

EEE 3352

Electromechanics & Electrical Machines



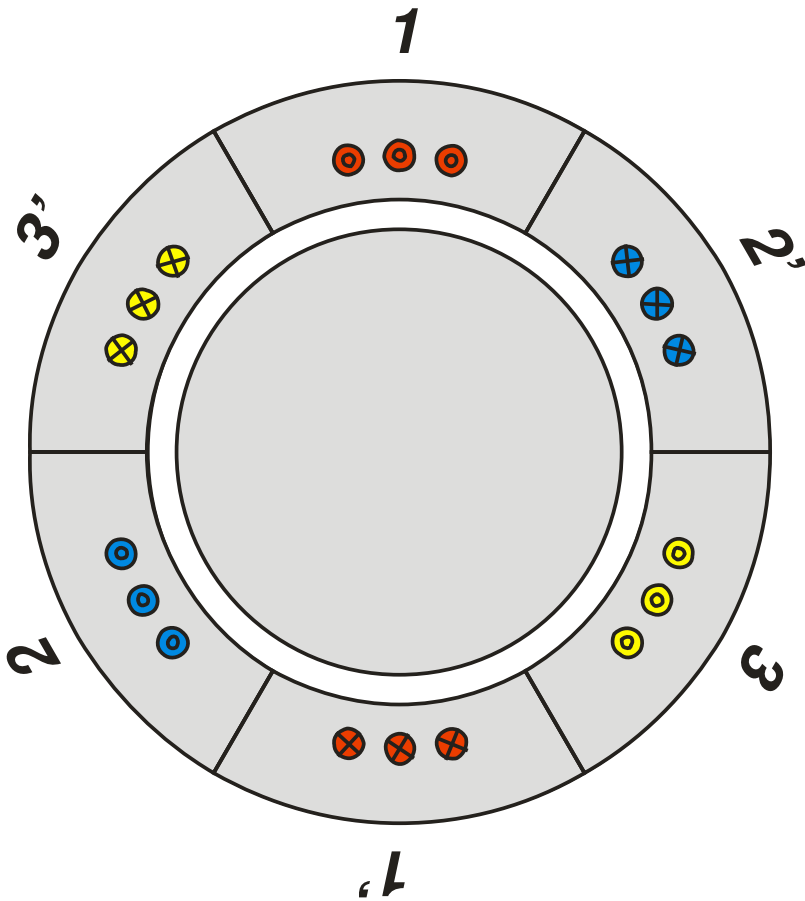
Lecture 8: Three phase AC machines



8. 3-Phase AC Machines

1. Production of rotating magnetic field
2. Induction machine
3. Synchronous machine

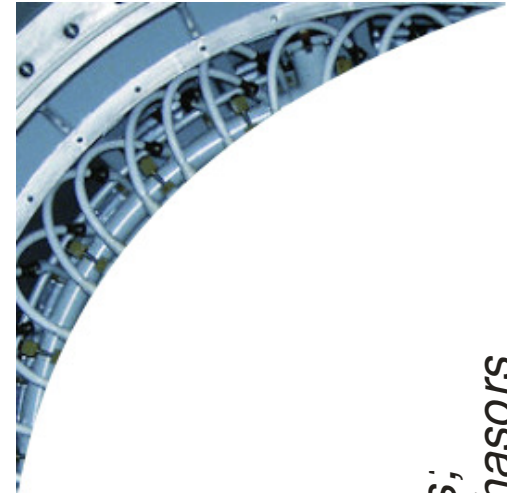
8.1. Rotating magnetic field



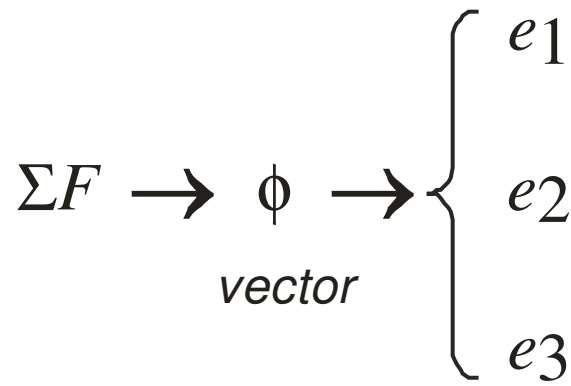
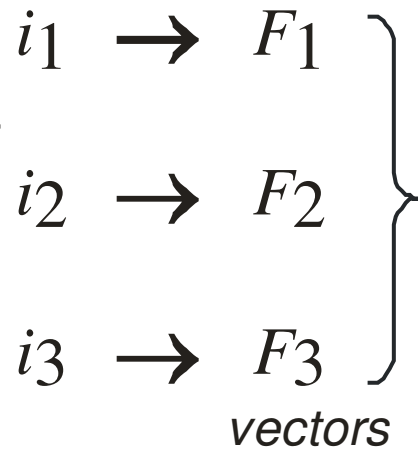
⇒ balanced 3-phase supply

⇒ 3 identical coils 120° from each other

⇒ each coil has N -turns



*instantaneous;
as sinusoid, use phasors*

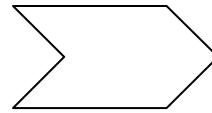


⇒ let

$$i_1 = I_m \cos \omega t$$

$$i_2 = I_m \cos \omega t - 120^\circ$$

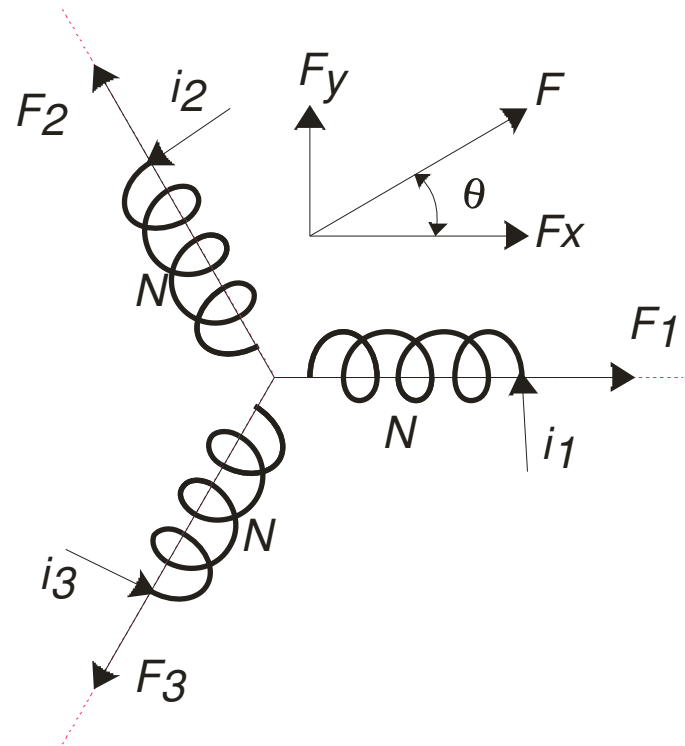
$$i_3 = I_m \cos \omega t + 120^\circ$$

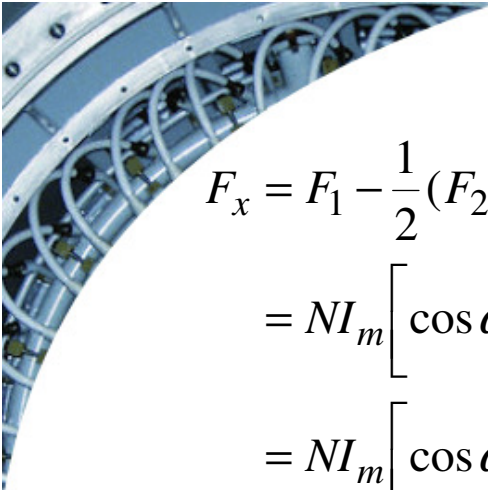


$$F_1 = NI_m \cos \omega t$$

$$F_2 = NI_m \cos \omega t - 120^\circ$$

$$F_3 = NI_m \cos \omega t + 120^\circ$$





$$\begin{aligned}
 F_x &= F_1 - \frac{1}{2}(F_2 + F_3) \\
 &= NI_m \left[\cos \omega t - \frac{1}{2}(\cos \omega t - 120^\circ + \cos \omega t + 120^\circ) \right] \\
 &= NI_m \left[\cos \omega t - \frac{1}{2} 2 \cos \omega t \cos 120^\circ \right] \\
 F_x &= \frac{3}{2} NI_m \cos \omega t
 \end{aligned}$$

$$\begin{aligned}
 F_y &= \frac{\sqrt{3}}{2}(F_2 - F_3) \\
 &= \frac{\sqrt{3}}{2} NI_m \left[\cos \omega t - 120^\circ - \cos \omega t + 120^\circ \right] \\
 &= \frac{\sqrt{3}}{2} NI_m 2 \sin \omega t \sin 120^\circ \\
 F_y &= \frac{3}{2} NI_m \sin \omega t
 \end{aligned}$$

$$|F| = \sqrt{F_x^2 + F_y^2} = \frac{3}{2} NI_m$$

⇒ the magnitude of the resultant mmf in space is independent of time and is **constant**

$$\begin{aligned}
 \tan \theta &= \frac{F_y}{F_x} = \tan \omega t \\
 \theta &= \omega t
 \end{aligned}$$

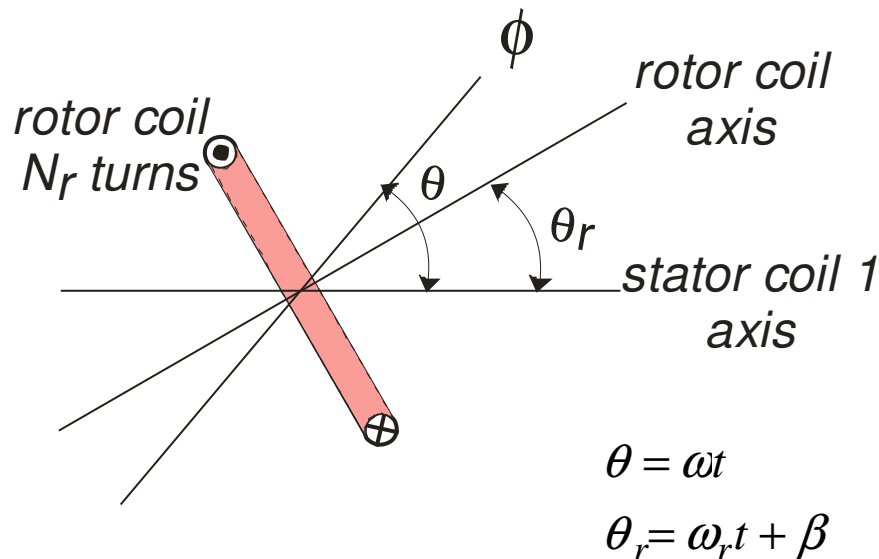
⇒ the direction of the resultant mmf in space is varying continuously in time with angular speed ω

⇒ resultant flux

$$\phi = \frac{F}{S} = \frac{3 NI_m}{2 S}$$

⇒ the reluctance S , though difficult to evaluate, is at least constant

⇒ ϕ has constant value and rotates in space at constant speed



⇒ flux linking stator coil 1 is

$$\phi_r = \phi \cos \overline{\theta - \theta_r} = \phi \cos(\overline{\omega - \omega_r t - \beta})$$

⇒ let $\frac{\omega_r}{\omega} = m$

$$\phi_r = \phi \cos(\overline{1 - m\omega t - \beta})$$

⇒ define slip, s $s = \frac{\omega - \omega_r}{\omega} = 1 - m$

⇒ induced voltage in rotor

$$\begin{aligned} e_r &= N_r \frac{d\phi_r}{dt} \\ &= N_r \phi \left[\sin(\overline{1 - m\omega t - \beta}) \right] (1 - m) \omega \\ e_r &= s N_r \phi \omega \sin(s\omega t - \beta) \end{aligned}$$



⇒ induced voltage in rotor coil has

- amplitude $\propto s$
- frequency $\propto s$

⇒ 2 special cases:

1) at synchronous speed, $m = 1$ or $s = 0$

$$e_r = 0$$

2) at standstill, $m = 0$ or $s = 1$

$$e_r = N_r \phi \omega \sin(\omega t - \beta)$$

voltages in stator & rotor windings

⇒ for stator coil 1

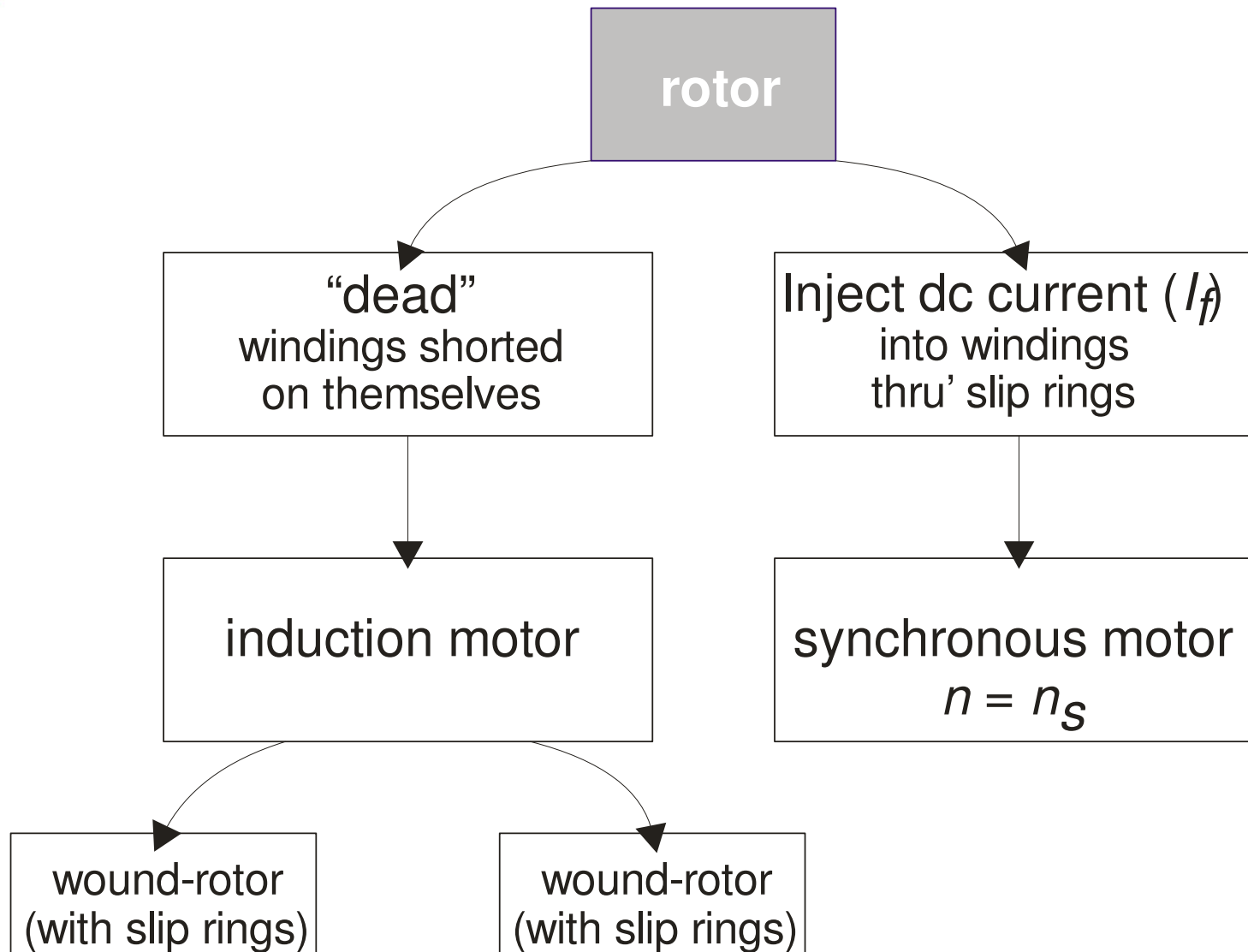
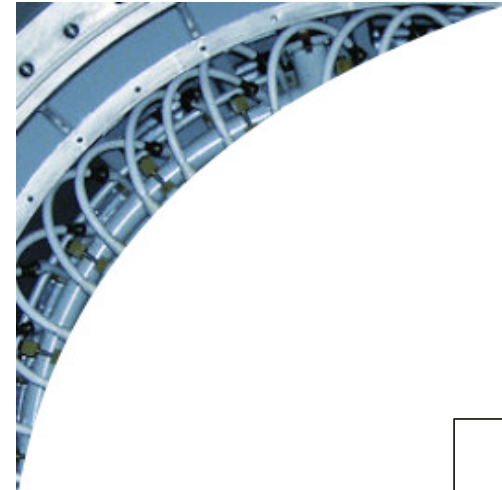
$$e_1 = N \frac{d(\phi \cos \theta)}{dt} = -N \phi \omega \sin \omega t$$

⇒ rms values

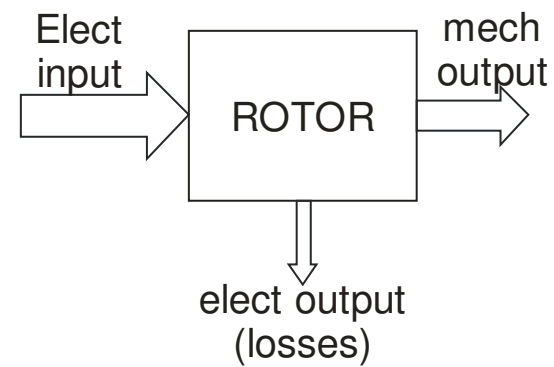
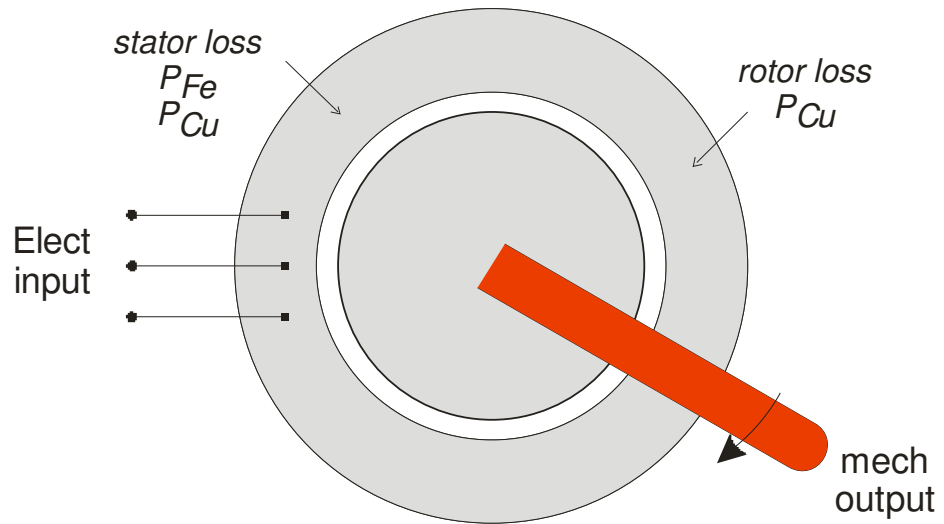
$$E_r = s \frac{N_r}{\sqrt{2}} \phi \omega$$

$$E_1 = \frac{N}{\sqrt{2}} \phi \omega$$

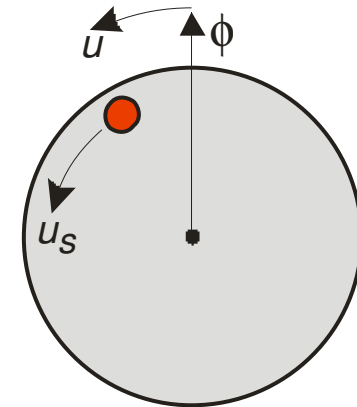
$$\frac{E_r}{E_1} = s \frac{N_r}{N}$$



8.2 Induction machines



- ⇒ single conductor of length l
- ⇒ carrying current i
- ⇒ at radius R
- ⇒ rotating magnetic field is moving at speed u_s
- ⇒ conductor is moving at speed u



⇒ define (fractional) slip s as
$$s = \frac{u_s - u}{u_s}$$

⇒ hence
$$s = \frac{2\pi R n_s - 2\pi R n}{2\pi R n_s} = \frac{n_s - n}{n_s}$$

⇒ Faraday's law gives induced voltage

$$V_{in} = Bl(u_s - u) = Blu_s s$$



⇒ rotor electrical power

$$V_{in}i = Blu_s si$$

⇒ mech force

$$Bli$$

⇒ mech power

$$(Bli)u = Bliu_s(1 - s)$$

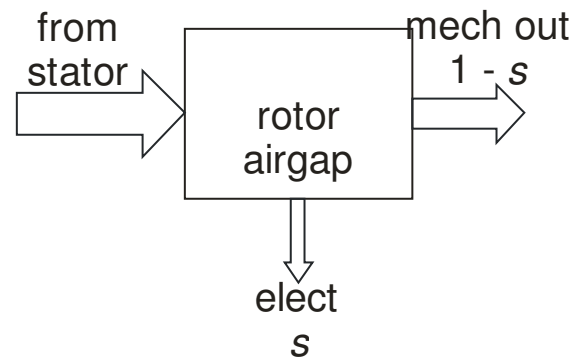
⇒ total input power

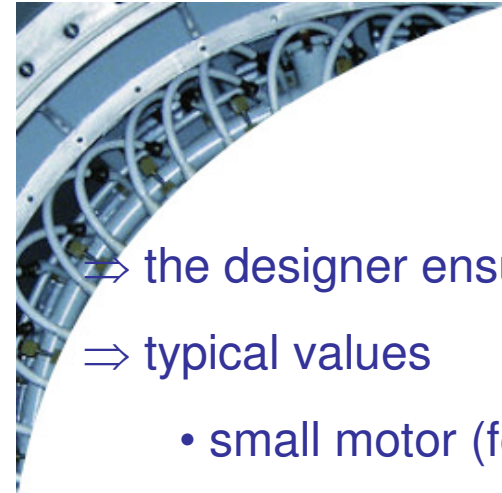
rotor elec power + rotor mech power

$$= Blu_s si + Bliu_s(1 - s)$$

$$= Blu_s i$$

⇒ thus





⇒ the designer ensures that s is small

⇒ typical values

- small motor (few kW) $s = 0.03$ (3%)
- large machines $s \leq 0.02$

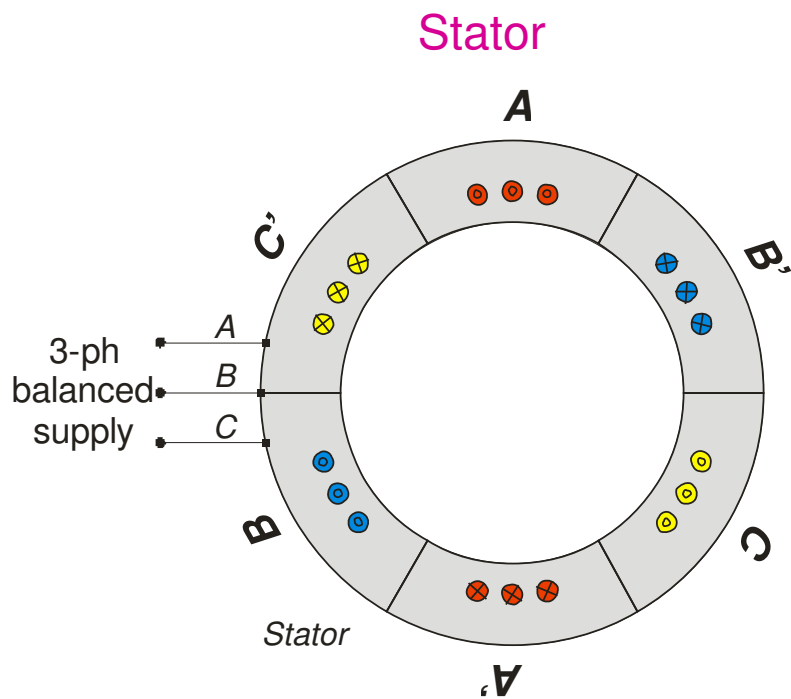


Examples

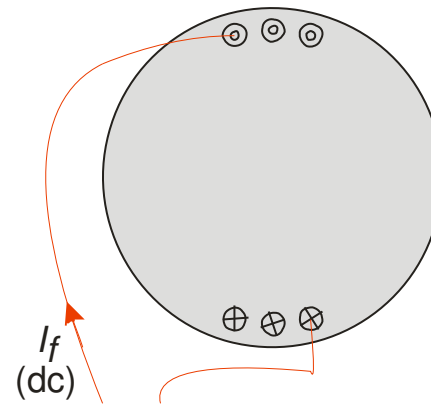
- 1) A 4-pole 50-Hz machine is running at $s = 0.025$ and 100 kW enters the rotor. What is the speed n , mechanical output and the rotor copper loss?
- 2) A 3-phase, 60-Hz, four-pole, 220-V, wound-rotor induction motor has a delta-connected stator winding and a star-connected rotor winding. The rotor has 40% as many turns as the stator. For a rotor speed of 1710 r/min, calculate the
 - a) slip
 - b) induced phase voltage in the rotor at standstill
 - c) induced phase voltage in the rotor at working speed
 - d) rotor terminal voltage on open circuit and at standstill
 - e) frequency of induced voltage in the rotor

8.3 Synchronous machines

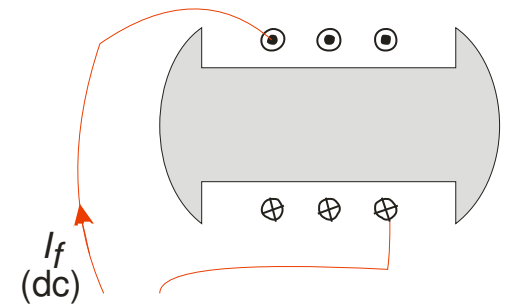
⇒ the synchronous machine has a 3-phase winding on the stator while the rotor is supplied with direct current



Rotor



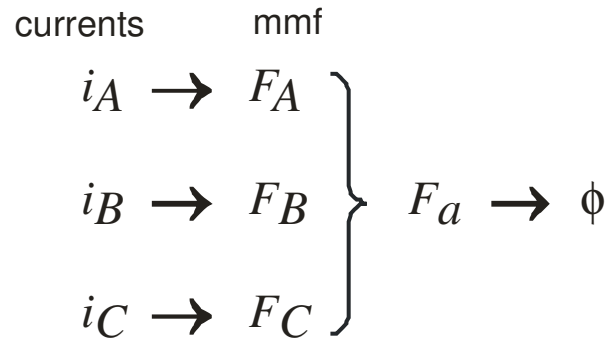
cylindrical (round) rotor



salient rotor



Stator:

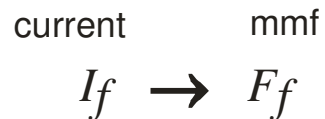


$\Rightarrow F_a$ has constant magnitude and rotates a constant speed ω_s

\Rightarrow for a $2p$ -pole machine

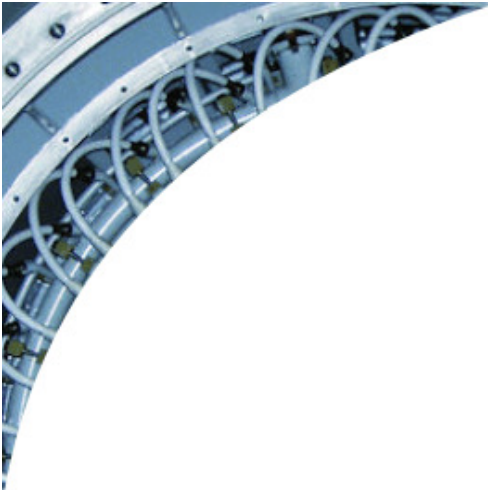
$$n_s = \frac{\omega_s}{2\pi} = \frac{f}{p}$$

Rotor:



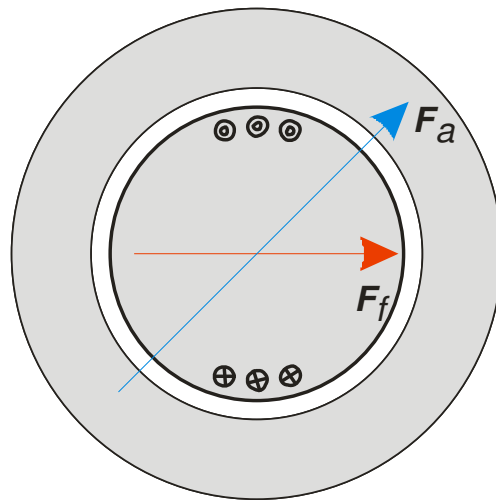
\Rightarrow the direction of the current through the brushes and slip rings to the winding is always in the same direction and so the polarity of on the rotor (N & S) never changes

$\Rightarrow F_f$ is along the axis of the rotor and rotates at ω_r



$$\overline{F}_a + \overline{F}_f = \overline{F}_r \rightarrow \phi_r \rightarrow \left\{ \begin{array}{l} e_A \\ e_B \\ e_C \end{array} \right\} E_r$$

⇒ in synchronous machine $\omega_r = \omega_s$




⇒ the machine functions as a motor or as a generator depending on whether the stator fields leads or lags the rotor field



Action of the ideal machine

⇒ assume:

- ideal cylindrical rotor
- connected to 'infinite' busbar
- stator windings have
 - negligible resistance
 - negligible leakage reactance
 - uniform air gap
 - high permeability magnetic circuit
 - no saturation
 - balanced load

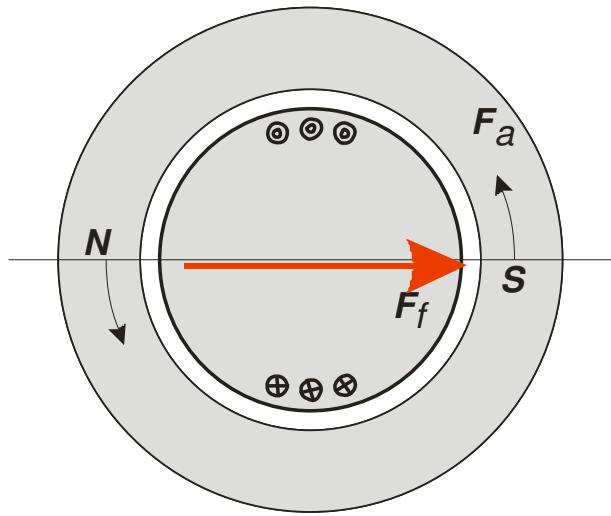
- 
- ⇒ to work at all the rotor must rotate at synchronous speed
 - ⇒ no torque can be developed if rotor is unexcited
 - ⇒ stator must take in lagging reactive power to magnetise the machine to a gap flux per pole of ϕ_r in order for the stator emf E_r to be induced to balance applied voltage V_a
 - ⇒ if rotor is given a small dc excitation it takes over part of the task of exciting the magnetic circuit, reducing the demand of stator magnetising power (under excitation)

⇒ λ = torque angle

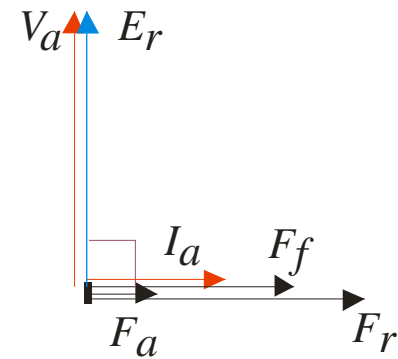
⇒ δ = load angle

- motor mode: δ is negative
- generator mode: δ is positive

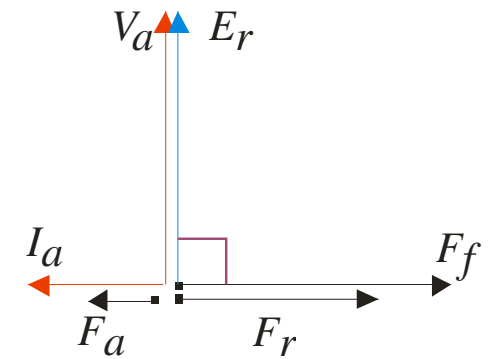
No-load mode



under excited



over excited



⇒ no torque is developed

- mmf axes are in alignment, torque angle is zero

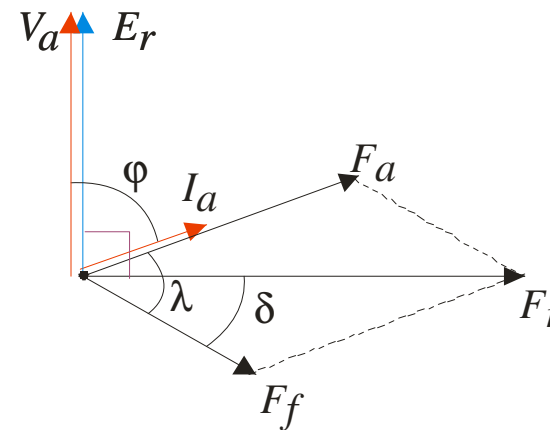
⇒ F_a and F_f combine to give F_r necessary to produce ϕ_r

⇒ if rotor mmf is increased very much into over excitation the stator must produce a demagnetising current so that F_r shall remain unchanged

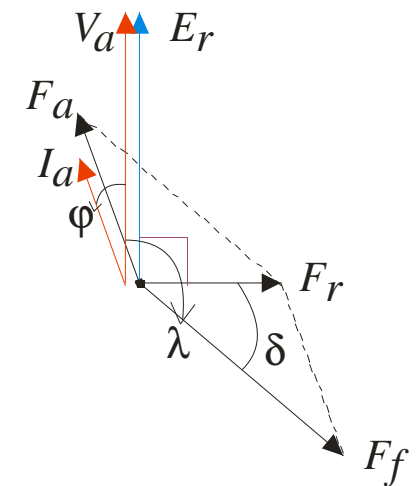
- i.e. stator must produce a leading current

Motor mode

under excited



over excited



⇒ depending on whether rotor is under or over excited, the stator takes an active current component

- thus accepting power from the supply and developing a forward torque on the rotor to balance the load torque

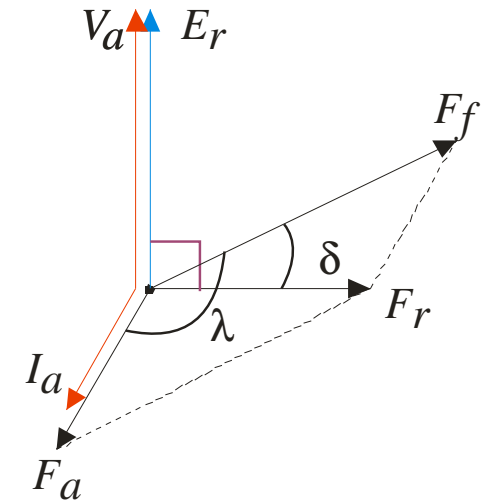
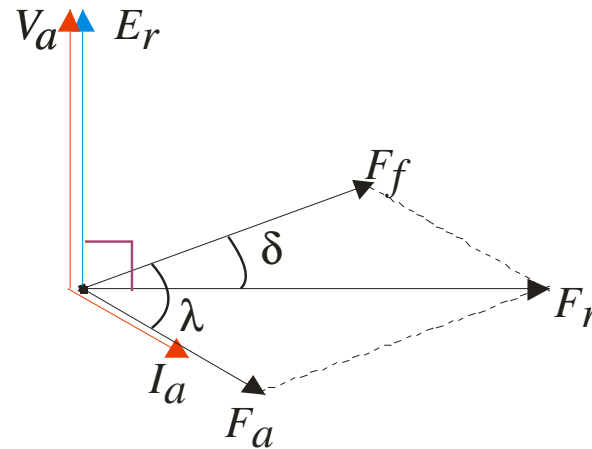
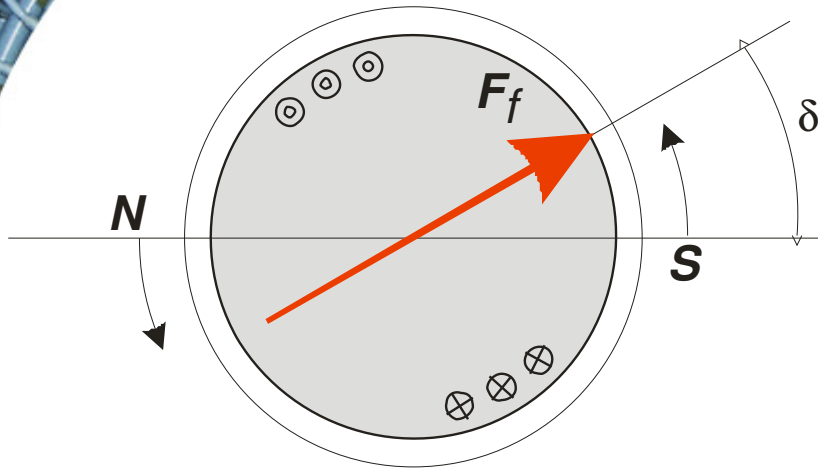
⇒ the reactive component of the current, as on no-load, compensates for under or over excitation

$$T \propto \sin \lambda$$

Generator mode

under excited

over excited



⇒ the active component of stator current reverses

- thus delivering power into the supply and developing a counter torque on the rotor to balance the driving torque

⇒ the reactive component of the current, as on no-load, compensates for under or over excitation



Compensator mode

⇒ a synchronous machine designed to run unloaded and the shaft is connected neither to mechanical load nor prime mover

⇒ variation of rotor excitation causes machine to take purely reactive power

- under excitation → lagging
- over excitation → leading

⇒ applied to control voltage in transmission systems by supplying or consuming reactive power

⇒ a.k.a. synchronous compensator, synchronous capacitor

⇒ in general, the machine has no torque at starting (zero speed) and therefore some other means must be used to bring it to synchronous speed, e.g.

- pony motor
- double-cage arrangement