

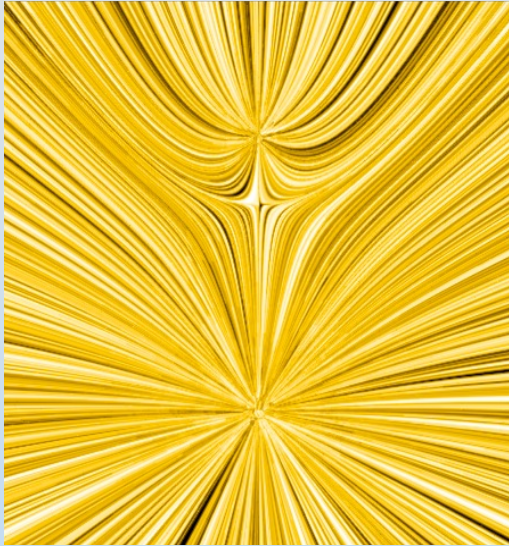
Exam 1 Topics

- Fields (visualizations)
- Electric Field & Potential
 - Discrete Point Charges
 - Continuous Charge Distributions
 - Symmetric Distributions – Gauss' s Law
- Conductors

General Exam Suggestions

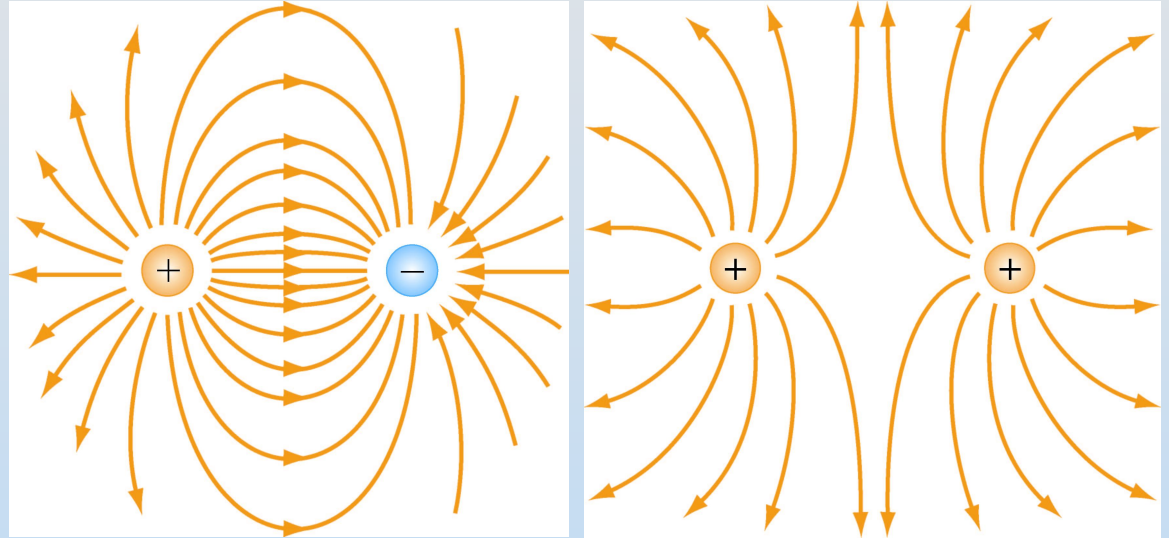
- You should be able to complete every problem
 - If you are confused, ask
 - If it seems too hard, think some more
 - Look for hints in other problems
 - If you are doing math, you're doing too much
- Read directions completely (before & after)
- Write down what you know before starting
- Draw pictures, define (label) variables
 - Make sure that unknowns drop out of solution
- Don't forget units!

Fields



Grass Seeds

Know how to read



Field Lines

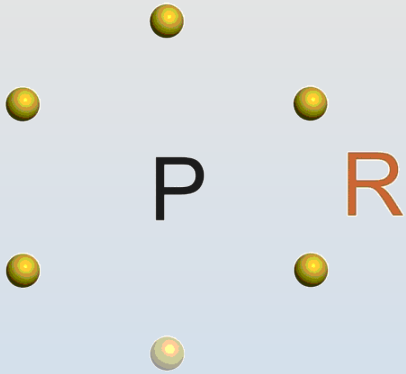
Know how to draw

- Field line density tells you field strength
- Lines have tension (want to be straight)
- Lines are repulsive (want to be far from other lines)
- Lines begin and end on sources (charges) or ∞

PRS Questions: Fields

Concept Question: 5

Equal Charges



Six equal positive charges q sit at the vertices of a regular hexagon with sides of length R . We remove the bottom charge. The electric field at the center of the hexagon (point P) is:

$$1. \vec{\mathbf{E}} = \frac{2kq}{R^2} \hat{\mathbf{j}}$$

$$2. \vec{\mathbf{E}} = -\frac{2kq}{R^2} \hat{\mathbf{j}}$$

$$5. \vec{\mathbf{E}} = 0$$

$$3. \vec{\mathbf{E}} = \frac{kq}{R^2} \hat{\mathbf{j}}$$

$$4. \vec{\mathbf{E}} = -\frac{kq}{R^2} \hat{\mathbf{j}}$$

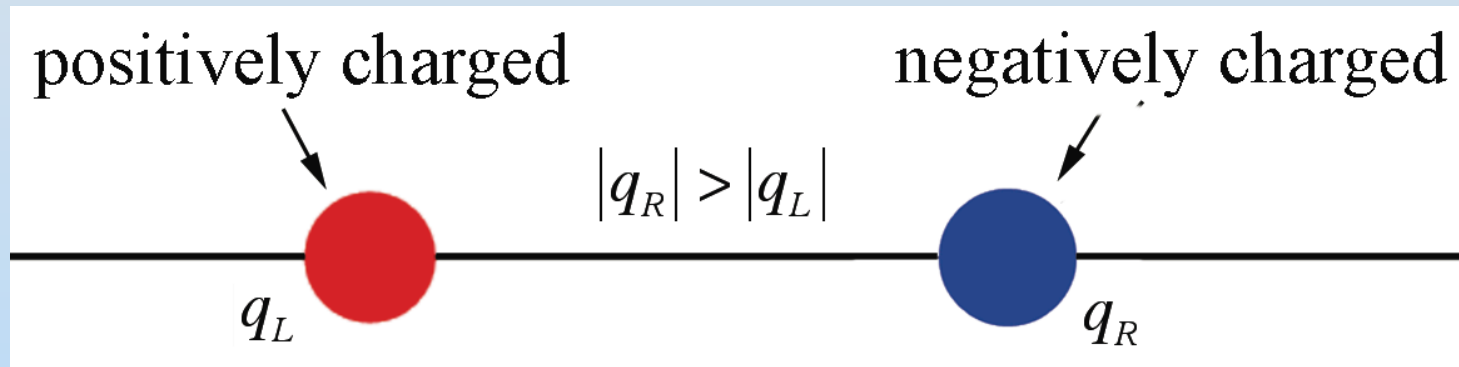
Concept Question: Field Lines

Electric field lines show:

1. Directions of forces that exist in space at all times.
2. Directions in which positive charges on those lines will accelerate.
3. Paths that charges will follow.
4. More than one of the above.
5. I don't know.

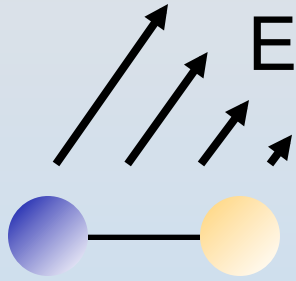
Concept Question: Electric Field

Two charged objects are placed on a line as shown below. The magnitude of the negative charge on the right is greater than the magnitude of the positive charge on the left, $|q_R| > |q_L|$. Other than at infinity, where is the electric field zero?



1. Between the two charged objects.
2. To the right of the charged object on the right.
3. To the left of the charged object on the left.
4. The electric field is nowhere zero.
5. Not enough info – need to know which is positive.

Concept Question: Dipole in Non-Uniform Field



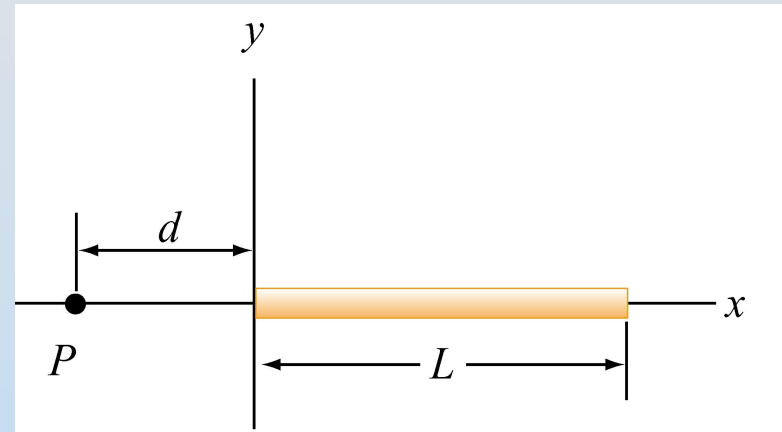
A dipole sits in a non-uniform electric field E

Due to the electric field this dipole will feel:

1. force but no torque
2. no force but a torque
3. both a force and a torque
4. neither a force nor a torque

Concept Question Electric Field of a Rod

A rod of length L lies along the x -axis with its left end at the origin. The rod has a *uniform* charge density λ . Which of the following expressions best describes the electric field at the point P



1. $\vec{\mathbf{E}}(P) = - \int_{x=0}^{x=L} \frac{\lambda dx}{(x-d)^3} \hat{\mathbf{i}}$

2. $\vec{\mathbf{E}}(P) = \int_{x=0}^{x=L} \frac{\lambda dx}{(x-d)^3} \hat{\mathbf{i}}$

3. $\vec{\mathbf{E}}(P) = - \int_{x=0}^{x=L} \frac{\lambda dx}{(x-d)^2} \hat{\mathbf{i}}$

4. $\vec{\mathbf{E}}(P) = \int_{x=0}^{x=L} \frac{\lambda dx}{(x-d)^2} \hat{\mathbf{i}}$

5. $\vec{\mathbf{E}}(P) = - \frac{\lambda L}{(d-x)^2} \hat{\mathbf{i}}$

6. $\vec{\mathbf{E}}(P) = \frac{\lambda L}{(d-x)^2} \hat{\mathbf{i}}$

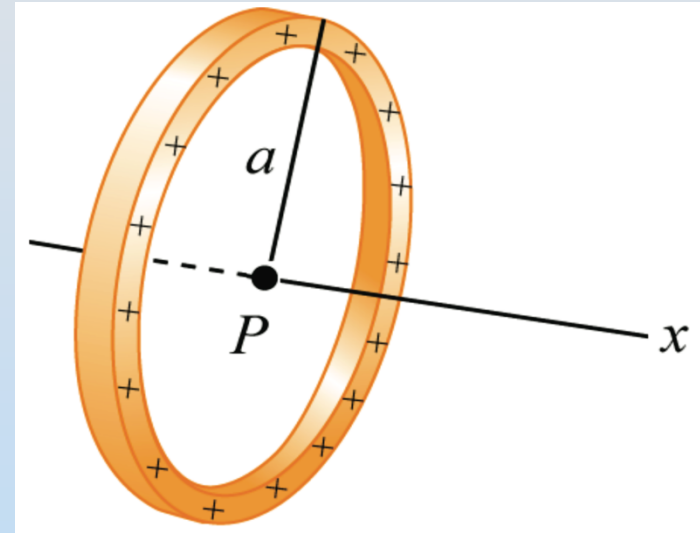
7. $\vec{\mathbf{E}}(P) = - \frac{\lambda L}{d^2} \hat{\mathbf{i}}$

8. $\vec{\mathbf{E}}(P) = \frac{\lambda L}{d^2} \hat{\mathbf{i}}$

9. None of the above.

Concept Question Electric Field of a Ring

A uniformly charged ring of radius a has total charge Q . Which of the following expressions best describes the electric field at the point P located at the center of the ring?



1. $\vec{\mathbf{E}}(P) = - \int_{\theta=0}^{\theta=2\pi} \frac{\lambda a d\theta}{a^3} \hat{\mathbf{i}}$

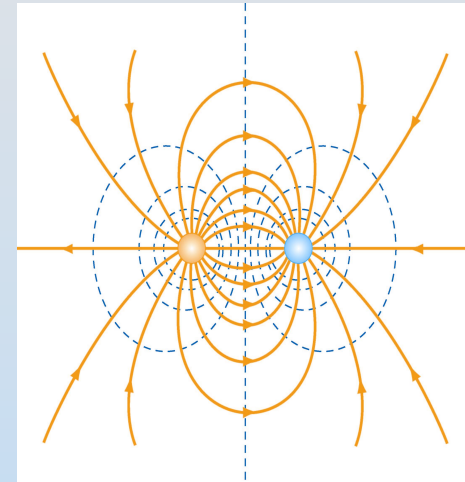
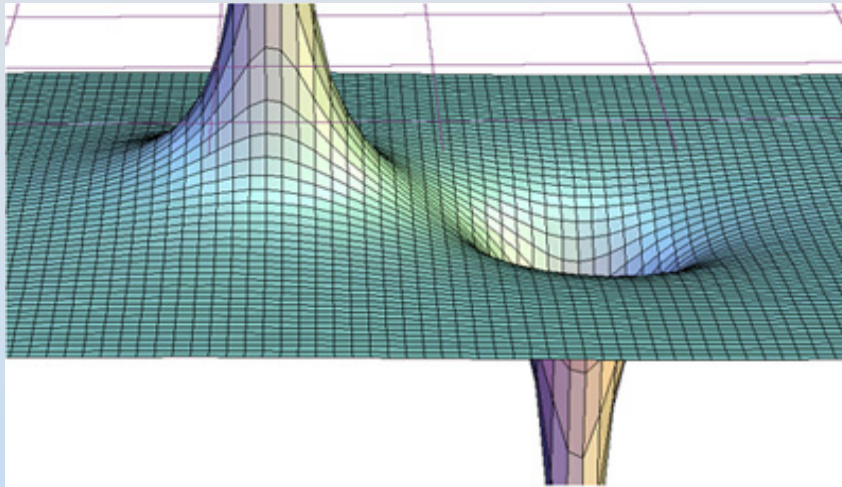
2. $\vec{\mathbf{E}}(P) = \int_{\theta=0}^{\theta=2\pi} \frac{\lambda a d\theta}{a^3} \hat{\mathbf{i}}$

3. $\vec{\mathbf{E}}(P) = -\frac{Q}{a^2} \hat{\mathbf{i}}$

4. $\vec{\mathbf{E}}(P) = +\frac{Q}{a^2} \hat{\mathbf{i}}$

5. $\vec{\mathbf{E}}(P) = \vec{\mathbf{0}}$

E Field and Potential: Creating



A point charge q creates a field and potential around it:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^3} \vec{\mathbf{r}}; \quad V = k_e \frac{q}{r}$$

Use superposition for systems of charges

They are related:

$$\vec{\mathbf{E}} = -\nabla V; \quad \Delta V \equiv V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

E Field and Potential: Creating

Discrete set of point charges:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^3} \vec{\mathbf{r}}; \quad V = k_e \frac{q}{r}$$

Add up from each point charge

Continuous charge distribution:

$$d\vec{\mathbf{E}} = k_e \frac{dq}{r^3} \vec{\mathbf{r}}; \quad dV = k_e \frac{dq}{r}$$

Break charged object into small pieces, dq , and integrate

Continuous Sources: Charge Density

Charge Densities:

$$\lambda = \frac{Q}{L}$$

$$\sigma = \frac{Q}{A}$$


$$\rho = \frac{Q}{V}$$

$$dQ = \lambda dL$$

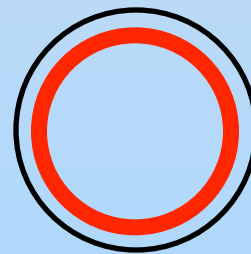
$$dQ = \sigma dA$$

$$dQ = \rho dV$$

Don't forget your geometry:

 $dL = dx$

 $dL = R d\theta$



$$dA = 2\pi r dr$$

$$dV_{cyl} = 2\pi r l dr$$

$$dV_{sphere} = 4\pi r^2 dr$$

E Field and Potential: Creating

Discrete set of point charges:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2} \hat{\mathbf{r}}; V = k_e \frac{q}{r}$$

Add up from each point charge

Continuous charge distribution:

$$d\vec{\mathbf{E}} = k_e \frac{dq}{r^2} \hat{\mathbf{r}}; dV = k_e \frac{dq}{r}$$

Break charged object into small pieces, dq , and integrate

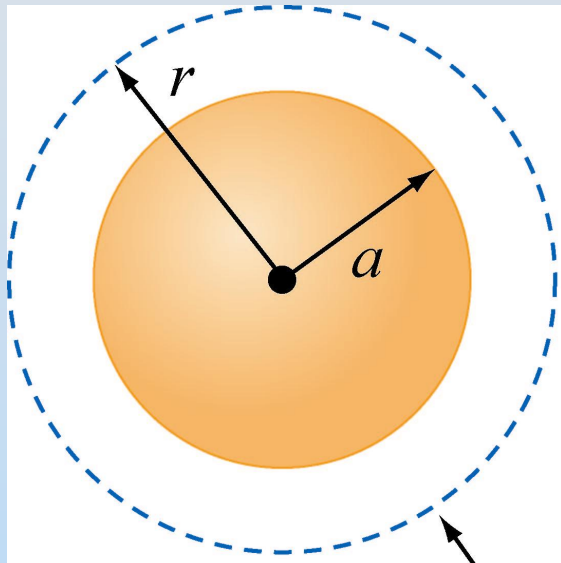
Symmetric charged object:

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\epsilon_0}; \Delta V \equiv - \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

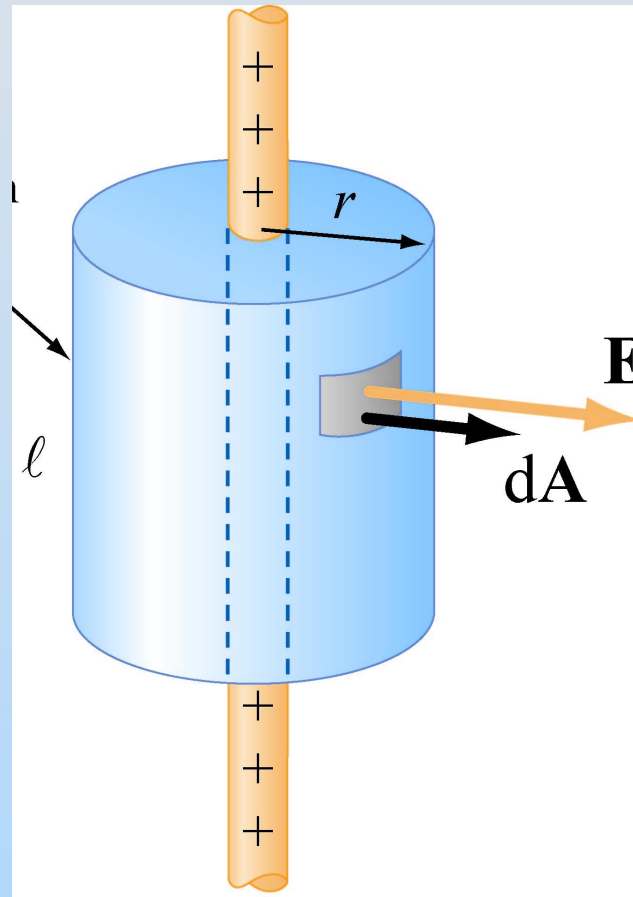
Use Gauss' law to get E everywhere, then integrate to get V

Gauss' s Law:

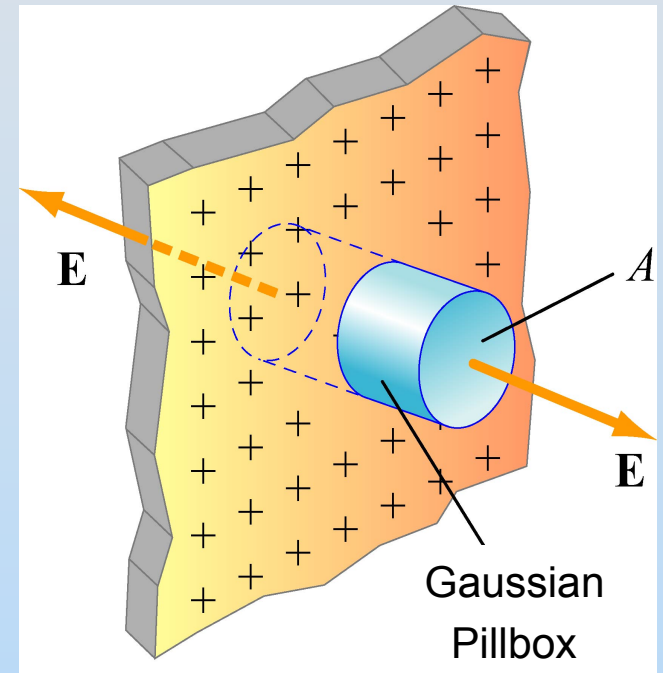
$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$



Spherical Symmetry



Cylindrical Symmetry



Planar Symmetry

E Field and Potential: Effects

If you put a charged particle, q , in a field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

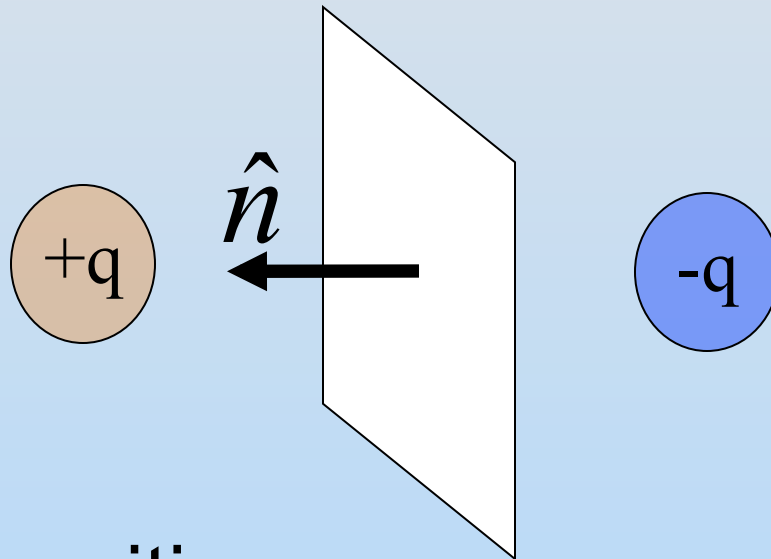
To move a charged particle, q , in a field:

$$W = \Delta U = q\Delta V$$

PRS Questions: Electric Fields and Potential

Concept Question: Flux

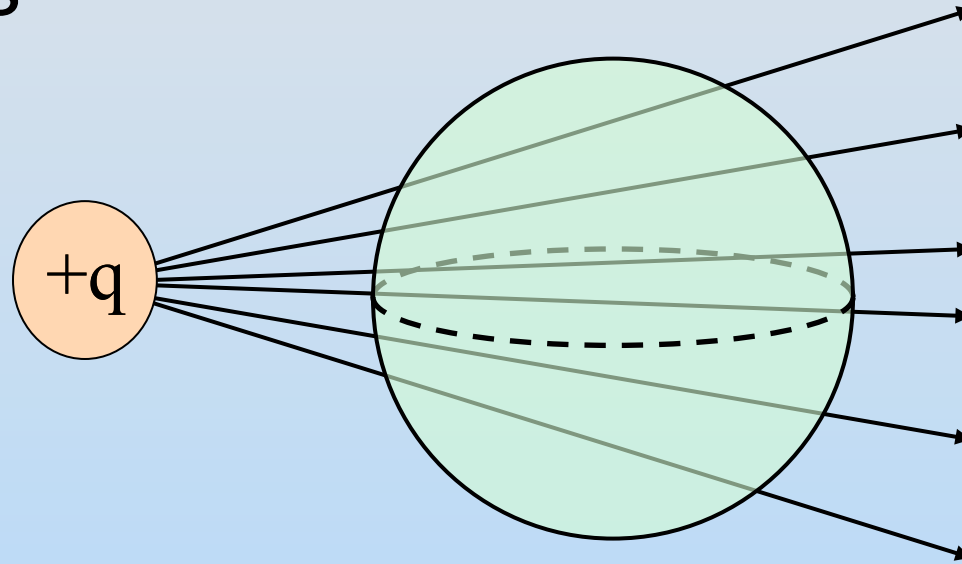
The electric flux through the planar surface below (positive unit normal to left) is:



1. positive.
2. negative.
3. zero.
4. Not well defined.

Concept Question: Flux thru Sphere

The total flux through the below spherical surface is

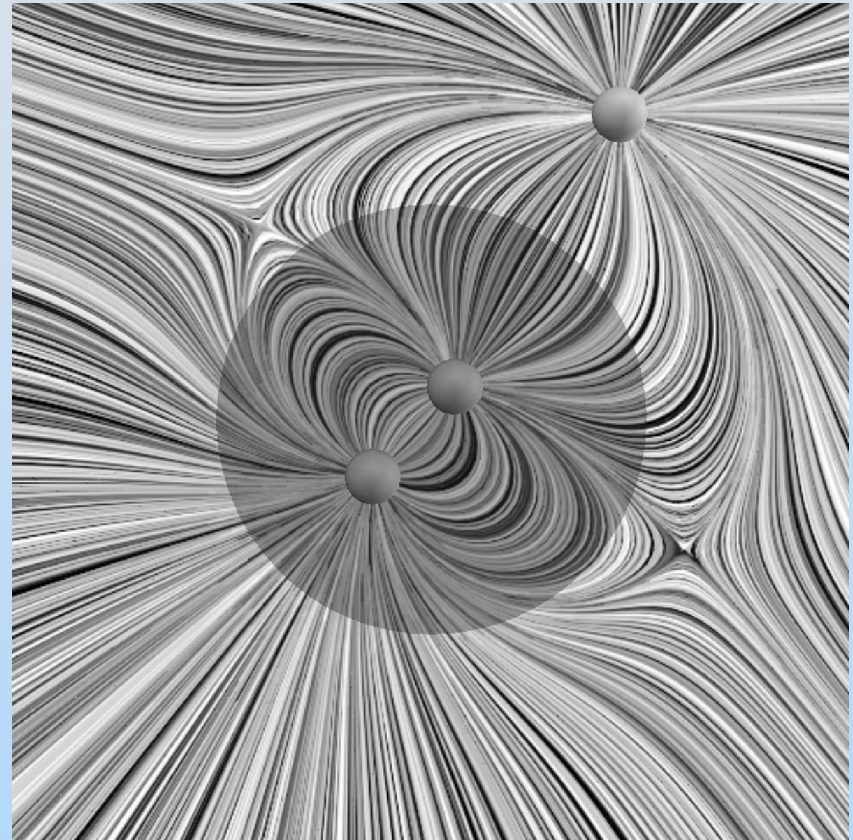


1. positive (net outward flux).
2. negative (net inward flux).
3. zero.
4. Not well defined.

Concept Question: Gauss' s Law

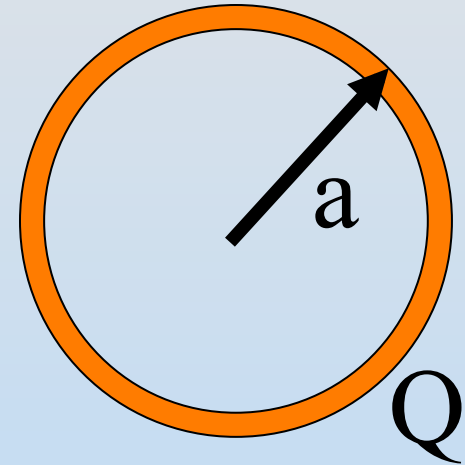
The grass seeds figure shows the electric field of three charges with charges $+1$, $+1$, and -1 . The Gaussian surface in the figure is a sphere containing two of the charges. The electric flux through the spherical Gaussian surface is

1. Positive
2. Negative
3. Zero
4. Impossible to determine without more information.



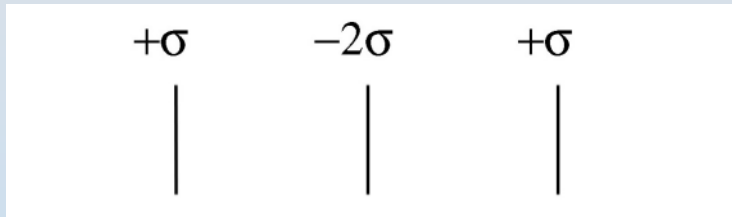
Concept Question: Spherical Shell

We just saw that in a solid sphere of charge the electric field grows linearly with distance. Inside the charged spherical shell at right ($r < a$) what does the electric field do?

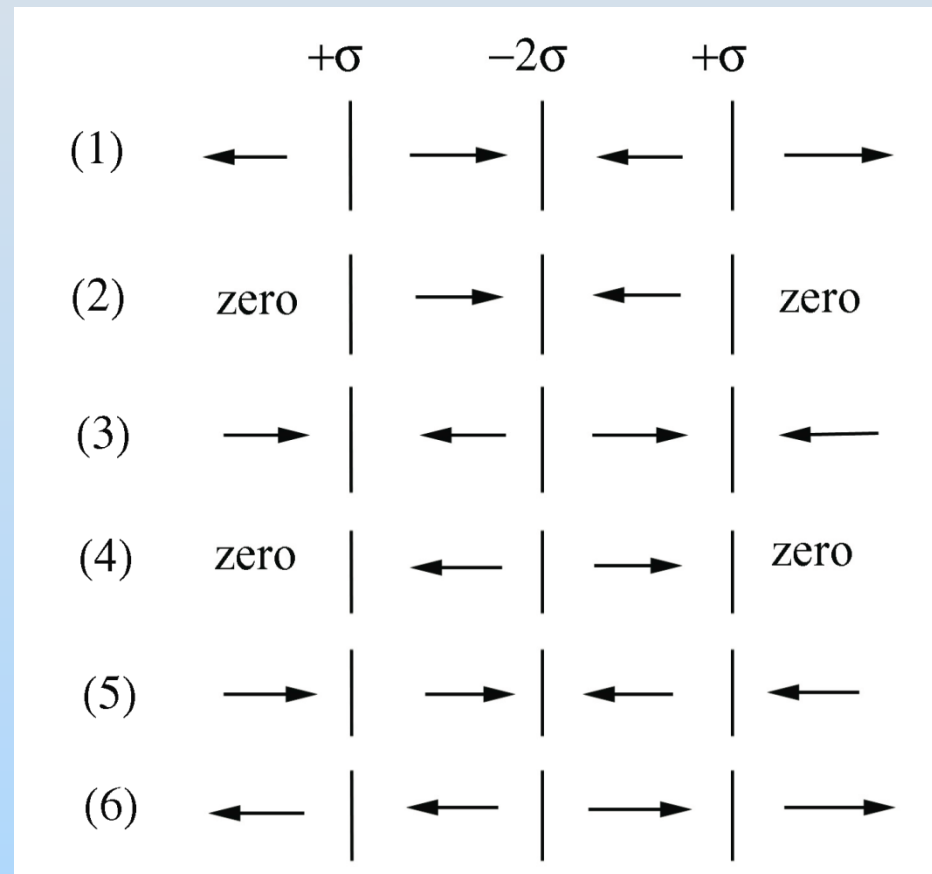


1. Zero
2. Uniform but Non-Zero
3. Still grows linearly
4. Some other functional form (use Gauss' Law)
5. Can't determine with Gauss Law

Concept Question: Superposition

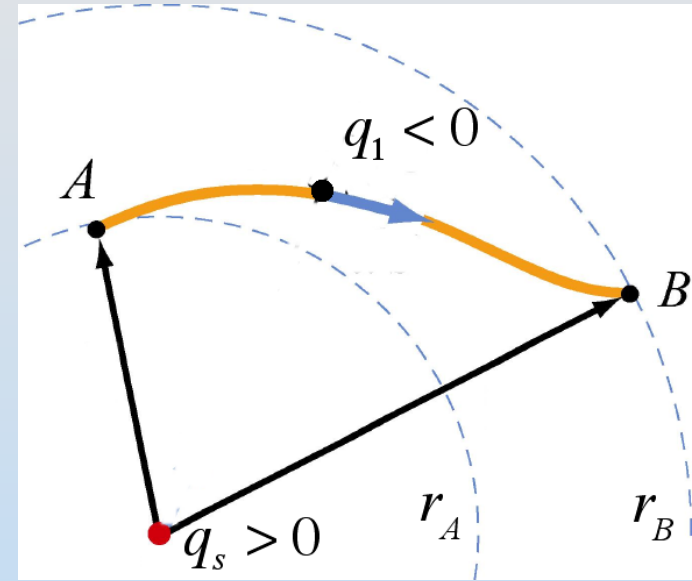


Three infinite sheets of charge are shown above. The sheet in the middle is negatively charged with charge per unit area -2σ , and the other two sheets are positively charged with charge per unit area $+\sigma$. Which set of arrows (and zeros) best describes the electric field?



Concept Question: Sign of W

Suppose a fixed positively charged object (charge $q_s > 0$) is at the origin and we move a negatively charged object (charge $q_1 < 0$) from A to B with $r_A < r_B$, where r is the distance from the origin.



1. Work done by the electrostatic force is positive and we do a positive amount of work
2. Work done by the electrostatic force is positive and we do a negative amount of work
3. Work done by the electrostatic force is negative and we do a positive amount of work
4. Work done by the electrostatic force is negative and we do a negative amount of work

Concept Question: Motion of Charged Objects

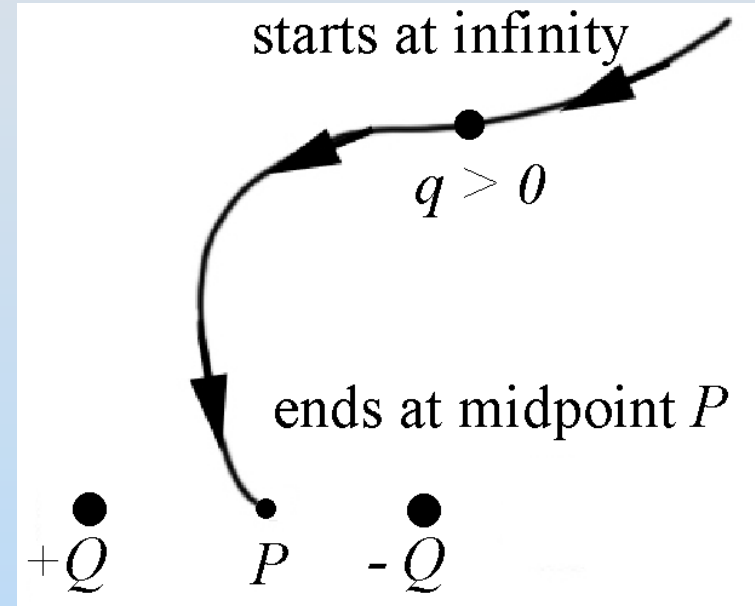
Two oppositely charged are released from rest in an electric field.

1. Both charged objects will move from lower to higher potential energy.
2. Both charged objects will move from higher to lower potential energy.
3. The positively charged object will move from higher to lower potential energy; the negatively charged object will move from lower to higher potential energy.
4. The positively charged object will move from lower to higher potential energy; the negatively charged object will move from higher to lower potential energy.

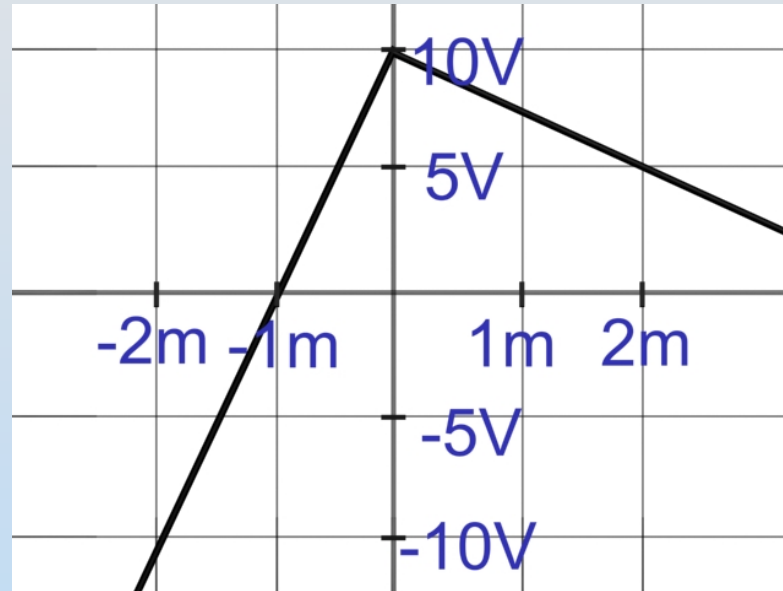
Concept Question: Two Point Charges

The work done in moving a positively charged object that starts from rest at infinity and ends at rest at the point P midway between two charges of magnitude $+Q$ and $-Q$

1. is positive.
2. is negative.
3. is zero.
4. can not be determined – not enough info is given.



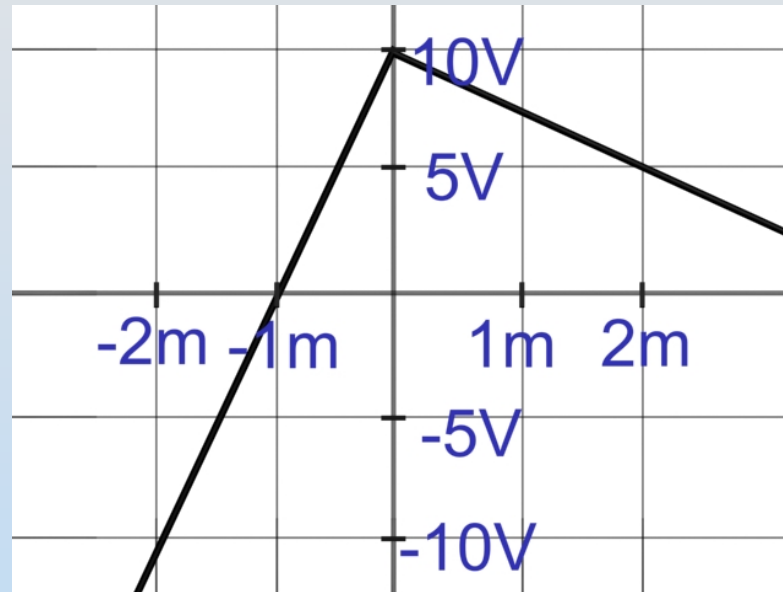
Concept Question: E from V



The graph above shows a potential V as a function of x . The *magnitude* of the electric field for $x > 0$ is

1. larger than that for $x < 0$
2. smaller than that for $x < 0$
3. equal to that for $x < 0$

Concept Question: E from V



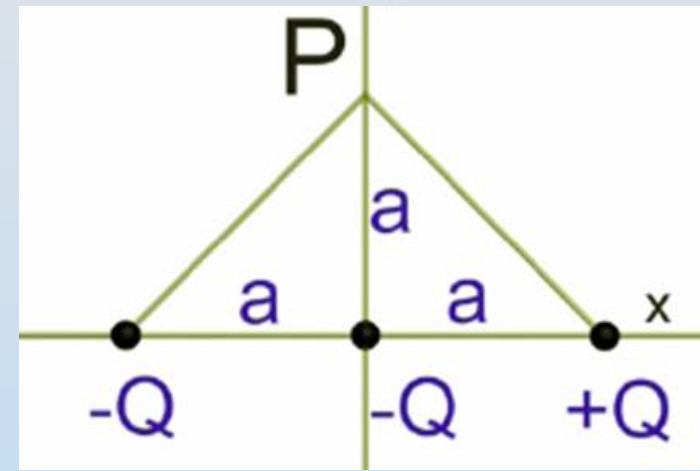
The above shows potential $V(x)$. Which is true?

1. $E_{x > 0}$ is positive and $E_{x < 0}$ is positive
2. $E_{x > 0}$ is positive and $E_{x < 0}$ is negative
3. $E_{x > 0}$ is negative and $E_{x < 0}$ is negative
4. $E_{x > 0}$ is negative and $E_{x < 0}$ is positive

Concept Question: E from V

Consider the point-like charged objects arranged in the figure below. The electric potential difference between the point P and infinity and is

$$V(P) = -kQ/a$$



From that can you derive $E(P)$?

1. Yes, its kQ/a^2 (up)
2. Yes, its kQ/a^2 (down)
3. Yes in theory, but I don't know how to take a gradient
4. No, you can't get $E(P)$ from $V(P)$

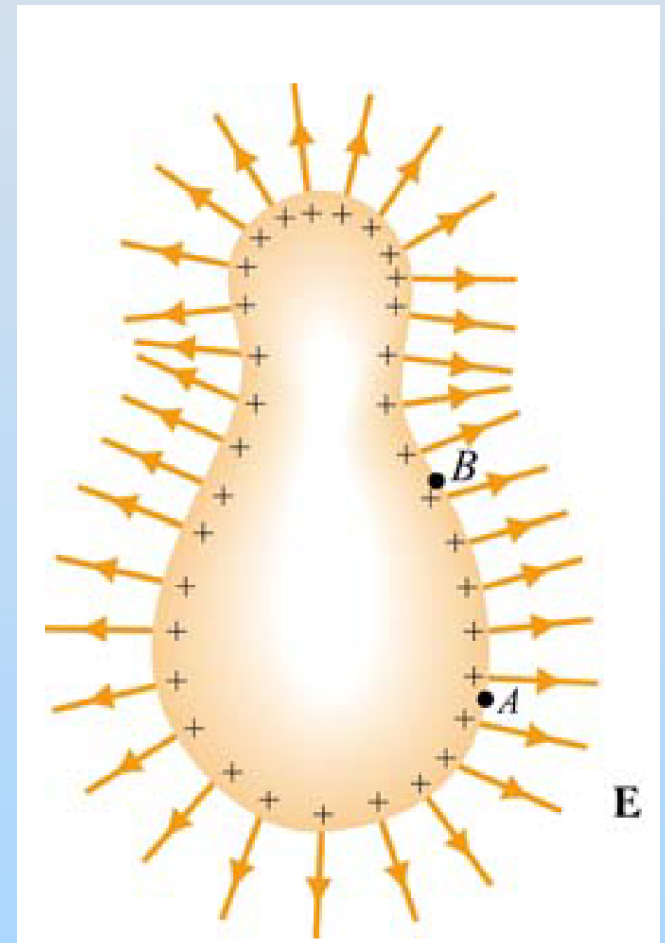
Conductors in Equilibrium

Conductors are equipotential objects:

- 1) $E = 0$ inside
- 2) E perpendicular to surface
- 3) Net charge inside is 0
- 4) Excess charge on surface

$$E = \frac{\sigma}{\epsilon_0}$$

- 5) Shielding – inside doesn't "talk" to outside

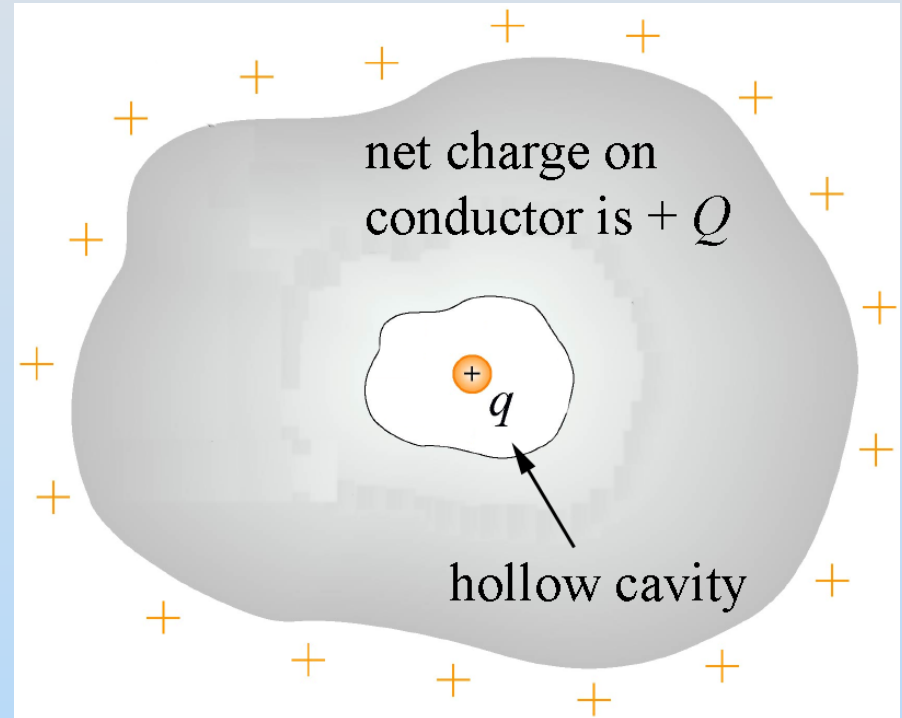


PRS Questions: Conductors

Concept Question: Point Charge in Conductor

A point charge $+q$ is placed inside a hollow cavity of a conductor that carries a net charge $+Q$. What is the total charge on the outer surface of the conductor?

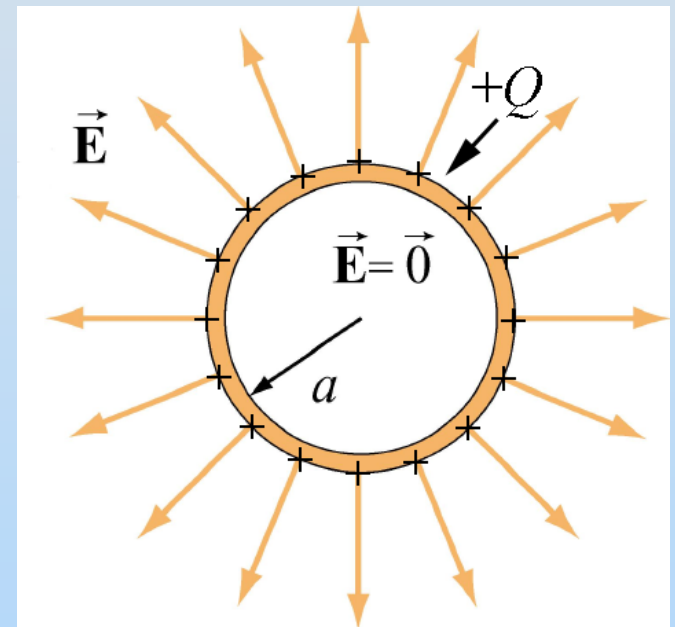
1. Q .
2. $Q + q$.
3. q .
4. $Q - q$.
5. Zero.



Concept Question: Isolated Spherical Conductor

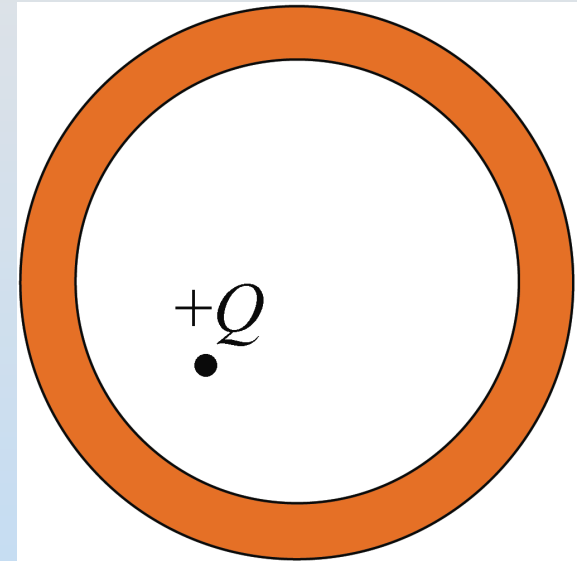
What is the capacitance of an isolated spherical conductor of radius a ?

1. Capacitance is not well defined.
2. Capacitance is $4\pi\epsilon_0 a$
3. Capacitance is infinite.
4. Capacitance is zero.



Concept Question: Point Charge in Conductor

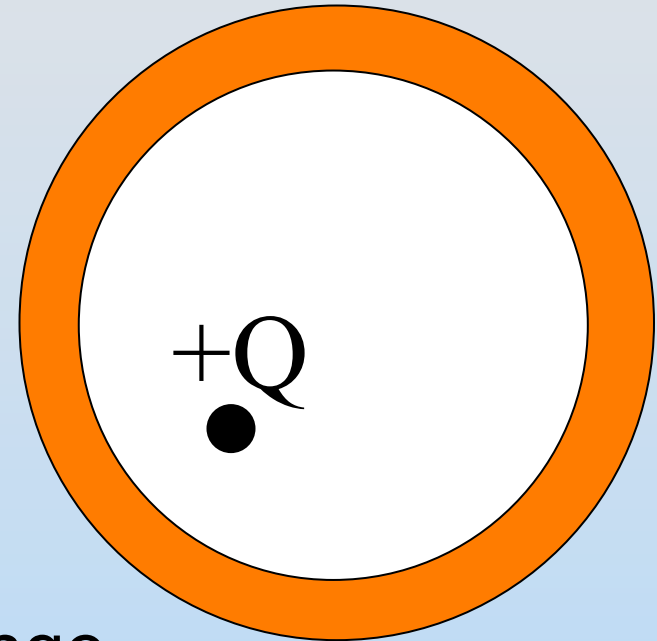
A point charge $+Q$ is placed inside a neutral, hollow, spherical conductor. As the charge is moved around *inside*, the surface charge density on the *outside*



1. is initially uniform and does not change when the charge is moved.
2. is initially uniform but does become non-uniform when the charge is moved.
3. is initially non-uniform but does not change when the charge is moved.
4. is initially non-uniform but does change when the charge is moved.

Concept Question: Point Charge in Conductor

A point charge $+Q$ is placed inside a neutral, hollow, spherical conductor. As the charge is moved around *inside*, the electric field *outside*

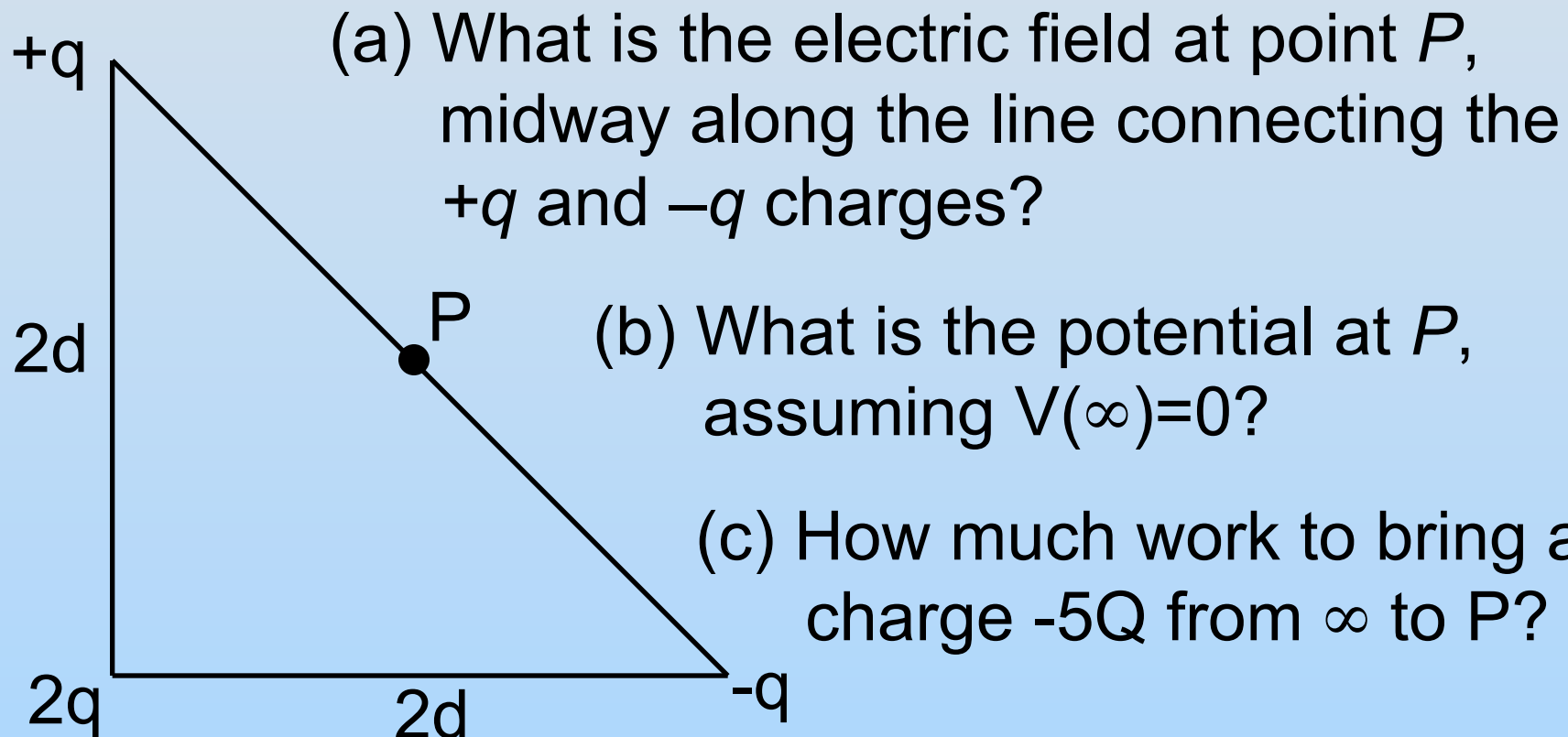


1. is zero and does not change
2. is non-zero but does not change
3. is zero when centered but changes
4. is non-zero and changes

SAMPLE EXAM:

Q: Point Charges

A right isosceles triangle of side $2d$ has charges q , $+2q$ and $-q$ arranged on its vertices (see sketch).



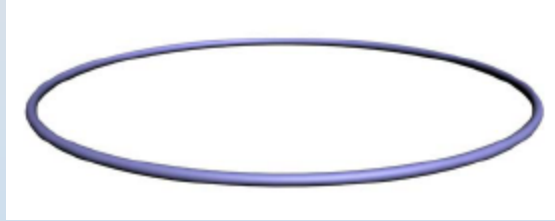
(a) What is the electric field at point P , midway along the line connecting the $+q$ and $-q$ charges?

(b) What is the potential at P , assuming $V(\infty)=0$?

(c) How much work to bring a charge $-5Q$ from ∞ to P ?

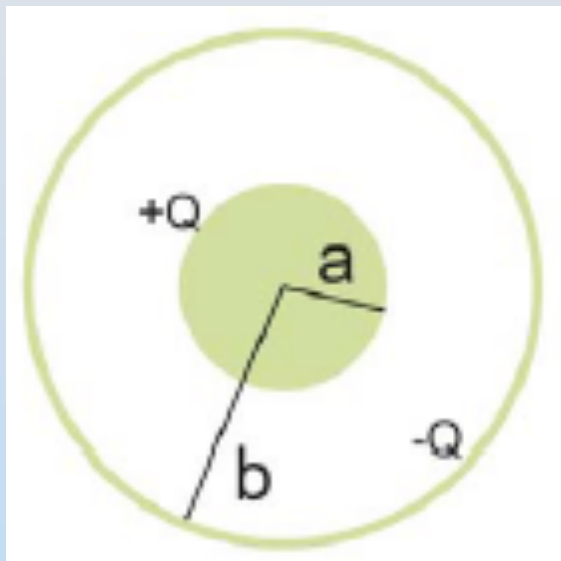
Q: Ring of Charge

A thin rod with a uniform charge per unit length λ is bent into the shape of a circle of radius R



- Choose a coordinate system for the rod. Clearly indicate your choice of origin, and axes on the diagram above.
- Choose an infinitesimal charge element dq . Find an expression relating dq , λ , and your choice of length for dq .
- Find the vector components for the contribution of dq to the electric field along an axis perpendicular to the plane of the circle, a distance d above the plane of the circle. The axis passes through the center of the circle. Express the vector components in terms of your choice of unit vectors.
- What is the direction and magnitude of the electric field along the axis that passes through the center of the circle, perpendicular to the plane of the circle, and a distance d above the plane of the circle.
- What is the potential at that point, assuming $V(\infty)=0$?

Q: Spherical Capacitor



A conducting solid sphere of radius a , carrying a charge $+Q$ is surrounded by a thin conducting spherical shell (inner radius b) with charge $-Q$.

a) What is the direction and magnitude of the electric field \mathbf{E} in the three regions below. Show how you obtain your expressions.

1. $r < a$

2. $a < r < b$

3. $r > b$

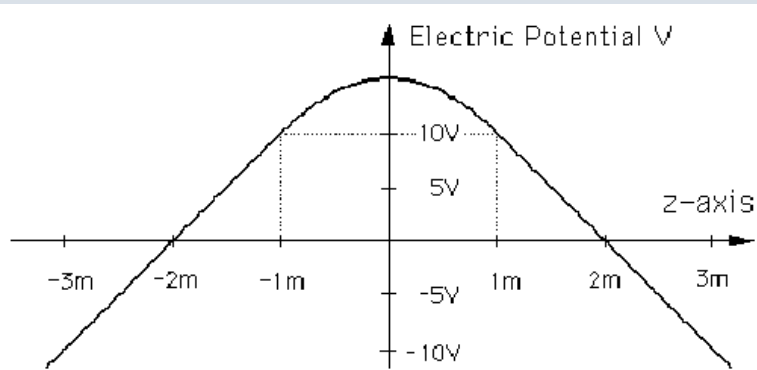
b) What is the electric potential $V(r)$ in these same three regions. Take the electric potential to be zero at ∞ .

c) What is the electric potential difference between the outer shell and the inner sphere, $\Delta V = V(b) - V(a)$?

d) What is the capacitance of this spherical capacitor?

e) If a positive charge $+2Q$ is placed anywhere on the inner sphere of radius a , what charge appears *on the outside surface* of the thin spherical shell of inner radius b ?

Q: Find E from V



The graph shows the variation of an electric potential V with distance z . The potential V does not depend on x or y . The potential V in the region $-1 \text{ m} < z < 1 \text{ m}$ is given in Volts by the expression $V(z) = 15 - 5z^2$. Outside of this region, the electric potential varies linearly with z , as indicated in the graph.

- Find an equation for the z -component of the electric field, E_z , in the region $-1 \text{ m} < z < 1 \text{ m}$.
- What is E_z in the region $z > 1 \text{ m}$? Be careful to indicate the sign
- What is E_z in the region $z < -1 \text{ m}$? Be careful to indicate the sign
- This potential is due a slab of charge with constant charge per unit volume ρ_0 . Where is this slab of charge located (give the z -coordinates that bound the slab)? What is the charge density ρ_0 of the slab in C/m^3 ? Be sure to give clearly both the sign and magnitude of ρ_0 .