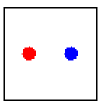
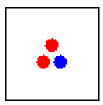
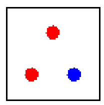
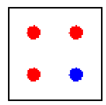
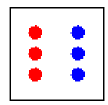
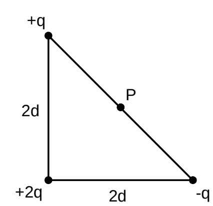
ELECTRIC FIELDS

1. Physlet-problem 22.7: An unknown green charge (with a #1 on it) is shot into a region containing four fixed charges (numbered 2--5), one of which is known to be positive (the red one). You can measure position using a mouse down (position is given in meters and time is given in seconds).
   Determine the signs of the unknown charges. You may consider neutral as a possible answer. Justify your response.
   Answer:
   charge 1 : + charge 2: + charge 3 : neutral charge 4 : -

2. Physlet-problem 23.3: Five animations show the electric field produced by a configuration of hidden charges. The arrows represent the direction of the electric field, and the color represents the intensity of the field.
   Which electric field would be produced by the charge configuration shown below? Red represents a charge of +Q and blue represents a charge of -Q.

   a.  b.  c.  d.  e. 

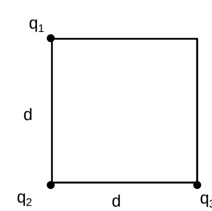
   Answer:
   Charge configuration a is E-Field IV
   Charge configuration b is E-Field II
   Charge configuration c is E-Field I
   Charge configuration d is E-Field III
   Charge configuration e is E-Field V



3. A right isosceles triangle of side 2d has charges q, +2q and -q arranged on its vertices (see sketch). What is the electric field at point P, midway along the line connecting the +q and –q charges?

   Solution:
   Note that all charges are at a distance $r = \sqrt{2}\,d$ from point P.

$$\vec{E}_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}(\hat{x} - \hat{y})\frac{1}{\sqrt{2}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2d^2}(\hat{x} - \hat{y})\frac{1}{\sqrt{2}}$$
$$\vec{E}_{2q} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}(\hat{x} + \hat{y})\frac{1}{\sqrt{2}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2d^2}(2\hat{x} + 2\hat{y})\frac{1}{\sqrt{2}}$$
$$\vec{E}_{-q} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}(-\hat{x} + \hat{y})\frac{1}{\sqrt{2}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2d^2}(+\hat{x} - \hat{y})\frac{1}{\sqrt{2}}$$
$$\vec{E} = \vec{E}_{+q} + \vec{E}_{2q} + \vec{E}_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{2\sqrt{2}d^2}(\hat{x} - \hat{y} + 2\hat{x} + 2\hat{y} + \hat{x} - \hat{y}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{2\sqrt{2}d^2}4\hat{x} = \frac{1}{2\pi\varepsilon_0}\frac{q}{\sqrt{2}d^2}\hat{x}$$



4. Charges q_1, q_2 and q_3 are placed at the corners of a square with sides of length d.
   a. Find an expression for the electric field components at the 4th corner (diagonally opposite the q_2 charge).
   b. Calculate the electric field for q_1=+20nC, q_2=-40nC, and q_3=+30 nC and d=2 m

   Solution:

$$\vec{E}_{q_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{d^2}\hat{x}$$
$$\vec{E}_{q_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(\sqrt{2}d\right)^2}(\hat{x} + \hat{y})\frac{1}{\sqrt{2}} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\sqrt{2}2d^2}(\hat{x} + \hat{y})$$
$$\vec{E}_{q_3} = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{d^2}\hat{y}$$

$$\vec{E} = \vec{E}_{q_1} + \vec{E}_{q_2} + \vec{E}_{q_3} = \frac{1}{4\pi\varepsilon_0}\frac{1}{d^2}\left(q_1\hat{x} + \frac{q_2}{2\sqrt{2}}(\hat{x}+\hat{y}) + q_3\hat{y}\right) = \frac{1}{4\pi\varepsilon_0}\frac{1}{d^2}\left((q_1 + \frac{q_2}{2\sqrt{2}})\hat{x} + (q_3 + \frac{q_2}{2\sqrt{2}})\hat{y}\right)$$

b) just use the numbers:

$$\vec{E} = 8.988\cdot 10^9 \frac{1}{4}\left(\left(20\cdot 10^{-9} - \frac{1}{\sqrt{2}}20\cdot 10^{-9}\right)\hat{x} + \left(30\cdot 10^{-9} - \frac{1}{\sqrt{2}}20\cdot 10^{-9}\right)\hat{y}\right)$$

$$= 8.988\frac{1}{4}\left(\left(20 - \frac{1}{\sqrt{2}}20\right)\hat{x} + \left(30 - \frac{1}{\sqrt{2}}20\right)\hat{y}\right) = 2.247(5.86\hat{x} + 15.86\hat{y}) = 13.2\hat{x} + 35.6\hat{y}$$

5. Starting from the expression $\vec{E}_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}(3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p})$ of the electric field of an electric dipole derive an expression for the magnitude of the electric field
   a. for a point on the axis of the electric dipole and
   b. for a point in the plane perpendicular to the axis of the electric dipole
   c. Sketch the electric field lines.

Solution:

a) For a point on the axis of the electric dipole $\hat{r}$ and $\vec{p}$ are either parallel or antiparallel thus we have for the magnitude of the electric field:
$E_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}(3p - p) = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ which is the same expression as derived in the lecture.

b) For a point in the plane perpendicular to the axis of the electric dipole $\hat{r}$ and $\vec{p}$ are perpendicular to each other and thus $\vec{p}\cdot\hat{r} = 0$, so the electric field is:
$\vec{E}_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{-\vec{p}}{r^3}$ so the electric field is oriented opposite to the electric dipole moment $\vec{p}$ in this plane.

c)

6. A rigid dipole with a dipole moment $\vec{p} = p\hat{z}$ is located at the origin. A second dipole is placed at point $P = (x_0, 0, z_0)$. The second dipole is free to rotate, find an expression for the equilibrium orientation of the second dipole using x_0 and z_0.

Solution:

In the lecture we learned that an electric dipole will align itself with the electric field, so all we need to do is to find an expression for the electric field at point P:

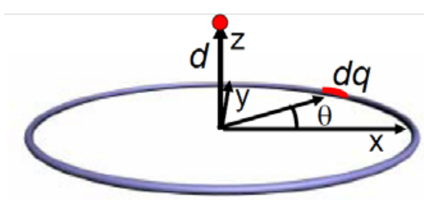
Use $\vec{p} = p\hat{z}$ and $\vec{r} = x_0\hat{x} + z_0\hat{z}$ in the equation for the electric field of a dipole:

$\vec{E}_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left(\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right)$ where $r = \sqrt{x_0^2 + z_0^2}$:

$$\vec{E}_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left(\frac{3(pz_0)(x_0\hat{x} + z_0\hat{z})}{r^2} - p\hat{z}\right)$$

$$\vec{E}_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\left(\frac{3z_0x_0\hat{x} + 3z_0x_0z_0\hat{z}}{r^2} - \hat{z}\right)$$

7. A thin rod with a uniform charge λ per unit length is bent into a circle with radius R. What is the direction and magnitude of the electric field along the axis that passes through the center of the circle perpendicular to the plane of the circle at a distance d above the plane of the circle?



Solution:

Because of the symmetry of the problem the electric field will only have a component in the z-direction:

$$E_z = \int dE_z = \int \frac{1}{4\pi\varepsilon_0}\frac{1}{\mathrm{r}^2}\frac{\mathrm{d}}{\mathrm{r}}\mathrm{dq} = \frac{1}{4\pi\varepsilon_0}\frac{d}{r^3}\int_0^{2\pi}\lambda R d\theta = \frac{\lambda R}{2\varepsilon_0}\frac{d}{r^3}$$

$$r = \sqrt{R^2 + d^2}$$