3/s]$, and

    H: height [m] .



- **Needs/Utilisation:** Heating, mechanical power, communication,lights,
- **Electrical network components:** Electricity supply systems have to deliver power to many types of load.
  - The greater the power supplied, for a given voltage, the greater the current.

GENERATION OF THREE PHASE E.M.F

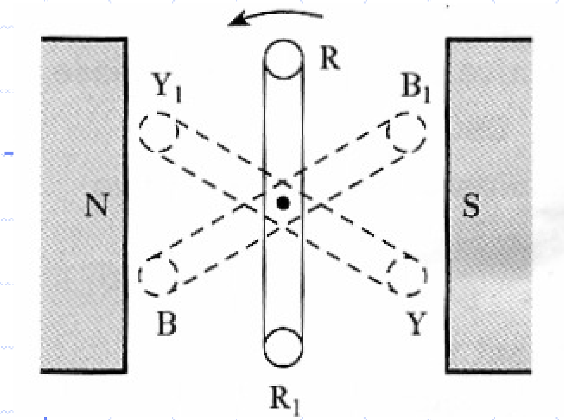


Fig.1: Generation of three-phase e.m.f.s

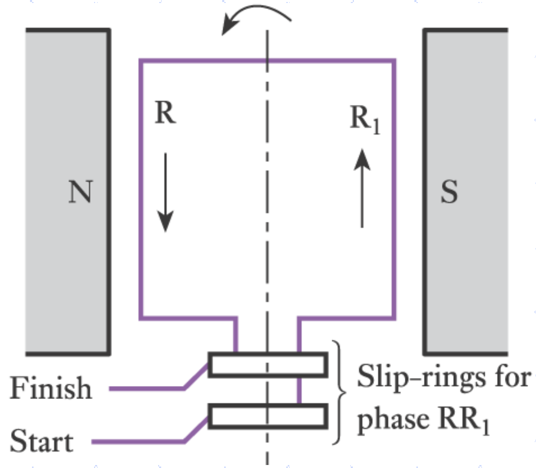
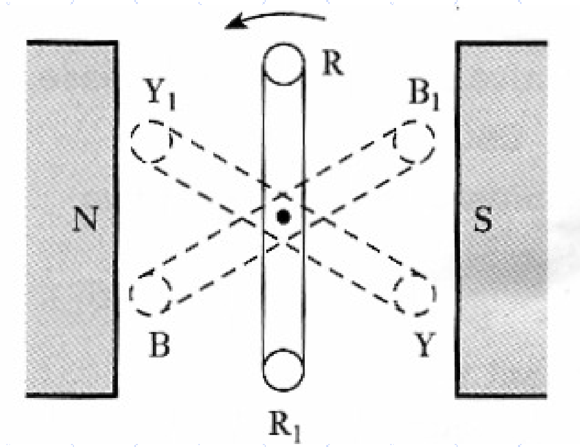
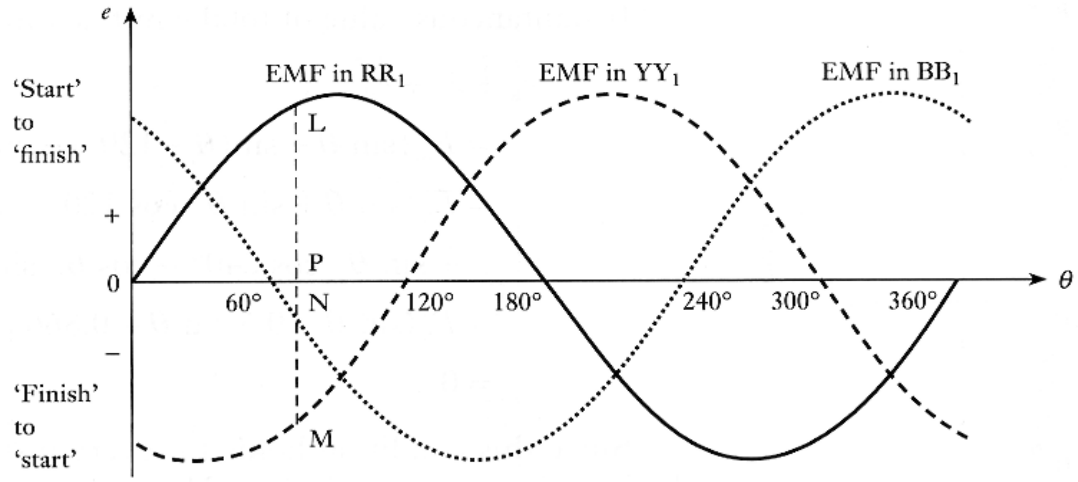


Fig.2: Loop RR_1 at instant of maximum e.m.f.

- In Fig. 4-3.1, RR_1, YY_1 and BB_1 represent three similar loops fixed to one another at angles of 120^0, each loop terminating in a pair of slip-rings carried on the shaft in Fig.4-3.2.
- We shall refer to the slip-rings connected to sides R, Y and B as the 'finishes' of the respective phases and those connected to R_1, Y_1 and B_1 as the 'starts'.

- The letters R, Y and B are abbreviations of 'red', 'yellow' and 'blue', namely the colors used to identify the three phases.
- Also, 'red-yellow-blue' is the sequence that is universally adopted to denote that the e.m.f. in the yellow phase lags that in the red phase by a third of a cycle (120^0), and the e.m.f. in the blue phase lags that in the yellow phase by another third of a cycle.





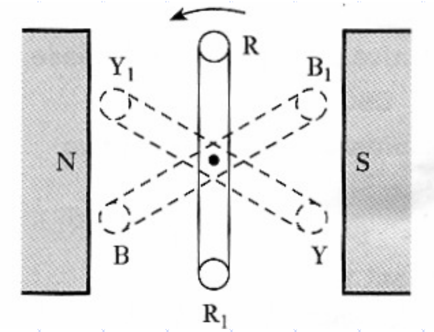


Fig1: Generation of three-phase e.m.f.s

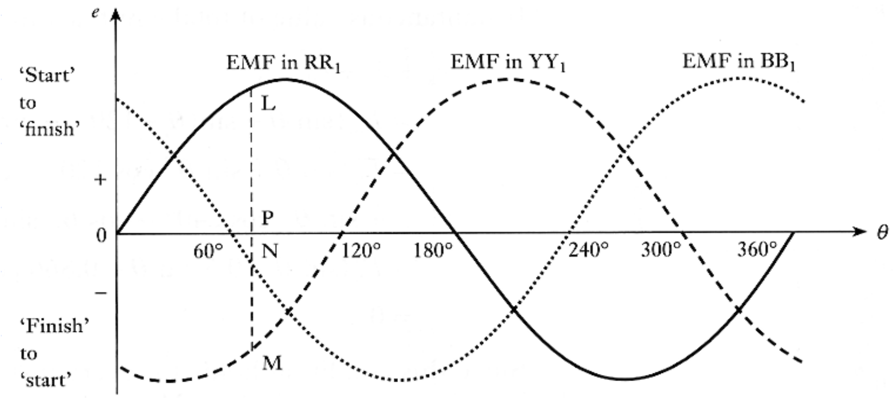


Fig.3: Waveforms of three phase e.m.f.s

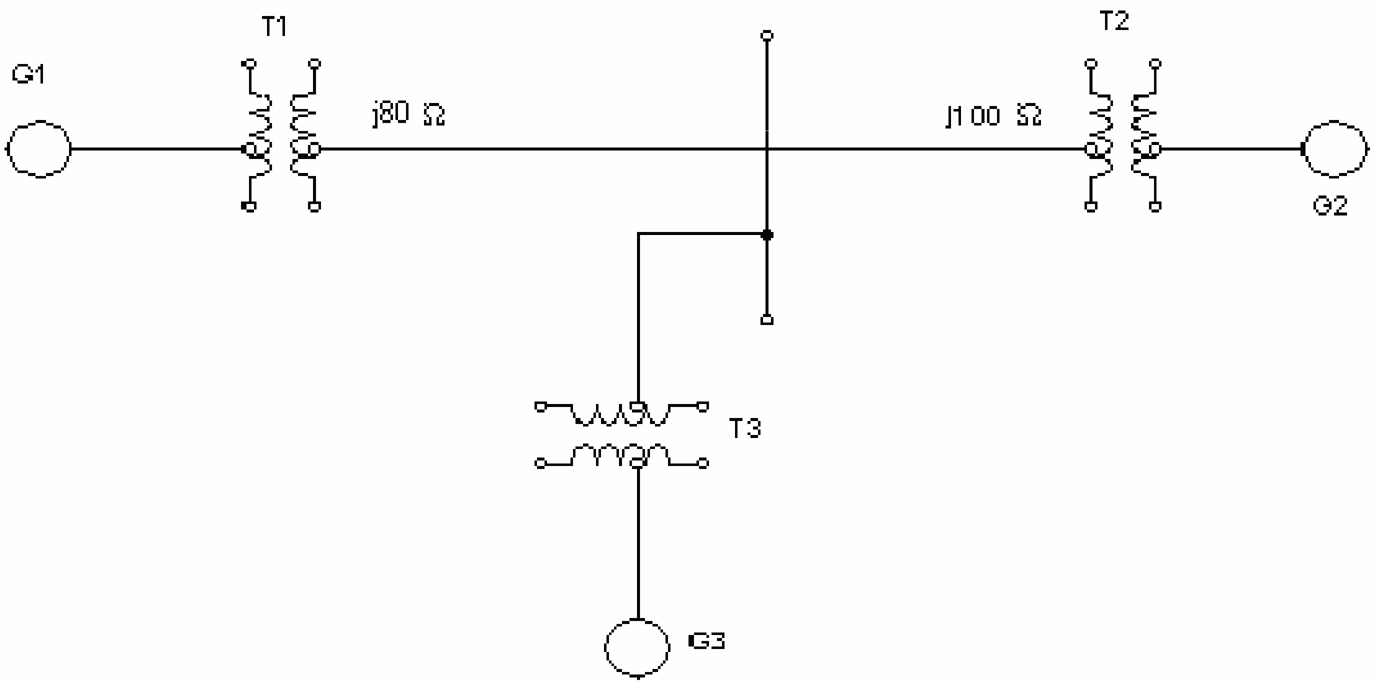
- Hence the e.m.f.s generated in loops RR_1, YY_1 and BB_1 are represented by the three equally spaced curves of Fig. 3, the e.m.f.s being assumed positive when their directions round the loops are from 'start' to 'finish' of their respective loops.
- If the instantaneous value of the e.m.f. generated in phase RR_1 is represented by $e_R = E_m \sin$

then instantaneous e.m.f. in YY_1 is $e_Y = E_m \sin(\;)$

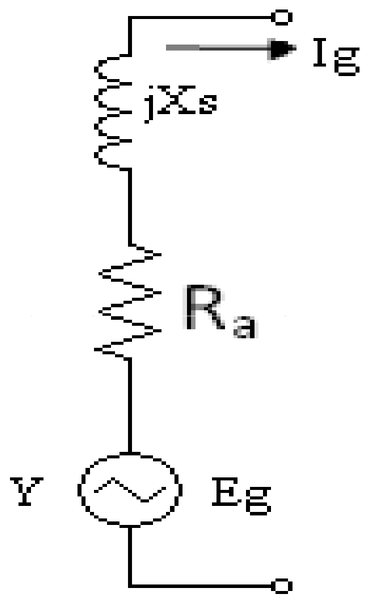
and instantaneous e.m.f. in BB_1 is $e_B = E_m \sin(\;)$

SINGLE LINE DIAGRAM

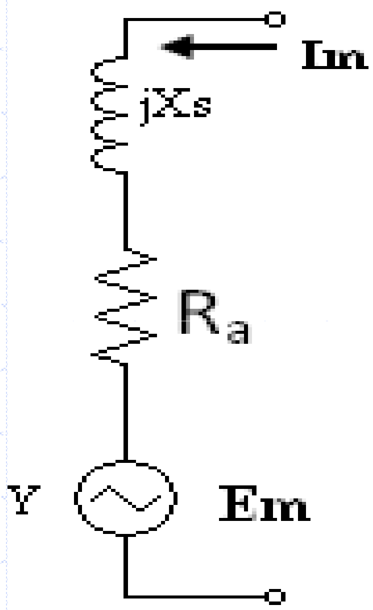
It is a diagrammatic representation of a power system in which the components are represented by their symbols.



MODELLING OF GENERATOR AND SYNCHRONOUS MOTOR

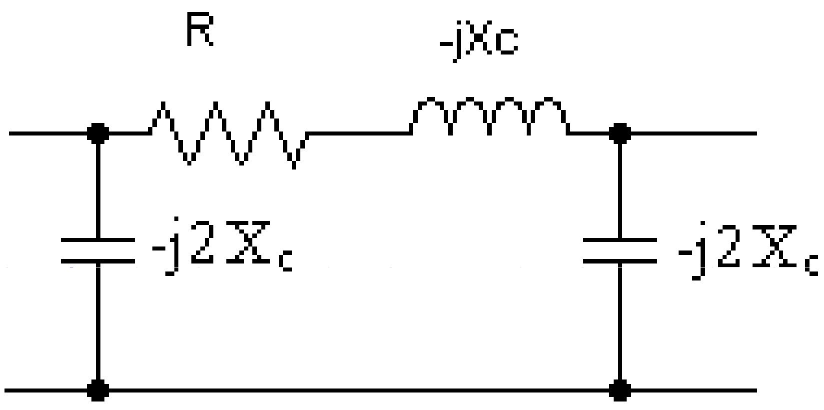


1Φ equivalent circuit of generator

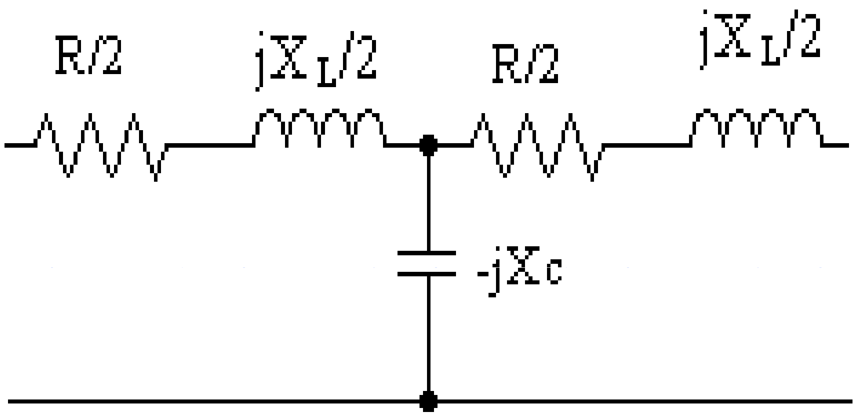


1Φ equivalent circuit of synchronous motor

MODELLING OF TRANSMISSION LINE

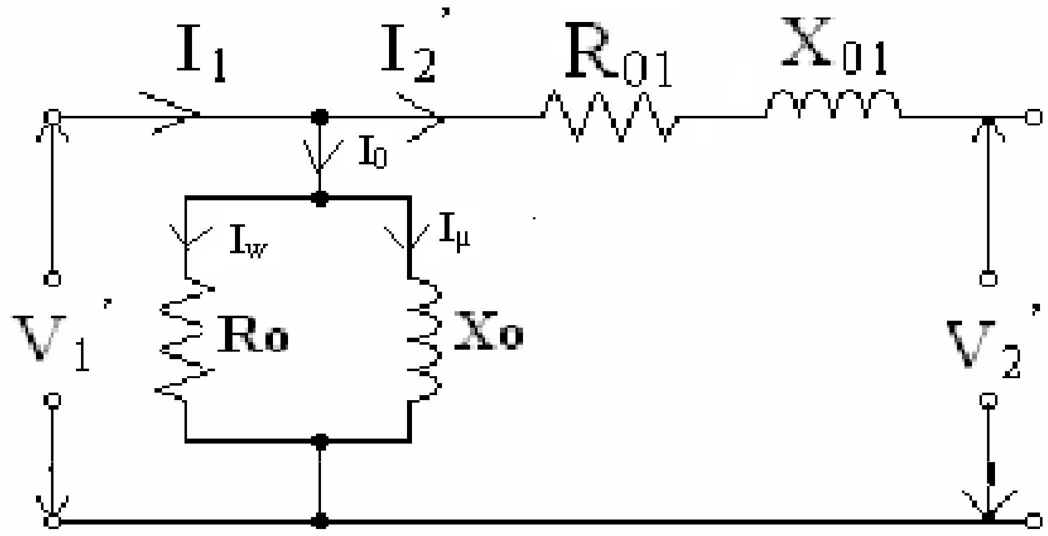


Π type



T type

MODELLING OF TRANSFORMER

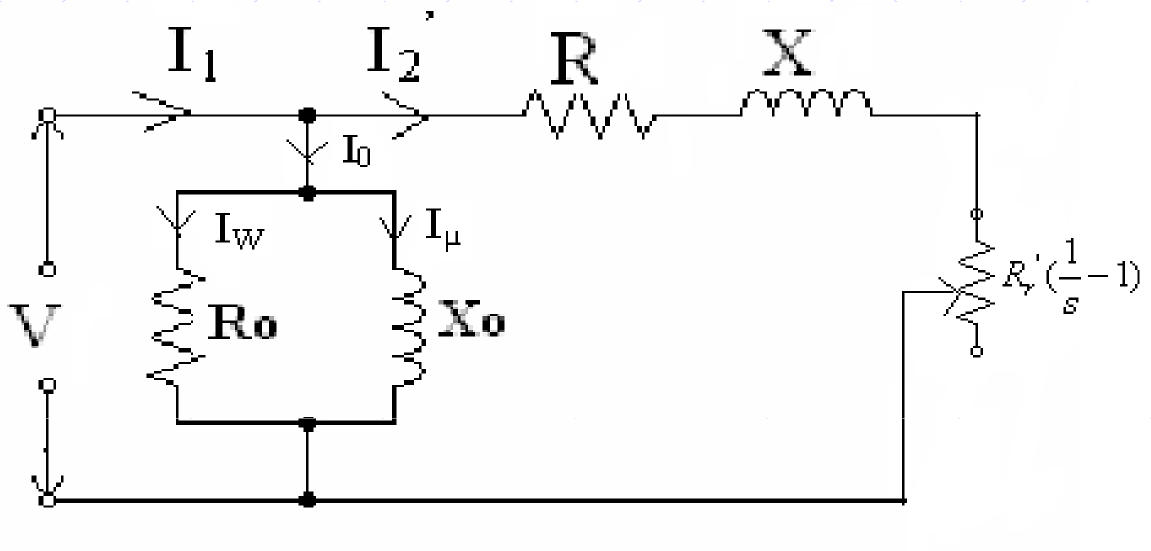


$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$ =Equivalent resistance referred to 1º

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$$ =Equivalent reactance referred to 1º

MODELLING OF INDUCTION MOTOR



$R_r'(\frac{1}{s} - 1)$ =Resistance representing load

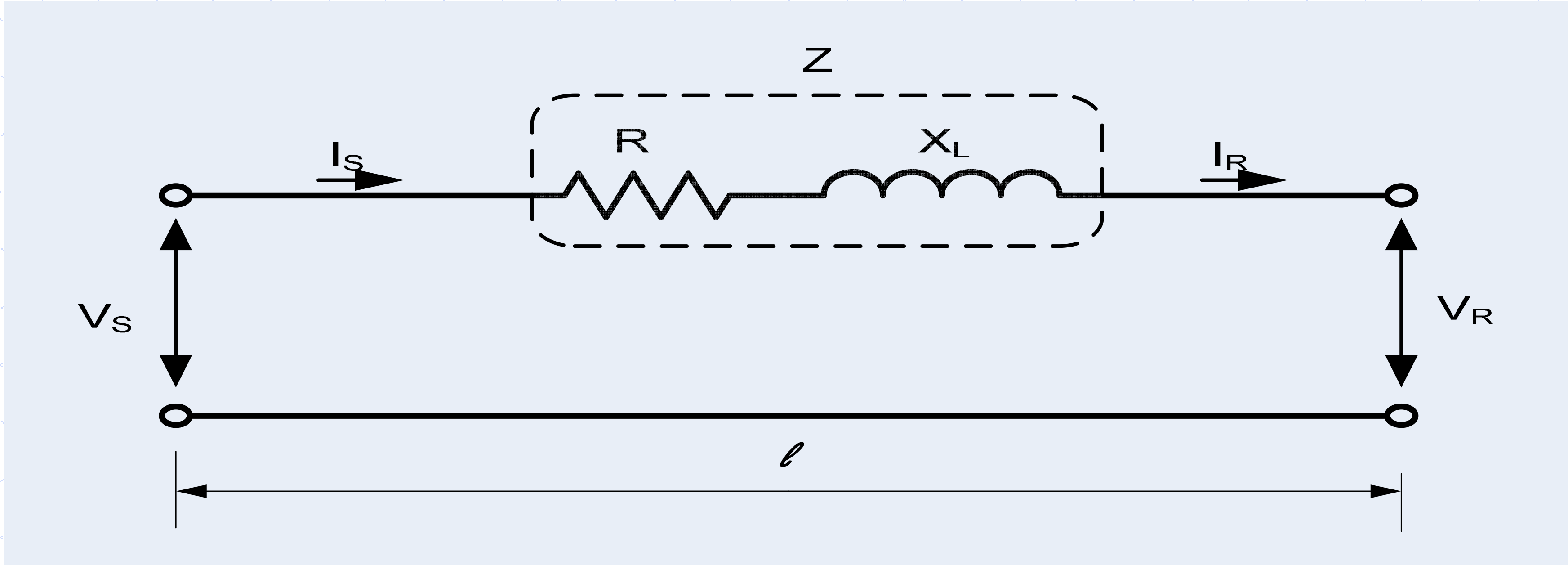
$R = R_s + R_r'$ =Equivalent resistance referred to stator

$X = X_s + X_r'$ =Equivalent reactance referred to stator

Transmission Line Representation

- Short Line Model
  - < 80 km in length
  - Shunt effects are neglected.
- Medium Line Model
  - Range from 80–240 km in length
  - Shunt capacitances are lumped at a few predetermined points along the line.
- Long Line Model
  - >240 km in length.
  - Uniformly distributed parameters.
  - Shunt branch consists of both capacitance and conductance.

Short Line



$$Z = z\ell = (r + j\omega L)\ell$$
$$= R + jX_L$$

$$V_s = I_R Z + V_R$$

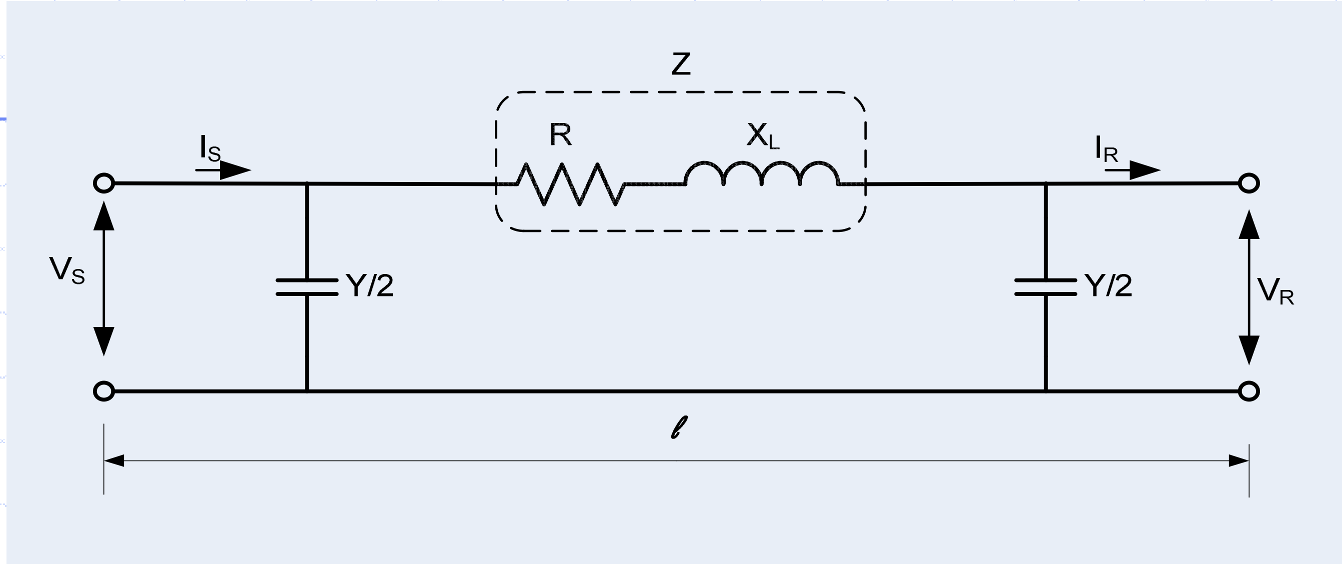
where:

r = per-phase resistance

L = per-phase inductance

ℓ = line length

Medium Line – Nominal π Circuit



- ➢Shunt capacitor is considered.
- ➢½ of shunt capacitor considered to be lumped at each end of the line – π circuit

Total shunt admittance, Y

$$Y = (g + j\omega C)\ell$$

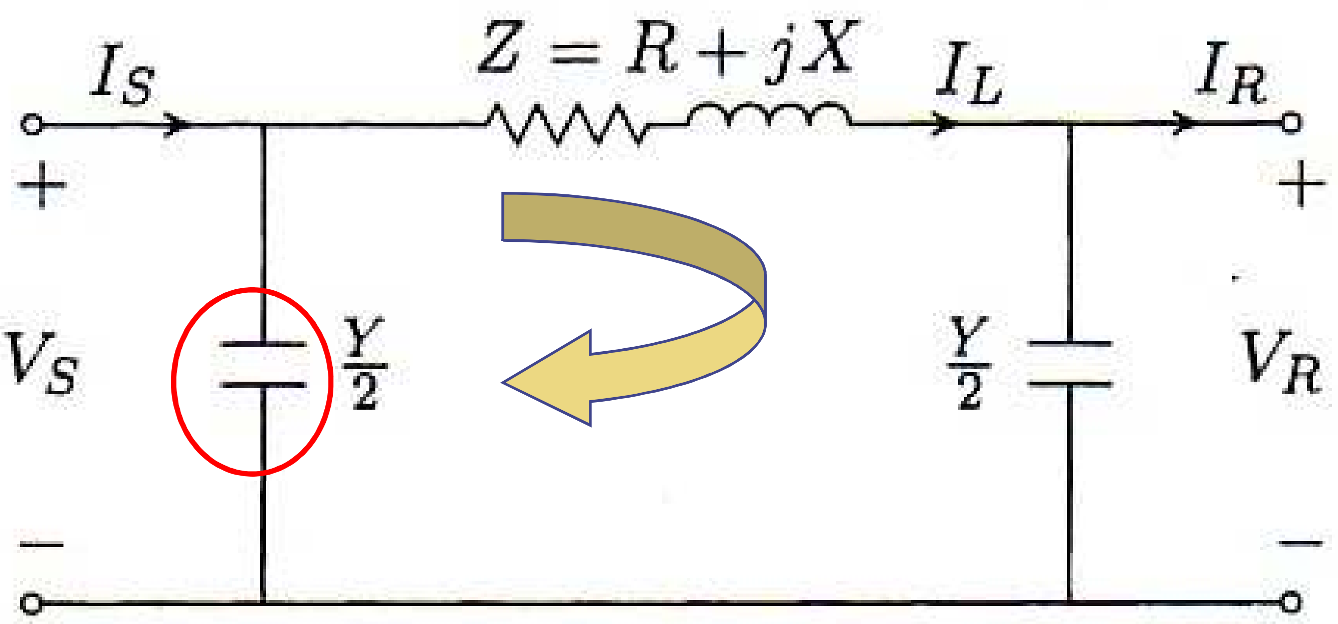
where :

C = line to neutral capacitance per km

g = line conductance per km

ℓ = line length

Medium Line Model



- Using KCL and KVL, the sending–end voltage is:

$$V_S = V_R + ZI_L \quad ..[1]$$

$$I_L = I_R + \frac{Y}{2}V_R \quad ..[2]$$

From [1] and [2]

$$V_S = V_R + Z\left(I_R + \frac{Y}{2}V_R\right)$$

$$= \left(1 + \frac{ZY}{2}\right)V_R + ZI_R \quad ..[3]$$

- Using KCL to obtain equation for sending–end current:

$$I_S = I_L + \frac{Y}{2} V_S \qquad ..[4]$$

Substitute [2] and [3] into [4]

$$I_S = I_R + \frac{V_R Y}{2} + \left[\left(1 + \frac{YZ}{2} \right) V_R + ZI_R \right] \frac{Y}{2}$$

$$= Y \left(1 + \frac{YZ}{4} \right) V_R + \left(1 + \frac{YZ}{2} \right) I_R \qquad ..[5]$$

Complex Power

Remember!

$$|V_{line}| = \sqrt{3}|V_{phase}|$$

- Sending end power

$$S_{S(3\phi)} = 3V_{S(phase)}I^*_{S(phase)}$$

or

$$S_{S(3\phi)} = \sqrt{3}V_{S(line)}I^*_{S(line)}$$

- Receiving end power

$$S_{R(3\phi)} = 3V_{R(phase)}I^*_{R(phase)}$$

or

$$S_{R(3\phi)} = \sqrt{3}V_{R(line)}I^*_{R(line)}$$

Transmission Line Efficiency

- Total Full–Load Line Losses

$$S_{L(3\phi)} = S_{S(3\phi)} - S_{R(3\phi)}$$

- Transmission Line Efficiency

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} \qquad \%\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} \times 100$$

- Note that only Real Power are taken into account!

Example

- A 220-kV, three-phase transmission line is 40 km long. The resistance per phase is 0.15 Ω/km and the inductance per phase is 1.5915 mH/km. The shunt capacitance is negligible.

  Use the line model to find the voltage and power at the sending end and efficiency when the line is supplying a three-phase load of

  a) 381 MVA at 0.8 pf lagging at 220 kV

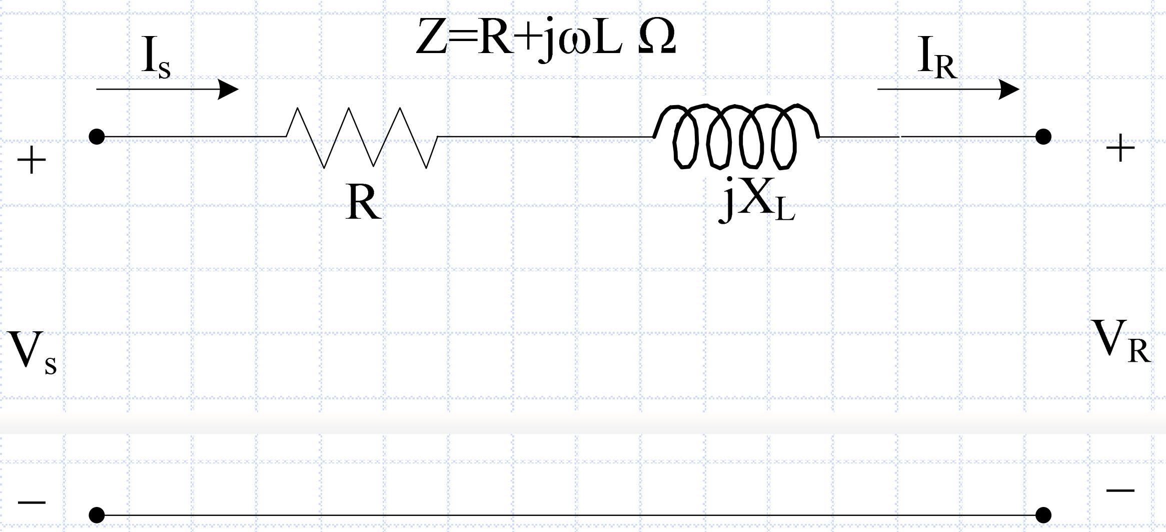
Solution

- Given

  $R = 0.15\ \Omega/km$, $L = 1.5915\ mH/km$

  $S = 381$ MVA with pf 0.8 lag

  $V_{R(line)} = 220$ kV



Find sending end voltage, $V_S = V_R + ZI_R$

Therefore, find V_R, Z, and I_R

$$V_{R(phase)} = \frac{V_{R(Line)}}{\sqrt{3}}$$

$$= \frac{220 \angle 0^o kV}{\sqrt{3}}$$

$$= 127 \angle 0^o kV$$

The series impedance per phase;

$$Z_{40km} = (r + j\omega L)l$$

$$= (0.15 + j(2\pi)(50)(1.5915m))40$$

$$= 6 + j20\Omega$$

$$S = 381 \text{ MVA}, \quad \theta = \cos^{-1} 0.8 = 36.87^{\circ}$$

Thus ,

$$S_R = 381 \angle 36.87^{\circ} MVA = 304.8 MW + j228.6 M\text{ var}$$

$$S_R = 3V_{R(Phase)} I_R^*$$

$$I_R^* = \frac{S_R}{3V_{R(Phase)}}$$

$$I_R = \frac{S_R^*}{3V_{R(Phase)}^*} = \frac{381 \angle -36.87^{\circ} MVA}{3(127 \angle 0^{\circ} kV)}$$

$$= 1000 \angle -36.87^{\circ} A$$

Therefore,

$$V_{S(Phase)} = V_{R(Phase)} + ZI_R$$

$$= 127\angle 0^o\, kV + (6 + j20\Omega)(1000\angle -36.87^o$$

$$= 144.3\angle 4.93^o\, kV$$

$$\left|V_{S(Line)}\right| = \sqrt{3}\left|V_{S(Phase)}\right|$$

$$= \sqrt{3}\left|144.3\right|$$

$$= 250V$$

Find Sending - end Power, $S_S = 3V_{S(Line)} I_S$

$$I_S = I_R = 1000 \angle -36.87^o A$$

$$S_S = 3V_{R(Phase)} \; I_R^*$$

$$= 3 (144.33 \angle 4.93^o V)(1000 \angle 36.87^o A)$$

$$= 322.8 MW + j288.6 M \text{ var}$$

$$= 433 \angle 41.8^o MVA$$

Effiency,η

$$\% \eta = \frac{P_R}{P_S} \times 100$$

$$= \frac{304.8}{322.8} \times 100$$

$$= 94.4\%$$