

# EEE 3352

## Electromechanics & Electrical Machines



## Lecture 4: Single phase transformer



# 4. Single phase transformer

1. introduction
2. ideal transformer
3. referred quantities
4. real transformer
5. determination of transformer parameters from tests
6. transformer rating
7. regulation
8. efficiency



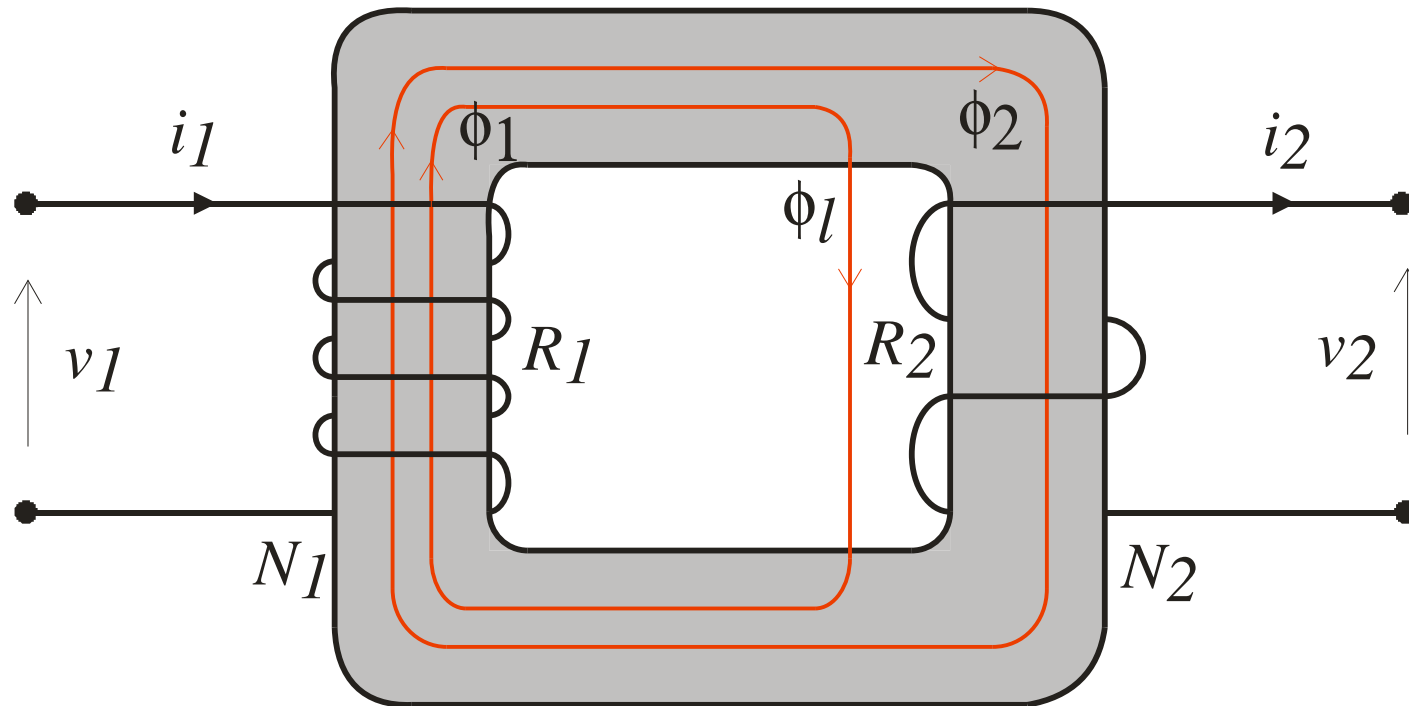
- At the end of the lecture, students should be able to
  - **describe** features of an ideal and real transformer
  - **derive** relations for voltage ratio and current ratio
  - **develop** an equivalent circuit of the real transformer
  - **express** the operating parameters of the transformer on a phasor diagram
  - **describe** the tests for determining parameters of the transformer
  - **determine** equivalent circuit parameters of the transformer from tests
  - **determine** rating of a transformer
  - **derive** expressions for performance of transformer: reg. and efficiency
  - **determine** the performance of transformer: regulation and efficiency

## 4.1 Introduction

*Demonstration transformer at KGRTC*



## Transformer schematic



- consists of  $\geq 2$  windings on a common magnetic circuit
- arranged so that all or nearly all flux linking 1<sup>st</sup> coil passes thru' 2<sup>nd</sup> coil
- achieved by well-defined low reluctance magnetic circuit linking the 2 coils

## 4.2 Ideal transformer

- ideal transformer applies 4 assumptions:
  - 1) No leakage flux
    - so  $\phi_1 = \phi_2$  ; or  $\phi_l = 0$
  - 2) No reluctance of magnetic circuit
    - $S = 0$  ; or  $\mu_r = \infty$
  - 3) No copper loss ( $P_{cu} = 0$ )
    - $R_1 = 0, R_2 = 0$
  - 4) No iron loss ( $P_{Fe} = 0$ )
    - $R_p = \infty$

- Faraday's law

$$\text{Coil 1: } v_1 = N_1 \frac{d\phi_1}{dt} \quad \Bigg| \quad \text{Coil 2: } v_2 = N_2 \frac{d\phi_2}{dt}$$

- with  $\phi_1 = \phi_2 = \phi$ , for ideal trx

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

- using rms values

$$\frac{V_{1i}}{V_{2i}} = \frac{N_1}{N_2}$$

- subscript *i* denotes 'ideal'

- voltage ratio = turns ratio

- analogous KVL round magnetic circuit

$$F_1 - F_2 = \phi S$$

$$N_1 i_1 - N_2 i_2 = \phi S$$

- for ideal trx,  $S = 0$

$$N_1 i_1 = N_2 i_2 \quad \text{or} \quad \frac{i_1}{i_2} = \frac{N_2}{N_1}$$

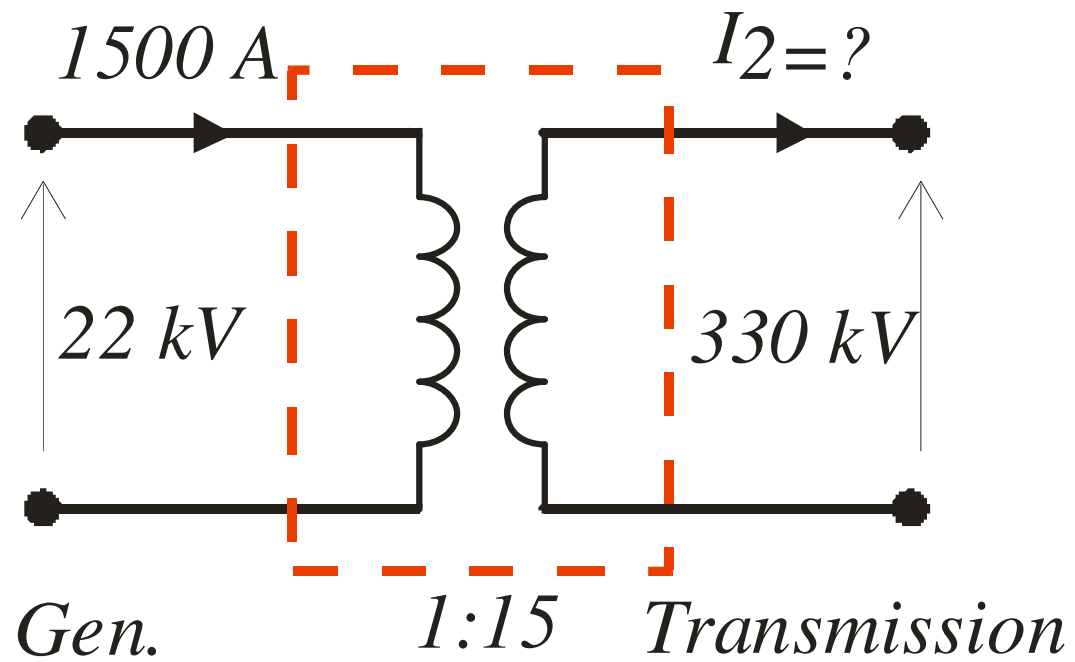
- using rms values

$$\frac{I_{1i}}{I_{2i}} = \frac{N_2}{N_1}$$

- hence

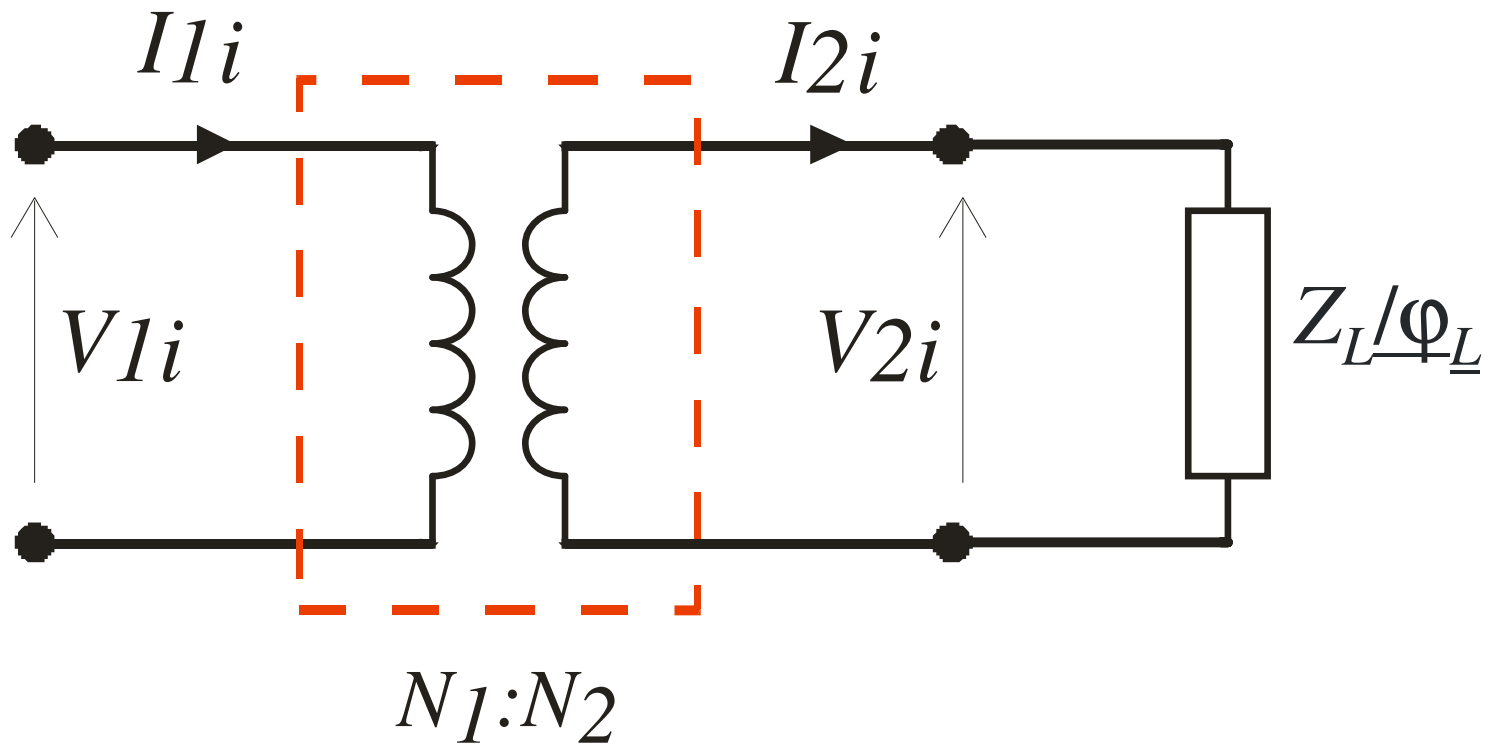
$$\frac{V_{1i}}{V_{2i}} = \frac{N_1}{N_2} = \frac{I_{2i}}{I_{1i}}$$

- example:

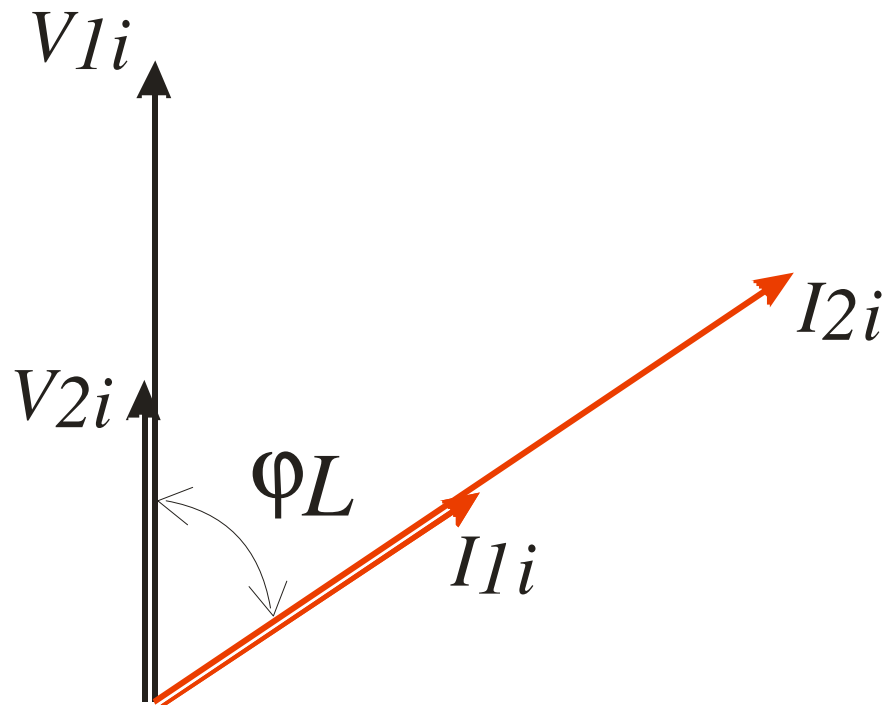


- on load:

*Circuit representation*

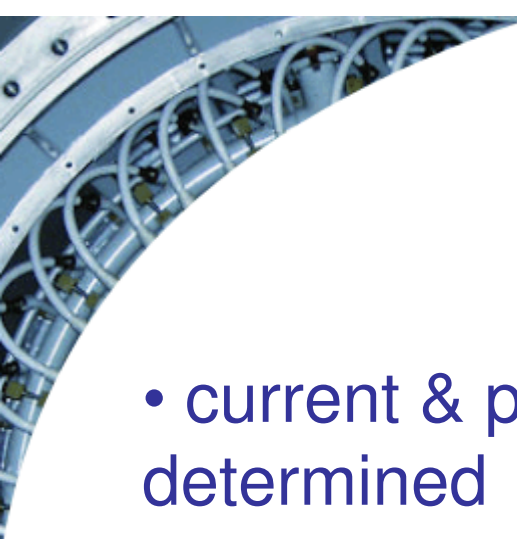


## Phasor diagram



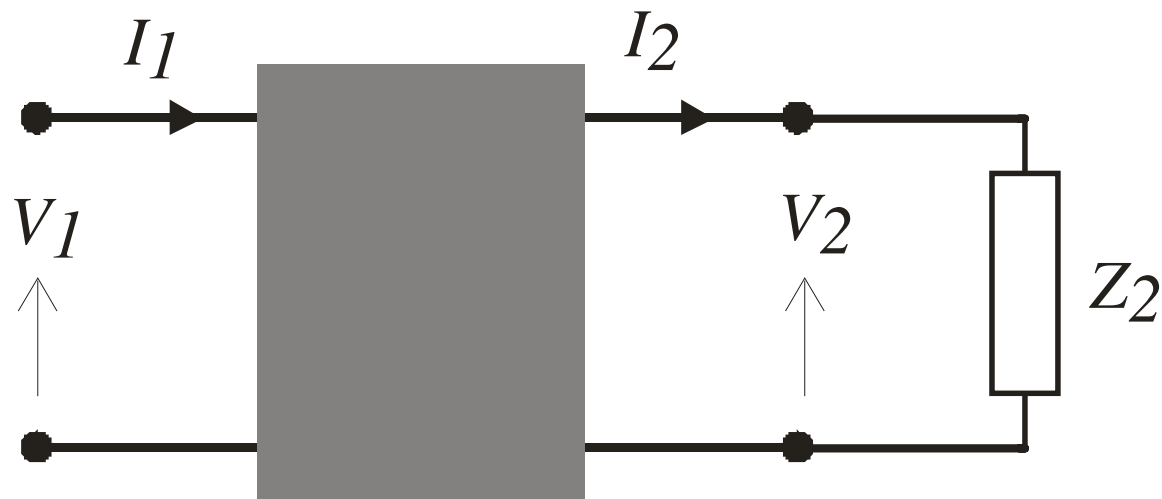
$$e.g. \frac{N_1}{N_2} = 2$$

$$I_{2i} = \frac{V_{2i}}{Z_L \angle \varphi_L}$$

- 
- current & phase angle both on primary and secondary are determined
    - by the load
    - not by the supply or power station
  - voltages are determined
    - by the supply or power station

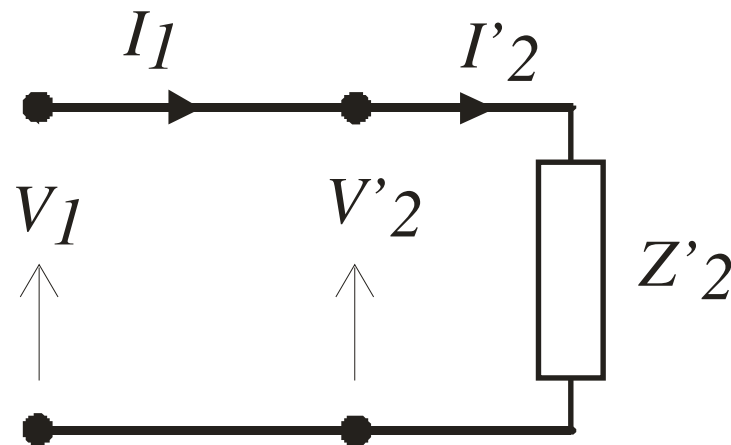
## 4.3 Referred quantities

Side 1 & 2 quantities



- referring  $V$ ,  $I$  and  $Z$  from one side of trx to the other:

All quantities are seen from side 1



- the dash ( ' ) is used to indicate a quantity has been referred from one side to the other

- what are  $V'_2$ ,  $I'_2$ , and  $Z'_2$  in terms of the original quantities?

$$V'_2 = V_1 = V_2 \frac{N_1}{N_2} \quad \longrightarrow \quad V'_2 = V_2 \frac{N_1}{N_2}$$

$$I'_2 = I_1 = I_2 \frac{N_2}{N_1} \quad \longrightarrow \quad I'_2 = I_2 \frac{N_2}{N_1}$$

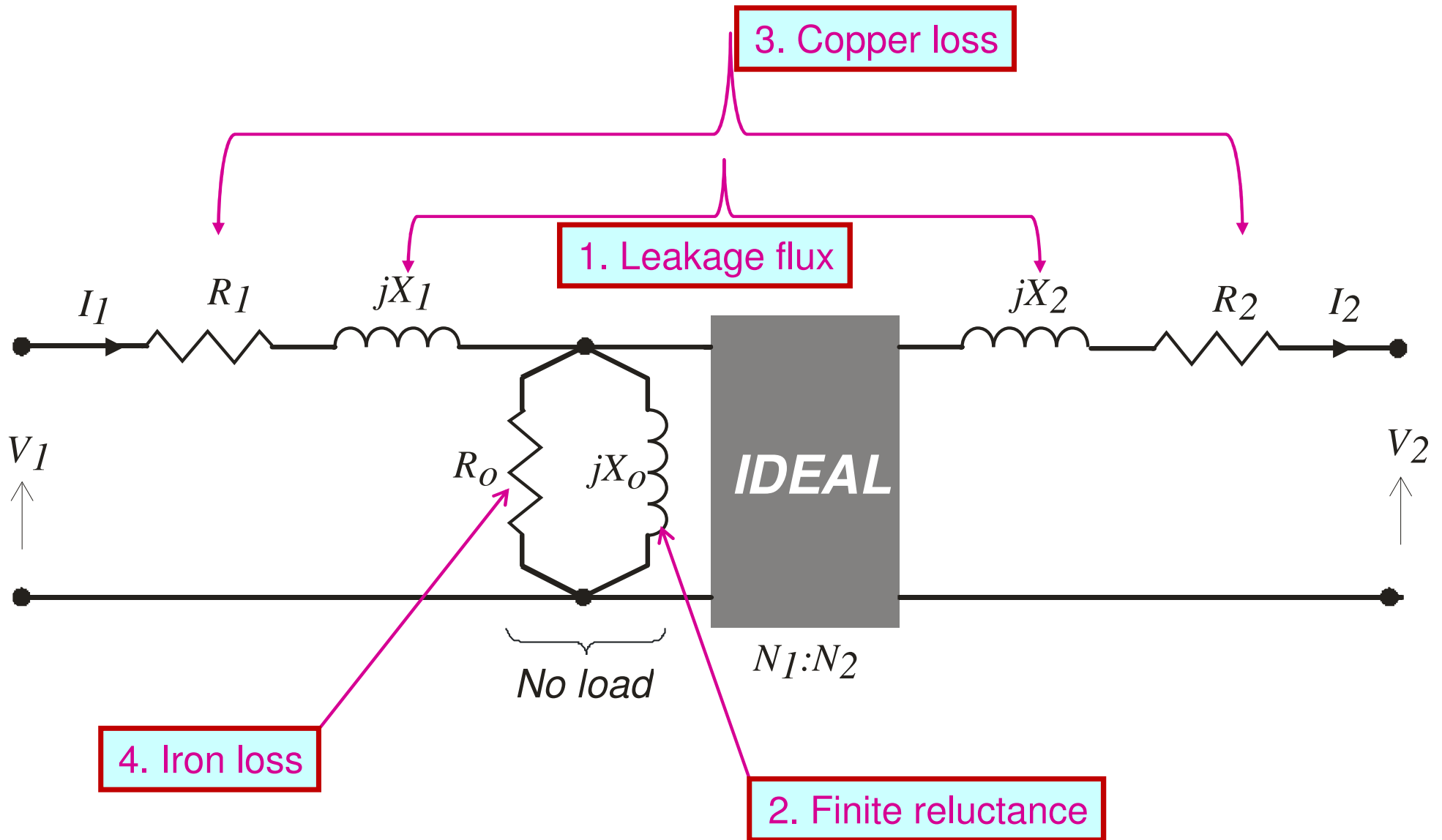
$$Z'_2 = \frac{V'_2}{I'_2} = \frac{V_2 \frac{N_1}{N_2}}{I_2 \frac{N_2}{N_1}} \quad \longrightarrow \quad Z'_2 = Z_2 \left( \frac{N_1}{N_2} \right)^2$$





## 4.4 Real transformer

- real (practical) transformer has:
  - 1) leakage flux,
    - $\phi_l \neq 0$
  - 2) finite reluctance of magnetic circuit
    - $S = \text{finite}$  ;
  - 3) copper loss ( $P_{cu} \neq 0$ )
    - $R_1 \neq 0, R_2 \neq 0$
  - 4) iron loss ( $P_{Fe} \neq 0$ )
    - $R_p = \text{finite}$

## 4.4.1 Equivalent circuit



- 
- **1:** leakage flux:
    - represented by two reactances in the **primary** and **secondary**  $X_1$  and  $X_2$ ;
    - they have small values compared to any other reactances
  
  - **2:** reluctance of magnetic circuit is not zero:
    - so there is a magnetising flux or mutual flux produced by magnetising current;
    - the effect is represented by a magnetising reactance  $X_o$


$$I_1 N_1 - I_2 N_2 = \phi_m S$$

- $\phi_m$  is the mutual flux produced by magnetising current

$$\phi_m S = I_{mag} N_1$$


$$I_1 N_1 = I_2 N_2 + \phi_m S$$

- on no-load,  $I_2 = 0$ , and

$$I_1 = I_{mag} = \frac{V_1}{jX_o} = \frac{V_1 S}{j\omega N_1^2}$$

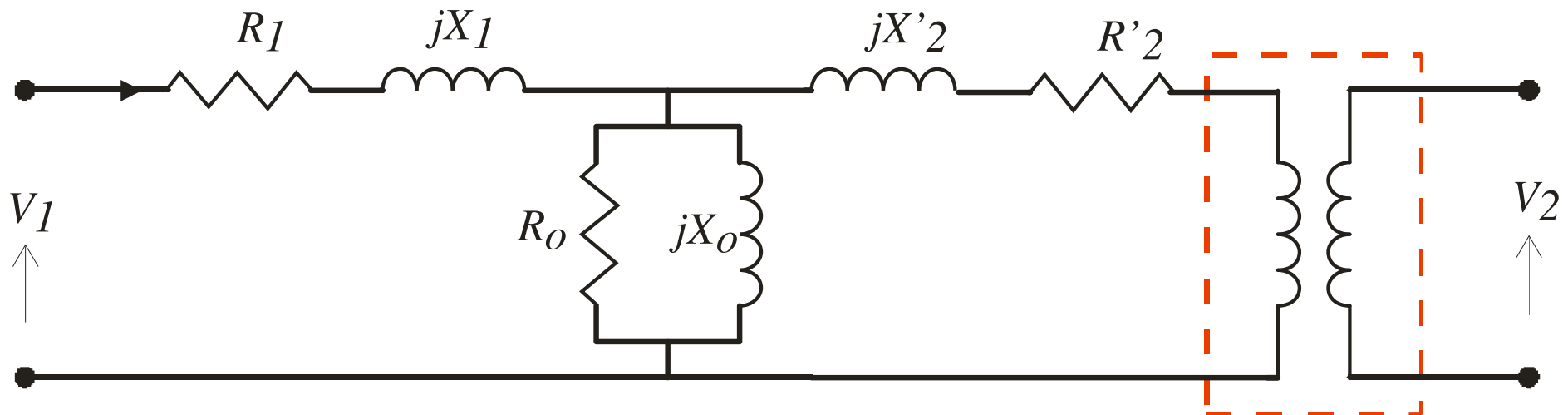
since

$$L_o = \frac{N_1^2}{S}$$

- 
- **3:** winding resistance, which incurs  $P_{cu}$ ,
    - is represented by the resistances  $R_1$  and  $R_2$ ,
    - for the **primary** and **secondary**, respectively
  
  - **4:** Iron loss,  $P_{Fe}$ ,
    - is represented by a resistance  $R_o$
    - connected across the supply terminals

## 4.4.2 Simplifying equivalent circuit

1) Move  $R_2$  and  $jX_2$  from the secondary to primary side



$$R'_2 \rightarrow R_2 \left( \frac{N_1}{N_2} \right)^2$$

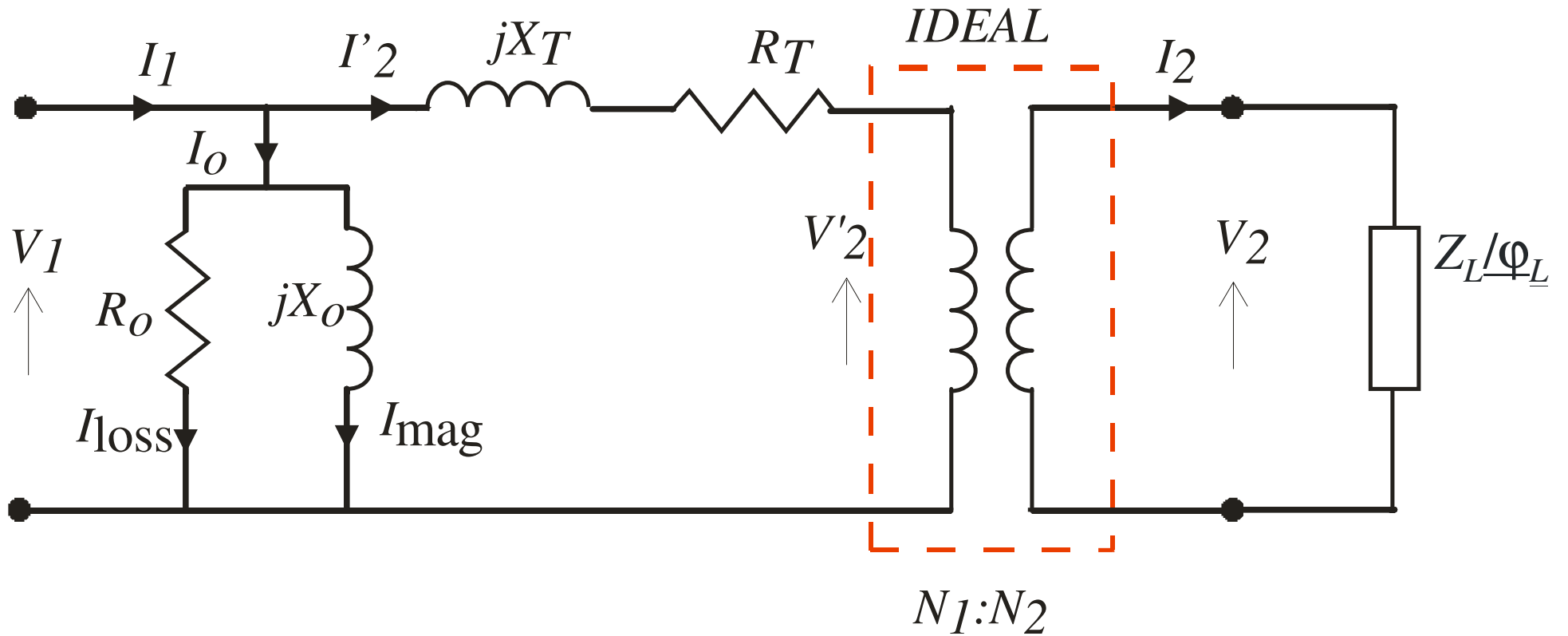
$$jX'_2 \rightarrow jX_2 \left( \frac{N_1}{N_2} \right)^2$$




2) Change  $R_o$  and  $jX_o$  across the supply terminals

- strictly not legitimate
- series impedances in the circuit are much smaller than parallel components
- $\therefore$  the differences (errors) are insignificant

## Final equivalent circuit



$$R_T = R_1 + R'_2$$
$$jX_T = j(X_1 + X'_2)$$

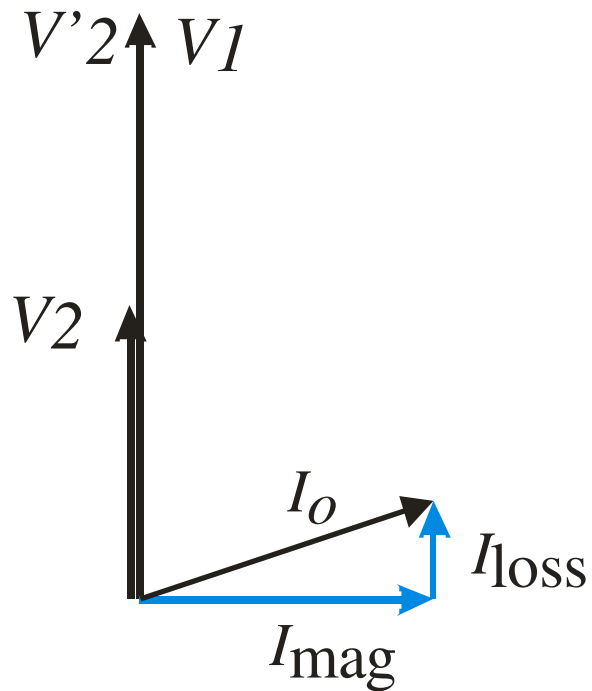
- 
- In final equivalent circuit, the designer ensures
    - $R_T$  and  $X_T$  are small compared to the load impedance  $Z'_L$
    - $I_o$  is small, so that  $R_o$  and  $X_o$  are large ( $\gg Z'_L$ )
      - small  $P_{Fe}$  gives large  $R_o$
      - large magnetising inductance  $L_m$ , or small  $S$ , gives large  $X_o$

- 
- Example:  $N_1/N_2 = 2$ , voltage equation is

$$\bar{V}_1 = \bar{V}'_2 + R_T \bar{I}'_2 + jX_T \bar{I}'_2$$

## 4.4.3 Phasor diagrams

(a) On no-load



$$I_{Loss} = \frac{V_1}{R_o}$$

$$I_{mag} = \frac{V_1}{X_o}$$

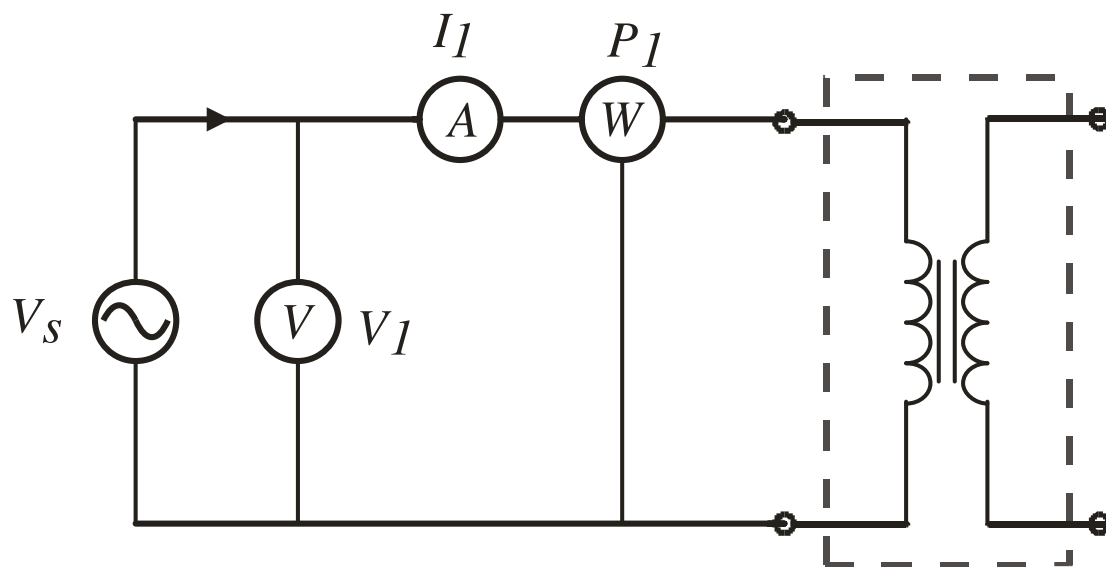
$$I_o^2 = I_{Loss}^2 + I_{mag}^2$$

$$P_{Fe} = \frac{V_1^2}{R_o}$$



## 4.4 Determination of transformer parameters by tests

### 4.4.1 Open circuit test



- 1) Apply  $V_1 = \text{rated voltage}$
- 2) Measure  $V_1, I_1, P_1$



1

$$P_1 = \frac{V_1^2}{R_o} \rightarrow R_o$$

- gives  $R_o$  and  $X_o$



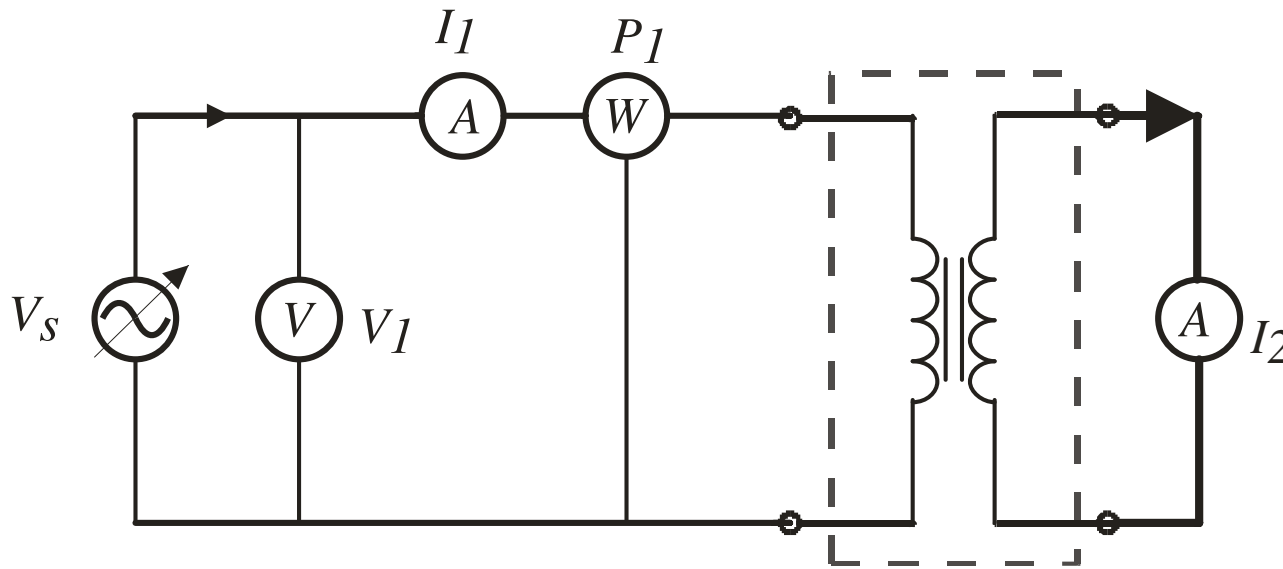
2

$$I_{Loss} = \frac{V_1}{R_o}$$

$$I_{mag}^2 + I_{Loss}^2 = I_o^2$$

$$X_o = \frac{V_1}{I_{mag}}$$

## 4.4.2 Short circuit test



- 1) Apply low  $V_1$ , gradually increasing until rated current flows
- 2) Measure  $V_1$ ,  $I_1$ ,  $P_1$



1

$$I_1^2 R_T = P_1 \rightarrow R_T$$

- gives  $R_T$  and  $X_T$

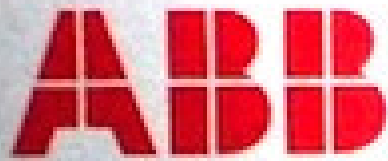
2

$$Z_T = \frac{V_1}{I_1}$$

$$Z_T^2 = R_T^2 + X_T^2$$

## 4.5 Transformer rating

$V_1$	$V_2$	$I_1$	$I_2$	$VA$
$4.44N_1AfB_m$				$VA = S_r = V_1I_1 = V_2I_2$
1	2	3	4	5



Made in New Zealand

ONAN TRANSFORMER TO **BS EN 60076-1997**

PHASE

FREQUENCY  Hz

POWER  KVA

H VOLTS  V

OIL  l

VECTOR

L VOLTS  V

OIL MASS  kg

IMP. (Z)  %

HV AMPS  A

CORE WDG  kg

DIAG No.

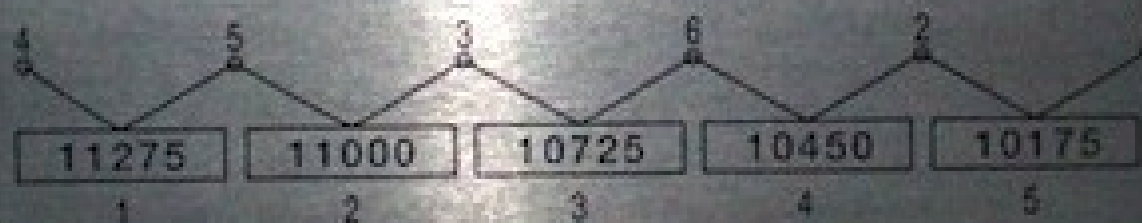
LV AMPS  A

TOTAL  kg


S/No.

MNFD

TAPPING SEQUENCE

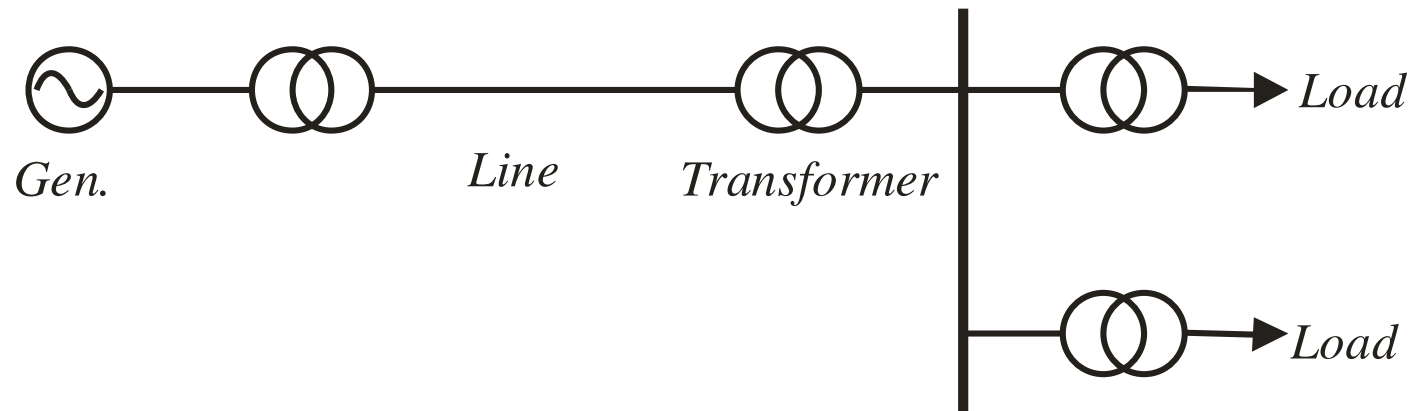


TAPPING SWITCH POSITION

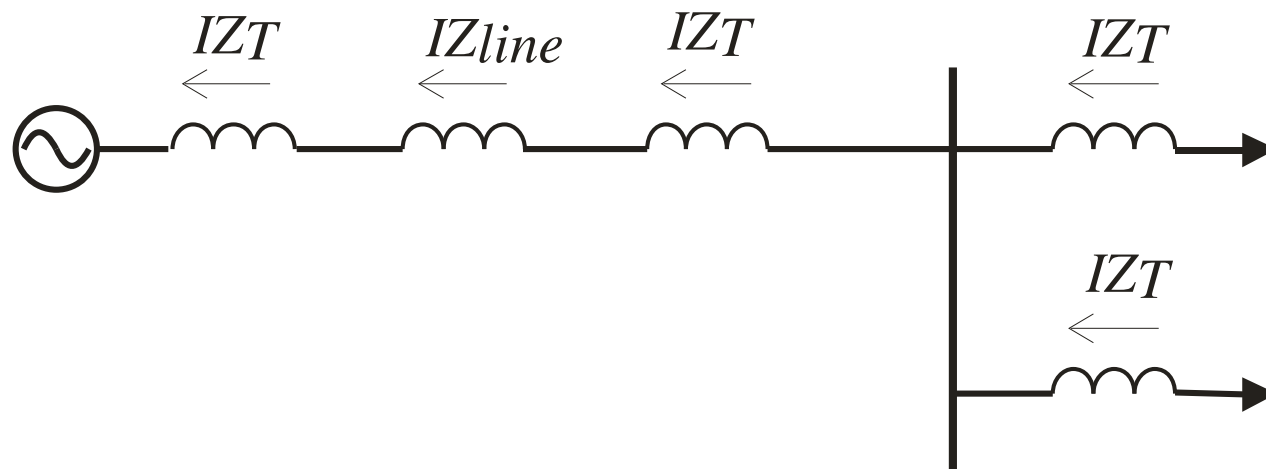
- 
- Any 3 of the 5 quantities are essential to define trx rating,
    - the other 2 can be calculated
  - $VA$  rating for ideal trx is the same on primary as on secondary
  - for real trx, the primary and secondary  $VA$  are slightly different, but so similar that there is no distinction between the two


# 4.6 Voltage regulation

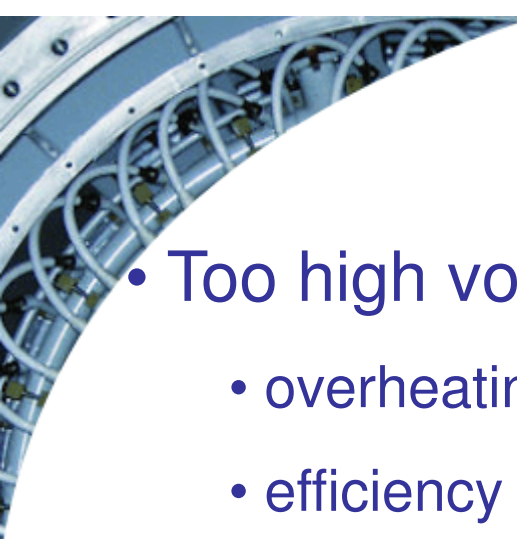
## Power system



## Equivalent circuit of power system



- 
- Primary voltage of trx must be such that
    - delivers rated output power
    - maintains rated voltage at secondary
  - Due to effect of transformer impedance
    - secondary voltage may vary from rated value
  - Voltage variations are important to consumer
    - virtually all equipment is designed for some rated voltage

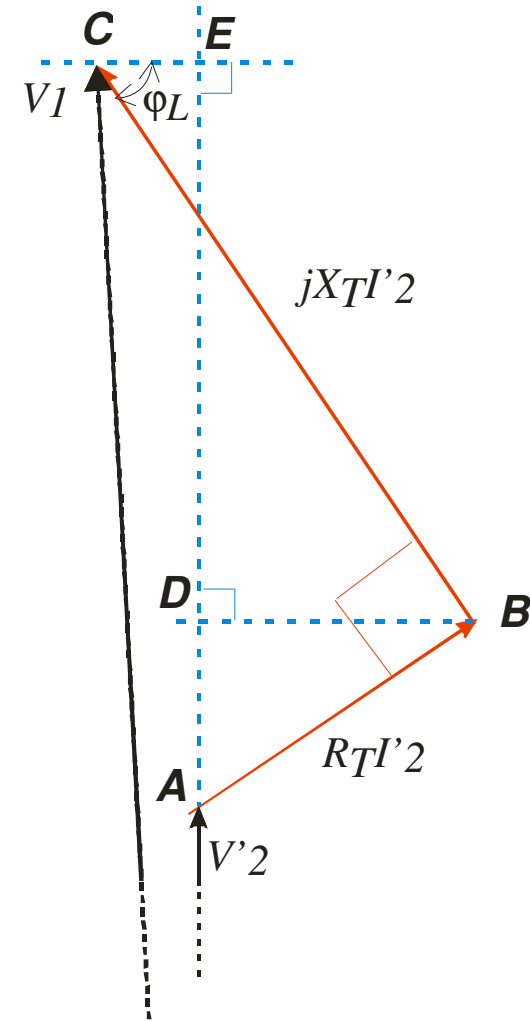
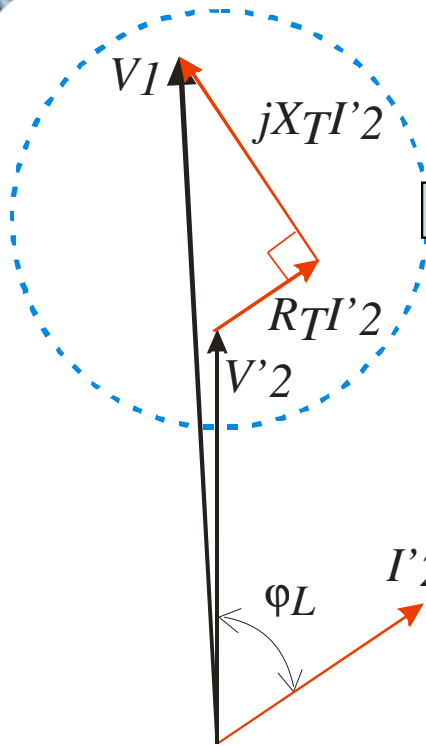
- 
- Too high voltage:
    - overheating ( shortening life of equipment)
    - efficiency may suffer
  - Too low voltage
    - low power output (light, heat, mechanical power)
    - drop in efficiency
  - In practice:
    - supply and consumer agree on tolerance of variation
      - e.g.  $\pm 5\%$
    - variation governed by rules
      - eg Zambian regulations / standards, ZS 387/397



- Regulation definition:

- fractional drop of voltage from no-load to some specified load, while keeping supply voltage constant

$$\begin{aligned} \text{Reg} &= \frac{|V_{2NL}| - |V_{2L}|}{|V_{2NL}|} \Big|_{V_1 = \text{const}} \\ &= \frac{|V'_{2NL}| - |V'_{2L}|}{|V'_{2NL}|} \Big|_{V_1 = \text{const}} \\ &= \frac{|V_1| - |V'_{2L}|}{|V_1|} \Big|_{V_1 = \text{const}} \end{aligned}$$



$$|V_1| - |V'_{2L}| = OC - OA$$

$$OC \approx OE$$

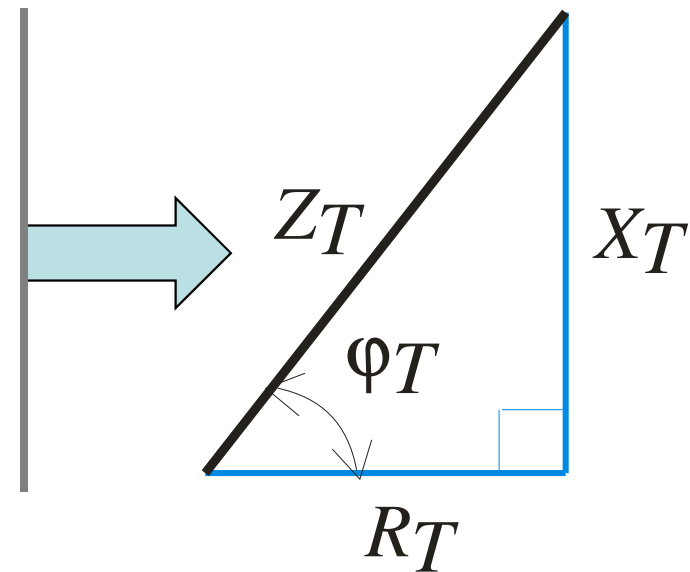
$$|V_1| - |V'_{2L}| = OE - OA$$

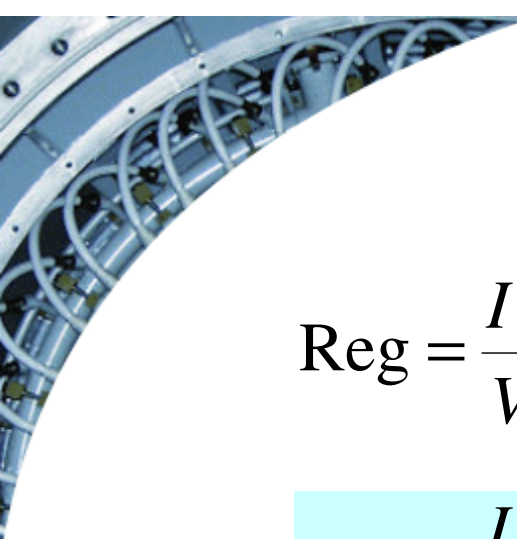
$$= AD + DE$$

$$= I'_2 R_T \cos \phi_L + I'_2 X_T \sin \phi_L$$

$$\begin{aligned} \text{Reg} &= \frac{|V_1| - |V'_{2L}|}{|V_1|} \\ &= \frac{I'_2 R_T \cos \varphi_L + I'_2 X_T \sin \varphi_L}{V_1} \\ &= \frac{I'_2}{V_1} (R_T \cos \varphi_L + X_T \sin \varphi_L) \end{aligned}$$

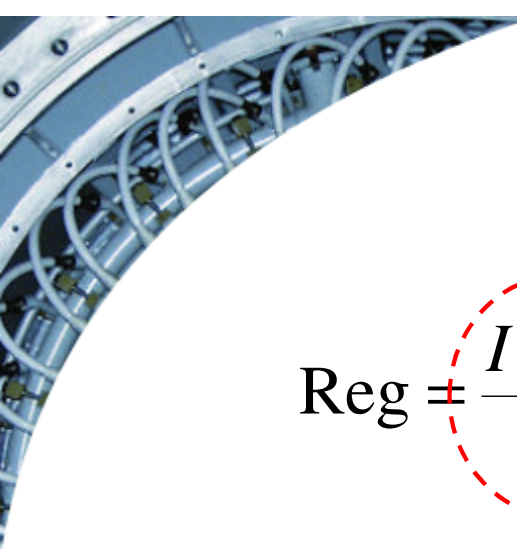
- Define  $Z_T$  and  $\varphi_T$ , using the impedance triangle
  - (these are constant for a given trx)




$$\text{Reg} = \frac{I_2'}{V_1} (Z_T \cos \varphi_T \cos \varphi_L + Z_T \sin \varphi_T \sin \varphi_L)$$

$$\text{Reg} = \frac{I_2' Z_T}{V_1} \cos(\varphi_T - \varphi_L)$$

- Reg depends on
  - trx parameters ( $R_T$ ,  $X_T$  or  $Z_T$ ,  $\varphi_T$ ) and
  - load conditions ( $I_2$ ,  $\varphi_L$ )


$$\text{Reg} = \frac{I_2' Z_T}{V_1} \cos(\phi_T - \phi_L)$$

all primary

From primary side

$$\text{Reg} = \frac{I_2 Z_{T \text{ sec}}}{V_2} \cos(\phi_{T \text{ sec}} - \phi_L)$$

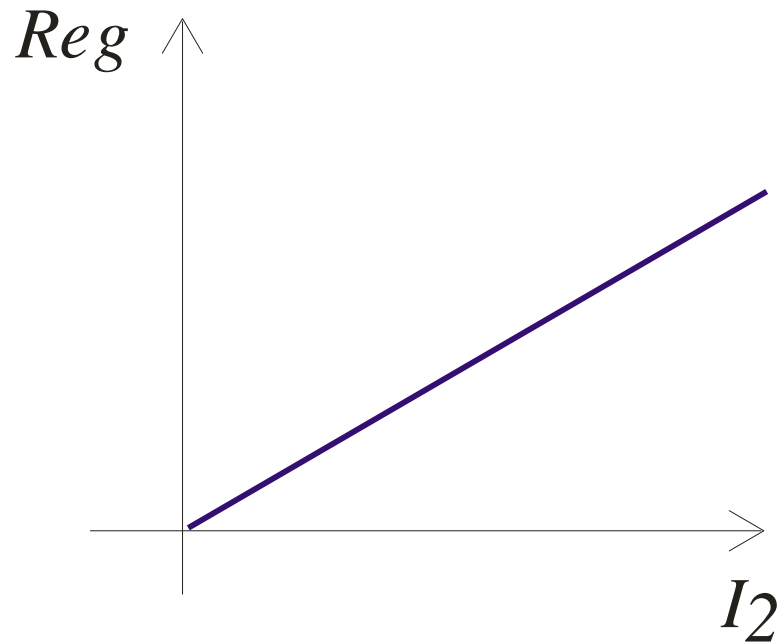
all secondary

From secondary side

## Variation of Reg with load current and p.f.

$$\text{Reg} = \frac{I_2' Z_T}{V_1} \cos(\varphi_T - \varphi_L)$$

- with p.f., const, Reg varies linearly with load current



- 
- at specified current, Reg varies with p.f., as given by fn

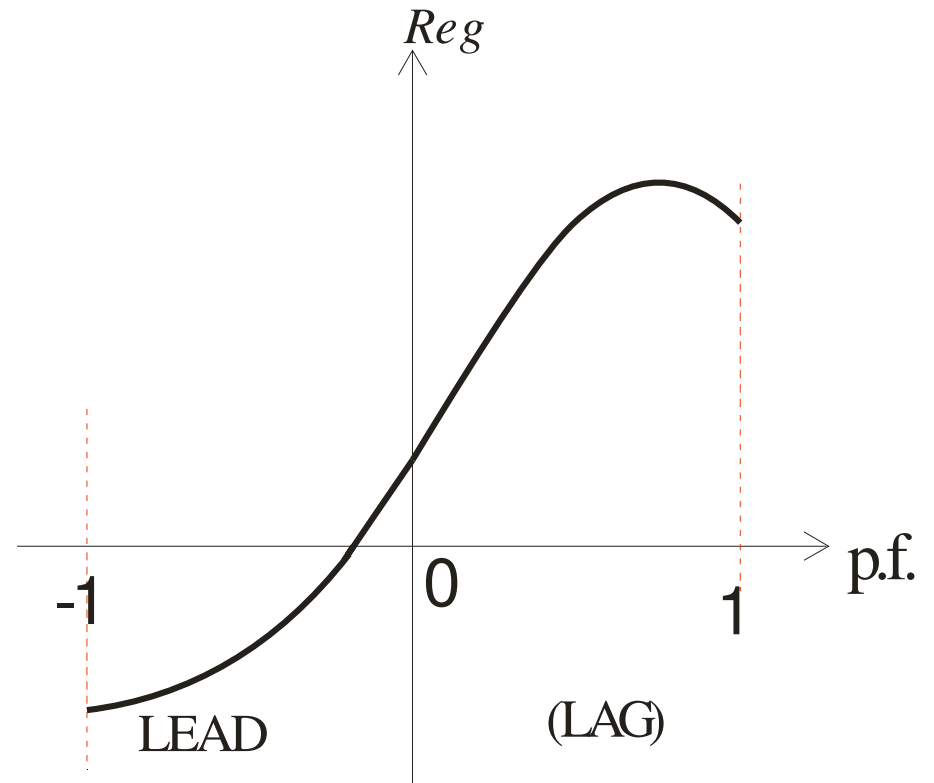
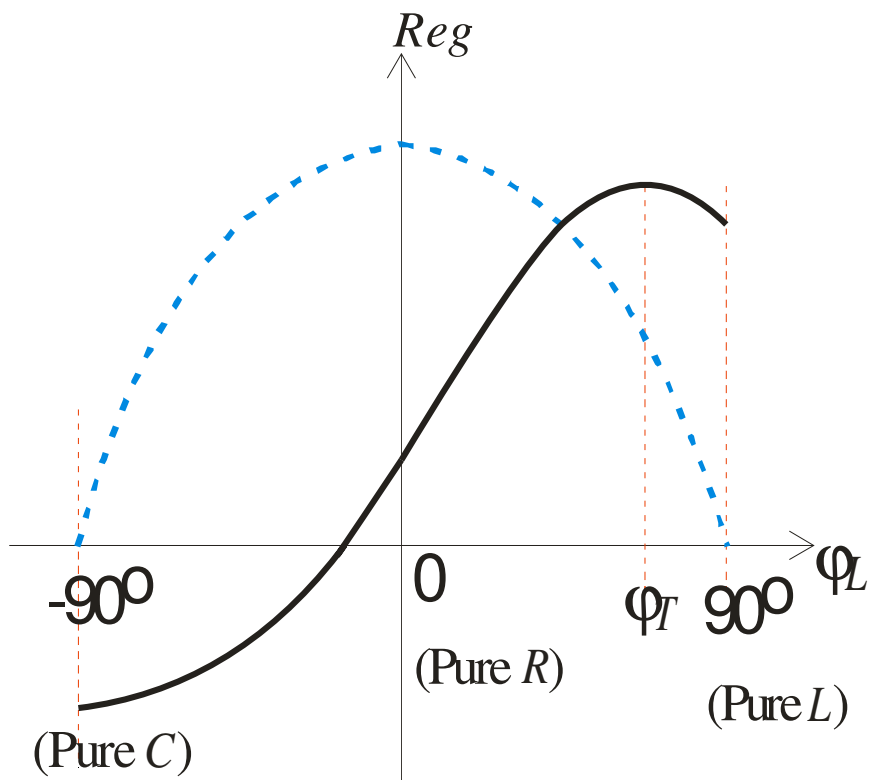
$$\text{Reg} = k_{reg} \cos(\varphi_T - \varphi_L)$$

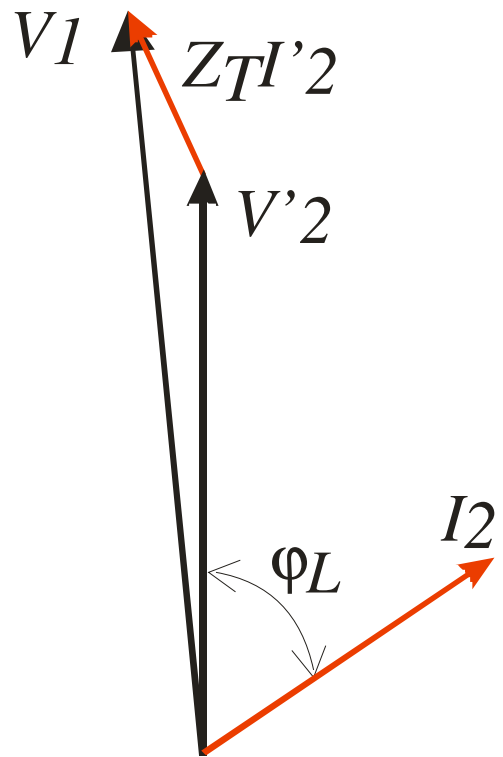
- Max. Reg occurs when  $\varphi_L = \varphi_T$  and has value

$$\text{Reg} = \frac{I'_2 Z_T}{V_1}$$

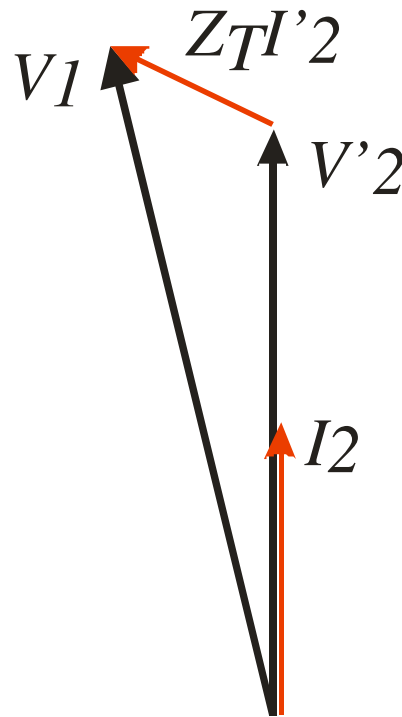
- Zero Reg occurs when  $\varphi_L - \varphi_T = \pm 90^\circ$

- in terms of p.f., graph is:

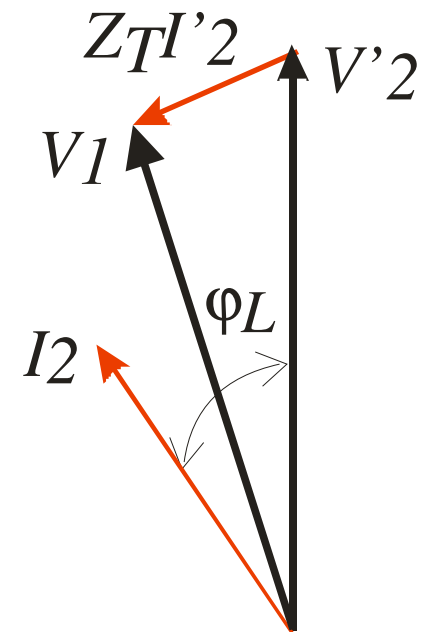




Inductive load



Resistive load



Capacitive load

- eg:  $\phi_L = 60^\circ \rightarrow \text{p.f.} = 0.5 \text{ (lag)}$   
 $\phi_L = -60^\circ \rightarrow \text{p.f.} = 0.5 \text{ (lead)}$

## 4.7 Efficiency

power efficiency,  $\eta = \frac{\text{Output power, } P_{out}}{\text{Input power, } P_{in}}$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + \text{Losses}} = \frac{V_2 I_2 \cos \phi_L}{V_2 I_2 \cos \phi + P_{Fe} + I_2^2 R_{T \text{ sec}}}$$

$$\eta = f(I_2, \phi_L)$$

- define ratio

$$x = \frac{I_2}{I_{2FL}} \quad I_2 = xI_{2FL}$$

- $x$  gives the function of load current

$$\eta = \frac{V_2(xI_{2FL}) \cos \varphi_L}{V_2(xI_{2FL}) \cos \varphi_L + P_{Fe} + (xI_{2FL})^2 R_{T \text{ sec}}}$$

$$\eta = \frac{xS_r \cos \varphi_L}{xS_r \cos \varphi_L + P_{Fe} + x^2 P_{CuFL}}$$

$S_r$  = rated VA

# Maximum efficiency

1) variable p.f., *current* constant:

$$\eta = \frac{xS_r}{xS_r + \frac{P_{Fe} + x^2 P_{CuFL}}{\cos \varphi_L}}$$

- denominator must be minimum
  - occurs when  $\cos \varphi_L$  is max.,
  - i.e.  $\cos \varphi_L = 1$ , unity p.f.

- variable load current,  $x$ , p.f. const.

$$\eta = \frac{S_r \cos \varphi_L}{S_r \cos \varphi_L + \frac{P_{Fe}}{x} + xP_{CuFL}}$$

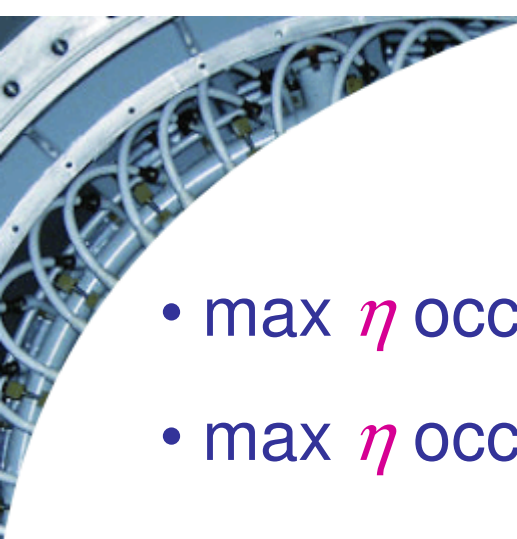
- denominator must be minimum

$$\frac{d}{dx} \left( S_r \cos \varphi_L + \frac{P_{Fe}}{x} + xP_{CuFL} \right) = 0$$

$$0 - \frac{P_{Fe}}{x^2} + P_{CuFL} = 0$$

$$x^2 P_{CuFL} = P_{Fe}$$

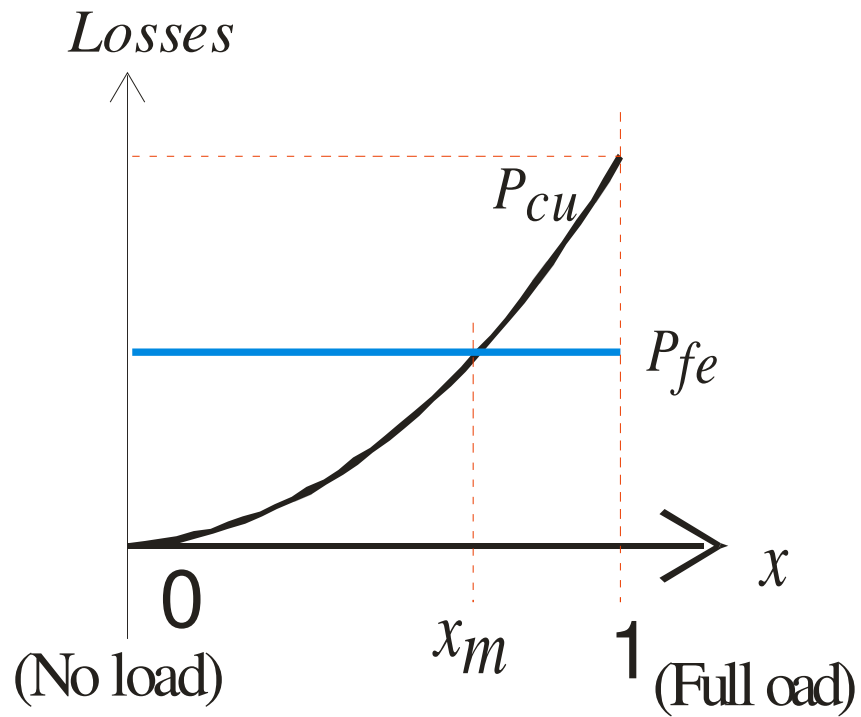
$$P_{Cu} = P_{Fe}$$

- 
- max  $\eta$  occurs when variable  $P_{Cu}$  has the value of the const  $P_{Fe}$
  - max  $\eta$  occurs when

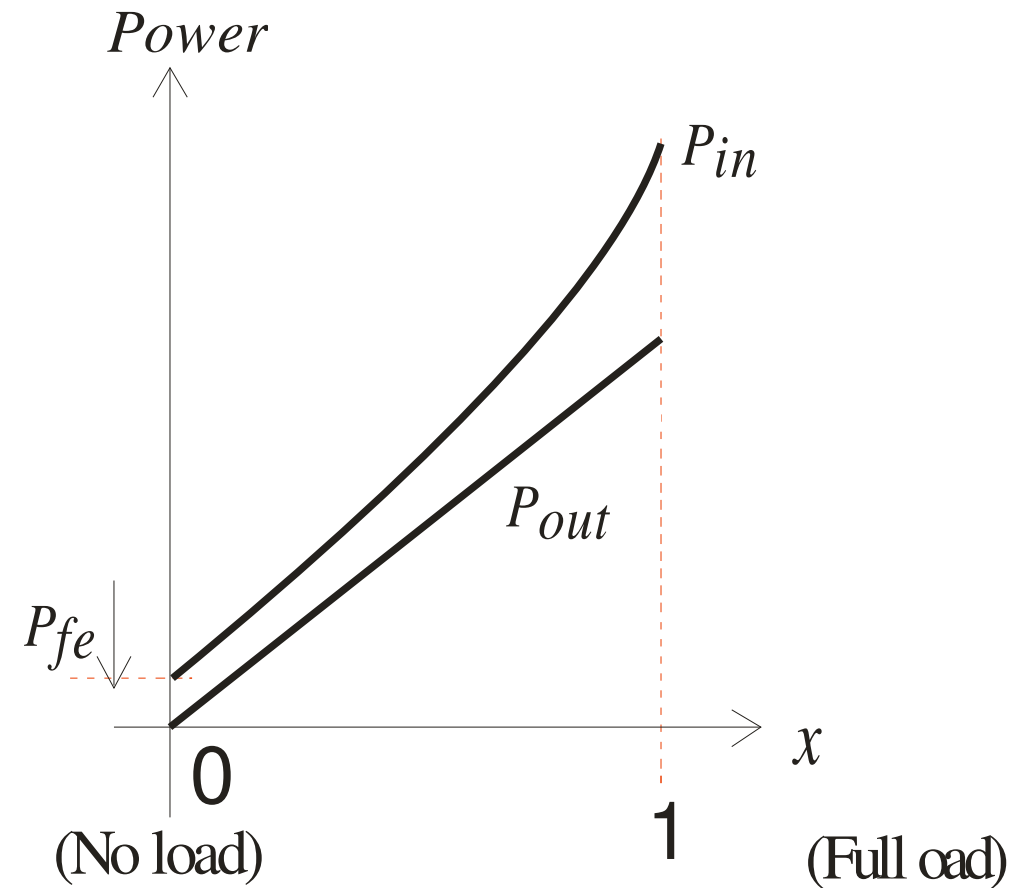
$$x^2 = \frac{P_{Fe}}{P_{CuFL}}$$

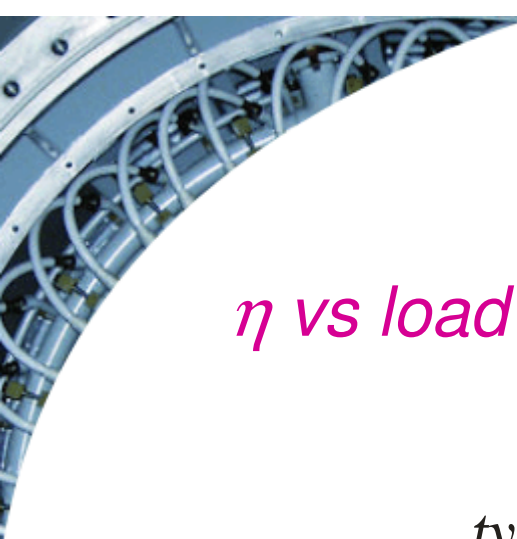
$$x = \sqrt{\frac{P_{Fe}}{P_{CuFL}}}$$

## Variation of losses vs load

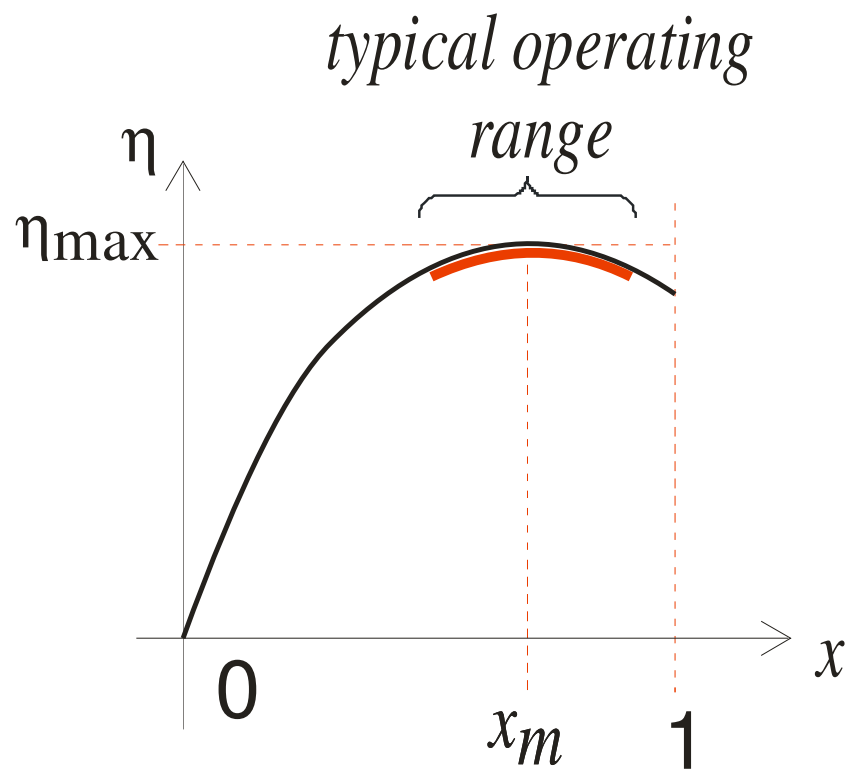


## Variation of power vs load

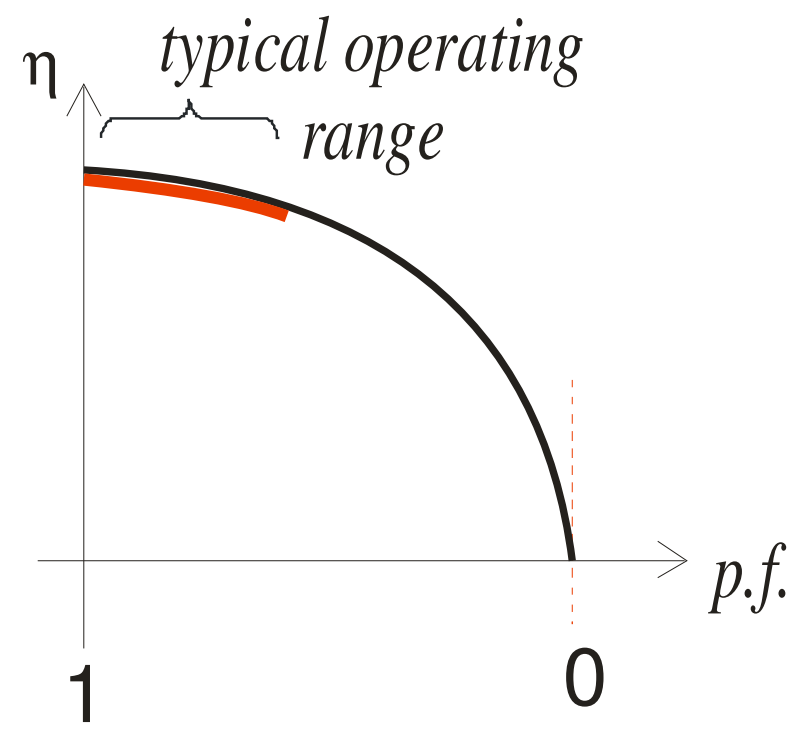




$\eta$  vs load



$\eta$  vs p.f.





## Examples:

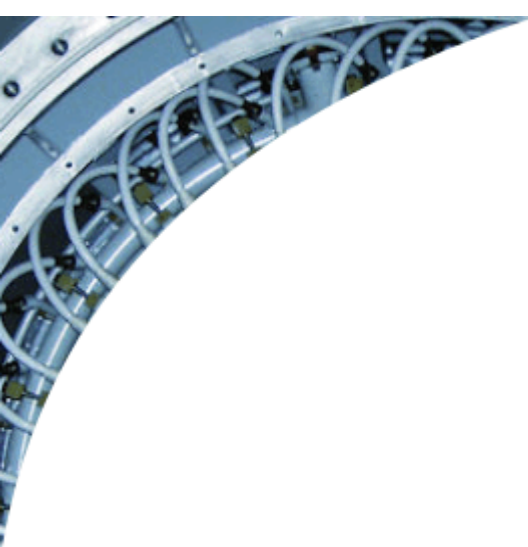
1.

- A 220/100-V transformer has a total resistance of  $2 \Omega$  and a total leakage reactance of  $3 \Omega$  both referred to the primary. The secondary current is 10 A.
- Find the
  - regulation at zero power factor lag
  - regulation at unity power factor
  - regulation at zero power factor lead
  - power factor which gives maximum regulation
  - value of the maximum regulation
  - power factor which gives zero regulation



2.

- A single-phase transformer is rated at 10 kVA, 240/100 V
- When the secondary terminals are open-circuited and the primary winding is supplied at normal voltage, the input current is 2.6 A at a power factor of 0.3 lag
- When the secondary terminals are short-circuited, a voltage of 18 V applied to the primary causes the full-load current to flow in the secondary, the power input to the primary being 240 W.
- Calculate, at full load unity power factor, the:
  - regulation
  - efficiency



- End of Lecture 4 -