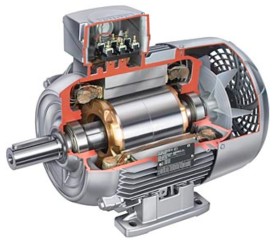
EE 321



Electromechanics & Electrical Machines

EE321

Course Outline

1. ***Introductory concepts***
2. ***Electromagnetic fields***
3. ***Magnetic circuits***
4. ***Transformers***
5. ***Introduction to electric power systems***
6. ***Generators and motors***
7. ***Illumination***

Recommended Text:

Hughes, E., *Electrical and Electronic Technology*, 10th Edition, 2008, Pearson Education Ltd, Essex, England

Hughes, E., *Electrical and Electronic Technology*, 9th Edition, 2008, Dorling Kindersley Publishing, Delhi, India

Assessment:

	Number	Timing	Weighting
Assignments	8	Every week	5%
Laboratories	6	As scheduled	15%
Tests	1	Mid-semester	20%
Exams	1	End of Semester	60%

Introductory Concepts

- Physical quantities, dimensions and units
- Scalars, vectors, phasors
- Faraday's law, Ampre's law, laws of conservation

Physical Quantities

-physical quantities in **electrical engineering** cover what happens in:

- conductors and insulators: electric
- magnetic materials: magnetism

} ⇒ Electrical

- mechanical engineering ⇒ Mechanics
- thermodynamics

Some of physical quantities are:

Mechanical	Electrical
Length [l or x]	Voltage [v, V]
Speed, velocity [u]	Current [i, I]
Force [F]	Resistance [R]
Momentum [Γ]	Capacitance [C]
Acceleration [a]	Inductance [L]
Torque [T]	Electric charge [q, Q]
Angular speed [ω]	Current density [J]
Mass [m]	

Dimensions and Units

⇒ are used to describe physical quantities as follows:

Dimension: ⇒ characteristic of a physical quantity

Units: ⇒ how the physical quantity is to be measured in terms of standard quantities

⇒ traditionally there are 3 reference dimensions in mechanics: ***length***, ***mass*** and ***time***.

⇒ one more dimension is needed for electricity and magnetism (e.g. current) and one more for thermodynamics (e.g. temperature)

⇒ a number of independent base units need to be chosen and carefully defined, the actual number being the same as that of the independent dimensions

⇒ The SI system has been adopted internationally, and comprises 7 base units and two supplementary units as follows

Quantity	Unit Name	Unit
Time	Second	s
Length	Metre	m
Mass	Kilogram	kg
Electric current	Ampere	A
Thermodynamic temperature	Kelvin	K
Luminous intensity	Candela	cd
Amount of substance	Mole	mol
Plane Angle	Radian	rad
Solid angle	Steradian	sr

Definition of Units

⇒ definitions are precise, but you need only remember for electric current

[see extract from IET Units and Symbols for Electrical and Electronic Engineering]

⇒ units of all physical quantities can be expressed in terms of these base and supplementary units

⇒ In practice it appears better to define derived units in cases where

- the quantity is used frequently eg Hz derived from s^{-1} for frequency
- the units are lengthy, eg N for kgm/s^2
- a new physical concept is introduced eg V for $kgm^2s^{-3}A^{-1}$

⇒ the numerical values of many quantities may be many orders of magnitude away from unity (bigger or smaller)

⇒ it may be convenient to either use power of 10 or to use decimal prefixes with the unit

quantity	name	symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k

quantity	name	symbol
10^{2}	hecto	h
10^{1}	deca	da
10^{-1}	deci	d
10^{-2}	centi	c

quan	name	sym
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Relationships

⇒ equations are a compact and effective way of stating relationships between various quantities

⇒ equations may arise in 4 different ways:

1. a statement of a basic law, tested and accepted e.g. Faraday's law, $v = N\frac{d\varphi}{dt}$
2. a definition of a new quantity or concept e.g. electric current density, $J = \frac{di}{dA}$
3. description of material property e.g. permeability, $\mu = \frac{B}{H}$
4. derivation from other equations, using mathematical processes

Scalars, Vectors and Phasors

Scalar

⇒ has magnitude only; is specified by a single numerical value together with its units, e.g. mass

Vector

⇒ has a magnitude and direction and so needs two values to specify it ,if it is confined to a plane, or three values, if in general space;

⇒ quoted values depend on system ie x,y for cartersian or r,θ for polar representation

Phasor

⇒ is a technique for displaying any sinusoidally varying quantity.

⇒ in general $y=y_m(\cos\omega t+\varphi)$ means the projection of a phasor of magnitude y_m rotating at angular speed ω onto a fixed reference unit, with angle φ between them at t=0 will give the instantaneous value of y.

Basic Laws

Maxwell's Equations

Faraday's Law $\nabla \times E = -\frac{\partial B}{\partial t} \rightarrow \left\{ v = N\frac{d\phi}{dt} \right\}$

Ampere's Law $\nabla \times H = \frac{\partial D}{\partial t} + J \rightarrow \{ Hl = NI \}$

Gauss's Laws

$\nabla \bullet D = \rho$ Conservation of electric charge

$\nabla \bullet B = 0$ Conservation of magnetic flux

Newton's Law $F = m\ddot{x}$

Law of conservation of mass

Law of conservation of energy

Electromagnetic fields

- Introduction to fields of
  - magnetostatics,
  - electrostatics and
  - conduction
- Total quantities:
  - reluctance,
  - capacitance,
  - resistance

Electrostatics

⇒ fields of electrostatic involve the quantities *q, ψ, D, E,* and *V* defined as:

Electric charge, (*q*)

Electric flux, (*ψ*)

$$\psi = q$$

Electric flux density, (*D*)

$$D = \frac{d\psi}{dA} = \frac{\psi}{A}$$

Electric field strength, (*E*)

⇒ *E*, at any point in the field, is defined as the force on a unit charge placed at that point

⇒ *E* is a vector quantity whose direction is that of the force on the unit charge

⇒ applying Gauss's law: at a distance *r*, from a fixed point charge *q*, the electric field strength in a radial direction from the point charge is

$$E = \frac{q}{4\pi\varepsilon r^2}$$

⇒ ε is the permittivity of the medium enclosing the charge

$$\varepsilon = \varepsilon_o \varepsilon_r$$

- ε_o = permittivity of free space,
- ε_r = relative permittivity of the medium to that of vacuum

⇒ knowing that $D = \frac{q}{A} = \frac{q}{4\pi r^2}$ then $D = \varepsilon E$

Electric potential, (*V*)

⇒ defined as the change in the potential of one coulomb of charge if one joule of work is required to move it from one point to another

⇒ If a **unit** charge moves from one point to another in an electric field, thereby changing its potential by *V* Volts, then the work done is *V* Joules

$\Rightarrow$ in general, work done is

$$W = qV$$

$\Rightarrow$ when a unit charge is moved a distance Δl in a direction opposite to the field strength E, the work done is

$$W = E\Delta l$$

$\Rightarrow$ if the associated change in potential of the unit charge is ΔV, then

$$\Delta V = E\Delta l$$

$$E = \frac{\Delta V}{\Delta l} = \frac{dV}{dl}$$

Electrical conduction

$\Rightarrow$ fields of electrical conduction involve the quantities *I, J, E,* and *V,* defined as:

Current, (*I*)

$\Rightarrow$ the flow of electric charge

Current density, (*J*)

$\Rightarrow$ if current is distributed uniformly throughout the X-section of a wire, the current density *J* is uniform, and is given by the total current divided by the X-section area *A* of the wire

$$J = \frac{I}{A}$$

$\Rightarrow$ if current density is not uniform, the local current density is given by

$$J = \lim_{\Delta A \to 0} \frac{\Delta I}{\Delta A} = \frac{dI}{dA}$$

Electric Field Strength & Potential in conduction

$\Rightarrow$ consider a small imaginary rectangle cell of length *l* and X-section *A*, with *J* normal at a point of consideration in the cell

$\Rightarrow$ let *V* be the potential difference between the ends of the cell

$\Rightarrow$ applying Ohm's law:

$V = IR$ but $V = El$ and $I = JA$ so that

$$El = JAR$$

$$J = \frac{l}{AR} E = \sigma E \quad \text{then} \quad J = \sigma E$$

where $\sigma = \frac{l}{AR}$

$\Rightarrow$ σ is the conductivity of the material; the reciprocal is resistivity, ρ

$$\rho = \frac{1}{\sigma} = \frac{AR}{l}$$

Magnetostatics

Magnetic field

$\Rightarrow$ that region in which a charged particle in motion or a magnetic material is acted upon by a magnetic force

Magnetic lines of force

- have direction
- form complete loops
- represent a tension about their length which tends to make them as short as possible
- repel one another
- cannot intercept but must always form an individual closed loop

⇒ fields of magnetostatics involve the quantities ϕ, *B*, *F* and *H*, defined as:

Magnetic flux, (ϕ)

⇒ total number of lines of force in a magnetic field, unit is Weber, [Wb].

⇒1 Wb = 10^8 lines of magnetic force, by definition

Magnetic flux density, (*B*)

⇒ magnetic flux per unit area, unit is [Wb/m^2] or Tesla [T]

$$B = \frac{d\phi}{dA}$$

⇒ in a uniform field

$$B = \frac{\phi}{A}$$

Magnetomotive force, mmf, (*F*)

⇒ magnetic flux is caused by a mmf, just as current in conductor is caused by an emf

⇒ if a coil of *N* turns carrying a current *I* is used as the source of mmf, then

$$mmf = F = NI$$

Magnetic field strength or magnetic field intensity, (*H*)

⇒ is defined as the mmf per unit length, and units are [A/m]

$$H = \frac{F}{l}$$

Permeability, (μ)

⇒ In a uniform field magnetic flux density *B* is related to magnetic filed intensity *H* by:

$$B = \mu H$$

where μ is the permeability of the medium

$$\mu = \mu_o \mu_r$$

μ_O = permeability of free space

μ_r = relative permeability

Electrical properties of matter and space

Capacitance, (C)

⇒ is a property of matter and space whereby electric charge *q* is accumulated when a potential difference *V* is applied to it

⇒ If the applied p.d. is *V* and accumulated charge is *q*, then capacitance is defined as

$$C = \frac{q}{V}$$

⇒ unit is Farad, [F]

Conductance, (G)

⇒ is the property of matter whereby electric current flows when a potential difference is applied to it

⇒ if the applied p.d. is *V* and the current that flows is *I*, then conductance is defined as

$$G = \frac{I}{V}$$

⇒ unit is Semiens, [S]

⇒ reciprocal of Conductance is Resistance, *R*

$$R = \frac{V}{I}$$

⇒ unit is Ohm, [Ω]

Permeance, (Λ)

⇒ is the property of matter whereby a magnetic flux is established when a magnetomotive force is applied to it

⇒ if the applied mmf is *F* and the flux established is ϕ, then permeance is defined as

$$\Lambda = \frac{\phi}{F}$$

⇒ unit is Henry, [H] or Weber per ampere [Wb/A]

⇒ in special types of matter such as conductors, inductance *L* is used to describe permeance

⇒ Inductance is the ability of a conductor to have a voltage induced when the current changes

⇒ in this case, if the total flux linkage, λ, causes a current, *I* to be induced in a conductor then inductance

$$L = \frac{\lambda}{I}$$

⇒ the reciprocal of *permeance* is *reluctance*, *S*

⇒ unit of reluctance is Ampere per Weber [A/Wb]

⇒ field is used to indicate a region of space throughout which the effect is appreciable

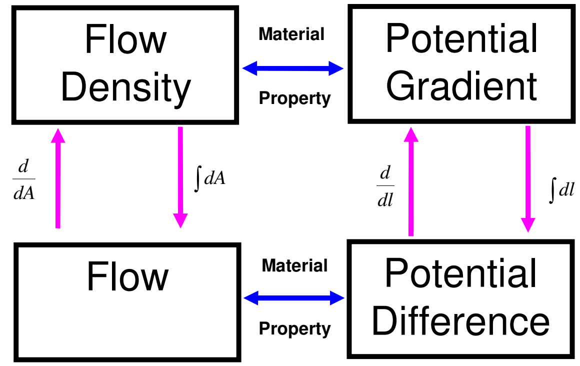
⇒ the region may be bounded or may extend to infinity

⇒ field to be studied are electric and magnetic

⇒ a field may be 0-D, 1-D, 2-D or 3-D

Lumped and field quantities

⇒ consider the relationships



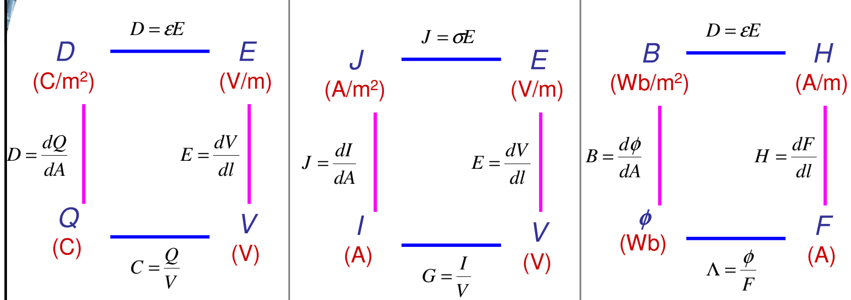
Field quantities:

⇒ flow density & potential gradient

Lumped quantities:

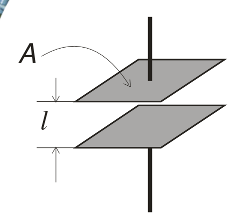
⇒ flow & potential difference

⇒ Electrostatics ⇒ Current conduction ⇒ Magnetostatics



0-D fields (Uniform fields)

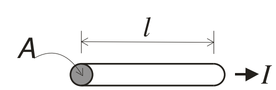
⇒ Electrostatics



$$C = \frac{Q}{V} = \frac{DA}{El}$$

$$C = \varepsilon \frac{A}{l}$$

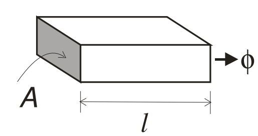
⇒ Current conduction



$$G = \frac{I}{V} = \frac{JA}{EA}$$

$$G = \sigma \frac{A}{l}$$

⇒ Magnetostatics

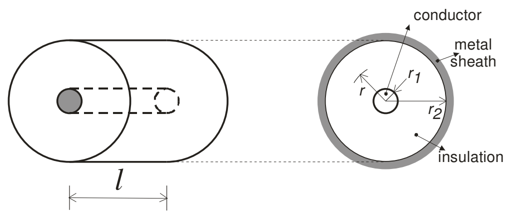


$$\Lambda = \frac{\phi}{F} = \frac{BA}{Hl}$$

$$\Lambda = \mu \frac{A}{l}$$

1-D fields: Examples

1. Concentric cable (electrostatic field)



⇒ consider length l having charge q

⇒ metal sheath is earthed, so is at zero potential

⇒ total flux:

$$\psi = q$$

⇒ at radius r

$$D = \frac{\psi}{A} = \frac{q}{2\pi r l}$$

$$E = \frac{D}{\varepsilon} = \frac{q}{2\pi r l \varepsilon}$$

$$V = \int_{r_1}^{r_2} E dr = \int_{r_1}^{r_2} \frac{q}{2\pi r l \varepsilon} dr$$

$$= \frac{q}{2\pi l \varepsilon} \ln\frac{r_2}{r_1}$$

$$C = "\frac{Q}{V}"$$

$$C = \frac{2\pi l \varepsilon}{\ln\frac{r_2}{r_1}}$$

⇒ consider E

$$E = \left(\frac{q}{2\pi l \varepsilon}\right)\frac{1}{r} = \left(\frac{V}{\ln\frac{r_2}{r_1}}\right)\frac{1}{r}$$

⇒ for max E, r is minimum, i.e., $r = r_1$

$$E_{max} = \frac{V}{r_1 \ln\frac{r_2}{r_1}}$$

⇒ for given V and r_2, what value of r_1 gives minimum E_{max}?

$$\frac{dE_{max}}{dr_1} = 0 = \frac{-V}{\left(r_1 \ln\frac{r_2}{r_1}\right)^2}\left[\ln\frac{r_2}{r_1} + r_1\left(\frac{r_1}{r_2}\right)\left(\frac{-r_2}{r_1^2}\right)\right]$$

$$\ln\frac{r_2}{r_1} - 1 = 0$$

$$\frac{r_2}{r_2} = e$$

$$r_1 = \frac{r_2}{e}$$

⇒ on a.c., capacitive current:

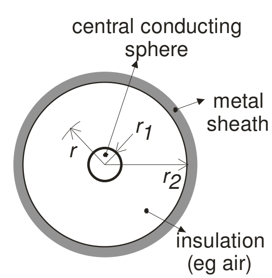
$$I = \frac{V}{Z}$$

$$Z = \frac{1}{j\omega C}$$

$$|I| = V\omega C = 2\pi f V C$$

⇒ on a.c., V is given as rms; use peak value to get ultimate stress E_{max}

2. Concentric sphere (electrostatic field)



$$D = \frac{\psi}{A} = \frac{q}{4\pi r^2}$$

$$E = \frac{D}{\varepsilon} = \frac{q}{4\pi r^2 \varepsilon}$$

$$V = \int E dr = \frac{q}{4\pi\varepsilon}\int_{r_1}^{r_2}\frac{dr}{r^2}$$

$$= \frac{q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

⇒ capacitance:

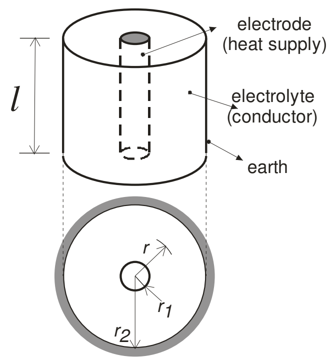
$$C = \frac{q}{V} = \frac{4\pi\varepsilon}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

⇒ electric field stress

$$E = \frac{V}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)r^2}$$

$$E_{max} = \frac{V}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)r_1^2}$$

3. Electrode boiler (conduction field)



⇒ assume total current I, depth of liquid l

$$J = \frac{I}{A} = \frac{I}{2\pi r l}$$

$$E = \frac{J}{\sigma} = \frac{I}{2\pi l \sigma}\frac{1}{r}$$

$$V = \int E dr = \frac{I}{2\pi l \sigma}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$= \frac{I}{2\pi l \sigma}\ln\frac{r_2}{r_1}$$

$$G = \frac{I}{V} = \frac{2\pi l \sigma}{\ln\frac{r_2}{r_1}}$$

⇒ for max J, $r = r_1$

$$J_{\max} = \frac{I}{2\pi l r_1} = \frac{\sigma}{G}\frac{I}{r_1 \ln\frac{r_2}{r_1}}$$

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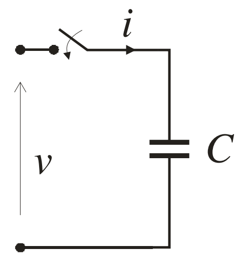
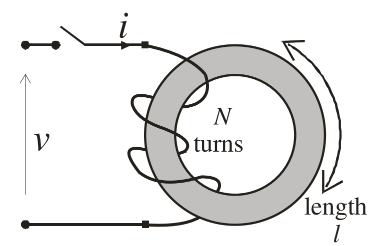
Stored energy in fields

I. Electrostatic field



⇒ parallel plate capacitor, capacitance, C

II. Magnetostatic field



⇒ toroidal coil, inductance L

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I. Electrostatic field

$$q = CV$$

$$i = C\frac{dv}{dt}$$

⇒ power $\quad P = vi = Cv\frac{dv}{dt}$

⇒ energy $\quad W = \int P dt = \int_0^V cv dv$

$$= C\frac{V^2}{2} = \frac{1}{2}qV$$

⇒ energy/volume $\quad w = \frac{\frac{1}{2}qV}{Al} = \frac{1}{2}\frac{q}{A}\frac{V}{l}$

$$w = \frac{1}{2}DE$$ [J/m³]

II. Magnetostatic field

$$Li = N\phi$$

$$v = L\frac{di}{dt}$$

$$P = vi = Li\frac{di}{dt}$$

$$W = \int P dt = \int_0^I Li di$$

$$= L\frac{I^2}{2} = \frac{1}{2}N\phi I$$

$$w = \frac{\frac{1}{2}N\phi I}{Al} = \frac{1}{2}\frac{\phi}{A}\frac{NI}{l}$$

$$w = \frac{1}{2}BH$$ [J/m³]

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I. Electrostatic field

$$w = \frac{1}{2}DE$$

II. Magnetostatic field

$$w = \frac{1}{2}BH$$

⇒ the expression are derived using a uniform field

⇒ as energy the expression apply ant any point in the field, whether uniform or not

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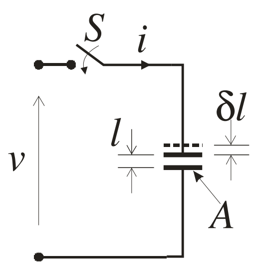
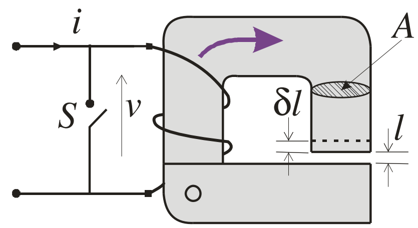
Stored energy in fields

I. Electrostatic field



II. Magnetostatic field



⇒ assume $\mu_{Fe} = \infty$, just consider airgap

I. Electrostatic field

1) close *S*, charge up *C* to voltage v, associated charge is

$$q = Cv$$

2) open *S*, charge q is trapped and is constant, ∴

$$q = \text{constant}$$
$$i = \frac{dq}{dt} = 0$$
$$\psi = \text{constant}$$
$$D = \text{constant}$$
$$E = \text{constant}$$

II. Magnetostatic field

1) open *S*, current *i* flows thru' coil, hence

$$\phi = \frac{Li}{N}$$

2) close *S*, making *v* = 0; since

$$v = N\frac{d\phi}{dt} = 0$$ (Faraday's law)

$$\phi = \text{constant}$$ (ϕ is 'trapped')

$$B = \text{constant}$$
$$H = \text{constant}$$

I. Electrostatic field

3) gap changes from *l* to *l* + δ*l* ; there are energy changes

a. (mech. energy ?) $= F_{\text{mech}} . \delta l$

b. (elect. energy ?) $= \int vidt = 0$

from $i = 0$

c. change of stored energy in electrostatic field:

$$= \delta\left(\tfrac{1}{2} DE \times \text{volume}\right)$$
$$= \tfrac{1}{2} DE(\delta l.A)$$

II. Magnetostatic field

3) gap changes from *l* to *l* + δ*l* ; there are energy changes

a. (mech. energy ?) $= F_{\text{mech}} . \delta l$

b. (elect. energy ?) $= \int vidt = 0$

from $v = 0$

c. change of stored energy in magnetostatic field (airgap):

$$= \delta\left(\tfrac{1}{2} BH \times \text{volume}\right)$$
$$= \tfrac{1}{2} BH(\delta l.A)$$

4) applying law of conservation of energy, the two forms of energy changes relevant to motion δ*l* can be equated

I. Electrostatic field

$$F_{\text{mech}} \delta l = \tfrac{1}{2} DE(\delta l.A)$$

$$F_{\text{mech}} = \tfrac{1}{2} DAE$$ [N]

Pressure = press

$$\text{press} = \frac{F_{\text{mech}}}{A}$$

$$\text{press} = \tfrac{1}{2} DE$$ [N/m²]

II. Magnetostatic field

$$F_{\text{mech}} \delta l = \tfrac{1}{2} BH(\delta l.A)$$

$$F_{\text{mech}} = \tfrac{1}{2} BAH$$ [N]

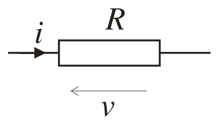
Pressure = press

$$\text{press} = \frac{F_{\text{mech}}}{A}$$

$$\text{press} = \tfrac{1}{2} BH$$ [N/m²]

III. Conduction field

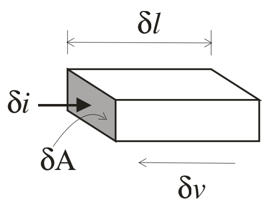
⇒ for a resistance, the electrical energy goes into heat energy



$$P = vi = i^2 R = \frac{v^2}{R}$$

⇒ no energy is stored

⇒ for field study:



local: $J = \frac{\delta i}{\delta A}$

local: $E = \frac{\delta v}{\delta l}$

material property: $J = \sigma E$

P in a small volume: $\delta P = \delta v \, \delta i$

power per unit volume (power density):

$$= \frac{\delta v \, \delta i}{\delta l \, \delta A} = \frac{\delta v}{\delta l} \cdot \frac{\delta i}{\delta A} = JE$$

$$= \sigma E^2 = \frac{J^2}{\sigma} \quad [\mathrm{J/m^3}]$$

Examples:

1) A 20-µF capacitor is charged at constant current of 5 µA for 10 min. Calculate the final p.d. and the corresponding stored charge?

2) An electrode boiler has an earthed metal cylinder of 0.8 m diameter with non conducting ends and a co-axial central electrode of 0.1 m diameter. The boiler is designed to absorb 8 kW when connected to a 240-V supply, the depth of the water being 0.5 m in the axial direction. To what specific conductivity should the water be treated in order to absorb this power?

3) A coil wound uniformly round a wooden ring having a mean circumference of 600 mm and uniform cross-section al area of 500 mm² produces an mmf of 0.8 kA. Find

   a) the magnetic field strength

   b) the flux density

   c) the total flux

   in the ring.

Examples:

4) What force can one pole of an electromagnet exert on a movable steel object if there is a uniform air gap of 5 mm between them, the area of the pole and end face is 10^4 mm² and the magnetic flux density is 0.5 T? Assume that the flux path has negligible reluctance apart from the airgap.