EEE 3352 - Transformers

Sample solutions to examples:

Example 1:

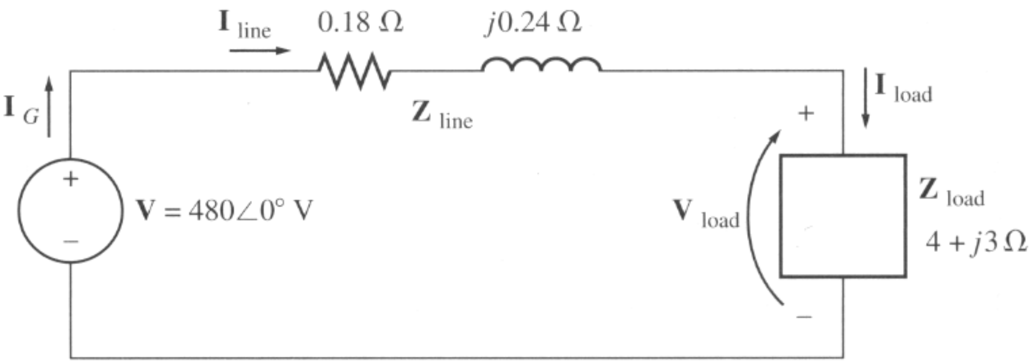
From the emf equation of a transformer, the number of turns that the primary winding must have is

$$N_1 = \frac{V_1}{4.44 f \varphi_m}$$
$$= \frac{240V}{4.44(60Hz)(5X10^{-3}Wb)} = 180\,turns$$

and the number of turns that the secondary winding must have is

$$N_2 = \frac{V_2}{4.44 f \varphi_m}$$
$$= \frac{120V}{4.44(60Hz)(5X10^{-3}Wb)} = 90\,turns$$

Example 2A:



Load current = line current

$$\mathbf{I}_G = \mathbf{I}_{line} = \mathbf{I}_{load} = \frac{\mathbf{V}}{\mathbf{Z}_{line} + \mathbf{Z}_{load}}$$

$$= \frac{480\angle 0^\circ}{0.18 + j0.24 + 4 + j3}$$

$$= \frac{480\angle 0^\circ}{5.29\angle 37.8^\circ} = 90.8\angle -37.8^\circ\ A$$

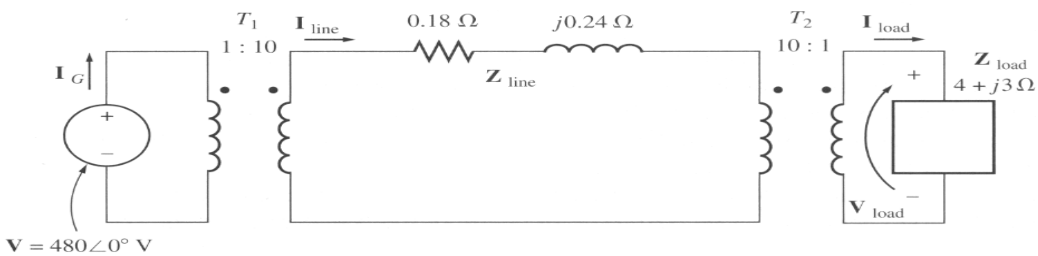
Load Voltage:

$$\mathbf{V}_{load} = \mathbf{I}_{load}\mathbf{Z}_{load} = (90.8\angle -37.8^\circ)(4 + j3) = (90.8\angle -37.8^\circ)(5\angle 36.9^\circ) = 454\angle -0.9^\circ\, V$$

Power losses:

$$P_{loss} = I_{line}^2 R_{line} = 90.8^2 \cdot 0.18 = 1484\ W$$

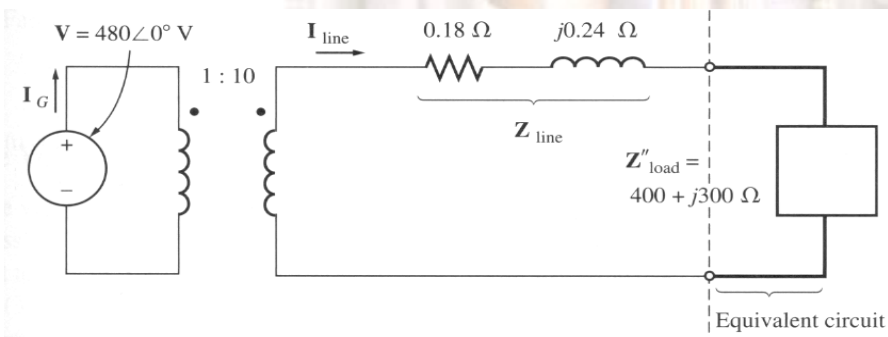
Example 2B:



Eliminating transformer T2 by referring the load over to the transmission line's voltage level

The load impedance when referred to the transmission line (while the transformer T_2 is eliminated) is:

$$\mathbf{Z}'_{load} = a_2^2 \mathbf{Z}_{load} = \left(\frac{10}{1}\right)^2 (4 + j3) = 400 + j300$$



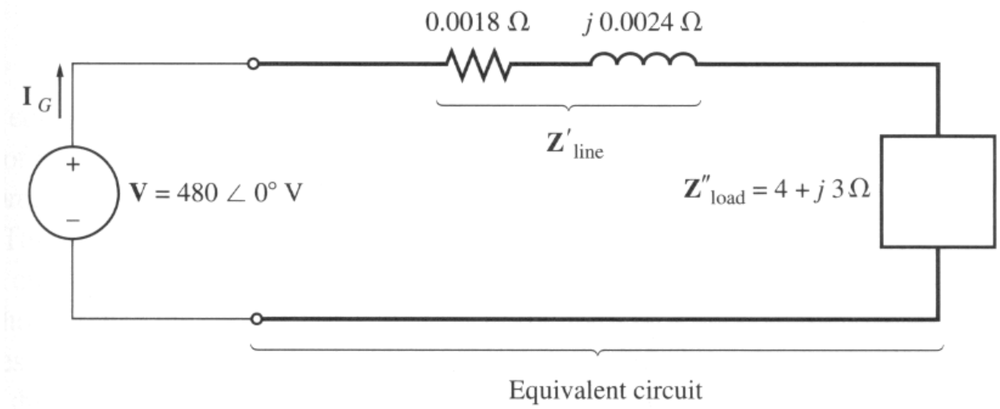
The total impedance on the transmission line level is

$$\begin{aligned} \mathbf{Z}_{eq} &= \mathbf{Z}_{line} + \mathbf{Z}'_{load} \\ &= 400.18 + j300.24 \\ &= 500.3\angle 36.88^\circ \ \Omega \end{aligned}$$

Eliminating transformer T_1 by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side

The total impedance is now referred across T_1 to the source's voltage level:

$$\mathbf{Z}'_{eq} = a_1^2 \mathbf{Z}_{eq} = \left(\frac{1}{10}\right)^2 (500.3\angle 36.88^\circ) = 5.003\angle 36.88^\circ \ \Omega$$



The generator's current is

$$\mathbf{I}_G = \frac{\mathbf{V}}{\mathbf{Z}'_{eq}} = \frac{480\angle 0^\circ}{5.003\angle 36.88^\circ}$$
$$= 95.94\angle -36.88^\circ \ A$$

Knowing transformers' turn ratios, we can determine line and load currents:

$$\mathbf{I}_{line} = a_1 \mathbf{I}_G = 0.1 \cdot (95.94\angle -36.88^\circ) = 9.594\angle -36.88^\circ \ A$$

$$\mathbf{I}_{load} = a_2 \mathbf{I}_{line} = 10 \cdot (9.594\angle -36.88^\circ) = 95.94\angle -36.88^\circ \ A$$

Therefore, the load voltage is:

$$\mathbf{V}_{load} = \mathbf{I}_{load} \mathbf{Z}_{load} = (95.94\angle -36.88^\circ)(5\angle -36.87^\circ) = 479.7\angle -0.01^\circ \ V$$

Power losses:

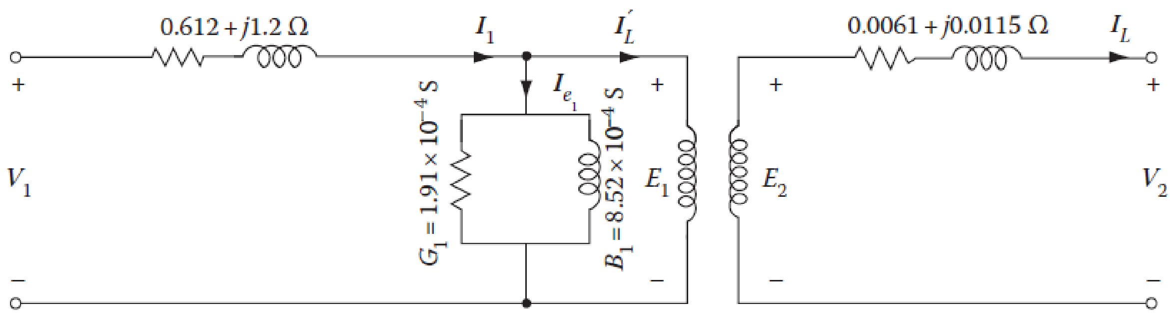
$$P_{loss} = I_{line}^2 R_{line} = 9.594^2 \cdot 0.18 = 16.7 \ W$$

Note: transmission line losses are reduced by a factor nearly 90, the load voltage is much closer to the generator's voltage – effects of increasing the line's voltage.

Example 3:

(a) The following circuit shows the ***equivalent circuit*** of the transformer, with the excitation admittance referred to the primary side. Therefore,

$$Y_{e1} = \frac{Y_{e2}}{a^2} = \frac{0.0191 - j0.0852\,S}{10^2} = 1.91X10^{-4} - j8.52X10^{-4}\ S$$



(b) Here, since the transformer delivers a rated load current (0.9 lagging pf) at rated V2 voltage;

We usually find rated values of a transformer from the kVA rating. Therefore,

$$S = I_2 V_2$$

$$I_L = I_2 = \frac{S}{V_2} = \frac{75,000VA}{240V} = 312.5\ A$$

$$@0.9\ pf\ \Rightarrow I_2 \text{ is } \textit{lagging} \text{ the voltage by } \cos^{-1}0.9 = -25.84^0$$

$$\therefore I_2 = 312.5\angle -25.84^0 = 312.5(0.9\text{ - j}0.4359) = 281.25\text{ - j}136.2156\text{ A}$$

Hence;

$$E_2 = V_2 + I_L\,(R_2 + jX_2\,) = 240\angle 0° + (281.25\text{ - j}136.2156)(0.0061 + j0.0115)$$
$$= 243.294\angle 0.57°V$$

$$E_1 = aE_2 = 10\left(243.294\angle 0.57°\right)V = 2432.94\angle 0.57°\,V$$

Therefore, the load current referred to the primary side is

$$I'_2 = I'_L = \frac{I_2}{a} = \frac{281.25 - j136.2156 \text{ A}}{10} = 28.125 - j13.62156\, A$$

Notice that

$$I_1 = I'_L + I_{e1}$$

$$\begin{aligned} I_{e1} = E_1 Y_{e1} &= \left(2432.94\angle 0.57^\circ \text{ V}\right) 1.91X10^{-4} - j8.52X10^{-4}\ S \\ &= 0.4851 - j2.0682\ A \end{aligned}$$

So that;

$$\begin{aligned} I_1 &= I'_L + I_{e1} \\ &= 28.125 - j13.62156 + 0.4851 - j2.0682\ A \\ &= 32.6299\angle \text{ -28.74}^\circ \text{ A} \end{aligned}$$

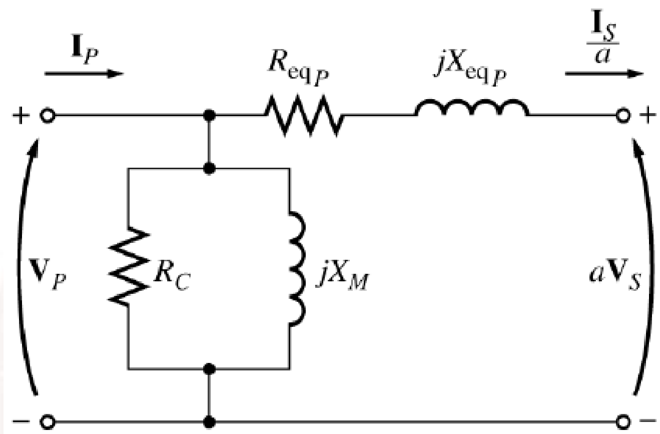
Therefore;

$$\begin{aligned} \text{V}_1 &= E_1 + I_1\left(R_1 + jX_1\right) \\ &= 2432.94\angle 0.57^\circ + \left(32.6299\angle \text{ -28.74}^\circ\ \right)\left(0.612 + j1.2\right) \\ &= 2469.6396\angle 1.13^\circ \text{ V} \end{aligned}$$

Tutorial Solutions

Q1:

The equivalent circuit of this transformer is shown below. (Since no particular equivalent circuit was specified, we are using the approximate equivalent circuit referred to the primary side.)

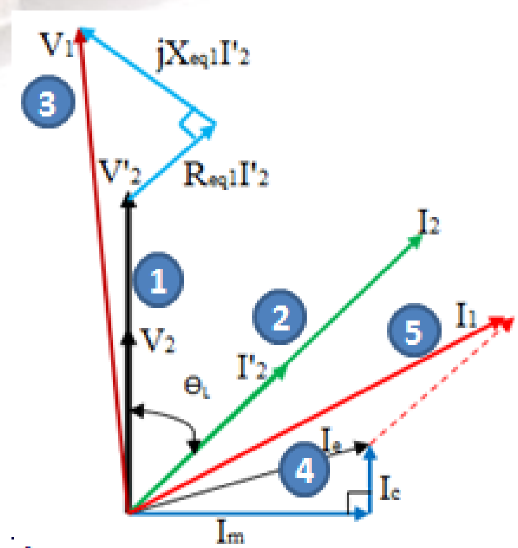


Where;

$$R_{eq\,p} = R_p + a^2 R_s$$

$$X_{eq\,p} = X_p + a^2 X_s$$

Following the steps outlined in the phasor diagram;



Step 1: The secondary voltage

$$V_s = \frac{282.8}{\sqrt{2}}\angle 0^0 = 200\angle 0^0 V$$

The secondary voltage referred to the primary side is

$$V'_s = aV_s = 0.25\left(200\angle 0^0\right) = 50\angle 0^0\ V$$

Step 2: The secondary current

$$I_s = \frac{7.07}{\sqrt{2}}\angle -36.87^0 = 5\angle -36.87^0\ A$$

The secondary current referred to the primary side is

$$I'_s = \frac{I_s}{a} = \frac{5\angle -36.87^0}{0.25} = 20\angle -36.87^0\ A$$

Step 3: The primary circuit voltage is given by

$$V_1 = V'_s + I'_s\left(R_{eq\,p} + jX_{eq\,p}\right)$$
$$= 50\angle 0^0 + \left(20\angle -36.87^0\right)\left(0.05 + j0.225\right) = 53.6\angle 3.2^0\,V$$

Step 4: The excitation current is given by

$$I_e = I_c + I_m = \frac{V_1}{R_c} + \frac{V_1}{jX_m} = \frac{53.6\angle 3.2^0}{75} + \frac{53.6\angle 3.2^0}{j20}$$
$$= 0.7145\angle 3.2^0 + 2.679\angle \text{-}86.8^0$$
$$= 2.77\angle -71.9^0\ A$$

Step 5: Therefore, the total primary current of this transformer is

$$I_p = I_e + I'_s = 2.77\angle -71.9^0 + 20\angle -36.87^0 = 22.3\angle -41.0^0\ A$$

Note: The primary current is lagging the applied primary voltage by

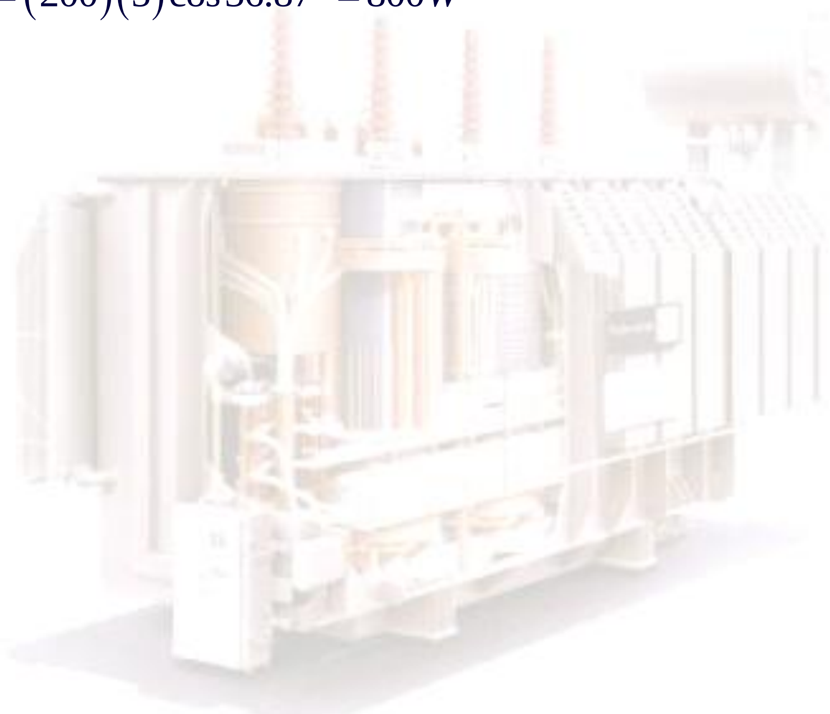
$3.2^0-(-41.0^0)=44.2^0$

Therefore, input power factor, pf = $\cos 44.2^0$

$$P_{in} = V_p I_p \cos\theta_p = (53.6)(22.3)\cos 44.2^0 = 857W$$

And

$$P_{out} = V_s I_s \cos\theta_s = (200)(5)\cos 36.87^0 = 800W$$



Q2.

	Open-Circuit Test (on Primary)	Short-Circuit Test (on Primary)
Voltmeter	$V_{oc} = 7500\,\text{V}$	$V_{sc} = 366\,\text{V}$
Ammeter	$I_{oc} = 0.2006\,\text{A}$	$I_{sc} = 2\,\text{A}$
Wattmeter	$P_{oc} = 180\,\text{W}$	$P_{sc} = 300\,\text{W}$

The power factor during the open-circuit test is

$$PF = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}} = \frac{180W}{(7500V)(0.2006A)} = 0.1196 \;\; lagging$$

The excitation admittance is

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1} PF = \frac{0.2006}{7500} \angle -\cos^{-1} 0.1196 = 0.0000032 - j0.0000265 = \frac{1}{R_C} - j\frac{1}{X_M}$$

Therefore: $R_C = \dfrac{1}{0.0000032} \cong 312.5\,k\Omega; \quad X_M = \dfrac{1}{0.0000265} \cong 37.3k\Omega$

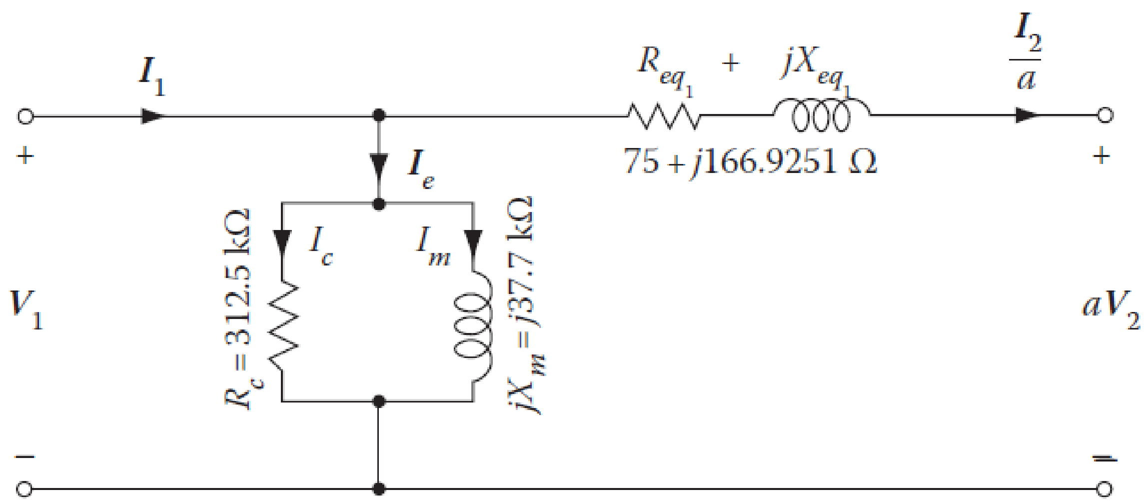
The power factor during the short-circuit test is

$$PF = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}} = \frac{300W}{(366V)(2A)} = 0.41 \;\; lagging$$

The series (i.e. equivalent) impedance is given by

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \cos^{-1} PF = \frac{366}{2} \angle 65.81^\circ$$
$$= 75 + j166.93\,\Omega$$

Therefore: $R_{eq} = 75\,\Omega; \quad X_{eq} = 166.93\Omega$



The approximate equivalent circuit

Q3.

$$\%Voltage\ Regulation = \frac{I_2 Z_{eq2}\cos(\phi-\theta)}{V_{2(full\ load)}}\cdot 100\%$$

The full-load current on the secondary side of the transformer is

$$I_{2,rated} = \frac{S_{rated}}{V_{2,rated}} = \frac{75\,000\,VA}{240\,V} = 312.5\,A$$

$$Z_{eq2} = \sqrt{R^2_{eq2} + X^2_{eq2}} = \sqrt{0.009318^2 + 0.058462^2} = 0.059195\Omega$$

$$\phi = \tan^{-1}\left(\frac{0.058462}{0.009318}\right) = 80.9^0$$

Therefore;

(a) At 0.85 lagging power factor,

$$\theta = \cos^{-1} pf = \cos^{-1}(0.85) = 31.79^0$$

$$\%Voltage\ Regulation = \frac{I_2 Z_{eq2} \cos(\phi - \theta)}{V_{2(full\ load)}} \cdot 100\%$$

$$= \frac{(312.5\ A)(0.059195\ \Omega)\cos(80.9^0 - 31.79^0)}{240\ V} \cdot 100\%$$

$$\cong 5\%$$

Since also,

$$\%Voltage\ Regulation, VR = \frac{V_{2(no\ load)} - V_{2(full\ load)}}{V_{2(full\ load)}} \cdot 100\% = 5\% = 0.05$$

Then

$$V_{2(no\ load)} = (VR)\left(V_{2(full\ load)}\right) + V_{2(full\ load)} = (0.05)(240V) + 240V = 252V$$

The meaning of 5% voltage regulation is that if the load is thrown off, the load terminal voltage will rise from 240 to 252 volts. In other words, when a full load at 0.85 lagging power factor is connected to the load terminals of the transformer, the voltage drops from 252 to 240 volts.

(b) At unity power factor,

$$pf = 1,\ thus,\ \theta = 0^0$$

$$\%Voltage\ Regulation = \frac{I_2 Z_{eq2} \cos(\phi - \theta)}{V_{2(full\ load)}} \cdot 100\%$$

$$= \frac{(312.5\ A)(0.059195\ \Omega)\cos(80.9^0)}{240\ V} \cdot 100\%$$

$$\cong 1.2\%$$

(c) At 0.85 leading power factor,

$$\theta = \cos^{-1} pf = \cos^{-1}(0.85) = 31.79^0$$

$$\begin{aligned}
\%Voltage\ Regulation &= \frac{I_2 Z_{eq2} \cos(\phi - \theta)}{V_{2(full\ load)}} \cdot 100\% \\
&= \frac{(312.5\ A)(0.059195\ \Omega)\cos(80.9^0 + 31.79^0)}{240\ V} \cdot 100\% \\
&\cong -2.97\%
\end{aligned}$$

Since also,

$$\%Voltage\ Regulation, VR = \frac{V_{2(no\ load)} - V_{2(full\ load)}}{V_{2(full\ load)}} \cdot 100\% = 5\% = -0.0297$$

Then

$$V_{2(no\ load)} = (VR)\left(V_{2(full\ load)}\right) + V_{2(full\ load)} = (-0.0297)(240V) + 240V = 232.86V$$

Note that the voltage regulation for this leading power factor load is negative.

The meaning of -2.97% voltage regulation is that if the load is thrown off, the load terminal voltage will decrease from 240 to 232.86 volts. Put differently, if a leading power factor load (0.85 pf) is connected to the load terminals of the transformer, the voltage increases from 232.86 to 240 volts.