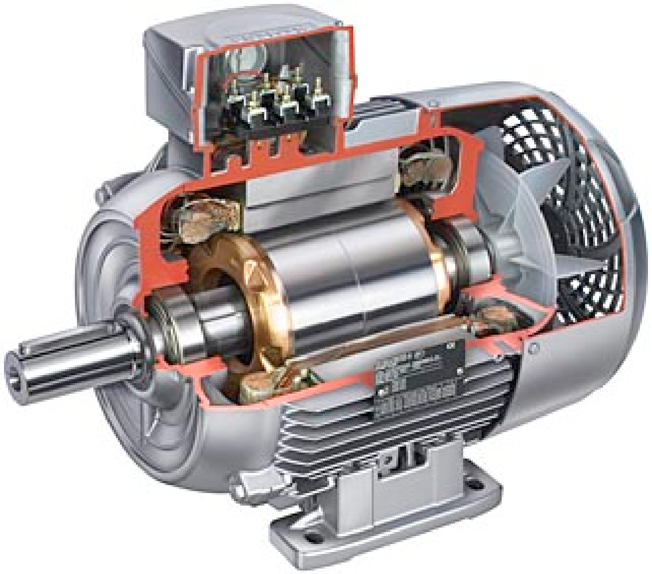
EEE 3352

Electromechanics & Electrical Machines



Lecture 8: Three phase AC machines

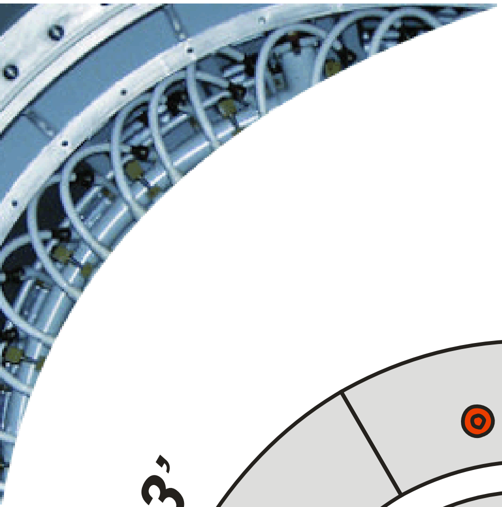
Dr A Zulu

8. 3-Phase AC Machines

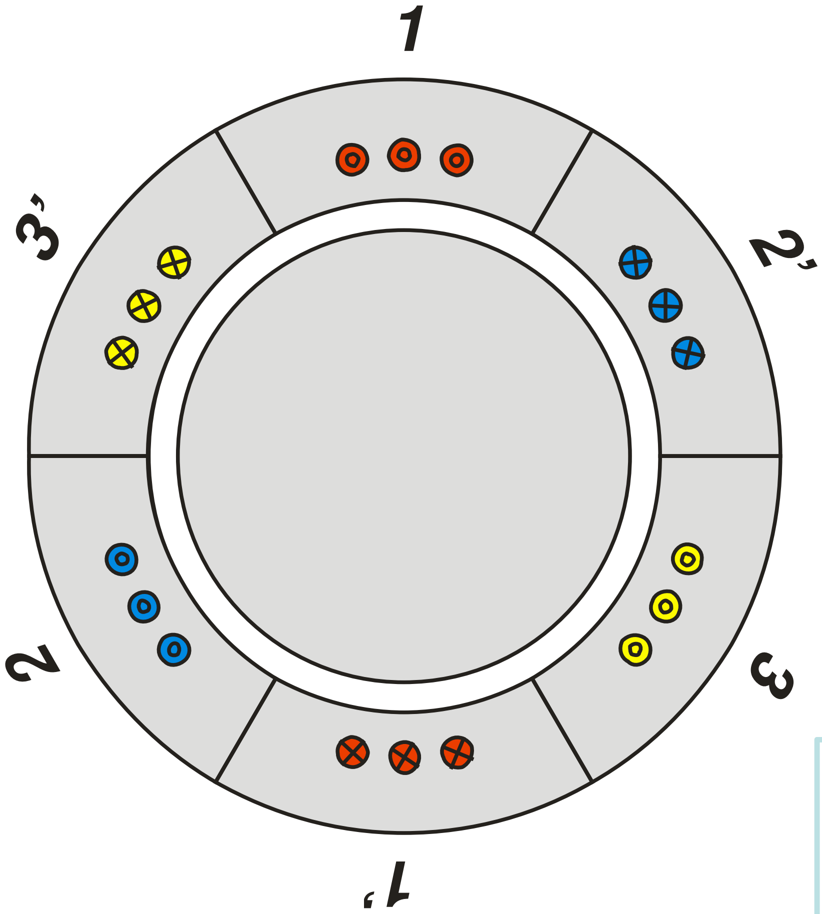
1. Production of rotating magnetic field
2. Induction machine
3. Synchronous machine

Objectives:

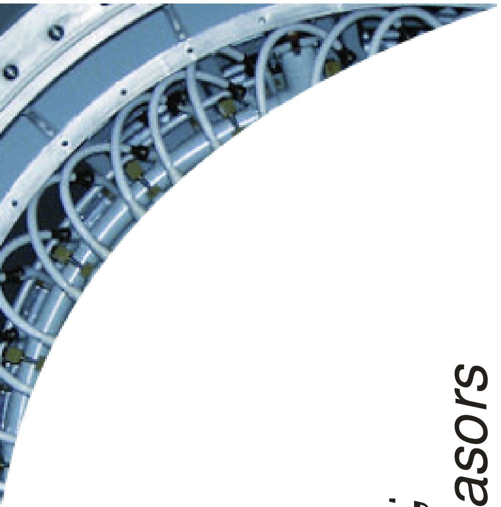
- at the end of the lecture, students should be able to
  - show the process for producing a rotating magnetic flux from three phase currents
  - explain the principle of operation of the three phase induction motor
  - determine the proportions of distribution of power in the elements of the induction motor
  - explain the principle of operation of the synchronous machine
  - differentiate the different modes of operation of the synchronous machine



8.1. Rotating magnetic field

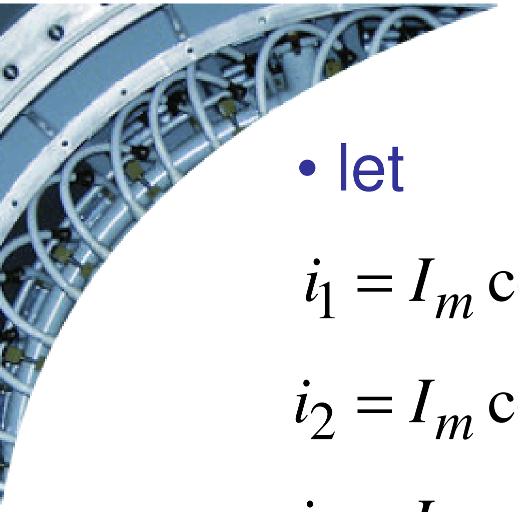


- balanced 3-phase supply
- 3 identical coils 120° from each other
- each coil has *N* -turns



instantaneous;
as sinusoid, use phasors

$$\left.\begin{array}{l} i_1 \xrightarrow{N} F_1 \\ i_2 \rightarrow F_2 \\ i_3 \rightarrow \underset{\text{vectors}}{F_3} \end{array}\right\} \Sigma F \xrightarrow{S} \underset{\text{vector}}{\phi} \rightarrow \left\{\begin{array}{l} e_1 \\ e_2 \\ e_3 \end{array}\right.$$



- let

$$i_1 = I_m \cos \omega t$$

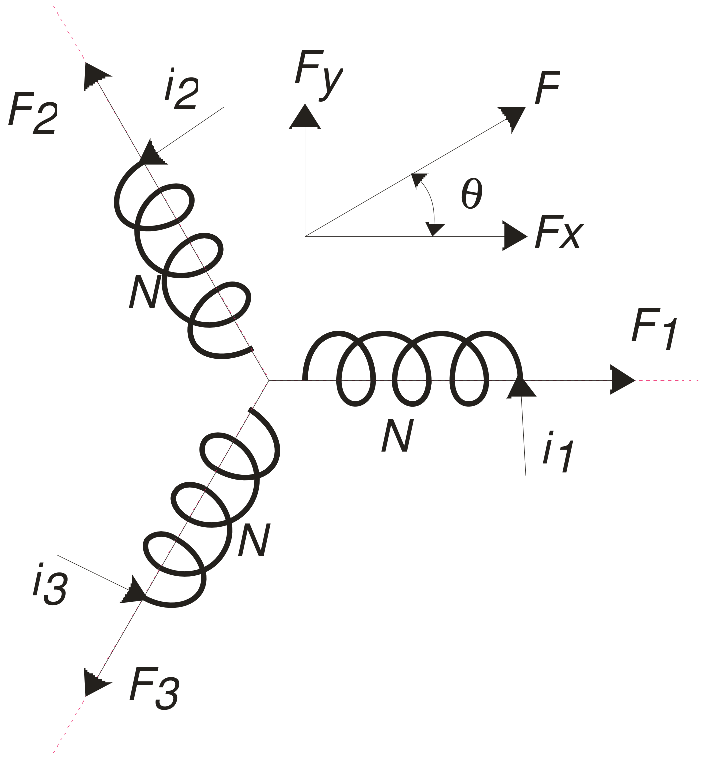
$$i_2 = I_m \cos(\omega t - 120^\circ)$$

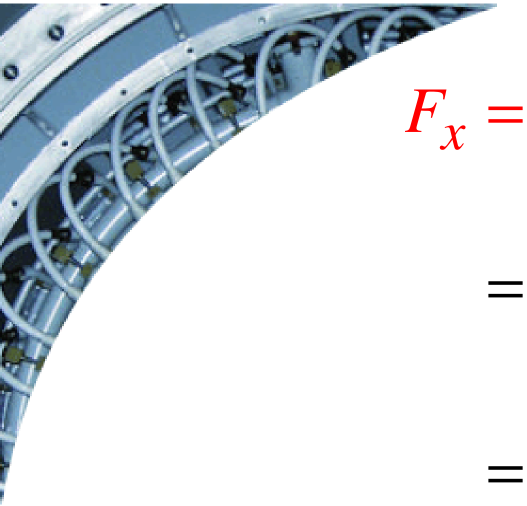
$$i_3 = I_m \cos(\omega t + 120^\circ)$$

$$F_1 = NI_m \cos \omega t$$

$$F_2 = NI_m \cos(\omega t - 120^\circ)$$

$$F_3 = NI_m \cos(\omega t + 120^\circ)$$





$$F_x = F_1 - \frac{1}{2}(F_2 + F_3)$$

$$= NI_m \left[\cos\omega t - \frac{1}{2}\left\{ \cos(\omega t - 120^\circ) + \cos(\omega t + 120^\circ) \right\} \right]$$

$$= NI_m \left[\cos\omega t - \frac{1}{2} 2\cos\omega t \cos 120^\circ \right]$$

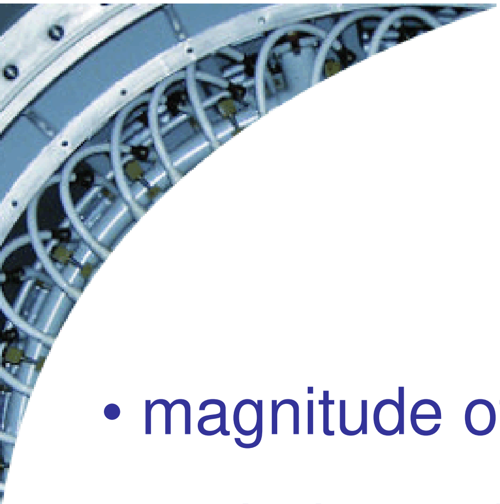
$$F_x = \frac{3}{2} NI_m \cos\omega t$$

$$F_y = \frac{\sqrt{3}}{2}(F_2 - F_3)$$

$$= \frac{\sqrt{3}}{2} NI_m \left[\cos(\omega t - 120^\circ) - \cos(\omega t + 120^\circ) \right]$$

$$= \frac{\sqrt{3}}{2} NI_m 2\sin\omega t \sin 120^\circ$$

$$F_y = \frac{3}{2} NI_m \sin\omega t$$



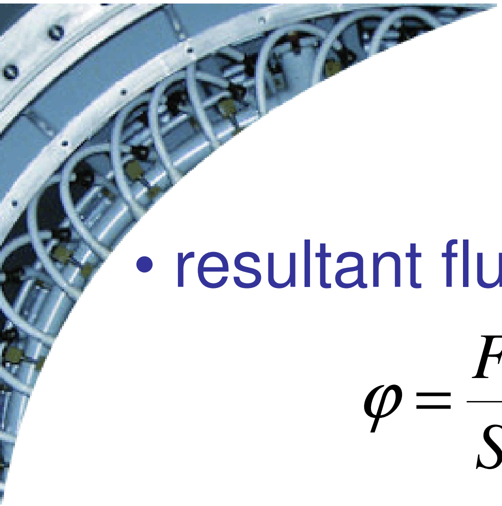
$$|F| = \sqrt{F_x^2 + F_y^2} = \frac{3}{2} N I_m$$

- magnitude of the resultant mmf in space is
    - independent of time
    - constant

$$\tan\theta = \frac{F_y}{F_x} = \tan\omega t$$

$$\theta = \omega t$$

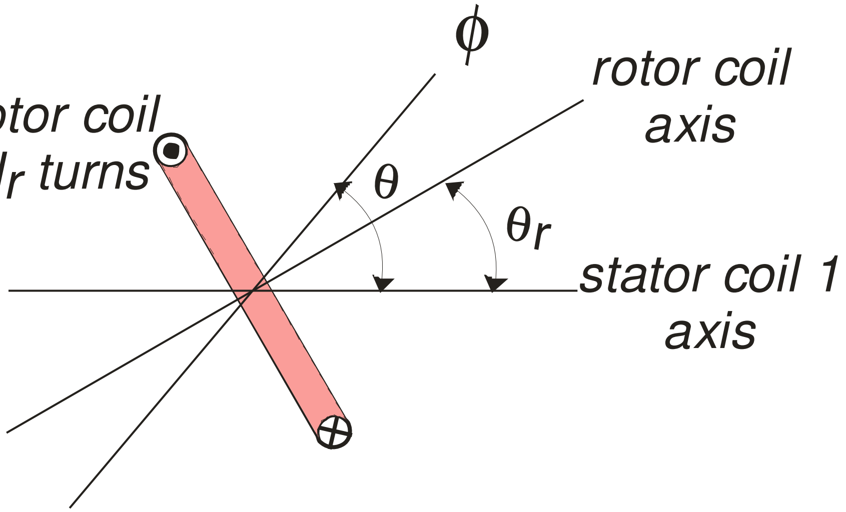
- direction of the resultant mmf in space
    - varies continuously in time
    - varies at angular speed ω



- resultant flux

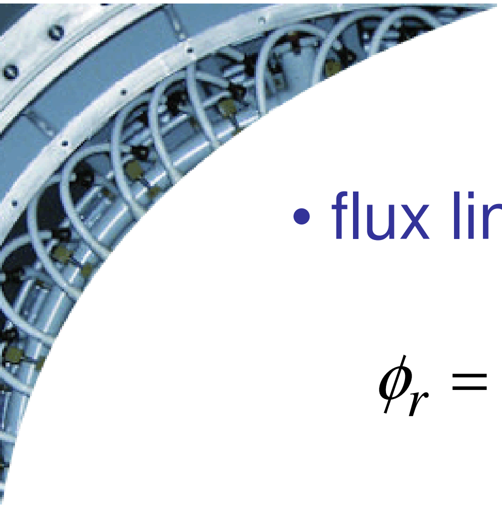
$$\varphi = \frac{F}{S} = \frac{3}{2}\frac{NI_m}{S}$$

- reluctance *S*, though difficult to evaluate, is at least constant
- ϕ has constant value and rotates in space at constant speed



$$\theta = \omega t$$

$$\theta_r = \omega_r t + \beta$$



- flux linking rotor coil *r* is

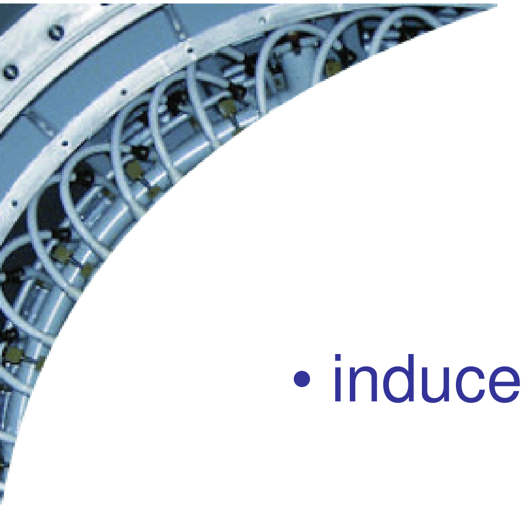
$$\phi_r = \phi\cos(\theta - \theta_r) = \phi\cos\left[(\omega - \omega_r t) - \beta\right]$$

- let $\dfrac{\omega_r}{\omega} = m$

$$\phi_r = \phi\cos\left[(1-m)\omega t - \beta\right]$$

- define slip, *s*

$$s = \frac{\omega - \omega_r}{\omega} = 1 - m$$



- induced voltage in rotor

$$
\begin{aligned}
e_r &= N_r \frac{d\phi_r}{dt} \\
&= N_r \phi \left[\sin\left\{(1-m)\omega t - \beta\right\}\right](1-m)\omega \\
e_r &= s N_r \phi \omega \sin(s\omega t - \beta)
\end{aligned}
$$

- induced voltage in rotor coil has
  - amplitude ∝ *s*
  - frequency ∝ *s*
- 2 special cases:

1) at synchronous speed, $m = 1$ or $s = 0$

$$e_r = 0$$

2) at standstill, $m = 0$ or $s = 1$

$$e_r = N_r \phi \omega \sin(\omega t - \beta)$$

voltages in stator & rotor windings

- for stator coil 1

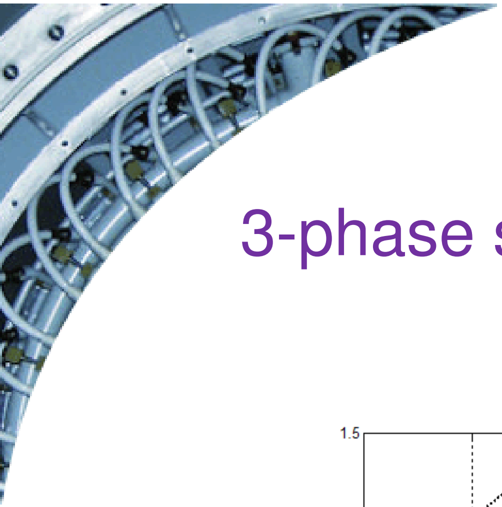
$$e_1 = N\frac{d(\phi\cos\theta)}{dt} = -N\phi\omega\sin\omega t$$

- rms values

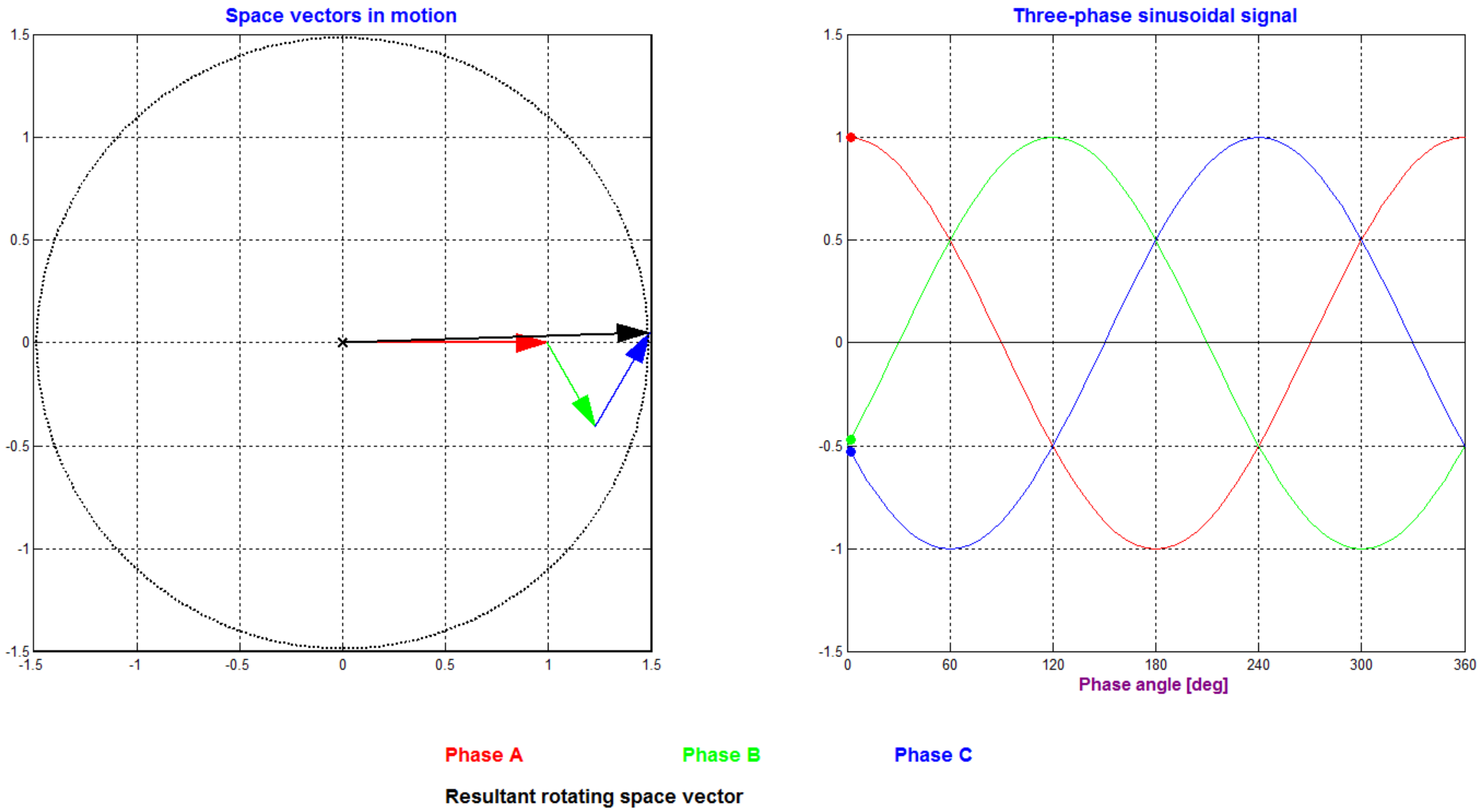
$$E_r = s\frac{N_r}{\sqrt{2}}\phi\omega$$

$$E_1 = \frac{N}{\sqrt{2}}\phi\omega$$

$$\frac{E_r}{E_1} = s\frac{N_r}{N}$$

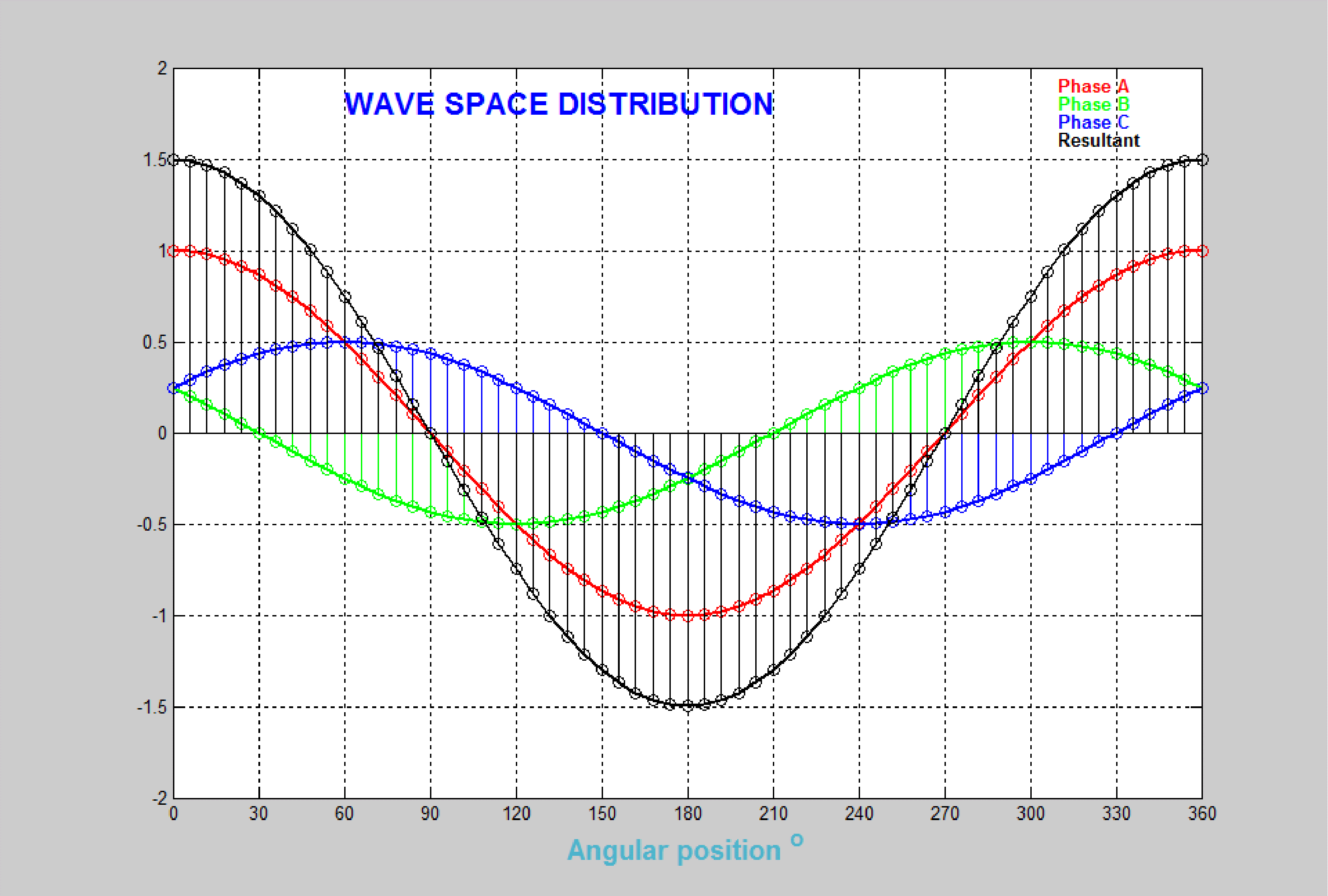


3-phase sinusoidal currents and their mmf wave vectors

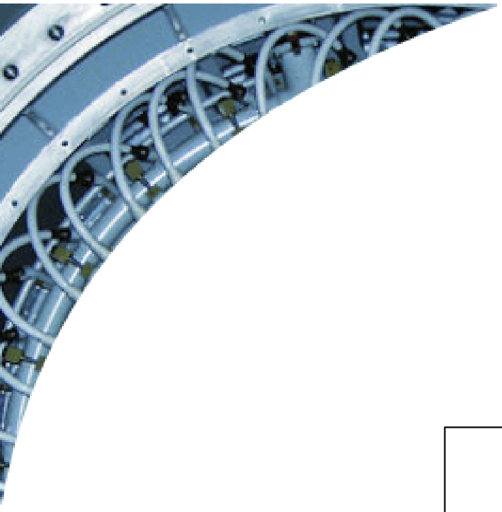


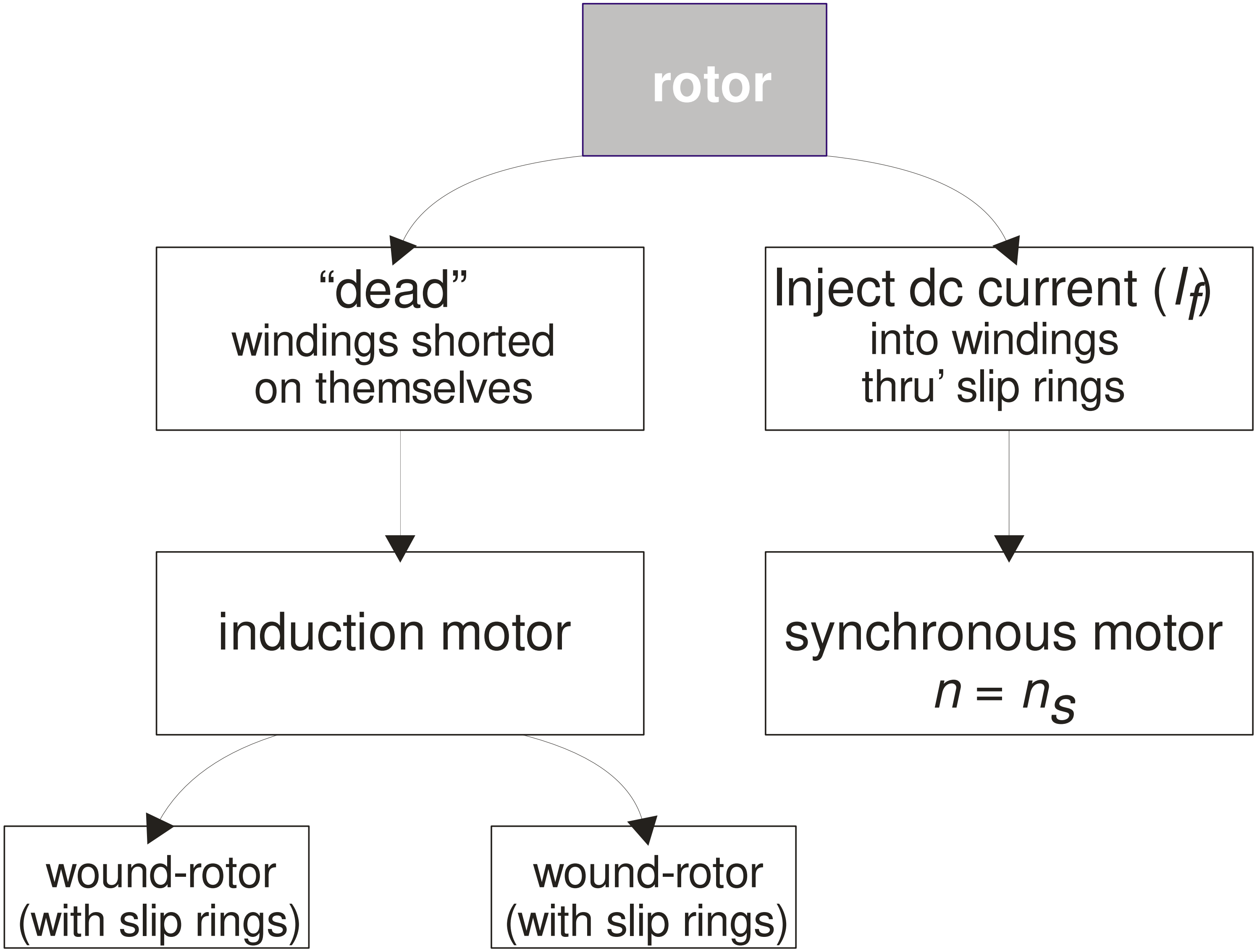
Credits: http://www.ece.umn.edu/users/riaz/animations/listanimations.html

Travelling wave F_a, F or ϕ_a, ϕ caused by 3-phase sinusoidal currents



Credits: http://www.ece.umn.edu/users/riaz/animations/listanimations.html





rotor

“dead”
windings shorted
on themselves

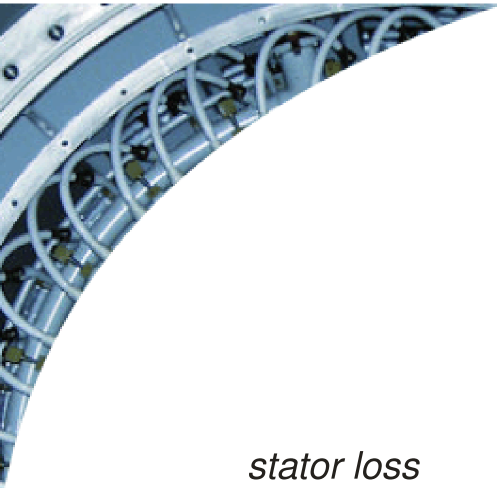
Inject dc current (I_f)
into windings
thru’ slip rings

induction motor

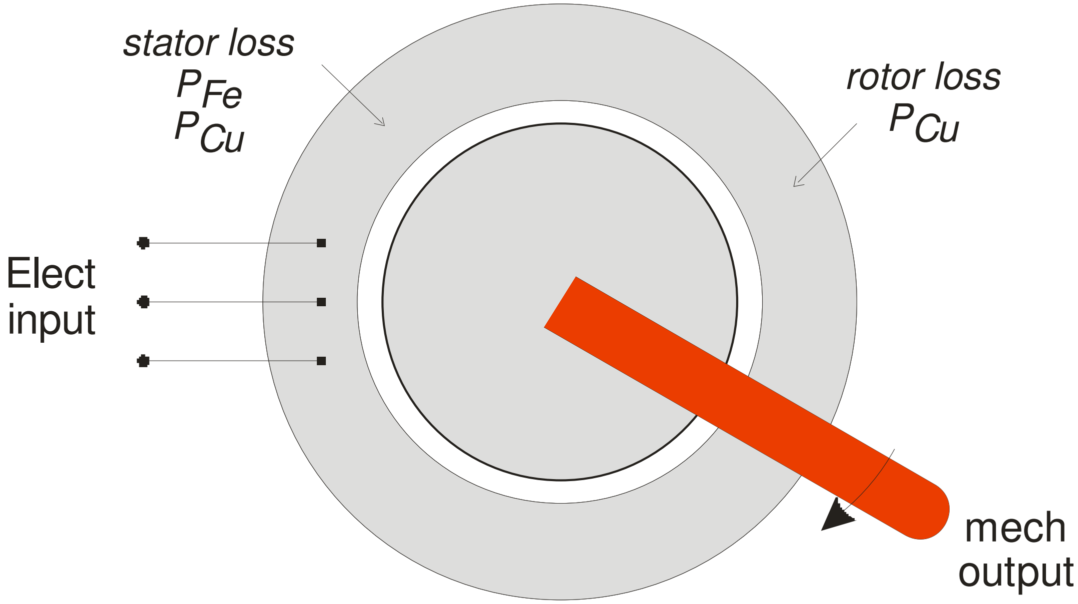
synchronous motor
$n = n_s$

wound-rotor
(with slip rings)

wound-rotor
(with slip rings)



8.2 Induction machines

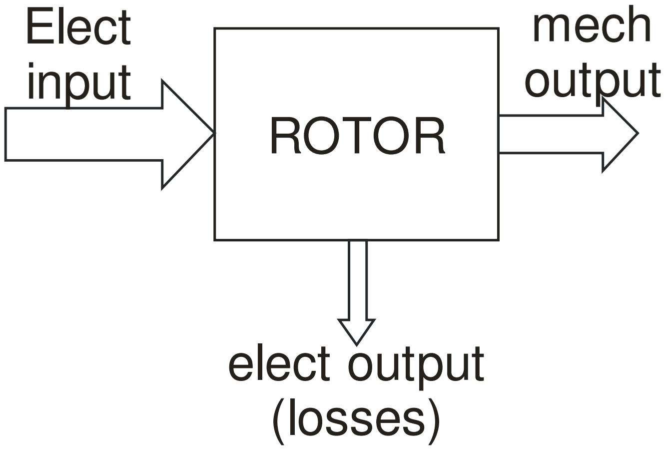


stator loss
P_{Fe}
P_{Cu}

rotor loss
P_{Cu}

Elect
input

mech
output

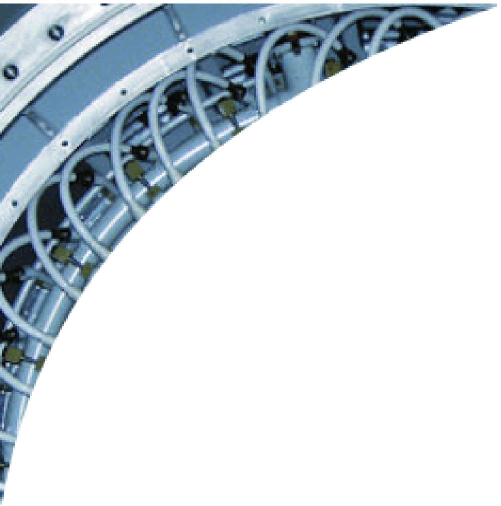


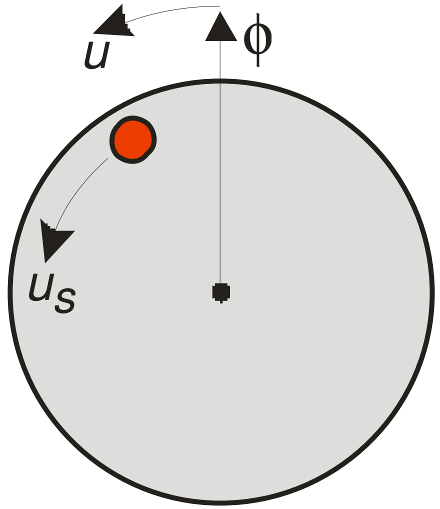
Elect
input

ROTOR

mech
output

elect output
(losses)





- single conductor of length l
- carrying current i
- at radius R
- rotating magnetic field is moving at speed u_s
- conductor is moving at speed u

- define (fractional) slip s as

$$s = \frac{u_s - u}{u_s}$$

- hence

$$s = \frac{2\pi R n_s - 2\pi R n}{2\pi R n_s} = \frac{n_s - n}{n_s}$$

- Faraday’s law gives induced voltage

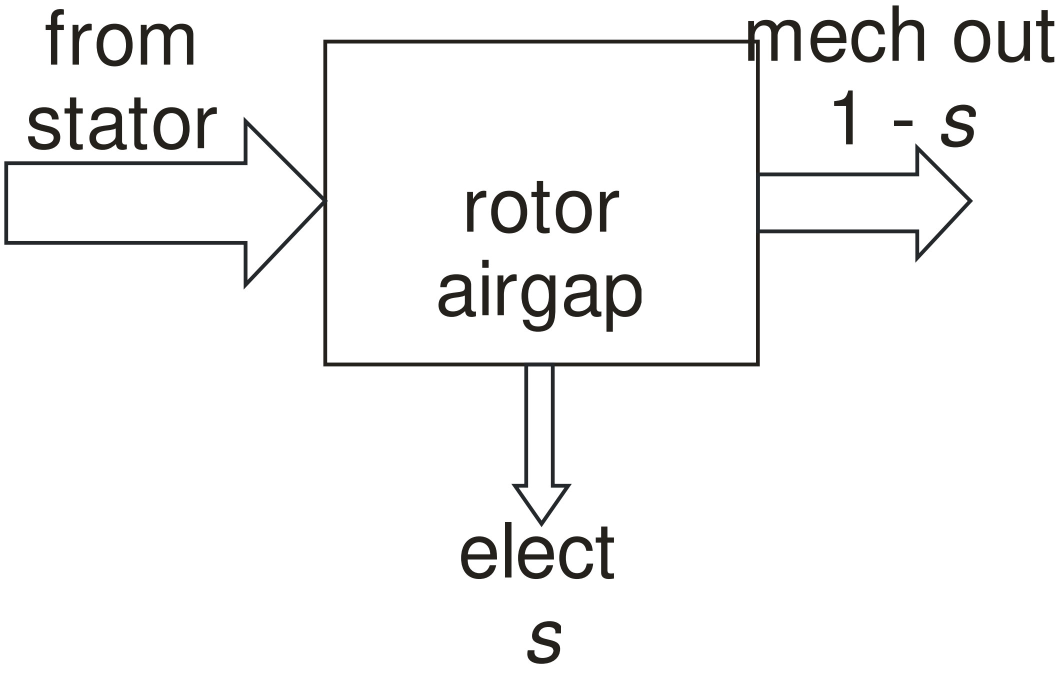
$$V_{in} = Bl(u_s - u) = Blu_s s$$

- rotor electrical power: $V_{in}i = Blu_s si$
- mech force: Bli
- mech power: $(Bli)u = Bliu_s(1-s)$
- total input power: rotor elec power + rotor mech power

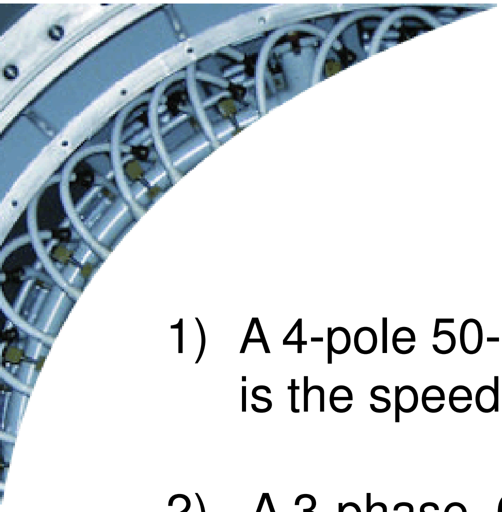
$$= Blu_s si + Bliu_s(1-s)$$

$$= Blu_s i$$

- thus



- designer ensures that s is small
- typical values
  - small motor (few kW) $s = 0.03$ (3%)
  - large machines $s \leq 0.02$



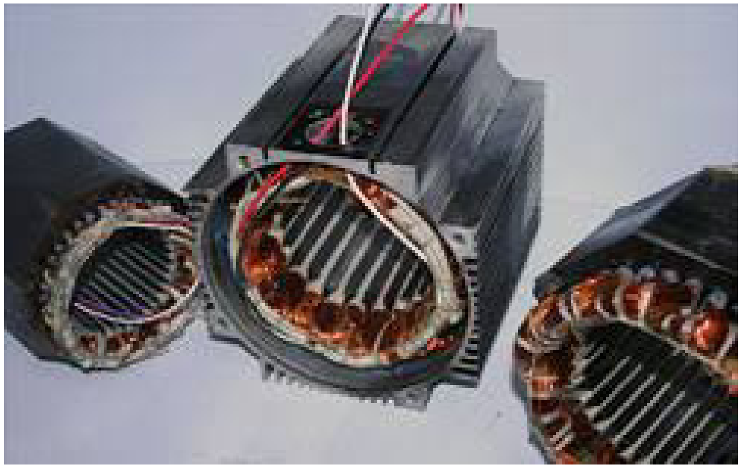
Examples

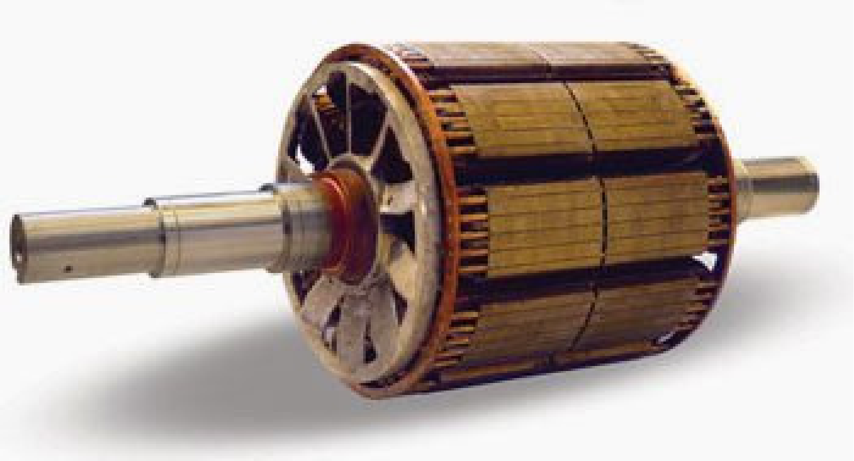
1) A 4-pole 50-Hz machine is running at $s = 0.025$ and 100 kW enters the rotor. What is the speed n , mechanical output and the rotor copper loss?

2) A 3-phase, 60-Hz, four-pole, 220-V, wound-rotor induction motor has a delta-connected stator winding and a star-connected rotor winding. The rotor has 40% as many turns as the stator. For a rotor speed of 1710 r/min, calculate the
   a) slip
   b) induced phase voltage in the rotor at standstill
   c) induced phase voltage in the rotor at working speed
   d) rotor terminal voltage on open circuit and at standstill
   e) frequency of induced voltage in the rotor

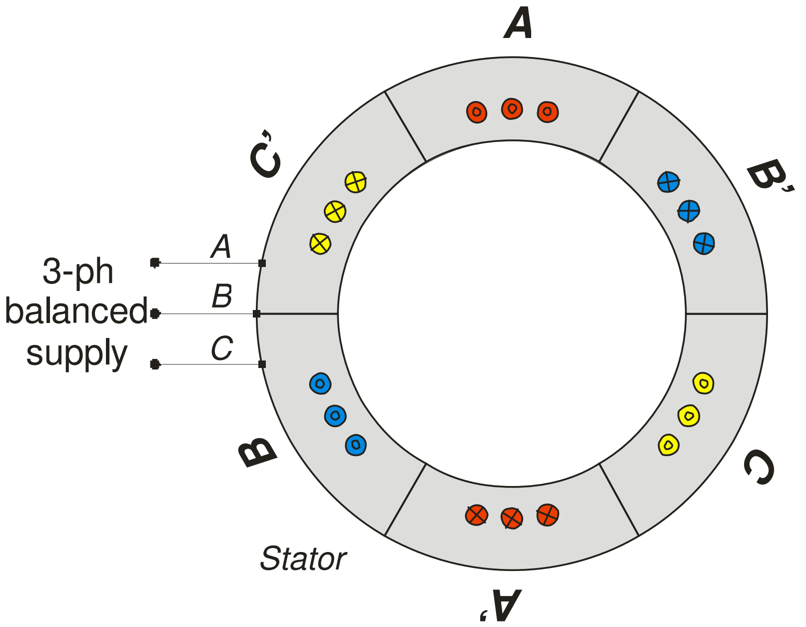
8.3 Synchronous machines

- the synchronous machine has
  - a 3-phase winding on the stator
  - a rotor supplied with direct current

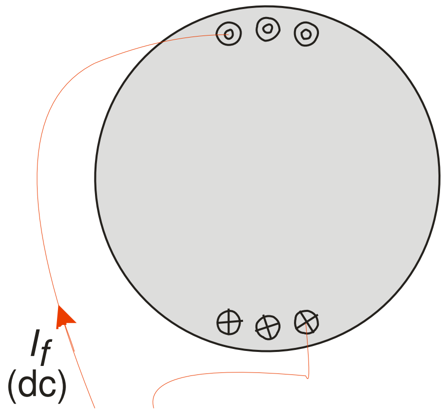




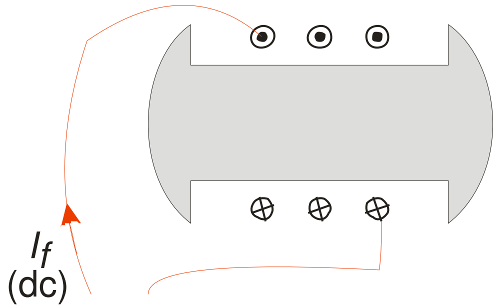
Stator



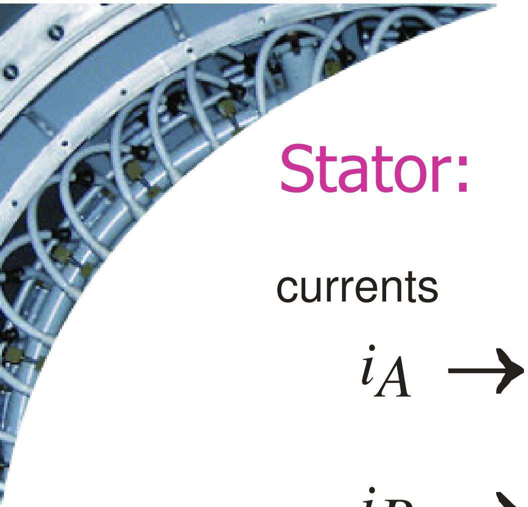
Rotor



cylindrical (round) rotor



salient rotor



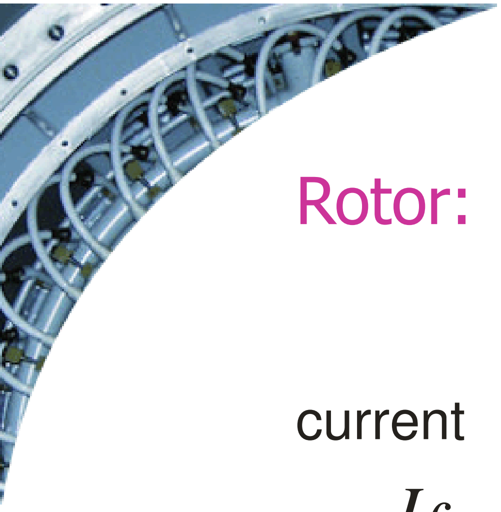
Stator:

currents mmf

$$\left.\begin{array}{l} i_A \rightarrow F_A \\ \\ i_B \rightarrow F_B \\ \\ i_C \rightarrow F_C \end{array}\right\} F_a \rightarrow \phi$$

- F_a has constant magnitude and rotates a constant speed ω_s
- for a 2p-pole machine

$$n_s = \frac{\omega_s}{2\pi} = \frac{f}{p}$$



Rotor:

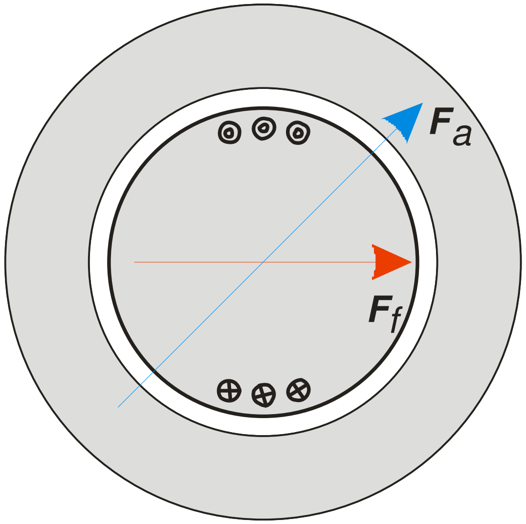
current mmf

$$I_f \rightarrow F_f$$

- direction of the current through the brushes and slip rings to the winding is always in the same direction
- polarity on the rotor (N & S) never changes
- F_f is along the axis of the rotor and rotates at ω_r

$$\overline{F}_a + \overline{F}_f = \overline{F}_r \rightarrow \phi_r \rightarrow \left\{ \begin{matrix} e_A \\ e_B \\ e_C \end{matrix} \right\} E_r$$

- in synchronous machine $\omega_r = \omega_s$

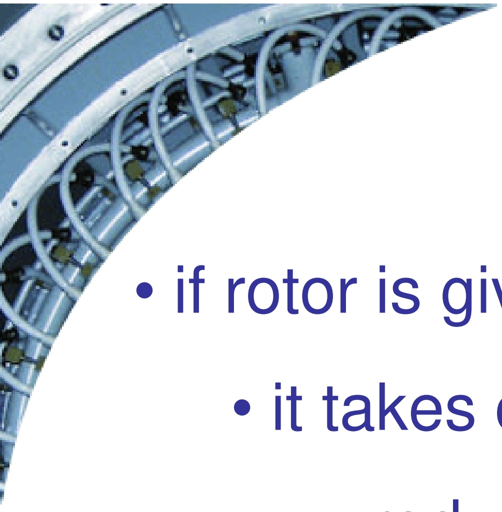


- the machine functions as a motor or as a generator depending on whether the stator (armature) field leads or lags the rotor field

Action of the ideal machine

- assume:
  - ideal cylindrical rotor
  - connected to ‘infinite’ busbar
  - stator windings have
    - negligible resistance
    - negligible leakage reactance
    - uniform air gap
    - high permeability magnetic circuit
    - no saturation
    - balanced load

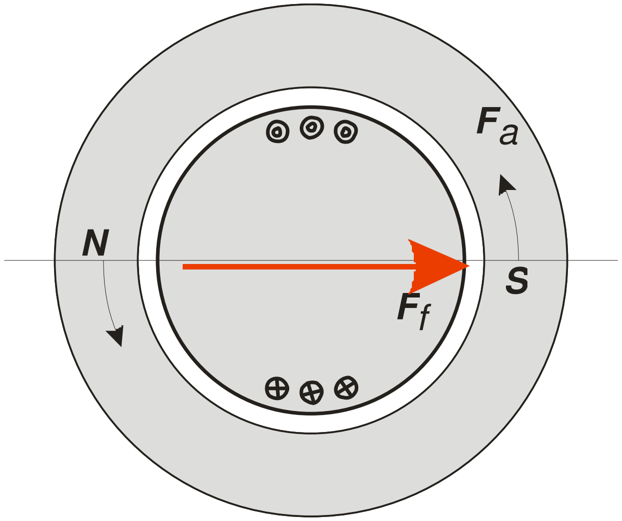
- to work at all the rotor must rotate at synchronous speed
- no torque can be developed if rotor is unexcited
- stator must draw lagging reactive power
  - to magnetise the machine to a gap flux per pole of ϕ_r
    - in order for the stator emf E_r to be induced
      - to balance applied voltage V_a (v_1, v_2, v_3)



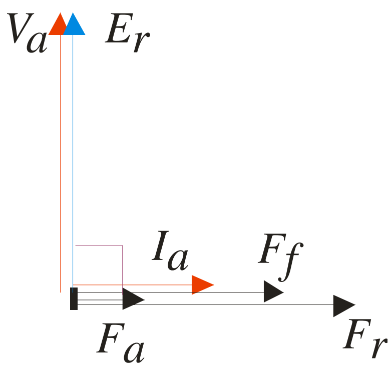
- if rotor is given a small dc excitation
  - it takes over part of the task of exciting the magnetic circuit,
    - reducing the demand of stator magnetising power (under excitation)

  - λ = torque angle
  - δ = load angle
    - motor mode: δ is negative
    - generator mode: δ is positive

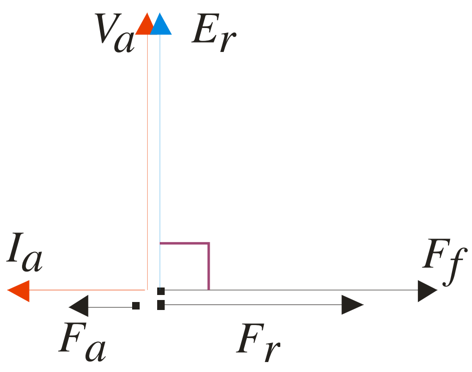
No-load mode



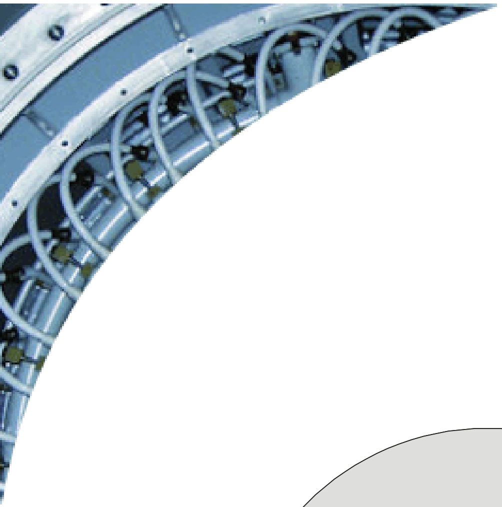
under excited



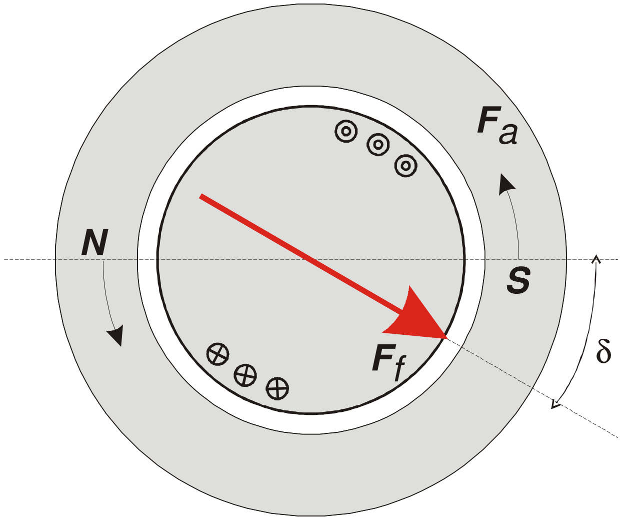
over excited



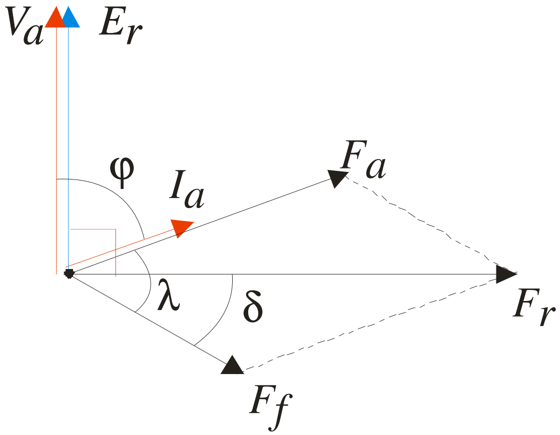
- no torque is developed
    - mmf axes are in alignment, torque angle is zero
- F_a and F_f combine to give F_r necessary to produce ϕ_r
- if rotor mmf is increased very much into over excitation
    - the stator must produce a demagnetising current so that F_r shall remain unchanged
    - i.e. stator must produce a leading current



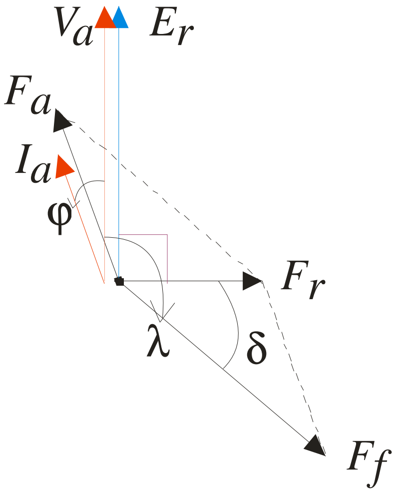
Motor mode



under excited

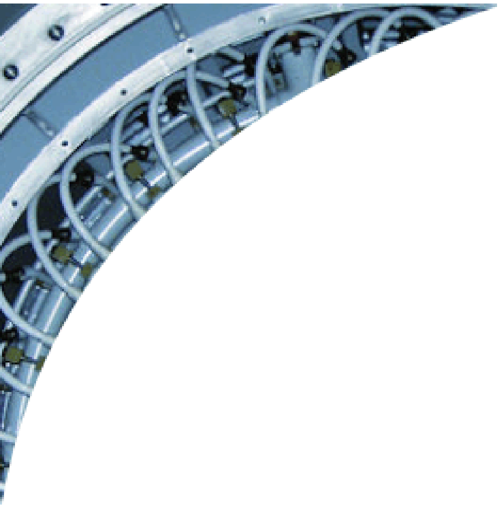


over excited

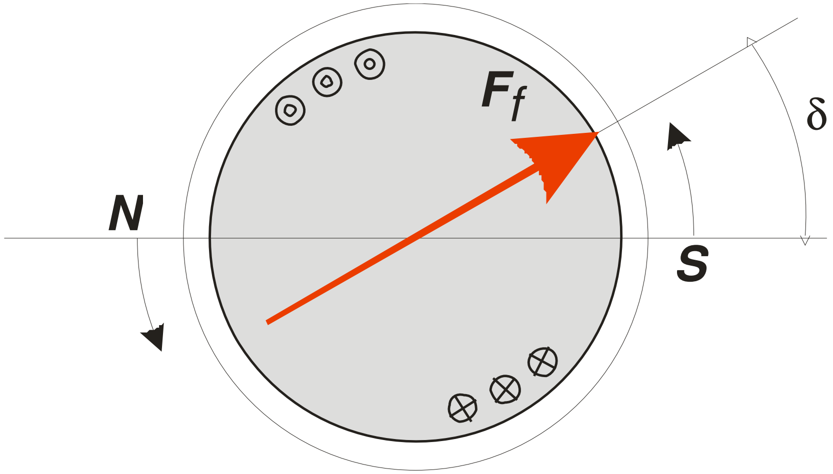


- depending on whether rotor is under or over excited
  - stator takes an active current component
    - accepting power from the supply and
    - developing a forward torque on the rotor
      - to balance the load torque
- the reactive component of the current, as on no-load,
  - compensates for under or over excitation

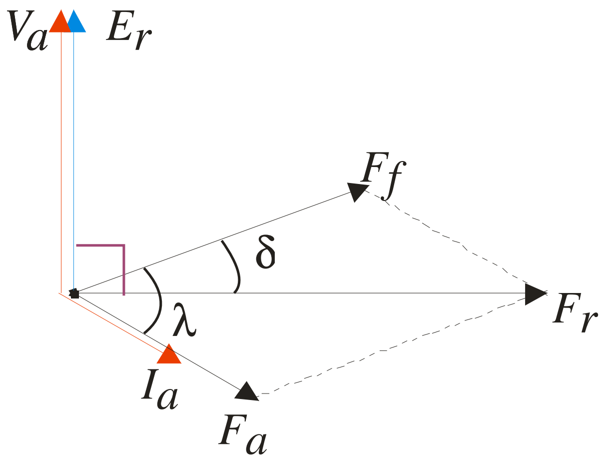
$$T \propto \sin \lambda$$



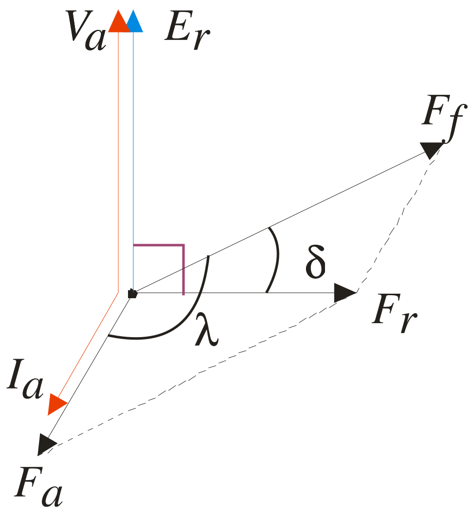
Generator mode



under excited



over excited



- the active component of stator current reverses
  - thus delivering power into the supply
  - developing a counter torque on the rotor
    - to balance the driving torque
- the reactive component of the current, as on no-load,
  - compensates for under or over excitation

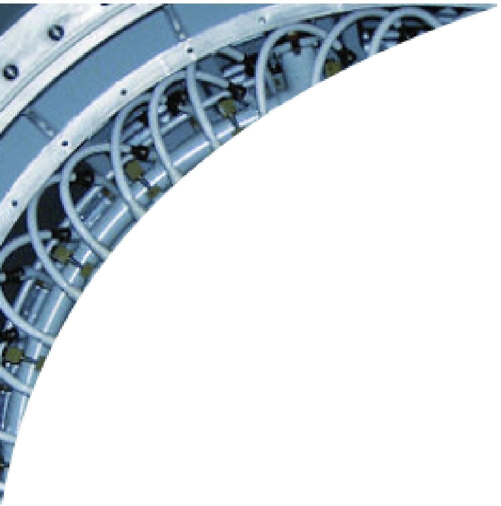
Compensator mode

- a synchronous machine designed
    - to run unloaded
    - the shaft is not connected to mechanical load or prime mover
- variation of rotor excitation causes machine to take purely reactive power
    - under excitation → lagging
    - over excitation → leading

- application in
  - control of voltage in transmission systems by
    - supplying reactive power
    - consuming reactive power
- a.k.a. synchronous compensator, synchronous capacitor
- in general, the machine has no torque at starting (zero speed)
  - some other means must be used to bring it to synchronous speed, e.g.
    - pony motor
    - double-cage arrangement

Starting of synchronous motor

- in general, the machine (motor, compensator) has no torque at starting (zero speed)
  - some other means must be used to bring it to synchronous speed, e.g.
    - pony motor
    - double-cage arrangement
      - cage 1: induction machine
      - cage 2: synchronous machine



- End of Lecture 8 -