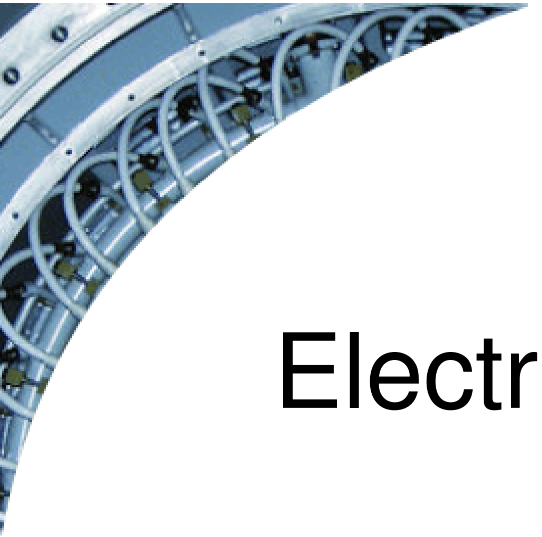
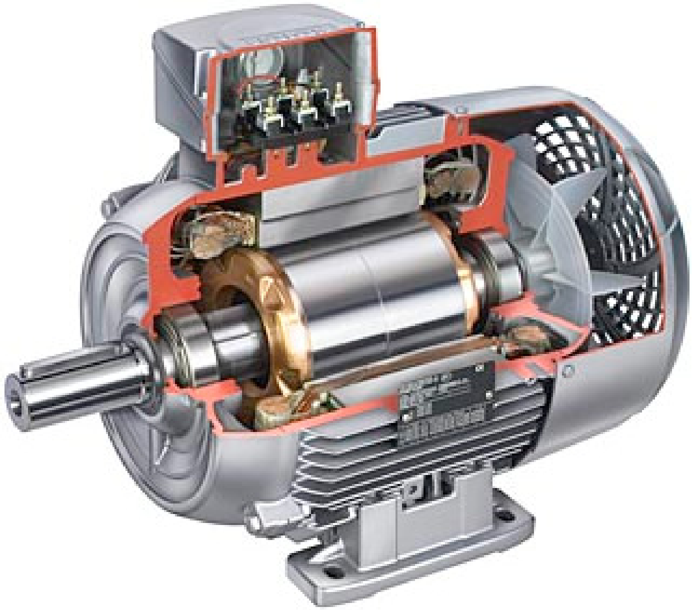


EEE 3352

Electromechanics & Electrical Machines



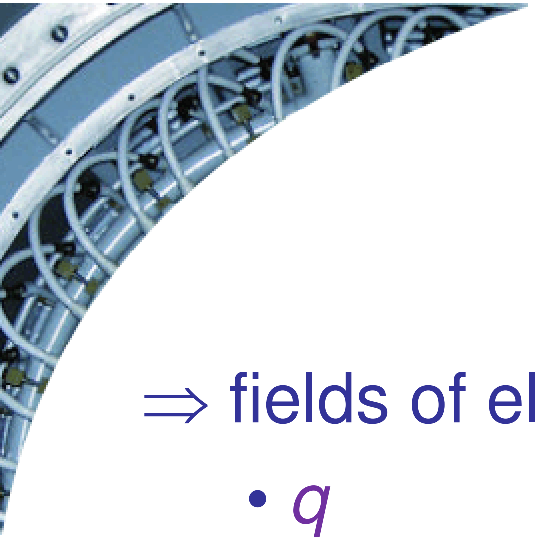
Lecture 2: Electromagnetic fields

2.Electromagnetic fields

1. Introduction to electromagnetic fields
2. Electrical properties of matter and space
3. Lumped and field quantities
4. 0-dimensional fields
5. 1-dimensional fields
6. Stored energy and force in EM fields

2.1 Introduction to EM fields

1. Fields of
    I. electrostatics
    II. conduction
    III. magnetostatics
2. Electrical properties of matter and space
    I. capacitance
    II. conductance/resistance
    III. reluctance

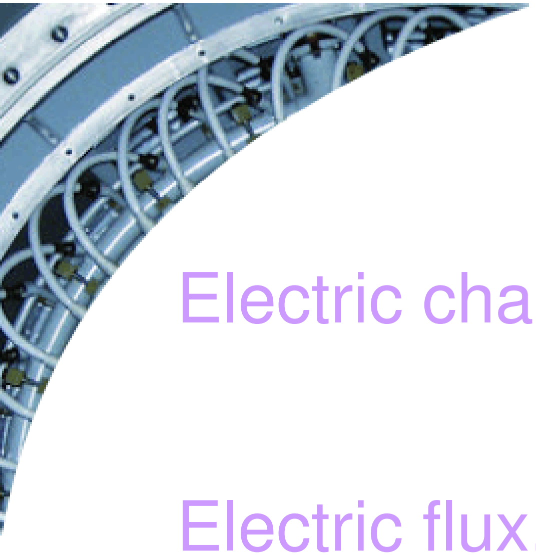


2.1.1 Electrostatics

⇒ fields of electrostatics involve the quantities

  - q
  - ψ
  - D
  - E
  - V

- defined as:



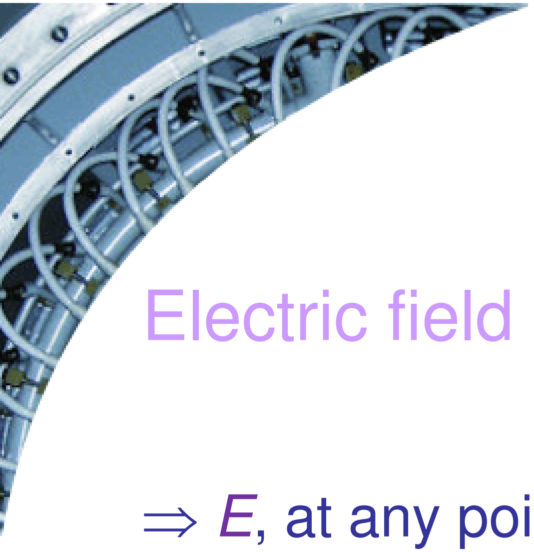
Electric charge, q

Electric flux, ψ

$$\psi = q$$

Electric flux density, D

$$D = \frac{d\psi}{dA} = \frac{\psi}{A}$$



Electric field strength, *E*

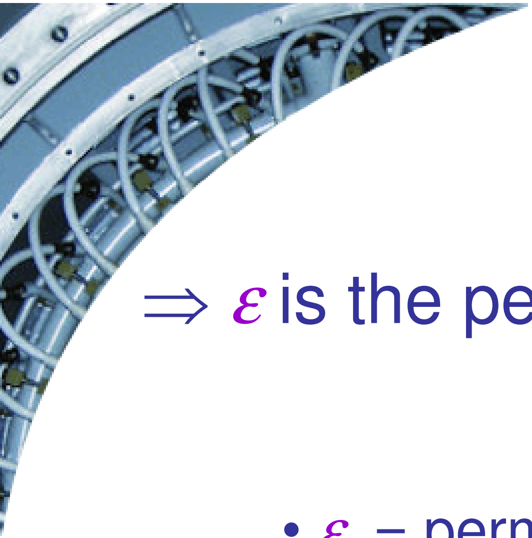
⇒ *E*, at any point in the field, is defined as the force on a unit charge placed at that point

⇒ *E* is a vector quantity whose direction is that of the force on the unit charge

⇒ applying Gauss's law:

- at a distance *r*, from a fixed point charge *q*, the electric field strength in a radial direction from the point charge is

$$E = \frac{q}{4\pi\varepsilon r^2}$$



⇒ ε is the permittivity of the medium enclosing the charge

$$\varepsilon = \varepsilon_o \varepsilon_r$$

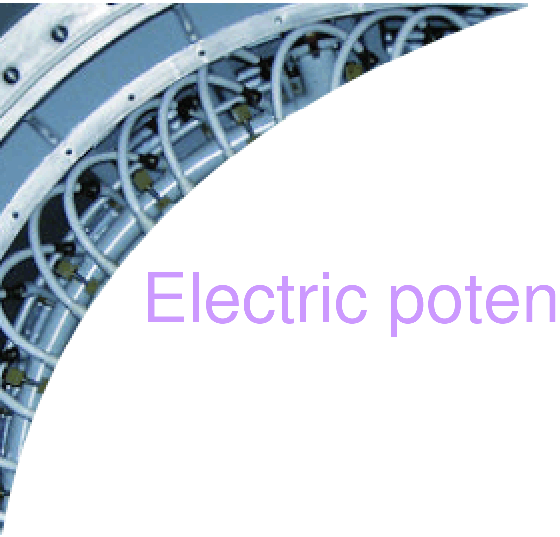
- ε_o = permittivity of free space,
- ε_r = relative permittivity of the medium to that of vacuum

⇒ knowing that

$$D = \frac{q}{A} = \frac{q}{4\pi r^2}$$

then

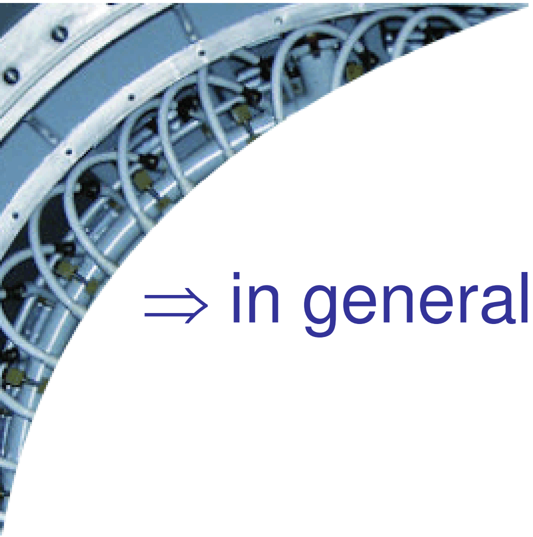
$$\boxed{D = \varepsilon E}$$



Electric potential, V

$\Rightarrow$ defined as the change in the potential of one coulomb of charge if one joule of work is required to move it from one point to another

$\Rightarrow$ if a unit charge moves from one point to another in an electric field, thereby changing its potential by V Volts, then the work done is V Joules



⇒ in general, work done is

$$W = qV$$

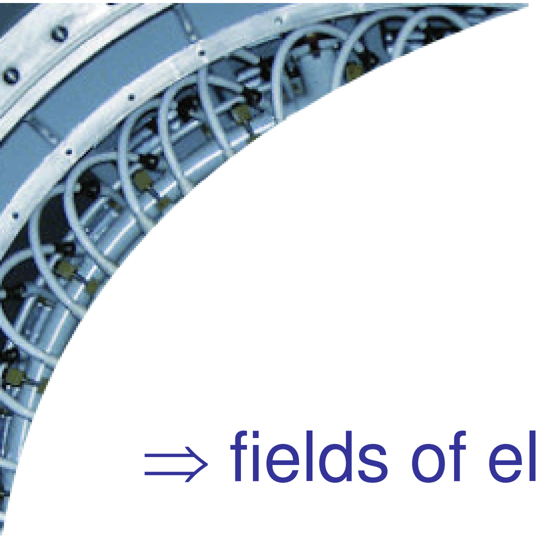
⇒ when a unit charge is moved a distance Δl in a direction opposite to the field strength E, the work done is

$$W = E\Delta l$$

⇒ if the associated change in potential of the unit charge is ΔV, then

$$\Delta V = E\Delta l$$

$$E = \frac{\Delta V}{\Delta l} = \frac{dV}{dl}$$

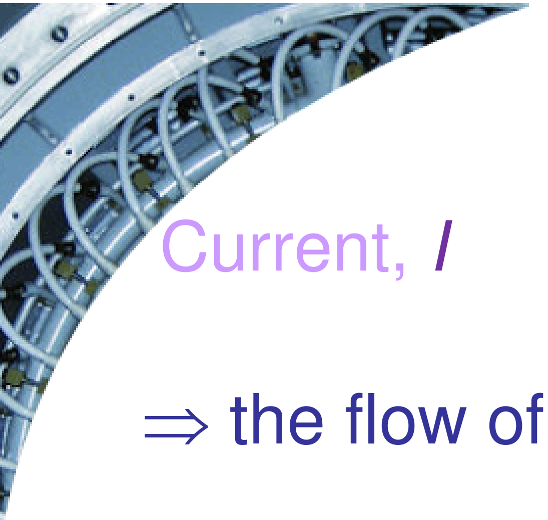


2.1.2 Electrical conduction

⇒ fields of electrical conduction involve the quantities

- I
- J
- E
- V

⇒ defined as:



Current, *I*

⇒ the flow of electric charge

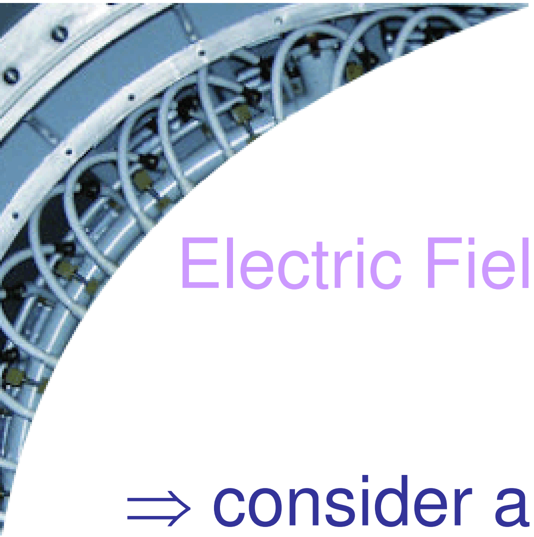
Current density, *J*

⇒ if current is distributed uniformly throughout the x-section of a wire, the current density *J* is uniform, and is given by the total current divided by the x-section area *A* of the wire

$$J = \frac{I}{A}$$

⇒ if current density is not uniform, the local current density is given by

$$J = \lim_{\Delta A \to 0} \frac{\Delta I}{\Delta A} = \frac{dI}{dA}$$



Electric Field Strength & Potential in conduction

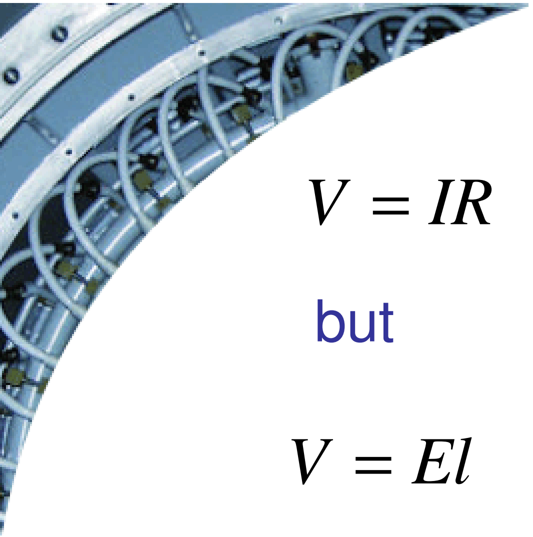
⇒ consider a small imaginary rectangle cell of

- length l
- x-section A,

⇒ with J normal at a point of consideration in the cell;

⇒ let V be the potential difference between the ends of the cell

⇒ applying Ohm’s law:



$$V = IR$$

but

$$V = El \qquad \text{and} \qquad I = JA$$

so that

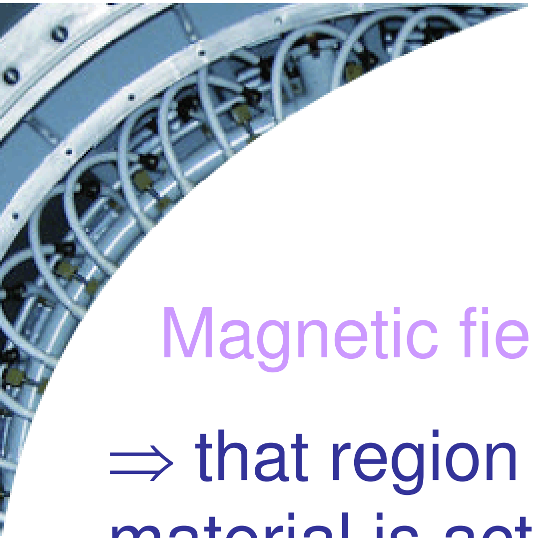
$$El = JAR \qquad \text{or} \qquad J = \frac{l}{AR}E = \sigma E$$

then

$$J = \sigma E \qquad \text{where} \qquad \sigma = \frac{l}{AR}$$

$\Rightarrow$ σ is the conductivity of the material; the reciprocal is resistivity, ρ

$$\rho = \frac{1}{\sigma} = \frac{AR}{l}$$



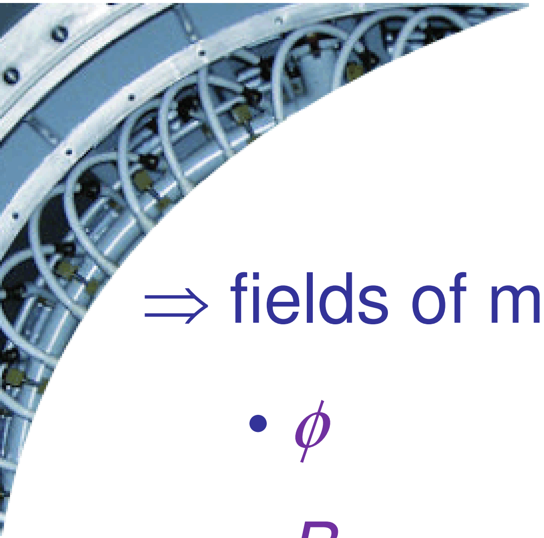
2.1.3 Magnetostatics

Magnetic field

⇒ that region in which a charged particle in motion or a magnetic material is acted upon by a magnetic force

Magnetic lines of force

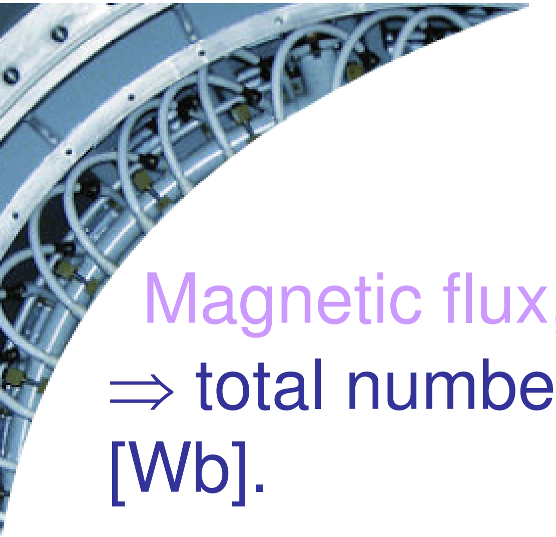
- have direction
- form complete loops
- represent a tension about their length which tends to make them as short as possible
- repel one another
- cannot intercept but must always form an individual closed loop



⇒ fields of magnetostatics involve the quantities

- ϕ
- B
- F
- H

⇒ defined as:



Magnetic flux, ϕ

⇒ total number of lines of force in a magnetic field, unit is Weber, [Wb].

⇒1 Wb = 10^8 lines of magnetic force, by definition

Magnetic flux density, (B)

⇒ magnetic flux per unit area, unit is [Wb/m^2] or Tesla [T]

$$B = \frac{d\phi}{dA}$$

⇒ in a uniform field

$$B = \frac{\phi}{A}$$

Magnetomotive force, mmf, *F*

⇒ magnetic flux is caused by a mmf, just as current in conductor is caused by an emf

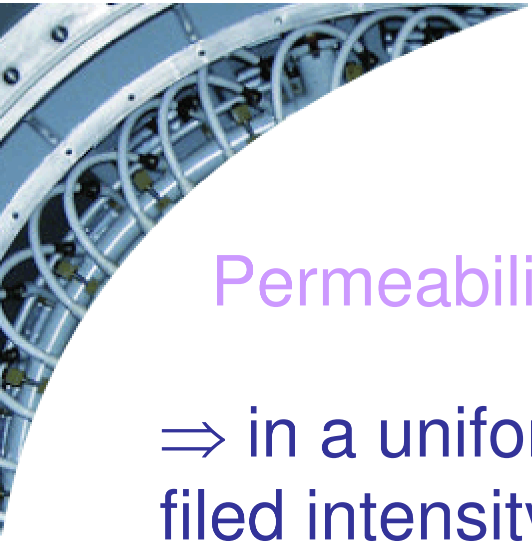
⇒ if a coil of *N* turns carrying a current I is used as the source of mmf, then

$$mmf = F = NI$$

Magnetic field strength or magnetic field intensity, *H*

⇒ is defined as the mmf per unit length, and units are [A/m]

$$H = \frac{F}{l}$$



Permeability, μ

$\Rightarrow$ in a uniform field magnetic flux density B is related to magnetic filed intensity H by:

$$\boxed{B = \mu H}$$

where μ is the permeability of the medium

$$\mu = \mu_o \mu_r$$

μ_O = permeability of free space

μ_r = relative permeability

2.2 Electrical properties of matter and space

I. capacitance

II. conductance/resistance

III. reluctance

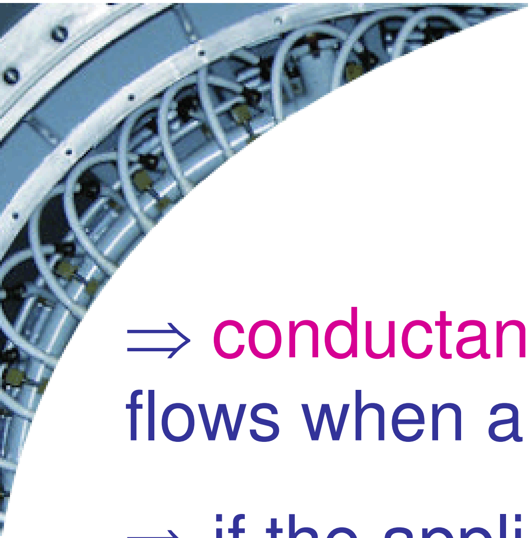
2.2.1 Capacitance

⇒ capacitance, C, is the property of matter and space whereby electric charge q is accumulated when a potential difference V is applied to it

⇒ if the applied p.d. is V and accumulated charge is q, then capacitance is defined as

$$C = \frac{q}{V}$$

⇒ unit is Farad, [F]



2.2.2 Conductance

⇒ conductance is the property of matter whereby electric current flows when a potential difference is applied to it

⇒ if the applied p.d. is *V* and the current that flows is I, then conductance is defined as

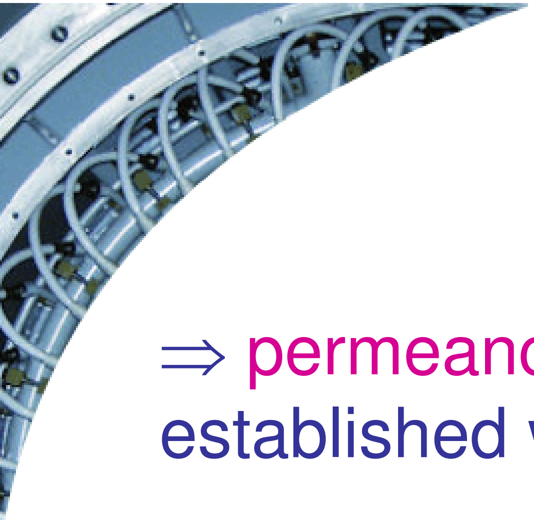
$$\boxed{G = \frac{I}{V}}$$

⇒ unit is Semiens, [S]

⇒ reciprocal of conductance is Resistance, *R*

$$R = \frac{V}{I}$$

⇒ unit is Ohm, [Ω]



2.2.3 Permeance

⇒ permeance is the property of matter whereby a magnetic flux is established when a magnetomotive force is applied to it

⇒ if the applied mmf is F and the flux established is ϕ, then permeance is defined as

$$\Lambda = \frac{\phi}{F}$$

⇒ unit is Henry, [H] or Weber per ampere [Wb/A]

⇒ in special types of matter such as conductors, inductance L is used to describe permeance

⇒ inductance is the ability of a conductor to have a voltage induced when the current changes

⇒ in this case, if the total flux linkage, λ, causes a current I to be induced in a conductor then inductance

$$L = \frac{\lambda}{I}$$

⇒ the reciprocal of *permeance* is *reluctance*, S

⇒ unit of reluctance is Ampere per Weber [A/Wb]

2.3 Lumped and field quantities

I. fields

II. Relationship between field and lumped quantities

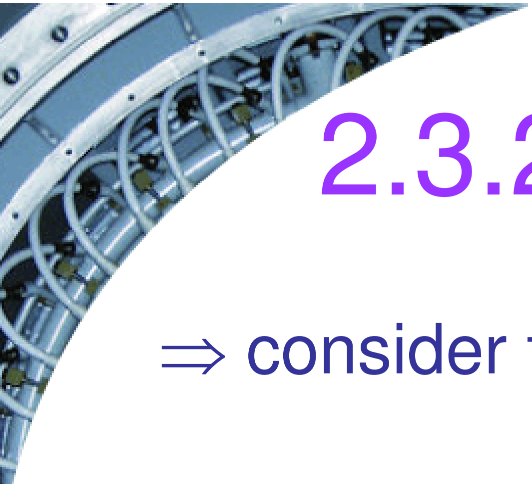
2.3.1 field

⇒ field is used to indicate a region of space throughout which the effect is appreciable

⇒ the region may be bounded or may extend to infinity

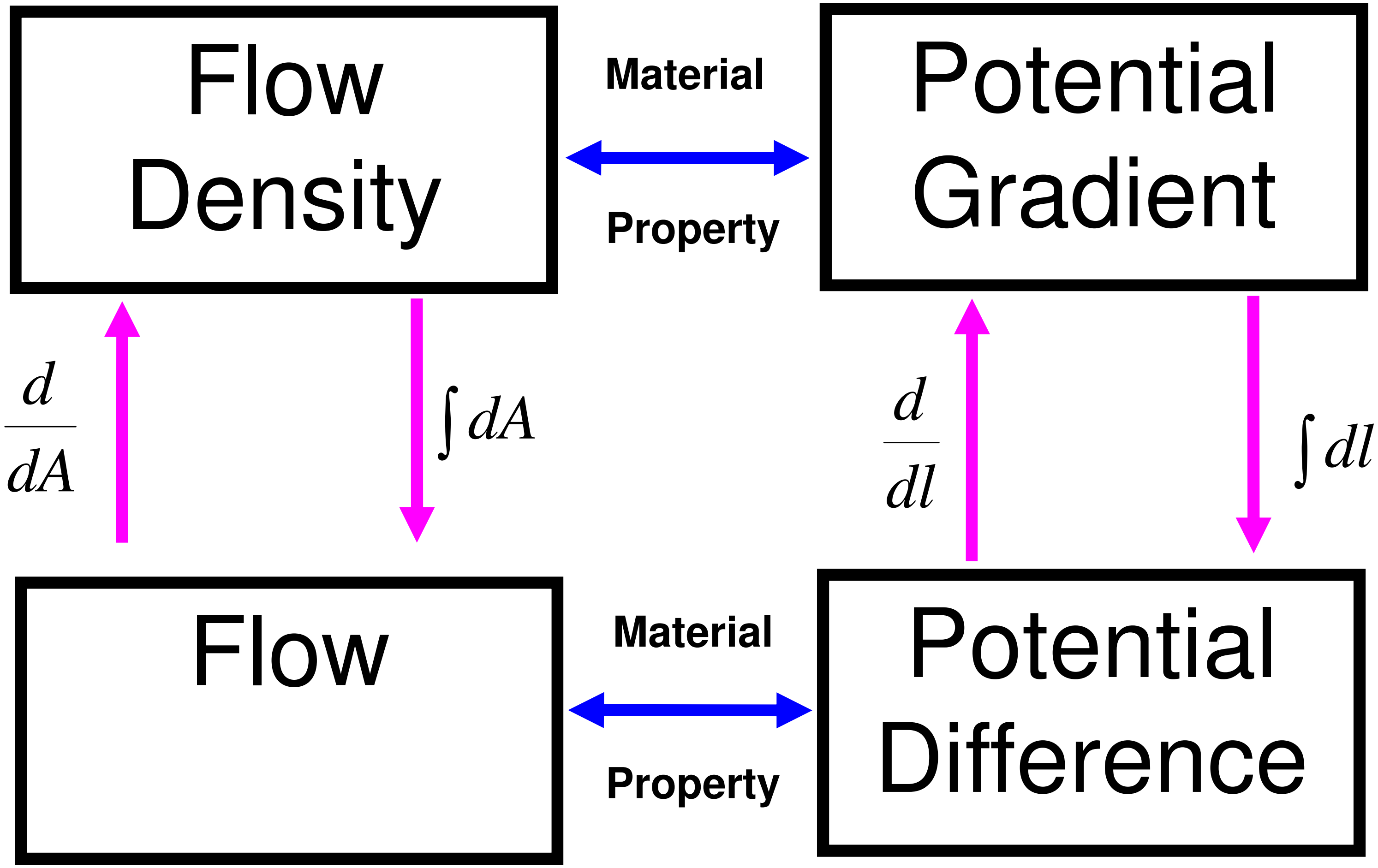
⇒ field to be studied are electric and magnetic

⇒ a field may be 0-D, 1-D, 2-D or 3-D



2.3.2 Lumped and field quantities

⇒ consider the relationships



Flow Density

Material

Property

Potential Gradient

$\frac{d}{dA}$

$\int dA$

$\frac{d}{dl}$

$\int dl$

Flow

Material

Property

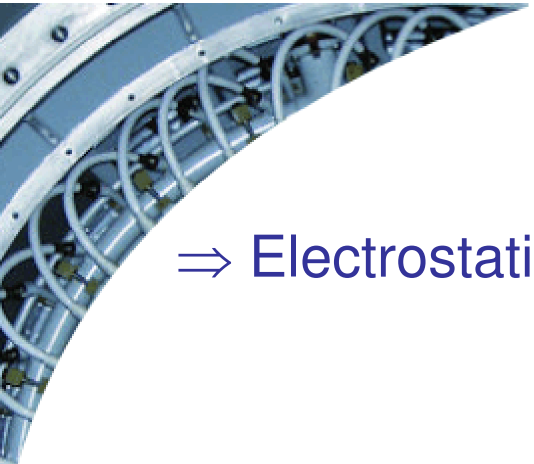
Potential Difference

Field quantities:

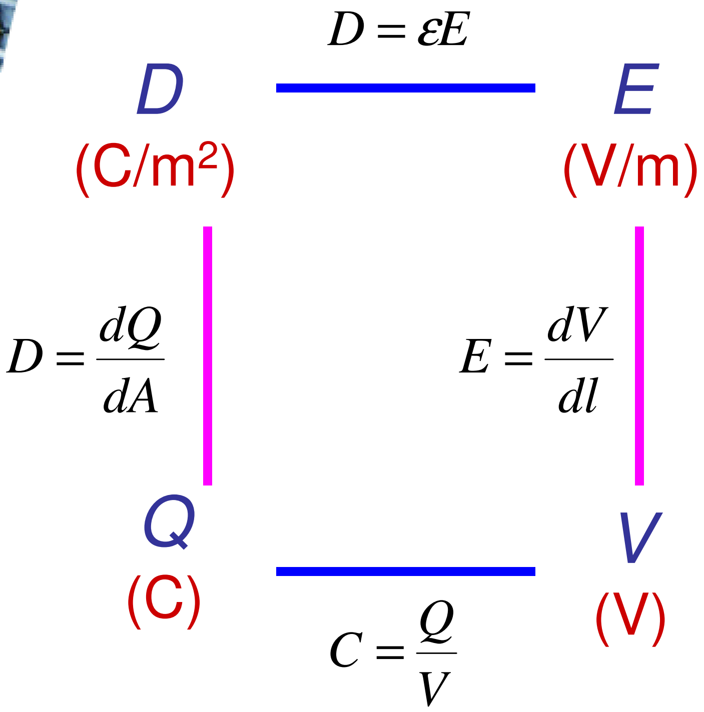
⇒ flow density & potential gradient

Lumped quantities:

⇒ flow & potential difference



⇒ Electrostatics



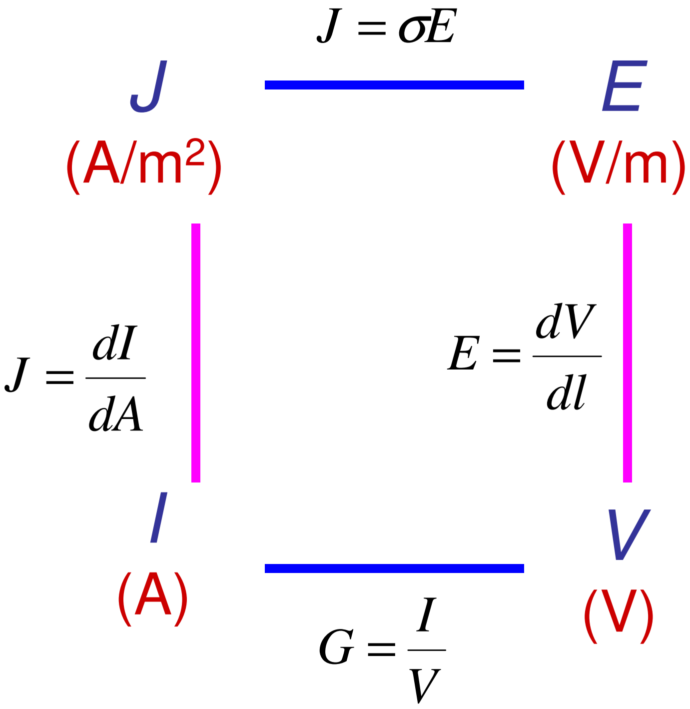
D (C/m²) — E (V/m): $D = \varepsilon E$

D — Q: $D = \dfrac{dQ}{dA}$

E — V: $E = \dfrac{dV}{dl}$

Q (C) — V (V): $C = \dfrac{Q}{V}$

⇒ Current conduction



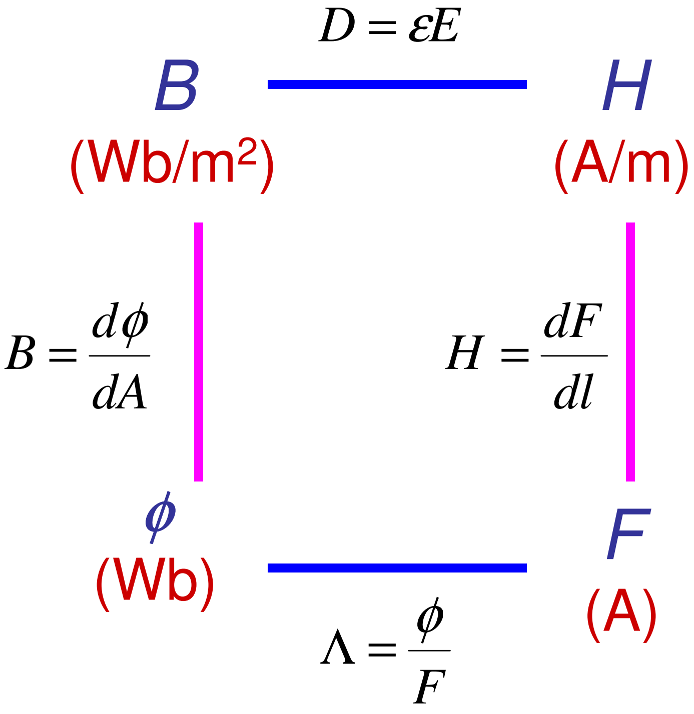
J (A/m²) — E (V/m): $J = \sigma E$

J — I: $J = \dfrac{dI}{dA}$

E — V: $E = \dfrac{dV}{dl}$

I (A) — V (V): $G = \dfrac{I}{V}$

⇒ Magnetostatics

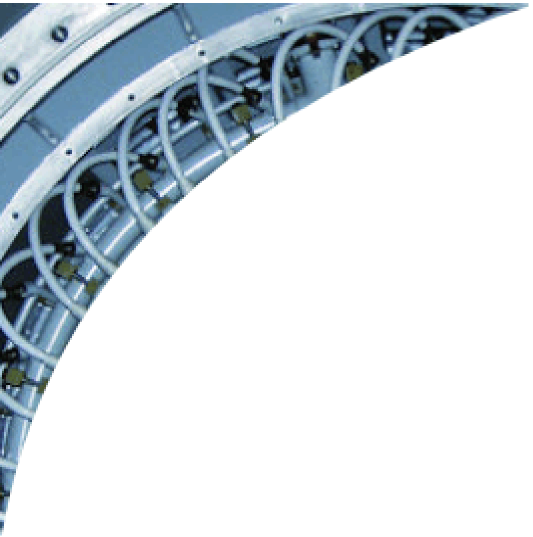


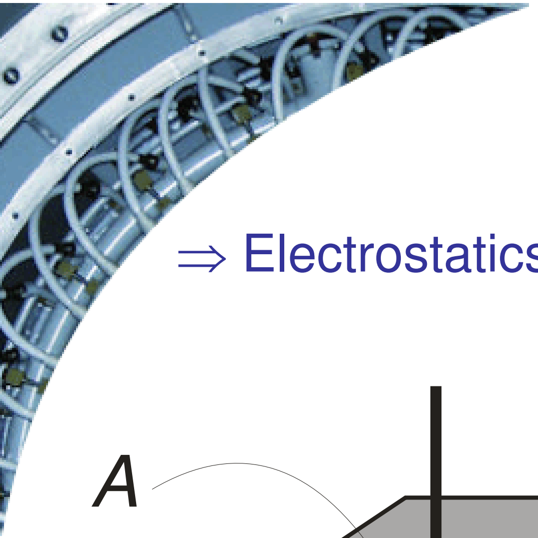
B (Wb/m²) — H (A/m): $D = \varepsilon E$

B — ϕ: $B = \dfrac{d\phi}{dA}$

H — F: $H = \dfrac{dF}{dl}$

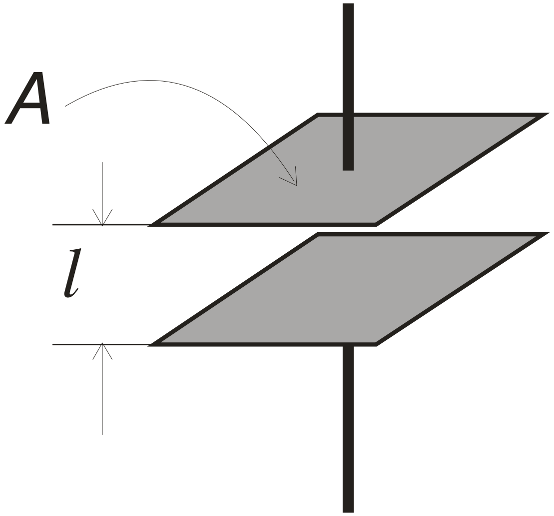
ϕ (Wb) — F (A): $\Lambda = \dfrac{\phi}{F}$





0-D fields (Uniform fields)

⇒ Electrostatics

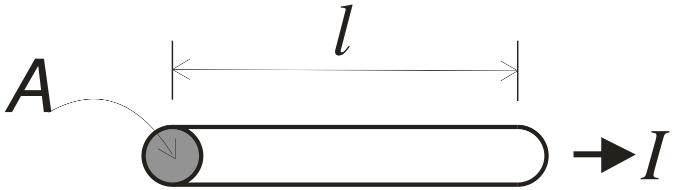


$$C = \frac{Q}{V}$$

$$= \frac{DA}{El}$$

$$C = \varepsilon \frac{A}{l}$$

⇒ Current conduction

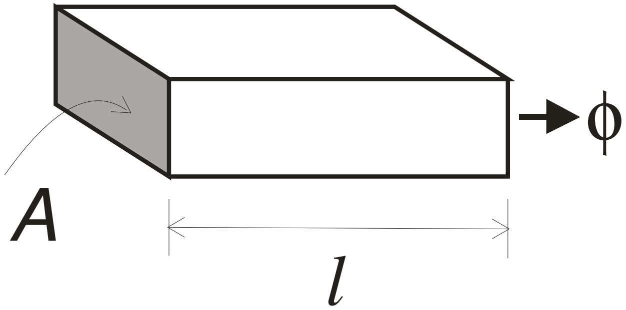


$$G = \frac{I}{V}$$

$$= \frac{JA}{EA}$$

$$G = \sigma \frac{A}{l}$$

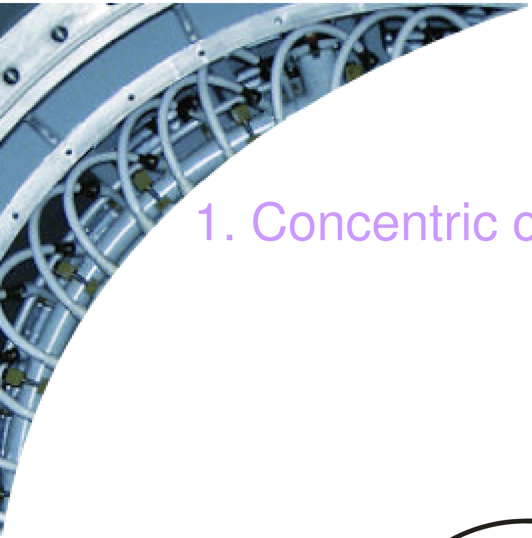
⇒ Magnetostatics



$$\Lambda = \frac{\phi}{F}$$

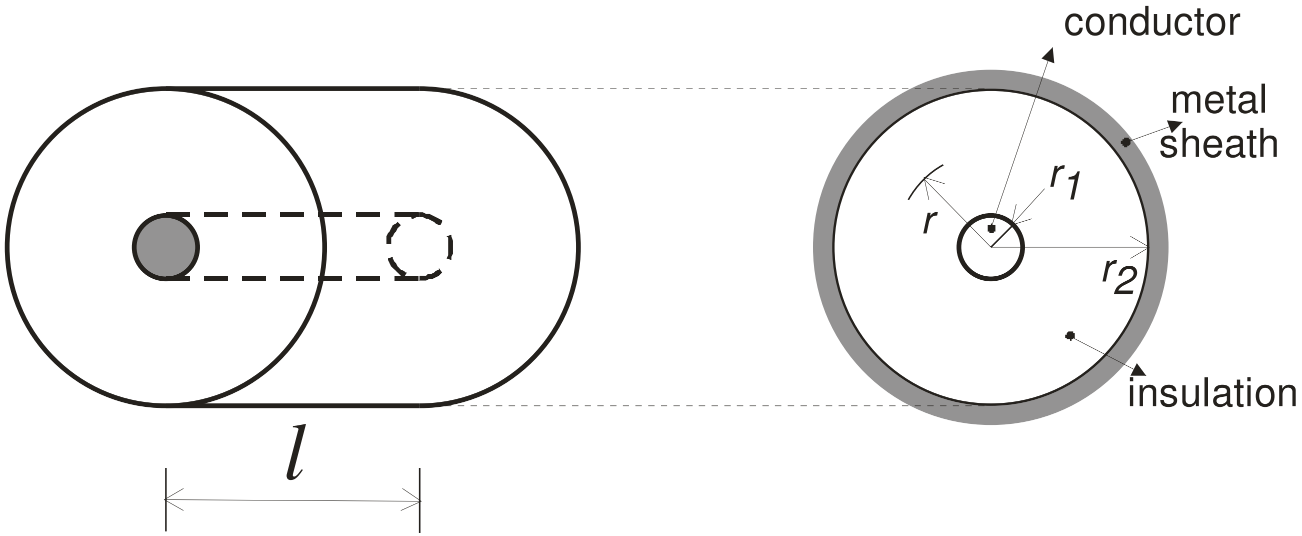
$$= \frac{BA}{Hl}$$

$$\Lambda = \mu \frac{A}{l}$$



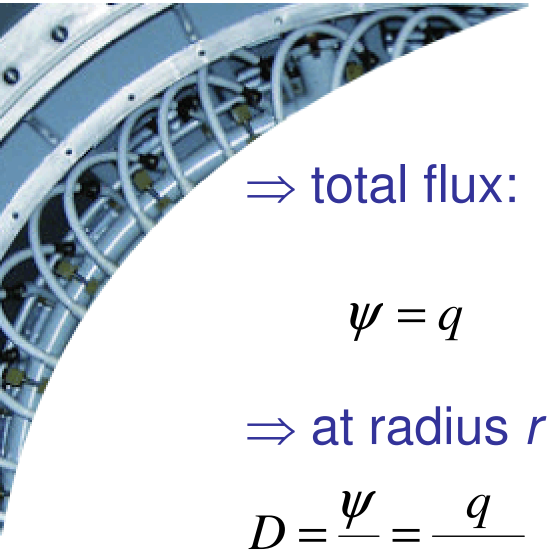
1-D fields: Examples

1. Concentric cable (electrostatic field)



⇒ consider length *l* having charge *q*

⇒ metal sheath is earthed, so is at zero potential



⇒ total flux:

$$\psi = q$$

⇒ at radius *r*

$$D = \frac{\psi}{A} = \frac{q}{2\pi rl}$$

$$E = \frac{D}{\varepsilon} = \frac{q}{2\pi rl\varepsilon}$$

$$V = \int_{r_1}^{r_2} Edr = \int_{r_1}^{r_2} \frac{q}{2\pi rl\varepsilon} dr$$

$$= \frac{q}{2\pi l\varepsilon} \ln \frac{r_2}{r_1}$$

$$C = "\frac{Q}{V}"$$

$$C = \frac{2\pi l\varepsilon}{\ln \frac{r_2}{r_1}}$$

⇒ consider *E*

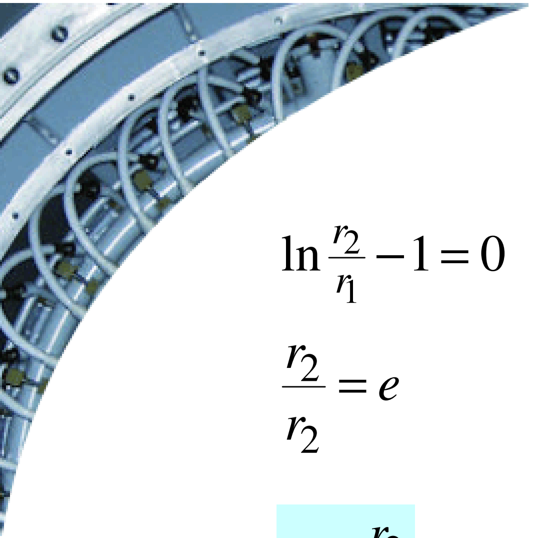
$$E = \left(\frac{q}{2\pi l\varepsilon}\right)\frac{1}{r} = \left(\frac{V}{\ln \frac{r_2}{r_1}}\right)\frac{1}{r}$$

⇒ for max *E* , *r* is minimum, i.e., $r = r_1$

$$E_{\max} = \frac{V}{r_1 \ln \frac{r_2}{r_1}}$$

⇒ for given *V* and r_2, what value of r_1 gives minimum E_{max}?

$$\frac{dE_{\max}}{dr_1} = 0 = \frac{-V}{\left(r_1 \ln \frac{r_2}{r_1}\right)^2}\left[\ln \frac{r_2}{r_1} + r_1 \left(\frac{r_1}{r_2}\right)\left(\frac{-r_2}{r_1^2}\right)\right]$$



$$\ln\frac{r_2}{r_1} - 1 = 0$$

$$\frac{r_2}{r_2} = e$$

$$\boxed{r_1 = \frac{r_2}{e}}$$

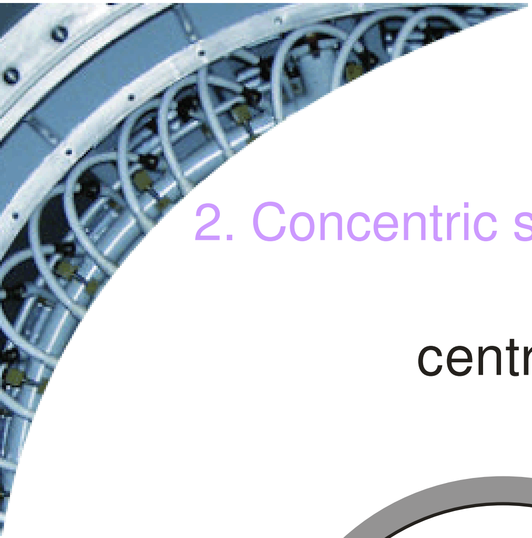
⇒ on a.c., capacitive current:

$$I = \frac{V}{Z}$$

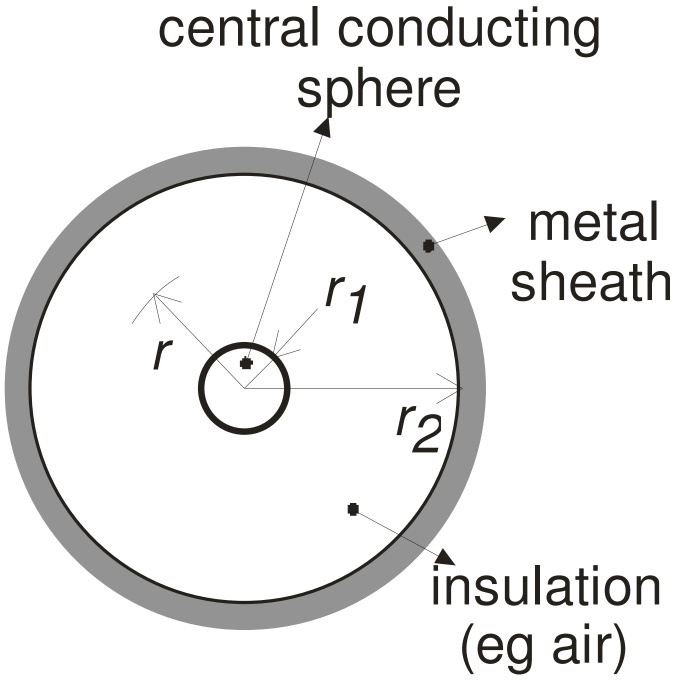
$$Z = \frac{1}{j\omega C}$$

$$|I| = V\omega C = 2\pi f V C$$

⇒ on a.c., *V* is given as rms; use peak value to get ultimate stress E_{max}



2. Concentric sphere (electrostatic field)



$$D = \frac{\psi}{A} = \frac{q}{4\pi r^2}$$

$$E = \frac{D}{\varepsilon} = \frac{q}{4\pi r^2 \varepsilon}$$

$$V = \int E dr = \frac{q}{4\pi\varepsilon} \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= \frac{q}{4\pi\varepsilon} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

⇒ capacitance:

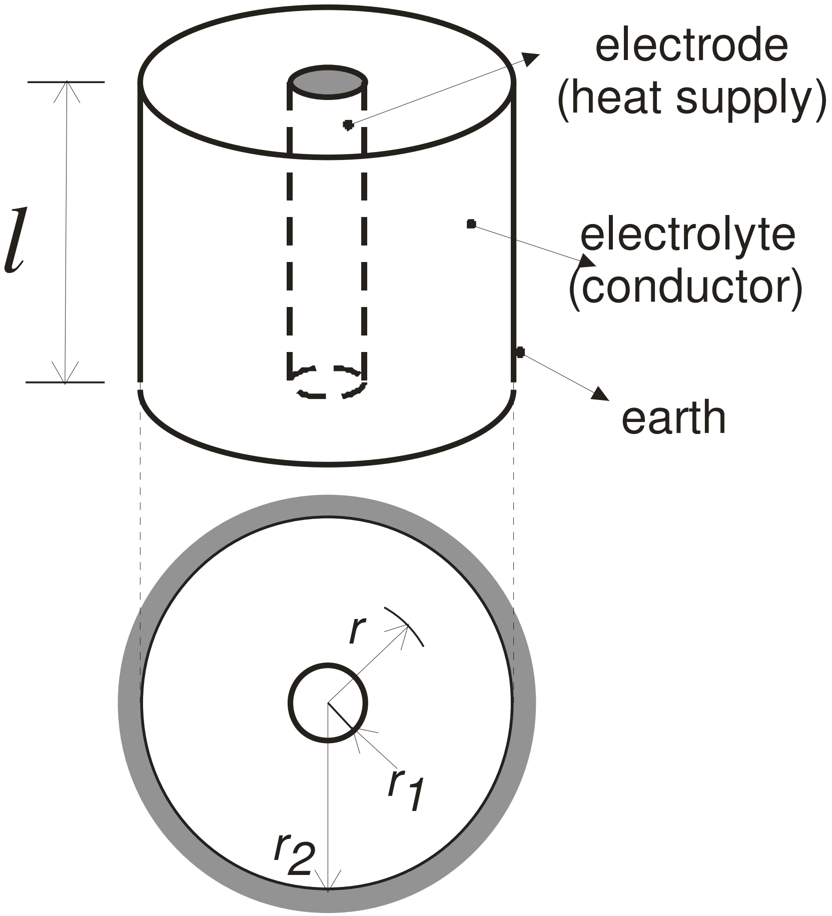
$$C = \frac{q}{V} = \frac{4\pi\varepsilon}{\left(\frac{1}{r_1} - \frac{1}{r_2} \right)}$$

⇒ electric field stress

$$E = \frac{V}{\left(\frac{1}{r_1} - \frac{1}{r_2} \right) r^2}$$

$$E_{\max} = \frac{V}{\left(\frac{1}{r_1} - \frac{1}{r_2} \right) r_1^2}$$

3. Electrode boiler (conduction field)



⇒ assume total current I,
depth of liquid l

$$J = \frac{I}{A} = \frac{I}{2\pi r l}$$

$$E = \frac{J}{\sigma} = \frac{I}{2\pi l \sigma}\frac{1}{r}$$

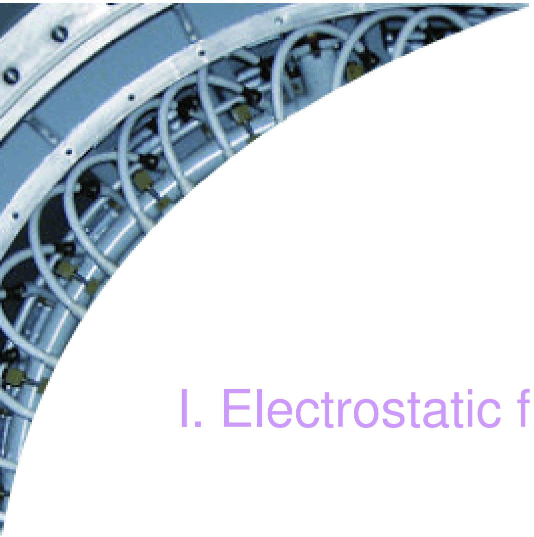
$$V = \int E dr = \frac{I}{2\pi l \sigma}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$= \frac{I}{2\pi l \sigma}\ln\frac{r_2}{r_1}$$

$$G = \frac{I}{V} = \frac{2\pi l \sigma}{\ln\frac{r_2}{r_1}}$$

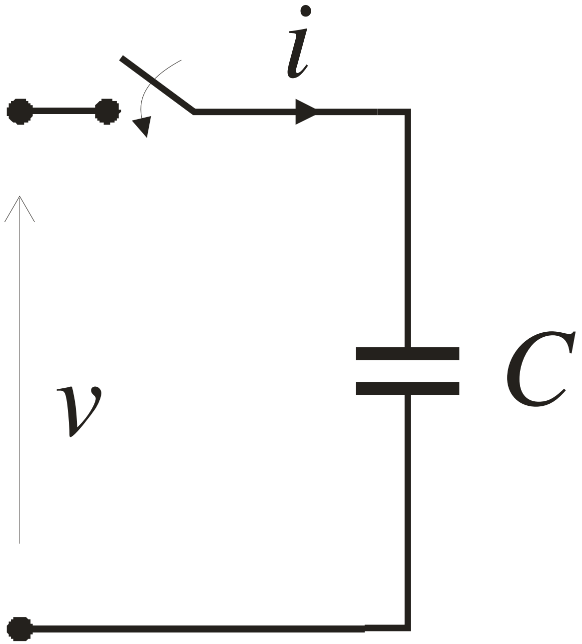
⇒ for max J, $r = r_1$

$$J_{\max} = \frac{I}{2\pi l r_1} = \frac{\sigma}{G}\frac{I}{r_1 \ln\frac{r_2}{r_1}}$$



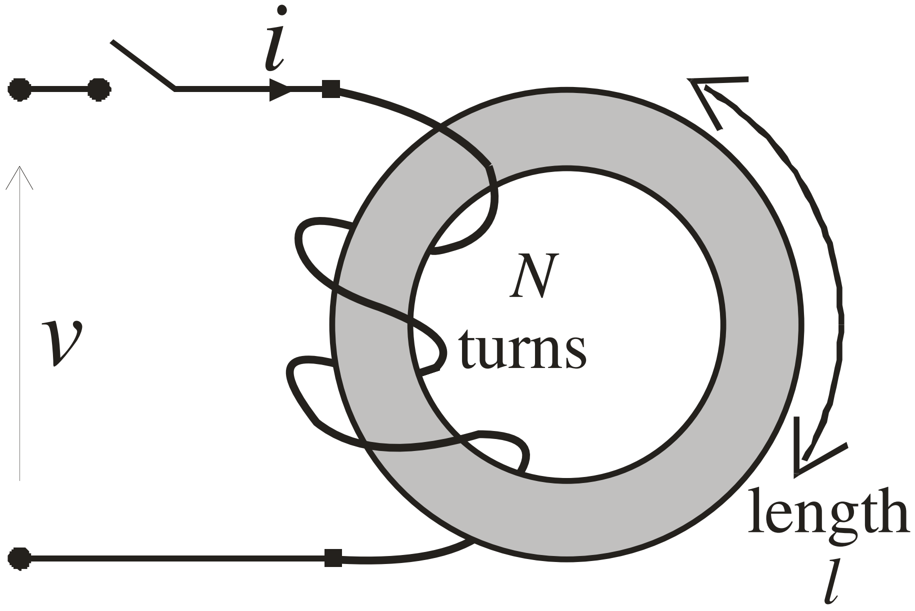
Stored energy in fields

I. Electrostatic field

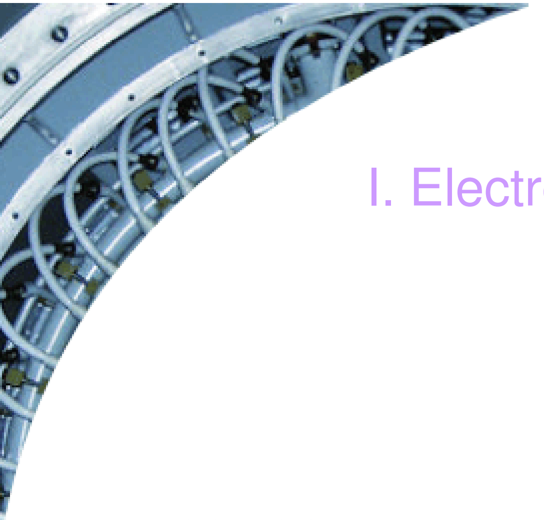


⇒ parallel plate capacitor, capacitance, *C*

II. Magnetostatic field



⇒ toroidal coil, inductance *L*



I. Electrostatic field

$$q = CV$$

$$i = C\frac{dv}{dt}$$

⇒ power

$$P = vi = Cv\frac{dv}{dt}$$

⇒ energy

$$W = \int Pdt = \int_0^V cvdv$$

$$= C\frac{V^2}{2} = \frac{1}{2}qV$$

⇒ energy/volume

$$w = \frac{\frac{1}{2}qV}{Al} = \frac{1}{2}\frac{q}{A}\frac{V}{l}$$

$$\boxed{w = \frac{1}{2}DE}$$ [J/m³]

II. Magnetostatic field

$$Li = N\phi$$

$$v = L\frac{di}{dt}$$

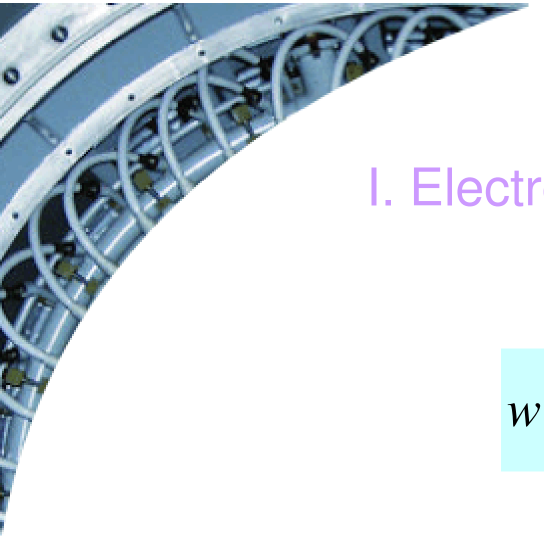
$$P = vi = Li\frac{di}{dt}$$

$$W = \int Pdt = \int_0^I Lidi$$

$$= L\frac{I^2}{2} = \frac{1}{2}N\phi I$$

$$w = \frac{\frac{1}{2}N\phi i}{Al} = \frac{1}{2}\frac{\phi}{A}\frac{NI}{l}$$

$$\boxed{w = \frac{1}{2}BH}$$ [J/m³]



I. Electrostatic field

$$w = \frac{1}{2} DE$$

II. Magnetostatic field

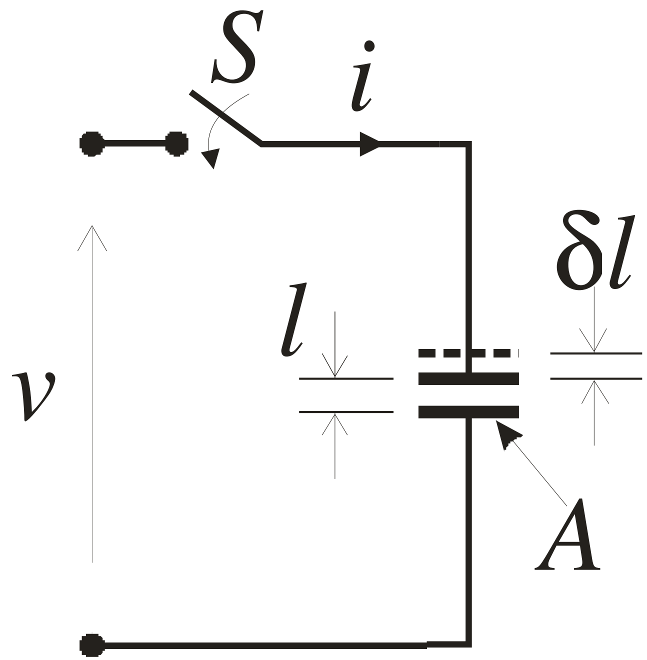
$$w = \frac{1}{2} BH$$

⇒ the expression are derived using a uniform field

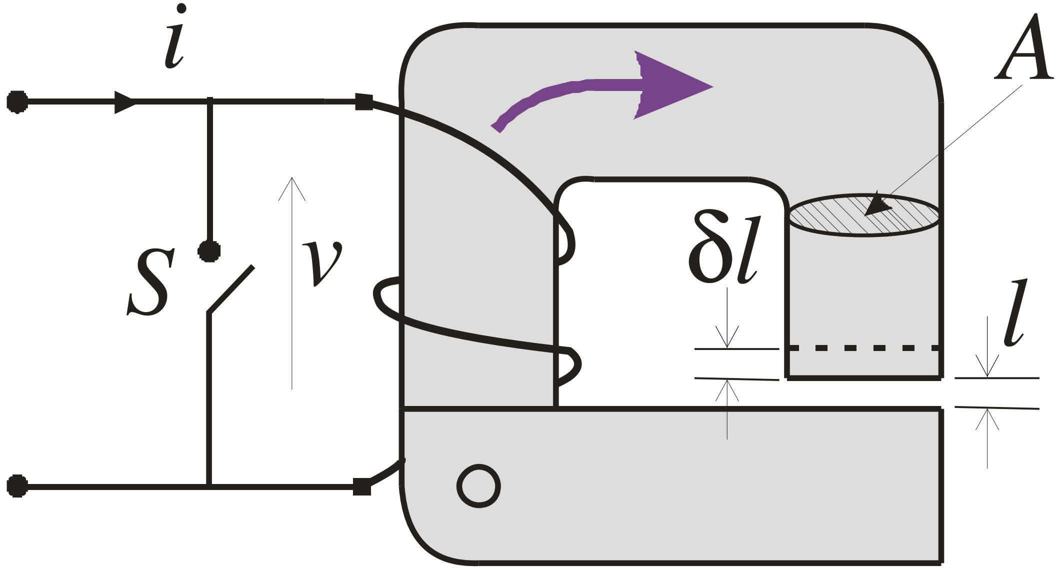
⇒ as energy the expression apply ant any point in the field, whether uniform or not

Stored energy in fields

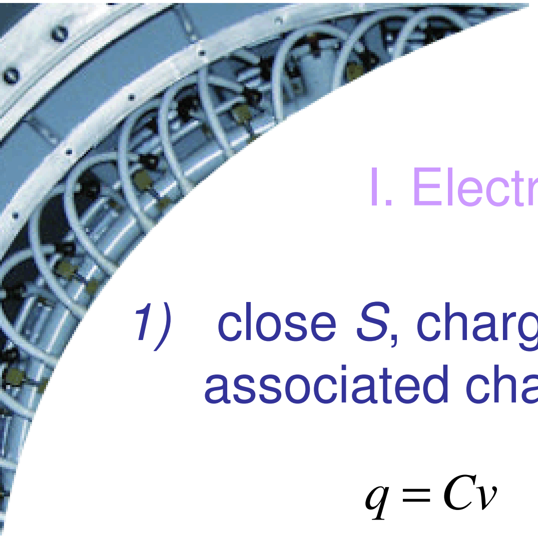
I. Electrostatic field



II. Magnetostatic field



⇒ assume $\mu_{Fe} = \infty$, just consider airgap



I. Electrostatic field

1) close *S*, charge up *C* to voltage v, associated charge is

$$q = Cv$$

2) open *S*, charge q is trapped and is constant, ∴

$$q = \text{constant}$$

$$i = \frac{dq}{dt} = 0$$

$$\psi = \text{constant}$$

$$D = \text{constant}$$

$$E = \text{constant}$$

II. Magnetostatic field

1) open *S*, current i flows thru' coil, hence

$$\phi = \frac{Li}{N}$$

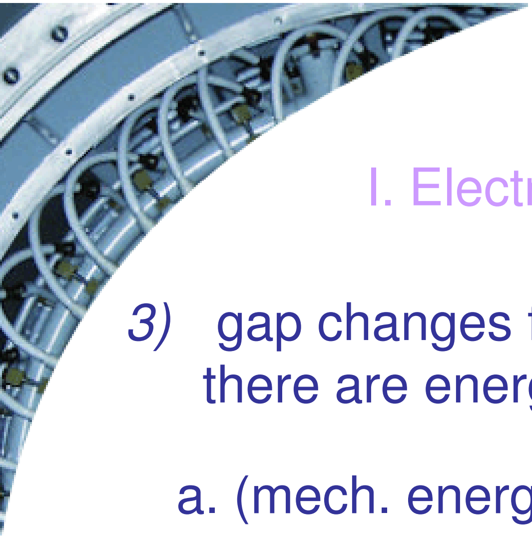
2) close *S*, making $v = 0$; since

$$v = N\frac{d\phi}{dt} = 0$$ (Faraday's law)

$$\phi = \text{constant}$$ (ϕ is 'trapped')

$$B = \text{constant}$$

$$H = \text{constant}$$



I. Electrostatic field

3) gap changes from l to $l + \delta l$; there are energy changes

a. (mech. energy ?) $= F_{\text{mech}} . \delta l$

b. (elect. energy ?) $= \int vi dt = 0$

from $i = 0$

c. change of stored energy in electrostatic field:

$$= \delta\left(\tfrac{1}{2} DE \times \text{volume}\right)$$
$$= \tfrac{1}{2} DE(\delta l . A)$$

II. Magnetostatic field

3) gap changes from l to $l + \delta l$; there are energy changes

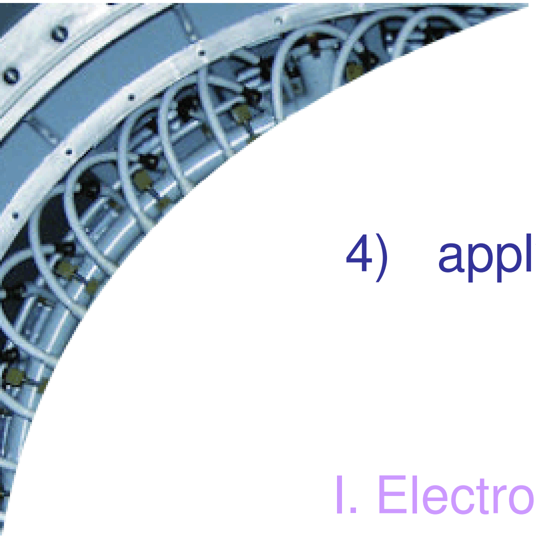
a. (mech. energy ?) $= F_{\text{mech}} . \delta l$

b. (elect. energy ?) $= \int vi dt = 0$

from $v = 0$

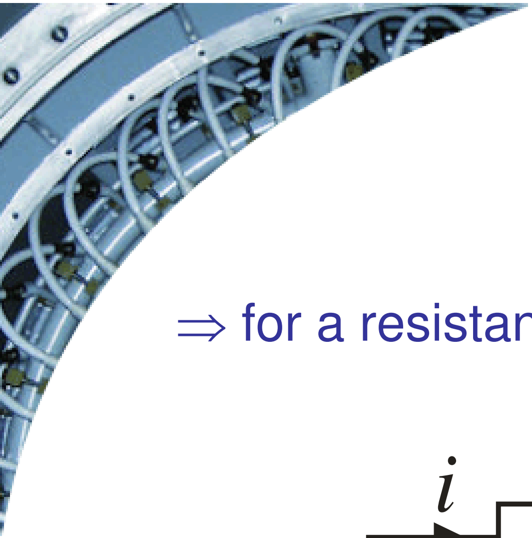
c. change of stored energy in magnetostatic field (airgap):

$$= \delta\left(\tfrac{1}{2} BH \times \text{volume}\right)$$
$$= \tfrac{1}{2} BH(\delta l . A)$$



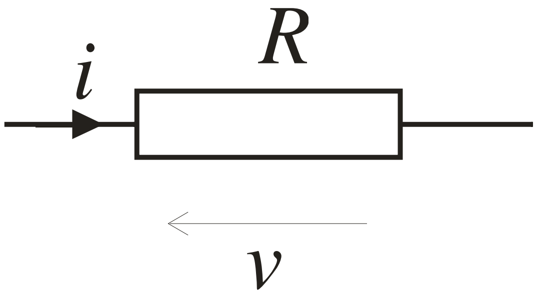
4) applying law of conservation of energy, the two forms of energy changes relevant to motion δl can be equated

I. Electrostatic field	II. Magnetostatic field
$F_{\text{mech}}\delta l = \frac{1}{2}DE(\delta l.A)$	$F_{\text{mech}}\delta l = \frac{1}{2}BH(\delta l.A)$
$F_{\text{mech}} = \frac{1}{2}DAE$ [N]	$F_{\text{mech}} = \frac{1}{2}BAH$ [N]
Pressure = press	Pressure = press
$\text{press} = \frac{F_{\text{mech}}}{A}$	$\text{press} = \frac{F_{\text{mech}}}{A}$
$\text{press} = \frac{1}{2}DE$ [N/m²]	$\text{press} = \frac{1}{2}BH$ [N/m²]



III. Conduction field

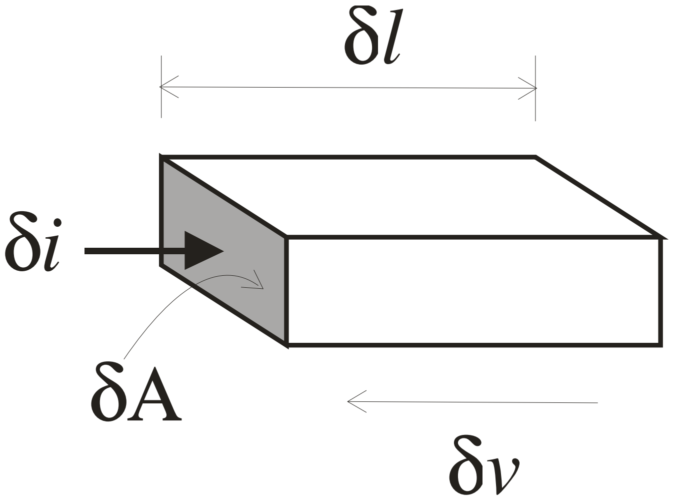
⇒ for a resistance, the electrical energy goes into heat energy



$$P = vi = i^2 R = \frac{v^2}{R}$$

⇒ no energy is stored

⇒ for field study:



local: $J = \frac{\delta i}{\delta A}$

local: $E = \frac{\delta v}{\delta l}$

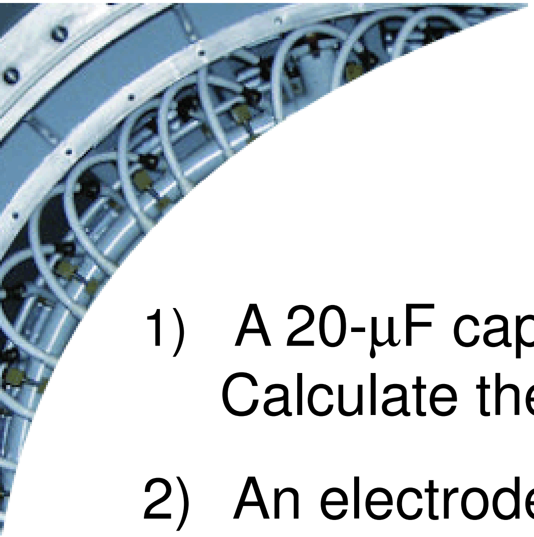
material property: $J = \sigma E$

P in a small volume: $\delta P = \delta v \, \delta i$

power per unit volume (power density):

$$= \frac{\delta v \, \delta i}{\delta l \, \delta A} = \frac{\delta v}{\delta l} \cdot \frac{\delta i}{\delta A} = JE$$

$$= \sigma E^2 = \frac{J^2}{\sigma}$$ [J/m³]



Examples:

1) A 20-μF capacitor is charged at constant current of 5 μA for 10 min. Calculate the final p.d. and the corresponding stored charge?

2) An electrode boiler has an earthed metal cylinder of 0.8 m diameter with non conducting ends and a co-axial central electrode of 0.1 m diameter. The boiler is designed to absorb 8 kW when connected to a 240-V supply, the depth of the water being 0.5 m in the axial direction. To what specific conductivity should the water be treated in order to absorb this power?

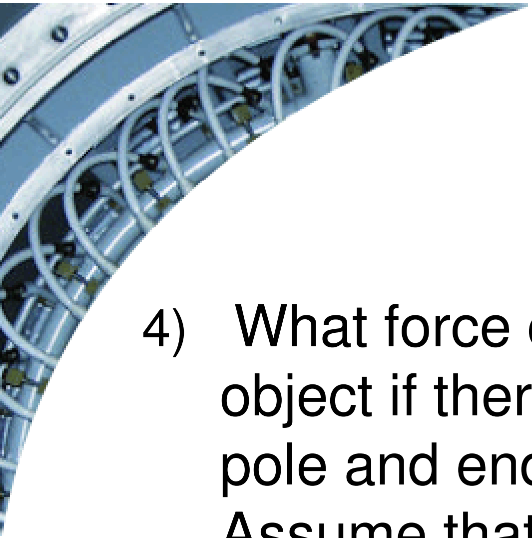
3) A coil wound uniformly round a wooden ring having a mean circumference of 600 mm and uniform cross-section al area of 500 mm^2 produces an mmf of 0.8 kA. Find

   a) the magnetic field strength

   b) the flux density

   c) the total flux

   in the ring.



Examples:

4) What force can one pole of an electromagnet exert on a movable steel object if there is a uniform air gap of 5 mm between them, the area of the pole and end face is 10^4 mm^2 and the magnetic flux density is 0.5 T? Assume that the flux path has negligible reluctance apart from the airgap.