EE310 ELECTRICAL ENGINEERING

SI units. Circuit elements, Kirchoffs laws, Network theorems for resistive network, Graphical analysis of simple non-linear resistive network; Multi-port network analysis, Steady state response of simple RLC networks to sinusoidal excitation; Laplace Transforms, Power and Energy, Electric fields and magnetism; Resonance in series and parallel RLC circuits; Power in ac circuits.

1.0 INTRODUCTION

1.1 UNITS

All systems of measurements depend upon the adoption of certain fundamental units i.e. the base units. The International System of Units (SI) is a metric system giving a fully coherent set of units involving non conversion factor. A coherent system of units is a system dependant on other systems i.e. the base units. In science and engineering the SI units form the basis of all units. There are seven 'base' units and two supplementary units of angles from which all the other units are derived called 'derived' units and 'supplementary' units.

Table 1.1 SI base units

Quantity	Unit Name	Unit symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	Ampere	A
Thermodynamic Temperature	Kelvin	K
Luminous Intensity	Candela	cd
Quantity of substance	mole	mol

Table 1.2 Supplementary units

Quantity	Unit Name	Unit symbol
Plane angle	Radian	rad
Solid angle	Steradian	sr

The advantages of the SI system are:

(a) It is the most widely used system in the world,

(b) It is a coherent system, thus removing problems with units in the formulae and equations,

(c) Multiples of units are in integral powers of 10, or, even more conveniently, integral powers of 10^3,

(d) The user is rightly forced to distinguish between mass and weight.

The base units are very carefully defined and the standards can be reproduced in the laboratory. The symbols and even the names that are recommended are based on the British and American standards, but they are sometimes influenced by prominent Authors and wide-spread practices.

1.2 Definitions

Length: The length of the path travelled by light in a vacuum during a time interval of $\frac{1}{299{,}792{,}458}$ of a second.

Mass (kg): The mass of the international prototype, block of platinum preserved at the International Bureau of Weights and Measures, at Servers near Paris.

Time(s): The duration of 9,192 ,631,770 periods of the radiation corresponding to the transition between the two hyper fine levels of the ground state of the Caesium-133 atom.

Electric Current(A): The current which is maintained in two straight parallel conductors of the infinite length, of negligible circular cross-section and one metre apart in vacuum, produces a force equal to $2\times10^{-7}\,Newton$ per metre of length.

Thermodynamic Temperature(K): The fraction of $\frac{1}{273.16}$ of thermodynamic (absolute) temperature of the triple point water.

Luminous Intensity(cd): The luminous intensity in the perpendicular direction of the surface of $\frac{1}{600{,}000}m^2$ of a block body at the temperature of freezing platinum under a pressure of 101,325 Newton per square.

Amount of Substance(mol): The amount of substance of a system which contains as many elementary entities as there are atoms in 0.021kg of carbon-12. The elementary entity must be specified and may be an atom, a molecule, an ion, an electron or a specified group of such entities.

Plane Angle (rad): The plane angle between two radii of a circle which cut off on the circumference of the circle an arc of length equal to the radius.

Solid Angle (sr): The solid angle which, having its vertex at the centre of a sphere, cuts off an area of surface of the sphere equal to a square having sides equal to the radius.

2.0 POWER SYSTEM FUNDAMENTALS

2.1 Mechanical

Power(P_{mech}) = Force × Speed
Force = mass × acceleration

$$speed = \frac{distance}{time} = \frac{dx}{dt}(m/s)$$

$$acceleration = \frac{speed}{time} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}(m/s^2)$$

$$Force = ma = (kg)(m/s^2) = N(Newton)$$

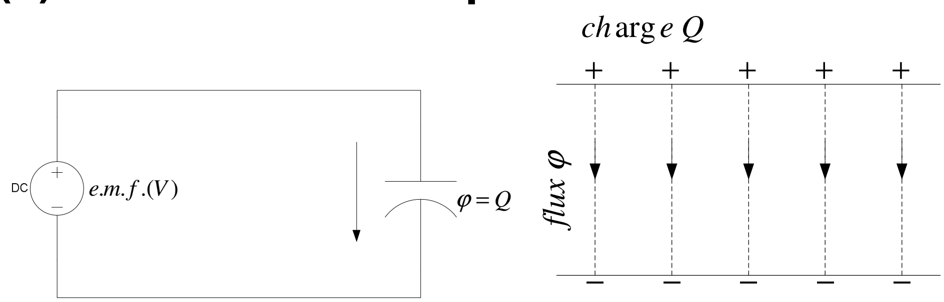
$$Hence Pmech = Force \times speed = (kg)(m/s^2)(m/s) = Nm/s = W(Watt)$$

$$Energy(W) = Force \times distance = Force \times speed \times time = (N)(m/s) = Nm = J(Joule)$$

2.2 Electrical

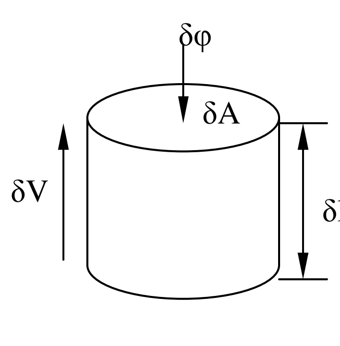
(i) Electrostatics

(a) Parallel Plate Capacitors



$Flow = Q = \varphi$ *(For uniform materials)*
Q = Charge (Coulomb, c)
φ = Electric flux (c)
Potential = V (e.m.f.) (v)

$Ratio = \frac{Q}{V} = \frac{\varphi}{V} = C$ *(Capacitance = Constant, c/v or F = Faraday)*



Flow density = $\frac{\partial \varphi}{\partial A} = \frac{\partial Q}{\partial A} = D(c/m^2)$ *For uniform materials:* $D = \frac{\varphi}{A}(c/m^2)$

D is the electric flux density

For $D = \frac{Q}{A}$ *is the charge density (c/m²)*

Potential gradient: $\frac{\partial V}{\partial l} = \frac{V}{l} = E(v/m)$

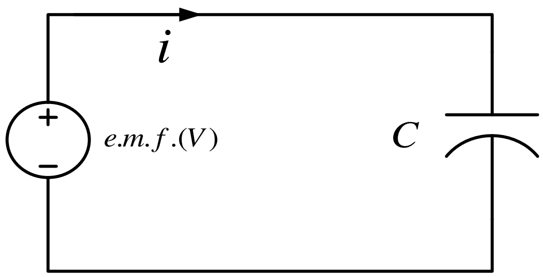
E is the electric field strength

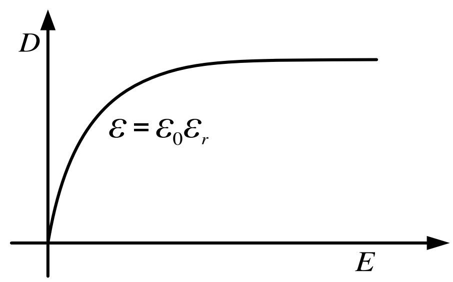
Ratio: $\frac{D}{E} = \varepsilon(F/m)$

ε is the absolute permittivity and $\varepsilon = \varepsilon_0 \varepsilon_r$

Where ε_0 *is the permittivity of free space also known as the electric constant and* $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} = 8.85 \times 10^{-12}(F/m)$

ε_r *is the relative permittivity and the value is 1.0 for air and vacuum.*





$$C = \frac{Q}{V} = \frac{\varphi}{V} = \frac{DA}{El} = \frac{\varepsilon EA}{El} = \frac{\varepsilon A}{l} = \frac{\varepsilon_0 \varepsilon_r A}{l}; C = \frac{\varepsilon_0 \varepsilon_r A}{l}(n \quad 1)$$

n= number of plates

Instantaneous electric power from supply, $p = vi$

Instantaneous electric energy from supply, $w = \int pdt = \int vidt$

$$idt = CdV = dq$$

$$\therefore w = \int Vdq = \int CVdV = C\int VdV = \tfrac{1}{2}CV^2(J)$$

The $\frac{1}{2}CV^2$ energy drawn from supply is stored in electric flux that has been established.

Stored energy per unit volume: $\frac{\frac{1}{2}CV^2}{Al} = \frac{\frac{1}{2}qV}{Al} = \frac{1}{2}DE = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}\frac{D^2}{\varepsilon}(J/m^3)$

For constant voltage across the capacitor;

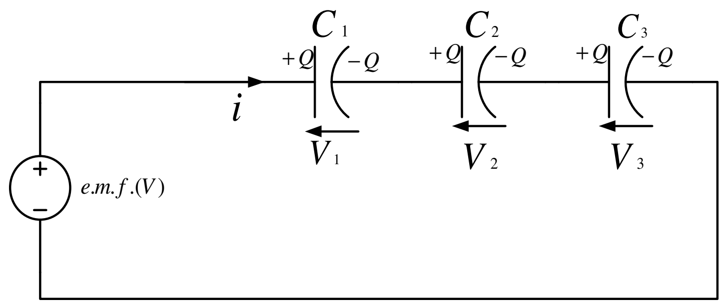
Instantaneous electric power from supply, $p = Vi$

Instantaneous electric energy from supply, $w = \int pdt = \int Vidt = V\int idt = Vq$

$$\therefore w = Vq\ (J)$$

(b) DC Capacitor Circuits

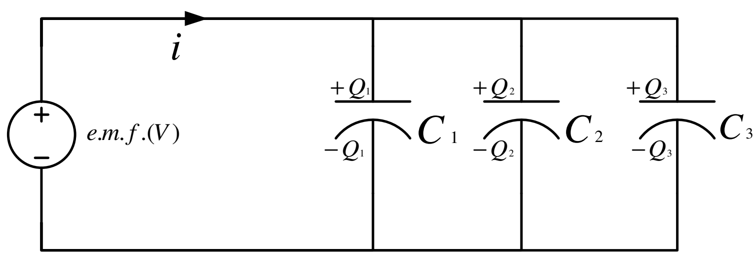
Series Capacitance Connections



$$V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3})$$

$$\therefore \frac{1}{C_{total}} = \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Parallel Capacitance Connections

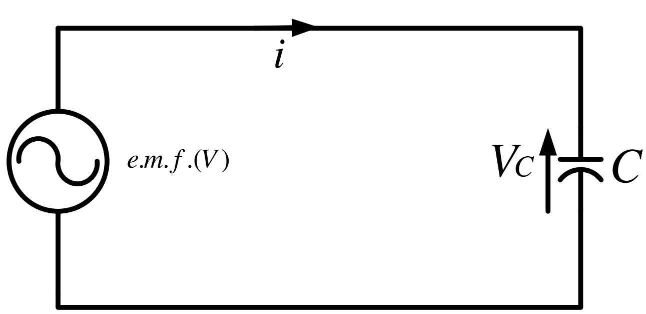


$$Q_1 = C_1V, Q_2 = C_2V \; and \; Q_3 = C_3V$$

$$Q_{total} = Q_1 + Q_2 + Q_3 = VC_{total} = VC_1 + VC_2 + VC_3 = V\left(C_1 + C_2 + C_3\right)$$

$$\therefore C_{total} = \frac{Q_{total}}{V} = C_1 + C_2 + C_3$$

(c) AC Capacitor Circuits



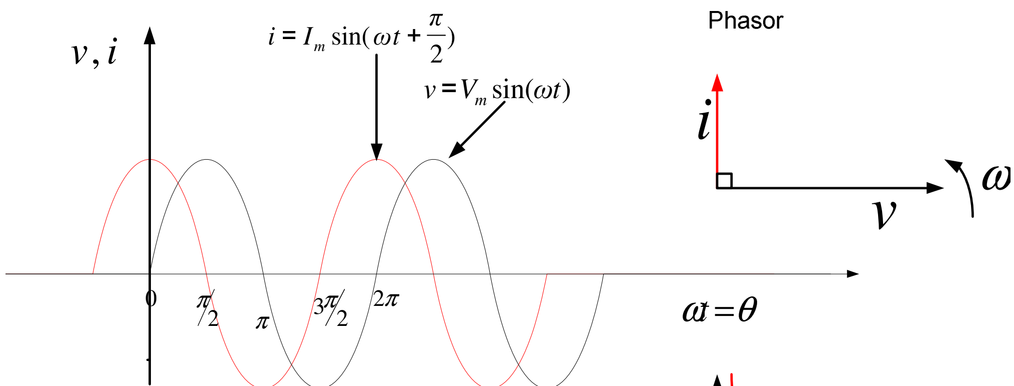
For $V = V_m \sin\omega t$, the capacitive current will be;

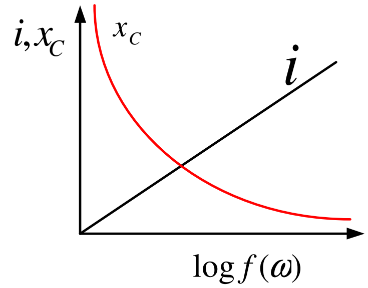
$$i = C\frac{dV}{dt} = C\frac{d}{dt}\left(V_m \sin \omega t\right) = CV_m \frac{d}{dt}\left(\sin \omega t\right) = \omega CV_m \cos \omega t = \omega CV_m \sin(\omega t + \frac{\pi}{2})$$

$$i = \omega CV_m \sin(\omega t + \frac{\pi}{2}) = I_m \sin(\omega t + \frac{\pi}{2})$$

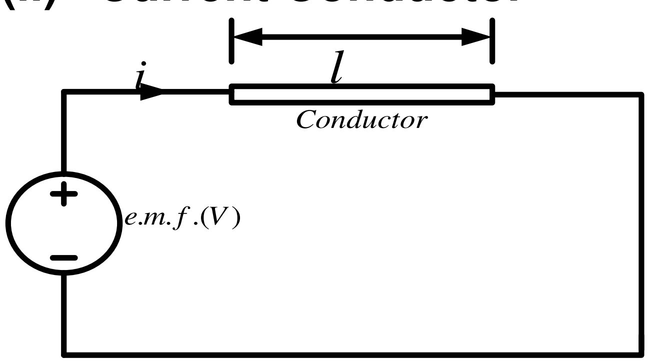
$$I_m = \omega CV_m = \frac{V_m}{x_C} \text{ and } x_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \text{ and } x_C \text{ is the capacitive reactance}$$

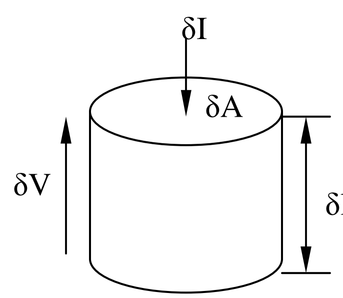
The voltage lags behind the current by $\frac{\pi}{2}$ i.e. the current leads the voltage by $\frac{\pi}{2}$. Hence $jI_m = \omega CV_m = \frac{V_m}{x_C}$ and $x_C = -j\frac{1}{\omega C} = -j\frac{1}{2\pi fC}$.





(ii) Current Conductor





Flow = Current I (A)
Potential = V (e.m.f.) (v)
$Ratio: \frac{I}{V} = G$ *(Constant) (s, A/v,* $\frac{1}{\Omega}$, $\mho$*)*

G = conductance

$G = \frac{1}{R}$ *and R = Resistance (Ω)*

Flow density = $\frac{\partial I}{\partial A} = J(A/m^2)$ *, For uniform material,* $J = \frac{I}{A}(A/m^2)$

J is the current density

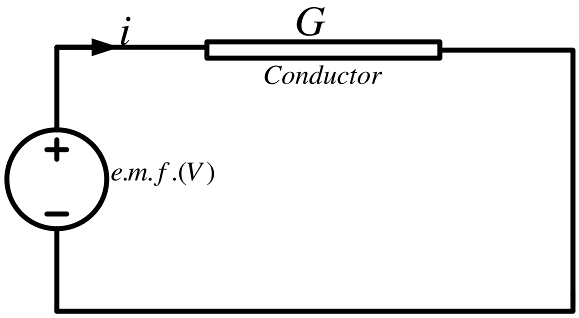
Potential gradient: $\frac{\partial V}{\partial l} = E(v/m)$ *, For uniform material,* $E = \frac{V}{l}(v/m)$

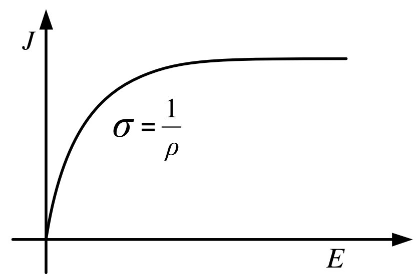
E is the electric field strength

$Ratio: \frac{J}{E} = \sigma(s/m, \mho/m)$ *and* $\sigma = \frac{1}{\rho}(s/m, \mho/m)$ *and* $\rho = \frac{1}{\sigma}(\Omega m)$

σ is the conductivity

ρ is the resistivity





$i = VG$

$G = \frac{i}{V} = \frac{JA}{El} = \frac{\sigma A}{l} = \frac{1}{\rho} \quad \frac{A}{l}$

$R = \frac{1}{G} = \rho\frac{l}{A}(\Omega)$

$\therefore R = \rho\frac{l}{A}(\Omega)$

The resistance is proportional to small temperature changes of between 0º to 150ºC. The resistance of all pure metals increases with an increase in temperature, where as the resistance of carbon, electrolytes and insulating materials decreases with an increase in temperature.

$$R_2 - R_1 = \alpha(\theta_2 - \theta_1)R_1$$

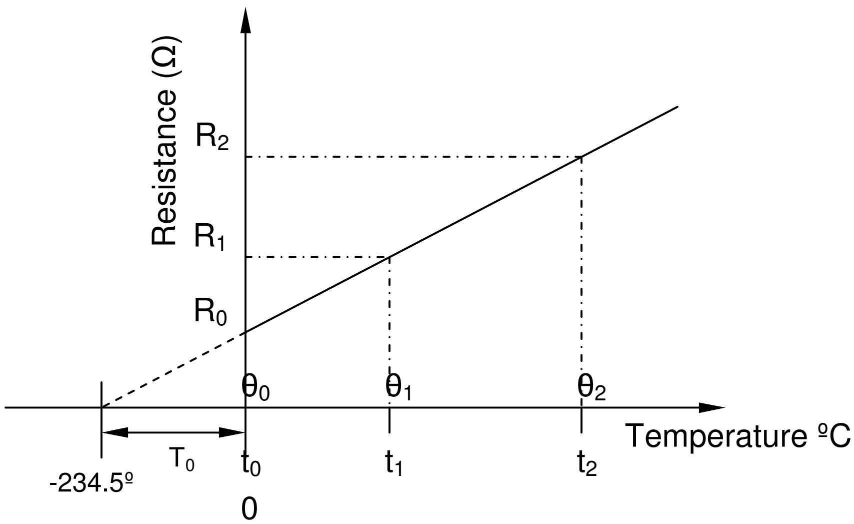
$$R_2 = R_1 + \alpha(\theta_2 - \theta_1)R_1 = R_1[1 + \alpha(\theta_2 - \theta_1)]$$

$$\therefore R_2 = R_1[1 + \alpha(\theta_2 - \theta_1)]$$

α is the temperature coefficient.

$$R_2 = R_1[(1 + \alpha(\theta_2 - \theta_1)) + \beta(\theta_2 - \theta_1)^2]$$

For temperature diffrence greater than $150K$



$R_1 = R_0[1+\alpha(\theta_1 - \theta_0)] = R_0(1+\alpha_0 t_1)$ Where $t_1 = (\theta_1 - \theta_0)$

$R_2 = R_0[1+\alpha(\theta_2 - \theta_0)] = R_0(1+\alpha_0 t_2)$ Where $t_1 = (\theta_2 - \theta_0)$

$$\frac{R_1}{R_2} = \frac{1+\alpha_0 t_1}{1+\alpha_0 t_2}$$

For a known resistance at say 20ºC (R_{20}), a conversion for a hot resistance R_w and cold resistance R_c can be calculated as follows:

$$R_w = R_{20}[1+\alpha_{20}(\theta_w - 20)]$$

$$R_c = R_{20}[1+\alpha_{20}(\theta_c - 20)]$$

$$\frac{R_w}{R_c} = \frac{1+\alpha_{20}(\theta_w - 20)}{1+\alpha_{20}(\theta_c - 20)}$$

A temperature constant T_0 at 20ºC yields the following constant

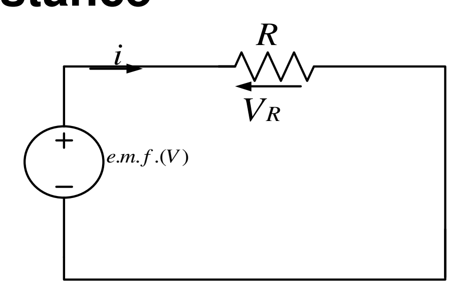
$$\alpha_{20}(T_0 + 20) = 1$$

$$\therefore \frac{R_w}{R_c} = \frac{\alpha_{20}(T_0+20) + \alpha_{20}(\theta_w - 20)}{\alpha_{20}(T_0+20) + \alpha_{20}(\theta_c - 20)} = \frac{T_0 + \theta_w}{T_0 + \theta_c}$$

The temperature coefficient α for any temperature is $\alpha = \dfrac{1}{T_0 + \theta}$.

(a) DC Resistance Circuits

Resistance



$V = V_R = iR = \dfrac{i}{G}$ *Where R = Resistance (Ω), G = Conductance (℧)*

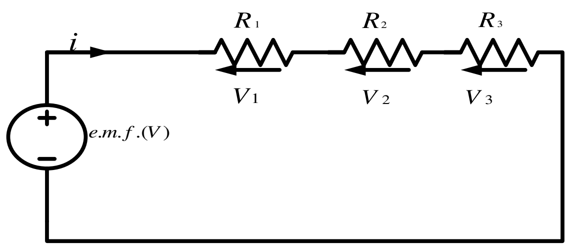
$Power = p = Vi = iRi = i^2 R$

$\therefore P_{out} = i^2 R$

For a pure resistance R carrying an instantaneous current I, the potential difference is $V = iR$ and the rate of heat production is $p = i^2 R$.

The heat generated (energy) is $W = i^2 Rt$ and 't' is time in seconds

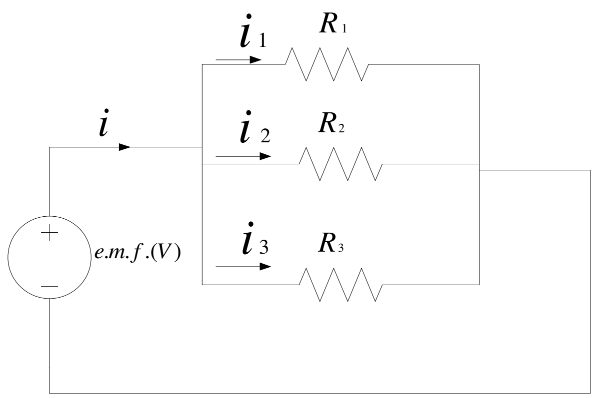
Series Resistance Connections



$$V = V_1 + V_2 + V_3 = iR_1 + iR_2 + iR_3 = i(R_1 + R_2 + R_3)$$

$$\therefore R_{total} = \frac{V}{i} = R_1 + R_2 + R_3$$

Parallel Resistance Connection



$$i = i_1 + i_2 + i_3$$

$$R_{total} = \frac{V}{i} = \frac{V}{i_1 + i_2 + i_3}$$

$$G = \frac{1}{R_{total}} = \frac{i}{V} = \frac{i_1 + i_2 + i_3}{V} = \frac{i_1}{V} + \frac{i_2}{V} + \frac{i_3}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

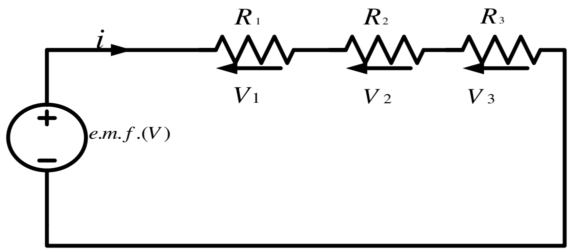
$$\therefore \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In particular for two resistors in parallel;

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

POTENTIAL DIVEDER RULE

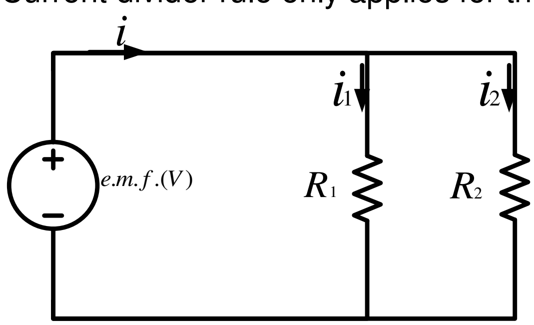


$$V = i(R_1 + R_2 + R_3)$$
$$V_1 = iR_1, V_2 = iR_2 \; and V_3 = iR_3$$
$$\therefore \left(\frac{V_1}{V}\right) = \left(\frac{R_1}{R_1 + R_2 + R_3}\right), \left(\frac{V_2}{V}\right) = \left(\frac{R_2}{R_1 + R_2 + R_3}\right) and \left(\frac{V_3}{V}\right) = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)$$

Potential divider rule works for any number of resistors in series.

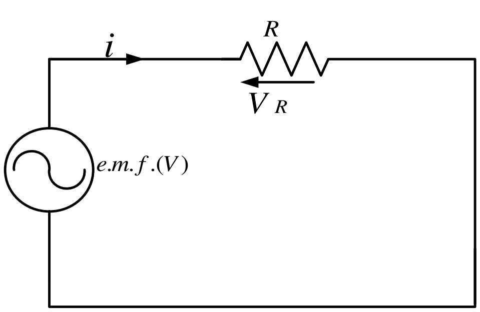
CURRENT DIVIDER RULE

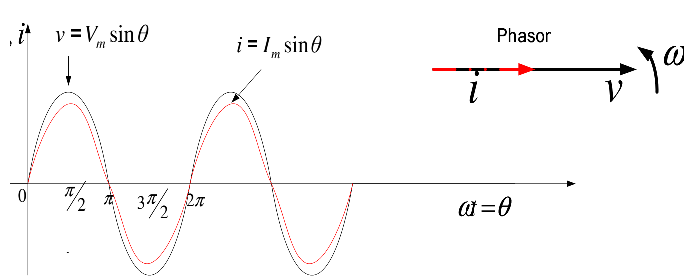
Current divider rule only applies for the case of two resistors in parallel.



$$i_1 = \left(\frac{V}{R_1}\right), i_2 = \left(\frac{V}{R_2}\right) and \; i = i_1 + i_2 = \left(\frac{V}{R_1}\right) + \left(\frac{V}{R_2}\right) = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$
$$V = i\left(\frac{R_1 R_2}{R_1 + R_2}\right) \Rightarrow \therefore i_1 = \left(\frac{V}{R_1}\right) = i\left(\frac{R_2}{R_1 + R_2}\right) and \; i_2 = \left(\frac{V}{R_2}\right) = i\left(\frac{R_1}{R_1 + R_2}\right)$$

(b) AC Resistance Circuits

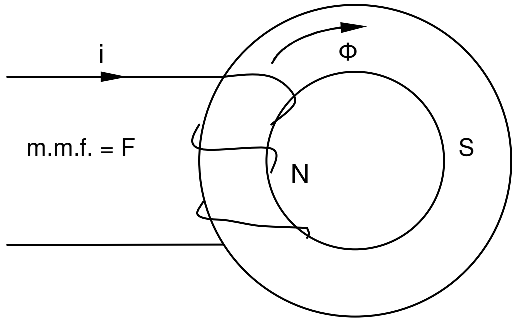


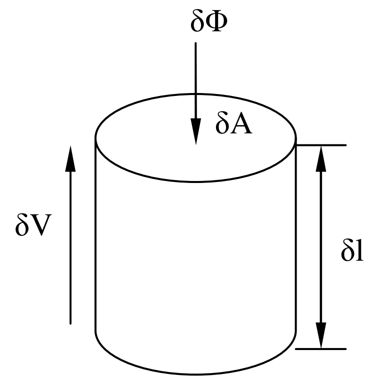


The current at any instant is $i_{rms} = \frac{v}{R}(A)$. Therefore the current shape will follow the voltage wave-shape. The two waves will cross zero point at the same time. Hence the voltage and current wave are said to be in phase. Ohms law can be applied without modifications to resistance circuits i.e.;

$i = \frac{v}{R} = \frac{V_m}{R}\sin\theta = I_m \sin\theta$ where $v = V_m \sin\theta$.

(iii) Magnetostatics





Flow = flux = Φ (Wb)
Potential: = m.m.f. = F = Ni (At)
*m.m.f. = **m**agne**m**otive **f**orce*
i = current (A)
N = number of turns

$Ratio: \frac{\Phi}{F} = \Lambda(Wb/At)$ *is a constant*

Λ = permeance and $S = \frac{1}{\Lambda}(At/Wb)$

S = reluctance

Flow density = $\frac{\partial\Phi}{\partial A} = B(Wb/m^2, T)$ *and for uniform material,* $B = \frac{\Phi}{A}(Wb/m^2, T)$.

B is the magnetic flux density.

Potential gradient: $\frac{\partial F}{\partial l} = H(At/m)$ and for uniform material, $H = \frac{F}{l}(At/m)$.

H is the magnetic field strength

$Ratio: \frac{B}{H} = \mu(H/m) and \mu = \mu_0\mu_r(H/m)$

μ *is the absolute permeability (H/m)*

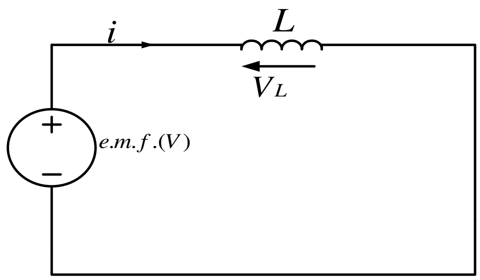
μ_0 *is the permeability of free space also known as magnetic constant and*

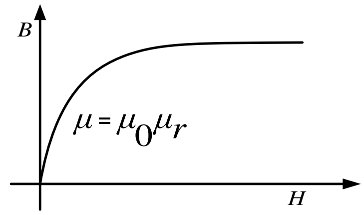
$\mu_0 = 4\pi \times 10^{-7}(H/m)$

μ_r *is the relative permeability and* $\mu_r = 1$ *in air and vacuum.*

$S = \frac{F}{\Phi} = \frac{Hl}{BA} = \frac{Bl}{\mu BA} = \frac{l}{\mu A} = \frac{l}{\mu_0\mu_r A} \Rightarrow \therefore S = \frac{l}{\mu_0\mu_r A}(At/Wb)$

(a) Self Inductance





$V = L\frac{di}{dt}$ *Faraday's Law and 'L' is the self inductance.*

$V = N\frac{d\Phi}{dt}$ *Faraday's Law*

$L\frac{di}{dt} = N\frac{d\Phi}{dt}$ and

$L = N\frac{d\Phi}{di}$

Assuming uniform material;

$$L = N\frac{d\Phi}{di} = N\frac{\Phi}{i} = \frac{N\Phi}{i} = \left(\frac{N\Phi}{i}\right)\left(\frac{N}{N}\right) = \frac{N^2\Phi}{Ni} = N^2\frac{\Phi}{F} = N^2\Lambda = \frac{N^2}{S}$$

$\therefore L = \frac{N^2}{S}$

Instantaneous electric power; $p = vi$

Instantaneous electric energy; $w = \int pdt = \int vidt = \int L\frac{di}{dt}idt = L\int idi = \frac{1}{2}Li^2$

$\therefore w = \frac{1}{2}Li^2$

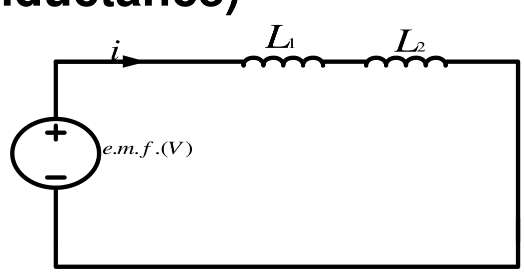
The ½Li² energy drawn from supply is stored in the magnetic field that has been established.

Stored energy per unit volume;

$$(\frac{1}{2})(\frac{Li^2}{Al}) = (\frac{1}{2})(\frac{N\Phi\Phi}{Al}) = (\frac{1}{2})(\frac{Ni}{l})(\frac{\Phi}{A}) = \frac{1}{2}BH = \mu H^2 = \frac{B^2}{\mu}$$

(b) Inductor Circuits

Series Inductance Connections (No Mutual Inductance)



Flux linkage = $N\Phi = Li$ *and* $L = N\frac{\Phi}{i}$.

Flux linkage due to $L_1 = iL_1$
Flux linkage due to $L_2 = iL_2$
Total flux linkage $Li = L_1 i + L_2 i = i(L_1 + L_2)$

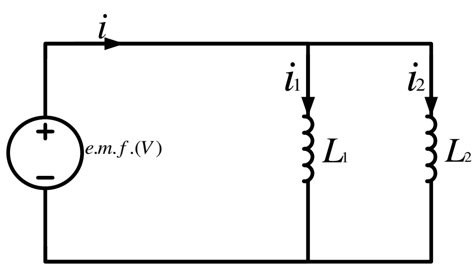
$$\therefore L_{total} = L_1 + L_2$$

Series Inductance Connections (With Mutual Inductance)

The effective inductance of the first coil is $L_1 + m$ if the self and mutual inductance are in the same direction and $L_1 - m$ if in opposition. Similarly the effective inductance of the second coil is $L_2 + m$ or $L_2 - m$.

The general expression is $L = L_1 + L_2 \pm 2m$.

Parallel Inductance Connections (No Mutual Inductance)



$$i = i_1 + i_2 \Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$V = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} \text{ and } V = L_{total}\frac{di}{dt} \Rightarrow \therefore \frac{V}{L_{total}} = \frac{di}{dt}$$

$$\frac{V}{L_1} = \frac{di_1}{dt} \text{ and } \frac{V}{L_2} = \frac{di_2}{dt} \Rightarrow \therefore \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{V}{L} = \frac{V}{L_1} + \frac{V}{L_2} = V\left(\frac{1}{L_1} + \frac{1}{L_2}\right) \Rightarrow \therefore \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

For two inductance in parallel, $L = \frac{L_1 L_2}{L_1 + L_2}$.

Parallel Inductance Connections (With Mutual Inductance)

$$V = L\frac{di}{dt} = L_1\frac{di_1}{dt} \pm m\frac{di_2}{dt} = L_2\frac{di_2}{dt} \pm m\frac{di_1}{dt}$$

$$L_1\frac{di_1}{dt} \pm m\frac{di_2}{dt} = L_2\frac{di_2}{dt} \pm m\frac{di_1}{dt}$$

$$L_1\frac{di_1}{dt} \mp m\frac{di_1}{dt} = L_2\frac{di_2}{dt} \mp m\frac{di_2}{dt}$$

$$(L_1 \mp m)\frac{di_1}{dt} = (L_2 \mp m)\frac{di_2}{dt}$$

$$\therefore \frac{di_1}{dt} = \left(\frac{L_2 \mp m}{L_1 \mp m}\right)\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \Rightarrow \frac{di}{dt} = \left(\frac{L_2 \mp m}{L_1 \mp m}\right)\frac{di_2}{dt} + \frac{di_2}{dt} = \left(\frac{L_2 \mp m}{L_1 \mp m} + 1\right)\frac{di_2}{dt} = \left(\frac{L_1 + L_2 \mp 2m}{L_1 \mp m}\right)\frac{di_2}{dt}$$

$$\therefore \frac{di}{dt} = \left(\frac{V}{L}\right) = \left(\frac{1}{L}\right)\left(L_1\frac{di_1}{dt} \pm m\frac{di_2}{dt}\right) = \left(\frac{1}{L}\right)\left(L_1\left\{\frac{L_2 \mp m}{L_1 \mp m}\right\} \pm m\right)\frac{di_2}{dt}$$

$$\therefore \left(\frac{L_1 + L_2 \mp 2m}{L_1 \mp m}\right) = \left(\frac{1}{L}\right)\left(L_1\left\{\frac{L_2 \mp m}{L_1 \mp m}\right\} \pm m\right) = \left(\frac{1}{L}\right)\left(\frac{L_1L_2 \mp L_1m}{L_1 \mp m} \pm m\right) = \left(\frac{1}{L}\right)\left(\frac{L_1L_2 \mp L_1m \pm m(L_1 \mp m)}{L_1 \mp m}\right)$$

$$\therefore \left(\frac{L_1 + L_2 \mp 2m}{L_1 \mp m}\right) = \left(\frac{1}{L}\right)\left(\frac{L_1L_2 \mp L_1m \pm L_1m - m^2}{L_1 \mp m}\right) = \left(\frac{1}{L}\right)\left(\frac{L_1L_2 - m^2}{L_1 \mp m}\right)$$

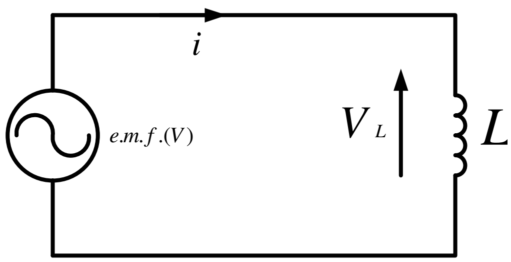
$$\therefore L = \left(\frac{L_1L_2 - m^2}{L_1 \mp m}\right)\left(\frac{L_1 \mp m}{L_1 + L_2 \mp 2m}\right) = \left(\frac{L_1L_2 - m^2}{L_1 + L_2 \mp 2m}\right)$$

When the mutual field is in the same direction, $L = \left(\frac{L_1L_2 - m^2}{L_1 + L_2 - 2m}\right)$

When the mutual field is in opposition, $L = \left(\frac{L_1L_2 - m^2}{L_1 + L_2 + 2m}\right)$

For general case; $L = \left(\frac{L_1L_2 - m^2}{L_1 + L_2 \mp 2m}\right)$

(c) AC Inductance Circuits



For $i = I_m \sin \omega t$,

$$V = -V_L = -L\frac{di}{dt} = -L\frac{d}{dt}\left(I_m \sin\theta\right) = -LI_m\frac{d}{dt}\sin\omega t = -\omega LI_m \cos\omega t$$

$$\sin(\omega t - \frac{\pi}{2}) = -\cos\omega t$$

$$\therefore V = -\omega LI_m \cos\omega t = \omega LI_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$V_m = \omega LI_m$$

$$I_m = \frac{V_m}{\omega L} = \frac{V_m}{2\pi fL} = \frac{V_m}{x_L}$$

$$x_L = \omega L = 2\pi fL$$

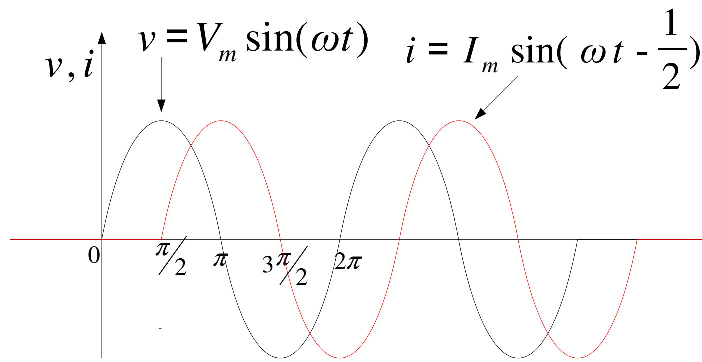
The current lags the voltage by $\frac{\pi}{2}$ i.e. the voltage leads the current by $\frac{\pi}{2}$.

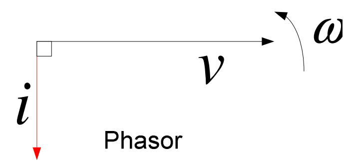
Hence $-jI_m = \frac{V}{\omega L} = \frac{V_m}{x_L}$ and $x_L = j\omega L = j2\pi fL$.

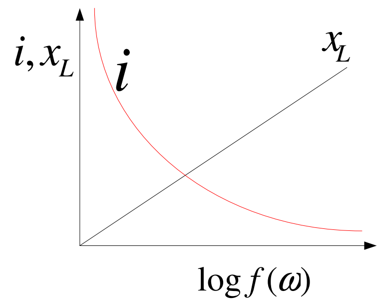
$$i = \frac{V}{\omega L} = \frac{\omega LI_m \sin\left(\omega t - \frac{\pi}{2}\right)}{\omega L} = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\therefore i = \frac{V}{\omega L} = I_m \sin(\omega t - \frac{\pi}{2})$$

$x_L = \omega L$ is the inductive reactance.

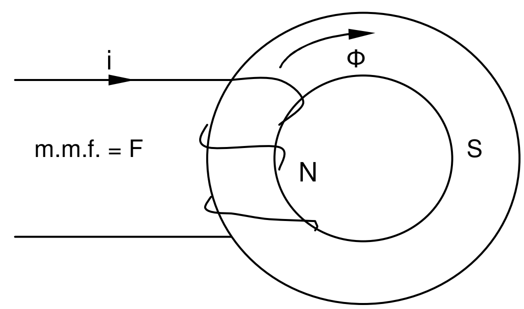






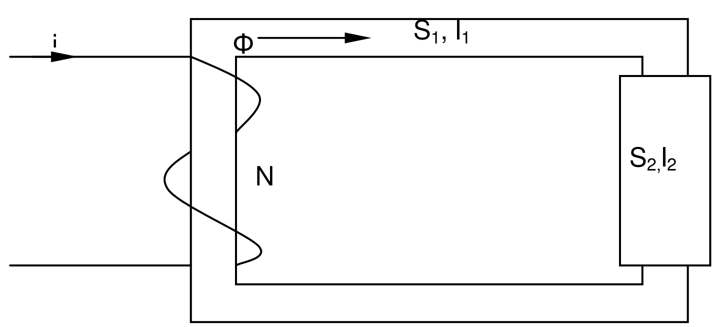
2.3 MAGNETIC CIRCUITS

(a) Series Reluctance



$$\Phi = \frac{F}{S} \text{ and } S = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}$$

F = m.m.f. = Ni (At/m),
S = reluctance (At/Wb) and
$l = \text{length of iron core}$.



$S_1 = \frac{l_1}{\mu_0 \mu_{r1} A_1}$ *and* $S_2 = \frac{l_2}{\mu_0 \mu_{r2} A_2}$

Let S be total reluctance, $F_{total} = F_1 + F_2$ *and* $F_1 = S_1\Phi$, $F_2 = S_2\Phi$

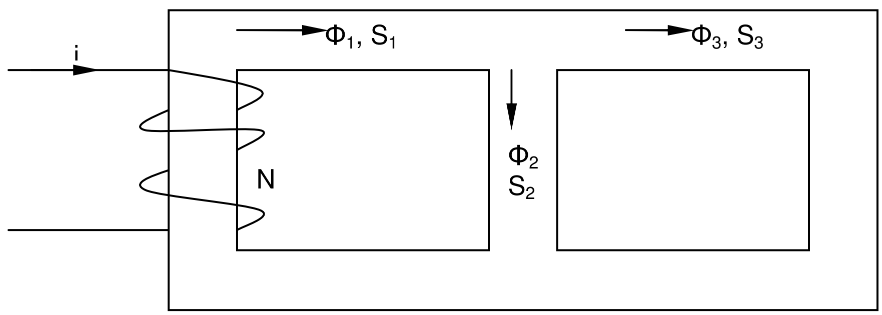
$$F_{total} = F_1 + F_2 = \Phi S_1 + \Phi S_2 = \Phi(S_1 + S_2) \text{ and } S = \frac{F_{total}}{\Phi} = \frac{\Phi(S_1 + S_2)}{\Phi} = S_1 + S_2$$

$$\therefore S = S_1 + S_2$$

$$\frac{F_1}{F_{total}} = \frac{\Phi S_1}{\Phi(S_1 + S_2)} = \frac{S_1}{S_1 + S_2}$$

$$F_1 = F_{total}\left(\frac{S_1}{S_1 + S_2}\right) \text{ and } F_2 = F_{total}\left(\frac{S_2}{S_1 + S_2}\right)$$

(b) Parallel Reluctance



Total Flux $\Phi = \Phi_1 = \Phi_2 + \Phi_3 = \frac{F_{total}}{S_2} + \frac{F_{total}}{S_3} = F_{total}(\frac{1}{S_2} + \frac{1}{S_3})$

$$\Phi_2 + \Phi_3 = F_{total}(\frac{1}{S_2} + \frac{1}{S_3}) = F_{total}(\frac{S_2 + S_3}{S_2 S_3})$$

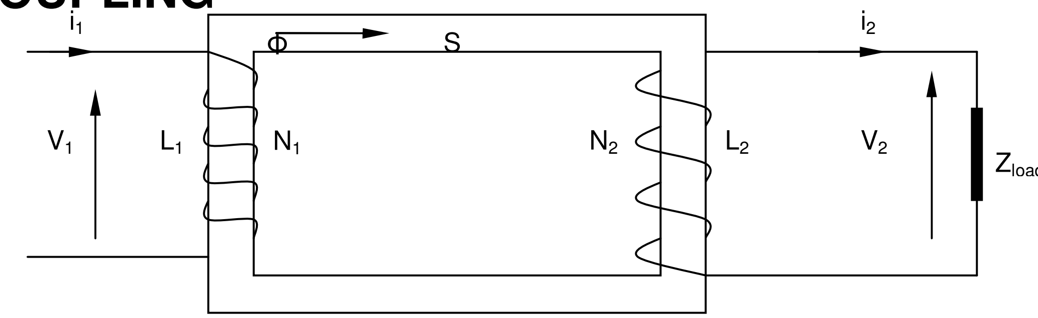
$$\therefore S_{total} = S_1 + S_{(S_2 // S_3)} = S_1 + \frac{S_2 S_3}{S_2 + S_3}$$

$$\therefore F_{total} = \Phi(\frac{S_2 S_3}{S_2 + S_3})$$

$$\Phi_2 = \frac{F_{total}}{S_2} = \Phi(\frac{S_3}{S_2 + S_3}) \text{ and } \Phi_3 = \frac{F_{total}}{S_3} = \Phi(\frac{S_2}{S_2 + S_3})$$

Reluctances in a magnetic circuit combine together in a similar way to resistances in an electric circuit.

(c) MUTUAL INDUCTANCE (m) FOR PERFECT COUPLING



$$V_1 = L_1 \frac{di_1}{dt} \; and \; V_2 = m\frac{di_1}{dt} \; where \; m = mutual \; induc\tan ce$$

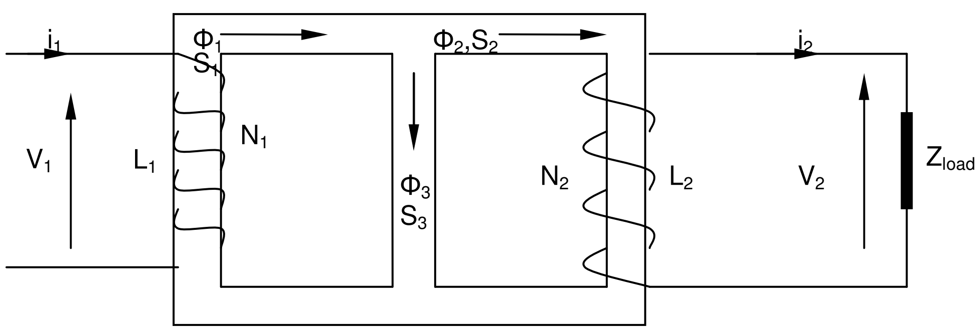
$$Also \; V_2 = N_2 \frac{d\Phi}{dt}$$

$$\Phi = \frac{F}{S} = \frac{N_1 i_1}{S} \Rightarrow \therefore V_2 = N_2 \frac{d\Phi}{dt} = N_2 \frac{d}{dt}\left(\frac{N_1 i_1}{S}\right) = \frac{N_1 N_2}{S}\left(\frac{di_1}{dt}\right) \Rightarrow \therefore m = \frac{N_1 N_2}{S}$$

$$L_1 = \frac{N_1^{\,2}}{S} \; and \; L_2 = \frac{N_2^{\,2}}{S} \Rightarrow \therefore L_1 L_2 = \frac{N_1^{\,2} N_2^{\,2}}{S^2}$$

$$\frac{N_1 N_2}{S} = \sqrt{L_1 L_2}$$

(d) MUTUAL INDUCTANCE (m) FOR IMPERFECT COUPLING



$$\Phi_1 \neq \Phi_2 \; and \; \Phi_3 = flux \; leakage$$

$$\Phi_1 = \frac{F}{S} \; and \; S = S_1 + \frac{S_2 S_3}{S_1 + S_3} = \frac{S_1 S_2 + S_1 S_3 + S_2 S_3}{S_2 + S_3}$$

$$\therefore \Phi_1 = \frac{F}{S} = \frac{N_1 i_1}{\left(\frac{S_1 S_2 + S_1 S_3 + S_2 S_3}{S_2 + S_3}\right)} = \frac{(S_2 + S_3)(N_1 i_1)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)}$$

$$\Phi_2 = \Phi_1 \left(\frac{S_3}{S_2 + S_3}\right) = \frac{(S_2 + S_3)(N_1 i_1)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)} \left(\frac{S_3}{S_2 + S_3}\right) = \frac{S_3 (N_1 i_1)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)}$$

$$V_2 = N_2 \frac{d\Phi_2}{dt}$$

$$\therefore V_2 = N_2 \frac{d}{dt}\left(\frac{S_3 (N_1 i_1)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)}\right) = \left[\frac{S_3 (N_1 N_2)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)}\right] \frac{di_1}{dt} = m \frac{di_1}{dt}$$

$$\therefore m = \left[\frac{S_3 (N_1 N_2)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)}\right]$$

$$L_1 = \frac{N_1^{\,2}}{S} = \frac{N_1^{\,2}(S_2 + S_3)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)}$$

$$L_2 = \frac{N_2^{\,2}}{S} \; \text{and} \; S = S_2 + \frac{S_1 S_3}{S_1 + S_3} = \frac{S_1 S_2 + S_2 S_3 + S_1 S_3}{S_1 + S_3}$$

$$\therefore L_2 = \frac{N_2^{\,2}(S_1 + S_3)}{S_1 S_2 + S_1 S_3 + S_2 S_3}$$

$$L_1 L_2 = \frac{N_1^{\,2} N_2^{\,2}}{S^2} = \left[\frac{N_1^{\,2}(S_2 + S_3)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)}\right]\left[\frac{N_2^{\,2}(S_1 + S_3)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)}\right] = \frac{(N_1 N_2)^2 (S_2 + S_3)(S_1 + S_3)}{(S_1 S_2 + S_1 S_3 + S_2 S_3)^2}$$

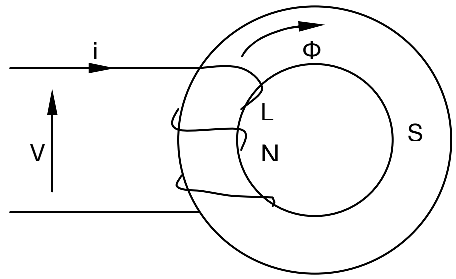
$$\therefore L_1 L_2 = \frac{m^2}{S_3^{\,2}} (S_2 + S_3)(S_1 + S_3)$$

$$m^2 = \frac{S_3^{\,2}(L_1 L_2)}{(S_2 + S_3)(S_1 + S_3)}$$

$$m = \frac{S_3}{\sqrt{(S_2 + S_3)(S_1 + S_3)}} \sqrt{L_1 L_2} = K \sqrt{L_1 L_2}$$

K is the coefficient of coupling. When K = 1, the case of perfect coupling is achieved where $S_3 = \infty$ i.e. the case for no leakage.

(e) MAGNETISING CURRENT



$$V = N\frac{d\Phi}{dt} = L\frac{di}{dt}.\ \ for\, i = I_m \sin\omega t, V = L\frac{di}{dt} = I_m \omega L \cos\omega t$$

$$V_m = I_m \omega L \cos 0 = I_m \omega L$$

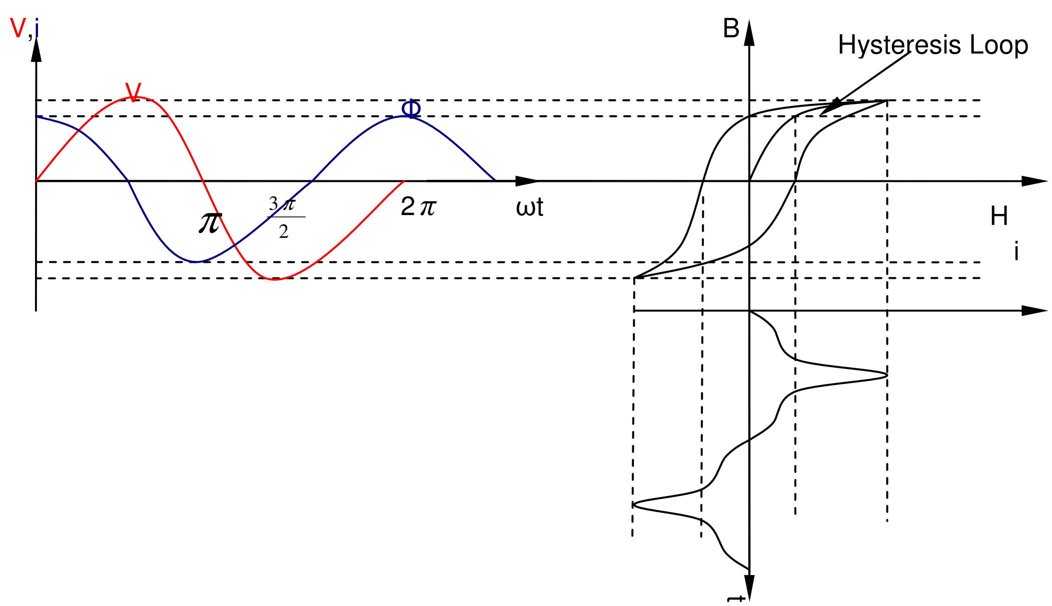
$$I_m = \frac{V_m}{\omega L}$$

$$For\ \Phi = \Phi_m \sin\omega t\ ,$$

$$V = N\frac{d\Phi}{dt} = N\Phi_m \omega \cos\omega t$$

$$V_m = N\Phi_m \omega \cos 0 = N\Phi_m \omega = 2\pi f N \Phi_m$$

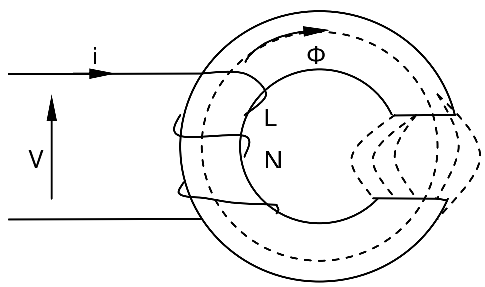
$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{2\pi f N \Phi_m}{\sqrt{2}} = 4.44 N\Phi_m f = 4.44 N B_m A f$$



(f) Hysteresis Loop

The area of the loop is proportional to the energy dissipated in the material due to the rapidly-changing magnetic flux where the magnetizing current is alternating. Although no energy is required to maintain a magnetic field, energy is required to build it up. This energy is stored in the field but it is found that, in the case of magnetic materials, the amount of energy returned to the circuit, when the current is reduced and the field allowed to collapse, is less than the energy supplied during the building-up period. Magnetization of a piece of material is due to revolving electrons having had their axes aligned by the magnetizing force. The energy required to change the direction of these axes is dissipated as heat, and represents the difference in energy supplied to the field and the energy returned from it during a cycle of magnetization, that is, the hysteresis loss.

(g) MAGNETIC FRINGING AND LEAKAGE



In the above figure, the flux continuity has been omitted for convenience. For a long air-gap, the flux passing across it bulges outwards, so that the effective area of the gap is increased, and the distribution of the flux in the gap is less uniform than in the iron. This effect is known as fringing, and allowance can be made for it by considering the area of the gap to be slightly increased, by an amount all round equal to half the length of the air-gap.

In a closed ring there is practically no magnetic leakage, but in other forms of magnetic circuits the magnetizing coil is wound over only part of the core, and this is often of a complicated form so that not all the lines induced by magnetizing coil pass round the magnetic circuit, but many complete their loop by shorter paths partly in the air. To allow for leakage of the lines in practice, the maximum flux to be induced must be greater by a factor known as the leakage coefficient and given by

$$Leakage\ coefficient = \frac{\Phi_{total}}{\Phi_{useful}}$$

In electrical machinery the value of the leakage coefficient is usually between 1.1 and 1.25.

2.4 Electric Circuits

Network Analysis

In an electrical network, electrical energy is conveyed from source to an array of interconnected branches in which energy is converted, dissipated or stored. Each branch has a characteristic voltage/current relationship that defines its parameters. The analysis of the network is concerned with the solution of the source and the branch currents/voltages in a given network configuration.

Network Elements

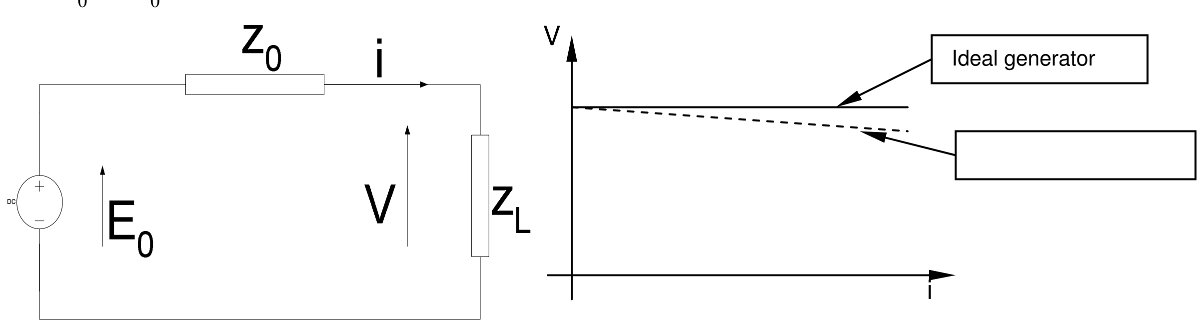
The elements are divided into passive and active elements. Passive elements consume or store energy while active elements pass out energy to passive elements. The three basic elements are resistor, inductor and capacitor. The resistor consumes ohmic or dissipative energy whereas the inductor and capacitor store in the positive half cycle and give away in the negative half cycle of the supply the magnetic field and electric field energies respectively.

2.4.1 Active Elements

Voltage Generator

A generator supplies voltage and current and the output voltage is relatively constant for a given constant current.

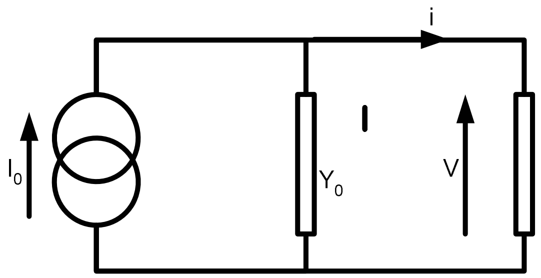
$V = E_0 - iZ_0$

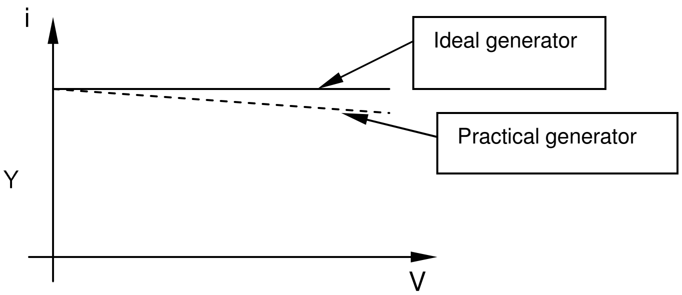


Current Generator

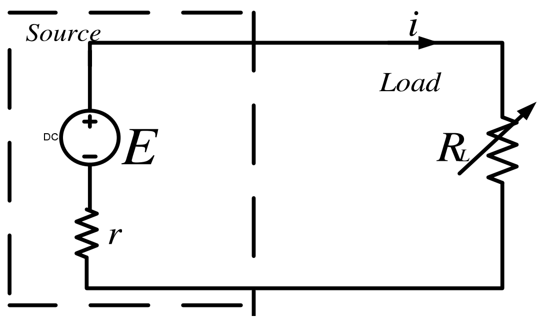
The voltage and current generator are identical provided that $I_0 = \dfrac{E_0}{Z_0}$ and $Y_0 = \dfrac{1}{Z_0}$. The identity applies only to the load terminal. Internally, the sources have quite different operating conditions.

$$V = E_0 - iZ_0 \Rightarrow i = \left(\frac{E_0 - V}{Z_0}\right) = \left(\frac{E_0}{Z_0} - \frac{V}{Z_0}\right) = \left(I_0 - VY_0\right)$$





Maximum Power Transfer



$$i = \left(\frac{E}{r+R_L}\right) \Rightarrow P = i^2 R_L = \left(\frac{E}{r+R_L}\right)^2 R_L = \frac{\left(E^2 R_L\right)}{(r+R)^2} = \frac{E^2 R_L}{r^2 + 2rR_L + R_L{}^2} = \frac{E^2}{\frac{r^2}{R_L} + 2r + R_L}$$

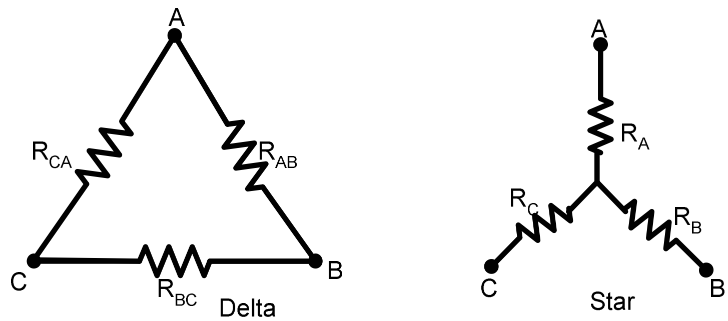
The maximum power P occurs when the denominator is minimum.

$$\frac{d}{dR_L}\left(\frac{r^2}{R_L} + 2r + R_L\right) = 0 = \frac{-r^2}{R_L{}^2} + 1 \Rightarrow \therefore \frac{r^2}{R_L{}^2} = 1 \Rightarrow r^2 = R_L{}^2 \Rightarrow r = R_L$$

A pure resistance load will abstract maximum power from a network when the load resistance is equal to the magnitude of the internal resistance of the network. This is known as Resistance Matching and is important in communication and electronics circuits where internal resistance ‘r’ is very high and the source e.m.f. is limited. In power circuits, the internal resistance ‘r’ is very low and so it is usually impossible to carry out resistance matching without wasting a lot of energy and overloading the source.

Resistance Transformation

Delta/Star Transformation



Resistance A-B in delta; $R_{AB} = \dfrac{R_{AB}(R_{BC}+R_{CA})}{R_{AB}+R_{BC}+R_{CA}}$

Resistance A-B in star; $R_{AB} = R_A + R_B$

$$\therefore R_A + R_B = \frac{R_{AB}(R_{BC}+R_{CA})}{R_{AB}+R_{BC}+R_{CA}} = \frac{R_{AB}R_{BC}+R_{AB}R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \text{........(1)}$$

Resistance B-C in star; $R_{BC} = R_B + R_C$

Resistance B-C in delta; $R_{BC} = \dfrac{R_{BC}(R_{CA}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}}$

$$\therefore R_B + R_C = \frac{R_{BC}(R_{CA}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}} = \frac{R_{BC}R_{CA}+R_{BC}R_{AB}}{R_{AB}+R_{BC}+R_{CA}} \text{........(2)}$$

Resistance C-A in star; $R_{CA} = R_C + R_A$

Resistance C-A in delta; $R_{CA} = \dfrac{R_{CA}(R_{BC}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}}$

$$\therefore R_C + R_A = \frac{R_{CA}(R_{BC}+R_{AB})}{R_{AB}+R_{BC}+R_{CA}} = \frac{R_{CA}R_{BC}+R_{CA}R_{AB}}{R_{AB}+R_{BC}+R_{CA}} \text{........(3)}$$

Subtracting equation (2) from equation (1):

$$R_A - R_C = \frac{R_{AB}R_{CA}-R_{BC}R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \text{........(4)}$$

Adding equations (3) and (4);

$$2R_A = \frac{2R_{CA}R_{AB}}{R_{AB}+R_{BC}+R_{CA}} \Rightarrow \therefore R_A = \frac{R_{AB}R_{CA}}{R_{AB}+R_{BC}+R_{CA}}$$

Subtracting equation (3) from equation (2);

$$R_B - R_A = \frac{R_{AB}R_{BC}-R_{AB}R_{CA}}{R_{AB}+R_{BC}+R_{CA}} \text{........(5)}$$

Adding equations (1) and (5);

$$2R_B = \frac{2R_{AB}R_{BC}}{R_{AB}+R_{BC}+R_{CA}} \Rightarrow \therefore R_B = \frac{R_{AB}R_{BC}}{R_{AB}+R_{BC}+R_{CA}}$$

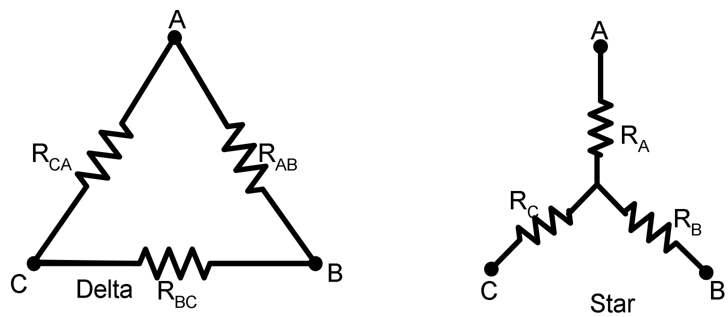
Subtracting equation (1) from equation (3);

$$R_C - R_B = \frac{R_{CA}R_{BC} - R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \text{........(6)}$$

Adding equations (2 and (6);

$$2R_C = \frac{2R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \Rightarrow \therefore R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

Star/Delta Transformation



$$\frac{R_A}{R_B} = \frac{R_{CA}}{R_{BC}} \Rightarrow \therefore R_{CA} = R_{BC}\left(\frac{R_A}{R_B}\right) \text{.......(1)}$$

$$\frac{R_A}{R_C} = \frac{R_{AB}}{R_{BC}} \Rightarrow \therefore R_{AB} = R_{BC}\left(\frac{R_A}{R_C}\right) \text{.......(2)}$$

But $R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$ *and substituting* R_A *in equations (1) and (2);*

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{R_{BC}\left(\frac{R_A}{R_C}\right)R_{BC}\left(\frac{R_A}{R_B}\right)}{R_{BC}\left(\frac{R_A}{R_C}\right) + R_{BC} + R_{BC}\left(\frac{R_A}{R_B}\right)} = \frac{R_{BC}R_A{}^2}{R_AR_B + R_BR_C + R_AR_C}$$

$$\therefore R_{BC} = \frac{R_A\left(R_AR_B + R_BR_C + R_AR_C\right)}{R_A{}^2} = \frac{R_AR_B + R_BR_C + R_AR_C}{R_A}$$

$$\frac{R_B}{R_C} = \frac{R_{AB}}{R_{CA}} \Rightarrow \therefore R_{AB} = R_{CA}\left(\frac{R_B}{R_C}\right) \text{.......(3)}$$

$$\frac{R_B}{R_A} = \frac{R_{BC}}{R_{CA}} \Rightarrow \therefore R_{BC} = R_{CA}\left(\frac{R_B}{R_A}\right) \text{.......(4)}$$

But $R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$ *and substituting* R_B *in equations (3) and (4);*

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} = \frac{R_{CA}\left(\frac{R_B}{R_C}\right)R_{CA}\left(\frac{R_B}{R_A}\right)}{R_{CA}\left(\frac{R_B}{R_C}\right) + R_{CA}\left(\frac{R_B}{R_A}\right) + R_{CA}} = \frac{R_{CA}R_B{}^2}{R_AR_B + R_BR_C + R_AR_C}$$

$$\therefore R_{CA} = \frac{R_B\left(R_A R_B + R_B R_C + R_A R_C\right)}{{R_B}^2} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_B}$$

$$\frac{R_C}{R_B} = \frac{R_{CA}}{R_{AB}} \Rightarrow \therefore R_{CA} = R_{AB}\left(\frac{R_C}{R_B}\right) ....(5)$$

$$\frac{R_C}{R_A} = \frac{R_{BC}}{R_{AB}} \Rightarrow \therefore R_{BC} = R_{AB}\left(\frac{R_C}{R_A}\right) ....(6)$$

But $R_C = \dfrac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$ *and substituting R_C in equations (5) and (6);*

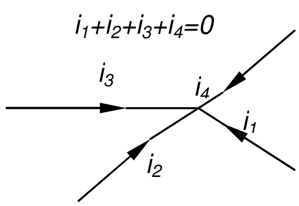
$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{R_{AB}\left(\frac{R_C}{R_A}\right)R_{AB}\left(\frac{R_C}{R_B}\right)}{R_{AB} + R_{AB}\left(\frac{R_C}{R_A}\right) + R_{AB}\left(\frac{R_C}{R_B}\right)} = \frac{R_{AB}{R_C}^2}{R_A R_B + R_B R_C + R_A R_C}$$

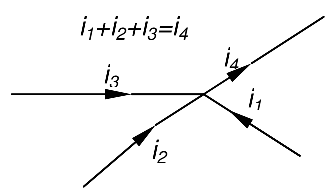
$$\therefore R_{AB} = \frac{R_C\left(R_A R_B + R_B R_C + R_A R_C\right)}{{R_C}^2} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_C}$$

2.4.2 Kirchoff's Laws

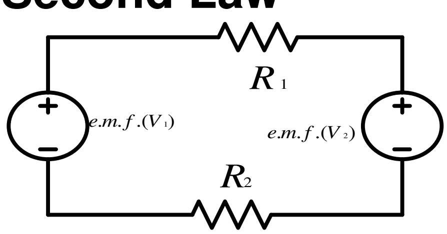
(a) First Law

The total current flowing into a node is zero, $\sum i = 0$. The sum of the branch currents flowing into a node is equal to the sum of the currents flowing out of a node.





(b) Second Law

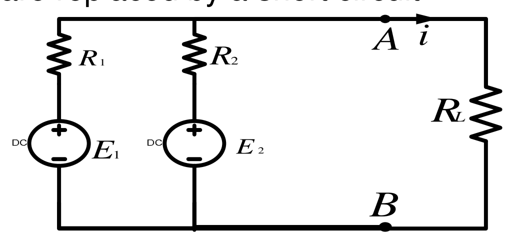


The sum of the voltages around a closed mesh or loop is zero, $\sum V = 0$. A rise of potential in the source is absorbed by a fall in potential in the successive branches forming the mesh.

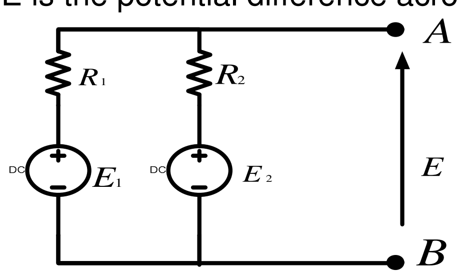
2.4.3 Network Theorems

Thevenin Theorem

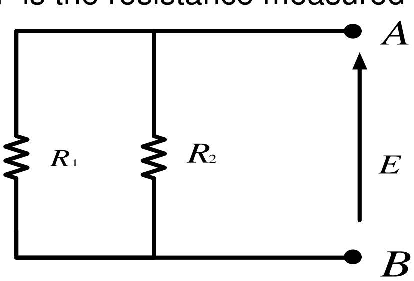
Thevenin's theorem states that any network containing sources and resistors can be replaced by a constant single source of e.m.f. 'E' and a series internal resistance 'r'. The value of 'E' is equal to the open-circuit terminal voltage of the network measured between A and B. The resistance 'r' is the resistance of the network measured between terminals A and B, with all network sources replaced by their internal resistances. If the sources internal network is ideal having zero internal resistance, these are replaced by a short circuit.



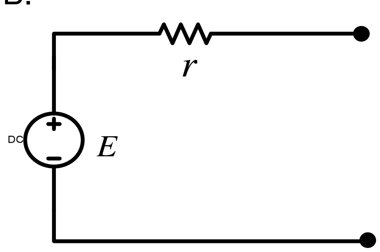
E is the potential difference across A-B with the load removed.



'r' is the resistance measured between A and B

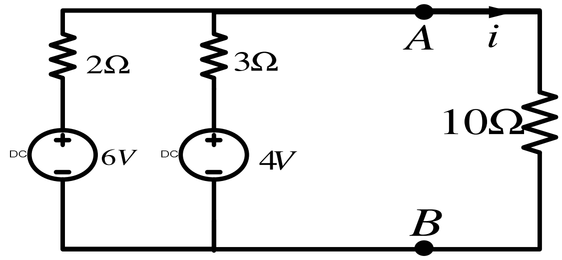


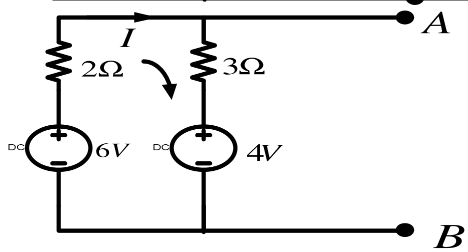
R_1 is in parallel with R_2, the combination that is measured between A and B.



Example 1

Calculate the current i using Thevenin,s theorem.

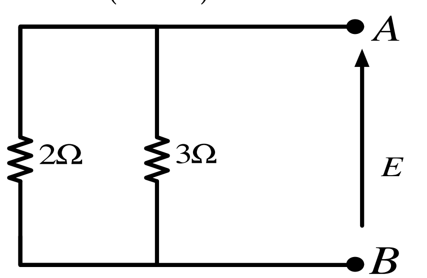




Circulating current I;

$I = (\frac{6-4}{2+3}) = \frac{2}{5} = 0.4A$ The potential difference across A-B;

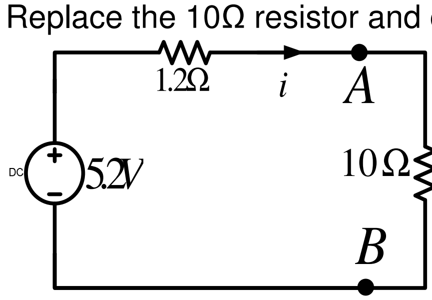
$E = 6 - (2 \times 0.4) = 5.2V$



'r' is the resistance of 2Ω in parallel with 3Ω;

$r = \left(\frac{2 \times 3}{2+3}\right) = \frac{6}{5} = 1.2\Omega$

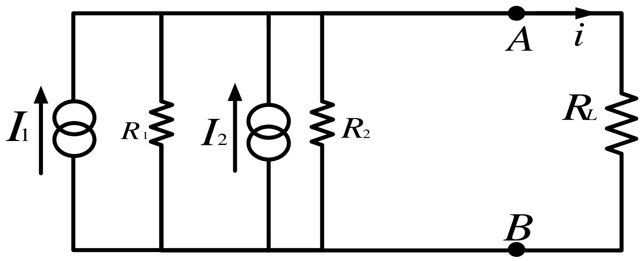
Replace the 10Ω resistor and calculate the circulating current;



$i = \frac{5.2}{(1.2+10)} = \frac{5.2}{11.2} = 0.4643A$

Norton's Theorem

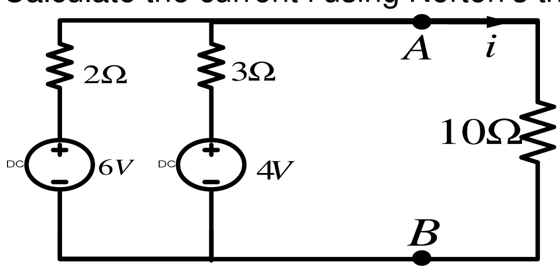
The Norton's theorem can be treated as a variation of the Thevenin's theorem in which a current generator replaces the Thevenin voltage generator.



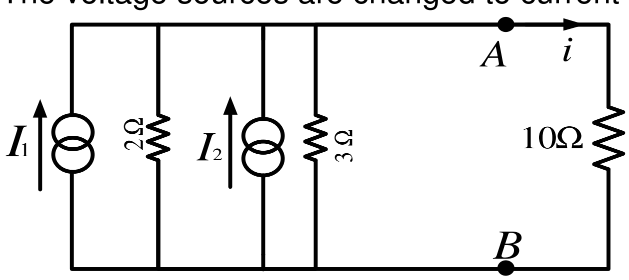
The two terminals A and B can be replaced by a constant current source I in parallel with the voltage generator internal resistance. The value of I is the current which flows from A and B when A and B are short circuited and 'r' is the resistance of the network measured from A and B with the load resistance disconnected and the current sources replaced by their internal resistances.

Example 2

Calculate the current i using Norton's theorem.

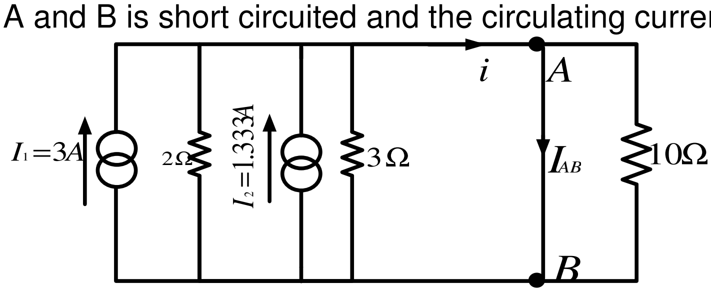


The voltage sources are changed to current sources.



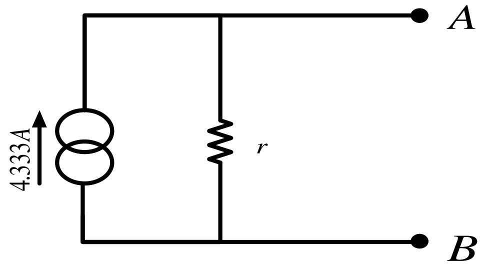
$$I_1 = \frac{E_1}{R_1} = \frac{6}{2} = 3A \text{ and } I_2 = \frac{E_2}{R_2} = \frac{4}{3} = 1.333A$$

A and B is short circuited and the circulating current calculated;



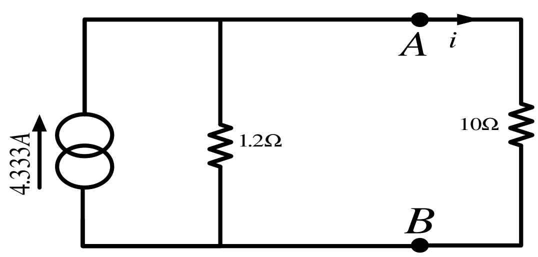
$I = I_1 + I_2 = 3 + 1.333 = 4.333A$

The internal resistance 'r' is the 2Ω resistor in parallel with the 3Ω.



$$r = \left(\frac{2\times 3}{2+3}\right) = 1.2\Omega$$

Norton's equivalent circuit



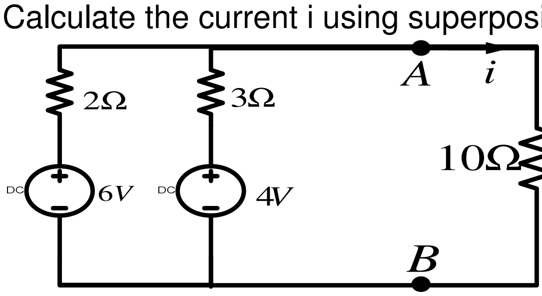
Using current divider rule, $i = \left(\frac{4.333\times 1.2}{10+1.2}\right) = 0.4643A$

Superposition Theorem

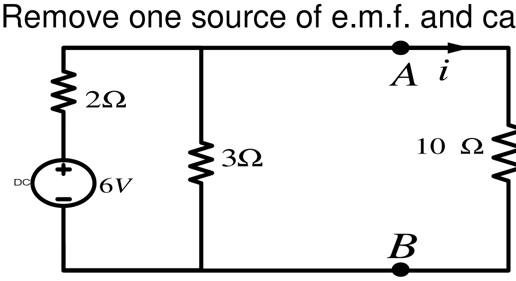
A network containing more than one source of e.m.f., the resultant current in any branch is the algebraic sum of the currents that would be produced by each e.m.f. acting alone, and all the other sources of e.m.f. being replaced mean while by their respective internal resistances.

Example 3

Calculate the current i using superposition method.



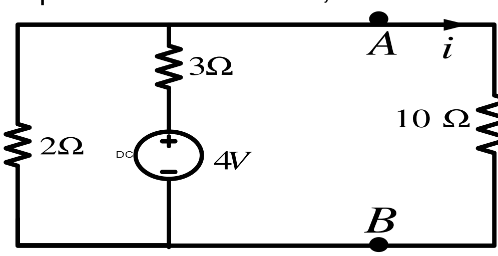
Remove one source of e.m.f. and calculate the current in a 10Ω resistor.



$$R_{total} = \left(\frac{10\times 3}{10+3}\right) + 2 = 4.308\Omega \ and \ I_{total} = \frac{6}{4.308} = 1.393A$$

Using current divider rule, $I_{10\Omega} = (1.393)\left(\frac{3}{10+3}\right) = 0.3214A$

Replace the removed e.m.f. source and remove the other source used in the previous calculation;

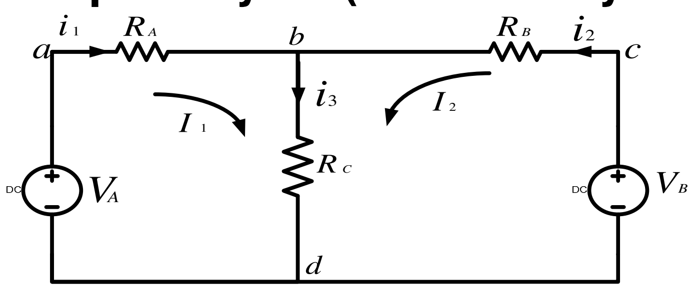


$$R_{total} = \left(\frac{2\times 10}{2+10}\right) + 3 = 4.667\Omega \ and \ I_{total} = \frac{4}{4.667} = 0.857A$$

Using current divider rule, $I_{10\Omega} = (0.857)\left(\frac{2}{2+10}\right) = 0.1429A$

With both e.m.f. sources in circuit, $I_{10\Omega} = 0.1429 + 0.3214 = 0.4643A$

Loop Analysis (Mesh Analysis)



The current in R_A resistor $i_1 = I_1$

The current in R_B resistor $i_2 = I_2$

The current in R_C resistor $i_3 = I_1 + I_2$

Applying Kirchoff's Voltage Law'

$$\sum V_{abd} = 0 = V_A - I_1R_A - R_C(I_1 + I_2)$$

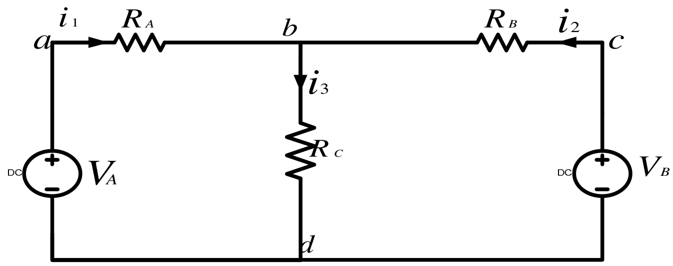
$$\therefore V_A = I_1R_A + R_C(I_1 + I_2) = I_1(R_A + R_C) + I_2R_C \text{.......(1)}$$

$$\sum V_{cbd} = 0 = V_B - I_2R_B - R_C(I_1 + I_2)$$

$$\therefore V_B = R_C(I_1 + I_2) + I_2R_B = I_1R_C + I_2(R_B + R_C) \text{........(2)}$$

Node Voltage Method

A reference node is chosen to which all the other node voltages are referred and then apply Kirchoff's Current Law to each independent node.



Node ‘d’ is chosen as the reference node,

$$\sum i = 0 = i_1 + i_2 + (-i_3)$$

$$\therefore i_3 = i_1 + i_2$$

$$i_1 = \frac{(V_a - V_b)}{R_A}, i_2 = \frac{(V_c - V_b)}{R_B} \; and \; i_3 = \frac{V_b}{R_C}$$

$$\therefore \frac{(V_a - V_b)}{R_A} + \frac{(V_c - V_b)}{R_B} + \left(-\frac{V_b}{R_C}\right) = 0$$

$$\frac{V_b}{R_C} = \frac{(V_a - V_b)}{R_A} + \frac{(V_c - V_b)}{R_B}$$

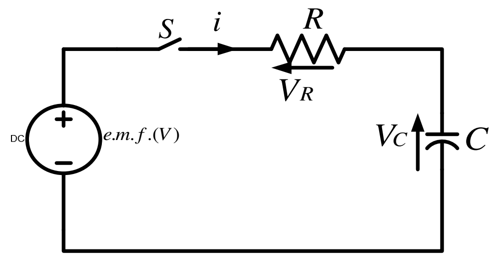
$$V_a = V_A$$

$$V_c = V_B$$

For known resistors (R_A, R_B and R_C), the voltage V_b can be calculated to enable the calculation of i_1, i_2 and i_3.

3.0 TRANSIENTS

3.1 RC NETWORK



When switch 'S' is closed, $V = V_R + V_C$

$$V_R = iR \ and \ i = C\frac{dV_c}{dt}$$

$$\therefore dV_C = \left(\frac{1}{C}\right)idt \Rightarrow V_C = \left(\frac{1}{C}\right)\int idt$$

$$\therefore V = iR + \left(\frac{1}{C}\right)\int idt \Rightarrow \frac{dV}{dt} = R\frac{di}{dt} + \frac{i}{C}..........(1)$$

For constant dc supply, $\frac{dV}{dt} = 0$ *since a steady state dc is a constant.*

$$0 = R\frac{di}{dt} + \frac{i}{C}$$

$$\frac{-i}{C} = R\frac{di}{dt}$$

$$\frac{-i}{RC} = \frac{di}{dt}$$

$$\left(\frac{-1}{RC}\right)dt = \frac{di}{i}$$

$$\int\left(\frac{-1}{RC}\right)dt = \int\frac{di}{i}$$

$$\frac{-t}{RC} = \log_e i + K.........(2)$$

K is the constant of integration.

At *t=0, when the switch is closed,* $i = \frac{V}{R}$.

$$\therefore \frac{0}{RC} = \log_e \frac{V}{R} + K$$

$$0 = \log_e \frac{V}{R} + K$$

But $I = \frac{V}{R}$, *the maximum current value in the circuit.*

$0 = \log_e I + K$

$\therefore K = -\log_e I$...........(3)

Substituting equation (3) into equation (2);

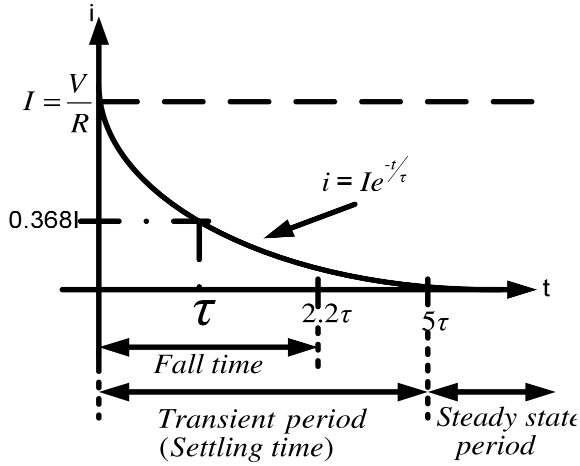
$$\frac{-t}{RC} = \log_e i - \log_e I = \log_e \frac{i}{I}$$

$$\therefore \frac{-t}{RC} = \log_e \frac{i}{I}$$

$$e^{-t/RC} = \frac{i}{I}$$

$$i = Ie^{-t/RC} = Ie^{-t/\tau}$$

Where $\tau = RC$ and is known as the time constant of the RC circuit.

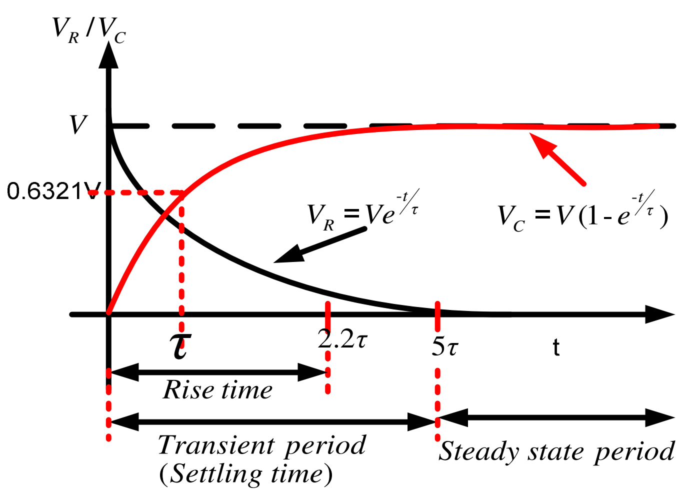


Fall time is defined as the time taken for the current to fall from 90% to 10% of its original value.

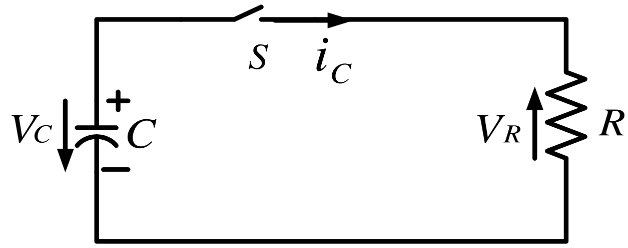
The voltage across the resistor is $V_R = iR = I\,\mathrm{R}e^{-t/RC} = Ve^{-t/RC} = Ve^{-t/\tau}$.

The voltage across the capacitor $V_C = V - V_R = V - Ve^{-t/\tau} = V\left(1 - e^{-t/\tau}\right)$.

At $\tau = RC$, $V_C = 0.6321V$.



3.2 CAPACITOR DISCHARGE



A capacitor which is originally charges to V_C volts is discharge through a resistor *R*.

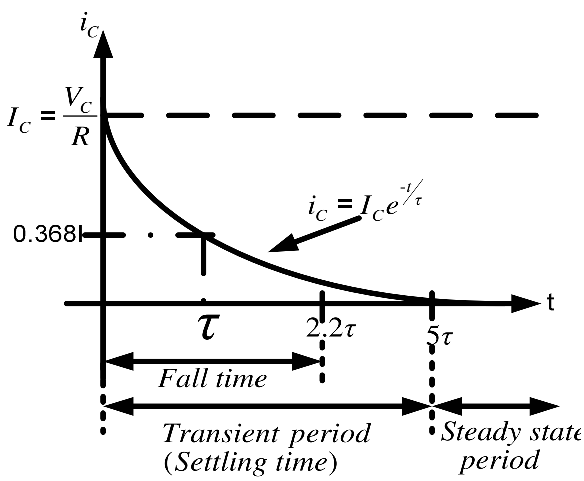
$$V_C = V\left(1 - e^{-t/\tau}\right)$$

$$i_C = \frac{V_C}{R} = \left(\frac{V}{R}\right)\left(1 - e^{-t/\tau}\right) = \frac{V}{R} - \frac{V_C}{R}e^{-t/\tau} = I - I_C e^{-t/\tau}$$

Since there is no voltage source in the circuit, the steady state current I must be zero.

$$\therefore i_C = I_C e^{-t/\tau}$$

The sign change is due to current reversal.



The voltage at any instant is the same across both the capacitor and resistor.

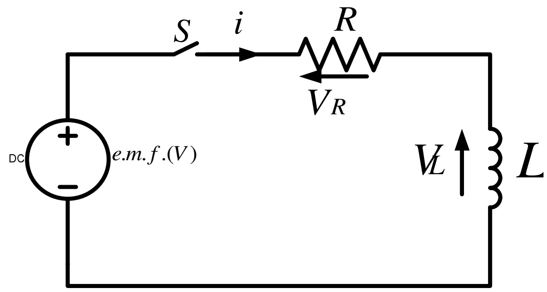
$$V = i_C R = V_C e^{-t/\tau}$$

The charge on the capacitor at any instant is;

$$q = CV = CV_C e^{-t/\tau} = Qe^{-t/\tau}$$

Where *Q* is the initial capacitor charge.

3.3 RL NETWORK



When switch 'S' is closed, $V = V_R + V_L$.

$$V_R = iR \text{ and } V_L = L\frac{di}{dt}$$

$$\therefore V = iR + L\frac{di}{dt} \Rightarrow L\frac{di}{dt} = V - iR \Rightarrow \left(\frac{L}{R}\right)\frac{di}{dt} = \frac{V}{R} - i$$

But $I = \frac{V}{R}$, *the maximum current value in the circuit.*

$$\therefore \left(\frac{L}{R}\right)\frac{di}{dt} = I - i$$

$$\frac{di}{I-i} = \frac{R}{L}dt$$

$$\int \frac{di}{I-i} = \left(\frac{R}{L}\right)\int dt$$

$$-\log_e(I-i) = \frac{Rt}{L} + K \text{..............(1)}$$

K is the constant of integration.

At the instant of closing the switch, t=0 and i=0,

$$-\log_e(I-0) = \frac{R(0)}{L} + K$$

$$\therefore K = -\log_e I \text{.......................(2)}$$

Substituting equation (2) into equation (1);

$$-\log_e(I-i) = \frac{Rt}{L} - \log_e I$$

$$\log_e I - \log_e (I \quad i) = \frac{Rt}{L}$$

$$\log_e\left(\frac{I}{I-i}\right) = \frac{Rt}{L}$$

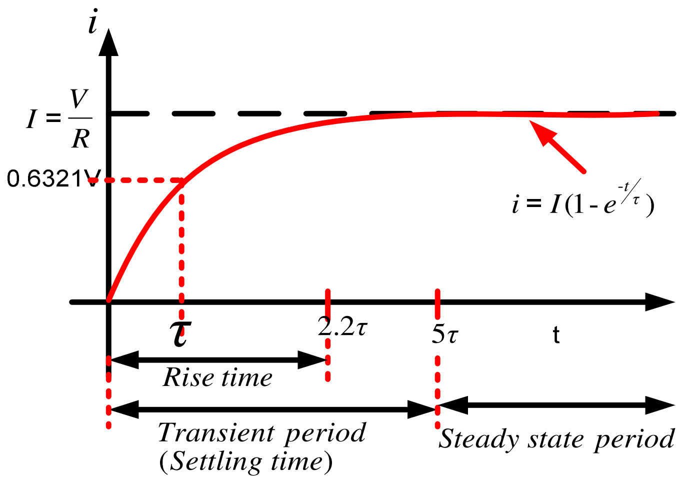
$$\left(\frac{I}{I-i}\right) = e^{Rt/L}$$

$$\left(\frac{I-i}{I}\right) = e^{-Rt/L}$$

$$(I - i) = Ie^{-Rt/L}$$

$$i = I - Ie^{-Rt/L} = I\left(1 - e^{-Rt/L}\right) = I\left(1 - e^{-t/\tau}\right)$$

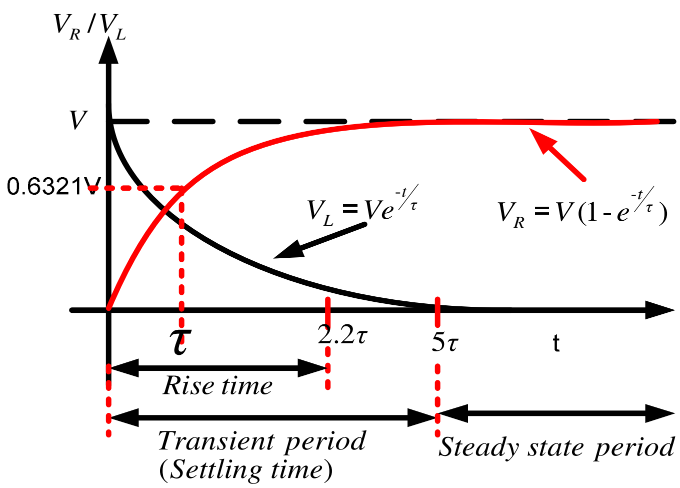
$\tau = \dfrac{L}{R}$ *and is known as the time constant of the RL circuit.*



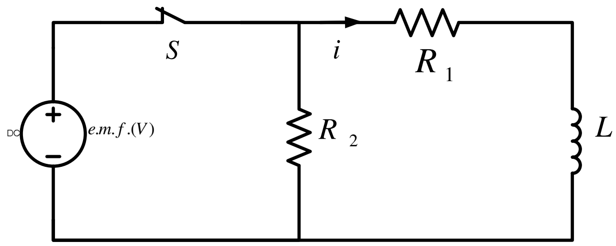
The voltage across the resistor $V_R = iR = IR\left(1 - e^{-t/\tau}\right) = V\left(1 - e^{-t/\tau}\right)$.

The voltage across the inductor at any instant is;

$$V_L = V - V_R = V - V\left(1 - e^{-t/\tau}\right) = Ve^{-t/\tau}$$



3.4 INDUCTIVE CURRENT DECAY



At time $t=0$, switch *'S'* is opened disconnecting the inductor L and resistor R_1 from the supply. The initial current just before disconnection is $i = I - Ie^{-t/\tau}$. When the switch *'S'* is disconnected, there will be no continuous source of e.m.f. in the circuit formed by L, R_1 and R_2. Therefore the steady state current I is zero i.e. $i = Ie^{-t/\tau}$ *where* $\tau = \dfrac{L}{R_1 + R_2}$.

The sign change is due to current reversal.

If R_2 is omitted, the energy stored $\left(\dfrac{1}{2}\right)Li^2$ in the magnetic circuit will cause a spark at the switch contacts and/or destroy the insulation of the coil owing to the large induced e.m.f. The energy is dissipated as heat in the circuit resistors.

3.5 DIFFERENTIAL EQUATIONS

Differential equations are classified according to the highest derivative which occurs in them i.e.

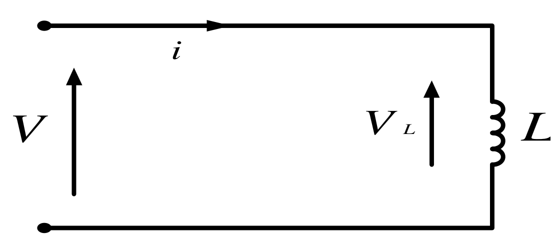
$\dfrac{dy}{dx} = 5x \Rightarrow$ First degree first order differential equation.

$\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 4y = 0 \Rightarrow$ First degree second order equation.

$\left(\dfrac{d^2s}{dt^2}\right)^3 + 2\left(\dfrac{ds}{dt}\right)^4 = 4 \Rightarrow$ Third degree second order equation.

Differential equations are useful for relating rates of change of variables and other parameters. The degree of a differential equation is that of the highest power of the highest differential which the equation contains after any necessary simplification.

INDUCTANCE



$$V = -V_L = -L\frac{di}{dt}$$

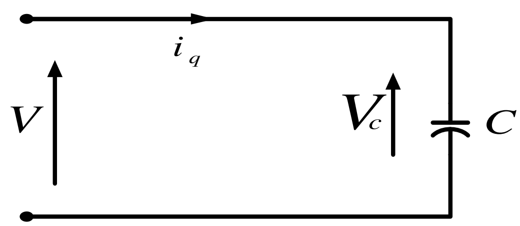
$$V_{(t)} = L\frac{di_{(t)}}{dt} \Rightarrow V_{(t)} = i_{(t)}x_L = j\omega L i_{(t)}$$

$$V_{(t)} = L\frac{di_{(t)}}{dt} \Rightarrow i_{(t)} = \frac{1}{L}\int V_{(t)}dt + I_0$$

$$V_{(t)} = L\frac{di_{(t)}}{dt} \Rightarrow i_{(t)} = \frac{1}{j\omega L}V_{(t)} + I_0$$

I_0 is the initial condition

CAPACITANCE



$$dq = i_q dt \Rightarrow dq = CdV \Rightarrow i_q = C\frac{dV}{dt} \Rightarrow V = \frac{1}{C}\int i_q dt$$

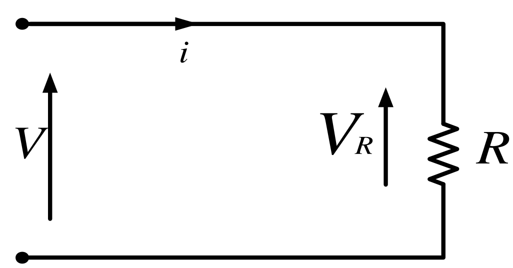
$$i_{(t)} = C\frac{dV_{(t)}}{dt} \Rightarrow i_{(t)} = \frac{V_{(t)}}{x_C} \Rightarrow i_{(t)} = \frac{V_{(t)}}{\frac{1}{j\omega C}} = j\omega C V_{(t)}$$

$$i_{(t)} = C\frac{dV_{(t)}}{dt} \Rightarrow V_{(t)} = \frac{1}{C}\int i_{(t)}dt + V_0$$

$$V_{(t)} = \frac{1}{j\omega C}i_{(t)} + V_0$$

V_0 is the initial condition

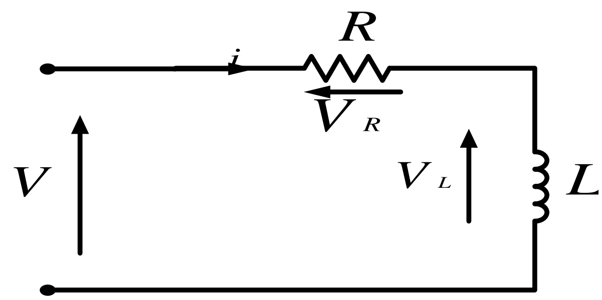
RESISTANCE



$V = iR$

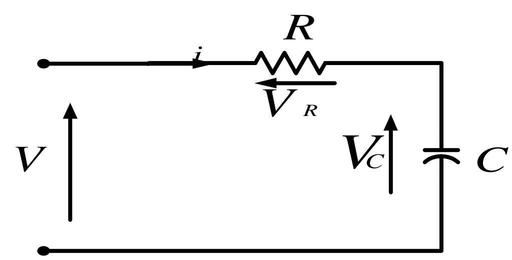
$V_{(t)} = i_{(t)}R$

RL CIRCUIT



$$\overline{V} = \overline{V_R} + \overline{V_L} = iR + L\frac{di}{dt} \Rightarrow \therefore L\frac{di}{dt} + iR - V = 0$$

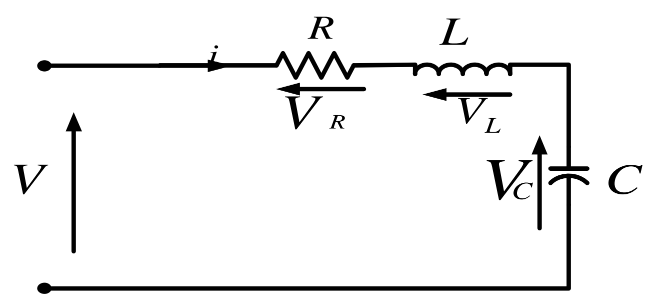
RC CIRCUIT



$$\overline{V} = \overline{V_R} + \overline{V_C} = iR + \frac{1}{C}\int idt \Rightarrow \frac{dV}{dt} = R\frac{di}{dt} + \frac{i}{C}$$

$$\therefore \frac{dV}{dt} - R\frac{di}{dt} - \frac{i}{C} = 0$$

RLC CIRCUIT

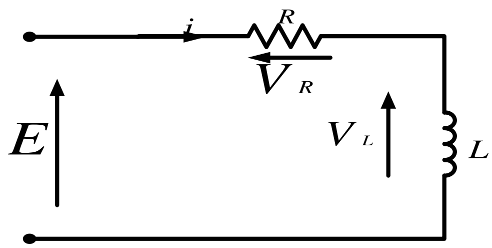


$$\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C} = iR + L\frac{di}{dt} + \frac{1}{C}\int idt \Rightarrow \frac{dV}{dt} = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

$$\therefore L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} - \frac{dV}{dt} = 0$$

Example

An electrical circuit contains inductance L and resistance R connected to a constant voltage source E. If the current i is given by the differential equation $E - L\frac{di}{dt} = iR$,calculate the current in terms of time *t* at *t = 0* *and i=0.*



$$E - L\frac{di}{dt} = iR$$

$$\frac{di}{dt} = \frac{E - iR}{L} \Rightarrow \frac{di}{(E - iR)} = \frac{dt}{L} \Rightarrow \int \frac{di}{(E - iR)} = \int \frac{dt}{L} \Rightarrow -\frac{1}{R}\log_e(E - iR) = \frac{t}{L} + C$$

t = 0 when $i = 0$

$$-\frac{1}{R}\log_e(E - 0) = \frac{0}{L} + C = C \Rightarrow -\frac{1}{R}\log_e E = C$$

$$\therefore -\frac{1}{R}\log_e(E - iR) = \frac{t}{L} - \frac{1}{R}\log_e E$$

$$\log_e\left(\frac{E}{E - iR}\right) = \frac{Rt}{L} \Rightarrow \frac{E}{E - iR} = e^{\frac{Rt}{L}}$$

$$\frac{E - iR}{E} = e^{-\frac{Rt}{L}} \Rightarrow E - iR = Ee^{-\frac{Rt}{L}} \Rightarrow iR = E - Ee^{-\frac{Rt}{L}} \Rightarrow i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

4.0 LAPLACE TRANSFORMS

The solution of most electrical circuit problems reduce ultimately to solution of differential equations. Methods of solving certain first order and second order differential equations exist. Laplace Transformation provides an alternative method of solving linear differential equations.

$$F(s) = \int_0^\infty e^{-st} f(t)dt$$

$F(s) \equiv$ *Laplace Transform on the original function* $f(t)$

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

$\mathcal{L}\{f(t)\} \equiv$ *Represents symbolically the operation of taking the Laplace Transform of whatever function that occurs inside the bracket. The operation on the original function is called Laplace Transformation.*

Examples:

1 $f(t) = 1 \quad (t \geq 0)$

$$F(s) = \int_0^\infty e^{-st}(1)dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty = -\frac{1}{s}\left[e^{-s\infty} - e^0\right] = -\frac{1}{s}(0-1) = \frac{1}{s}$$

$$\therefore F(s) = \frac{1}{s}$$

2 $f(t) = k$ *where k= constant*

$$F(s) = \int_0^\infty e^{-st}(k)dt = (k)\int_0^\infty e^{-st}dt = (k)\left[\frac{e^{-st}}{-s}\right]_0^\infty = -\frac{k}{s}\left[e^{-s\infty} - e^0\right] = -\frac{k}{s}(0-1) = \frac{k}{s}$$

$$\therefore F(s) = \frac{k}{s}$$

3 $f(t) = e^{at}$ *where 'a' is real constant* $\neq 0$

$$F(s) = \mathcal{L}\{f(e^{at})\} = \int_0^\infty e^{-st}e^{at}dt = \int_0^\infty e^{-(s-a)t}dt = \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty = \frac{1}{-(s-a)}[0-1]$$

$$\therefore F(s) = \mathcal{L}\{f(e^{at})\} = \frac{1}{(s-a)}$$

$s - a > 0, \; s > a$

4 $f(t) = \cosh at$ *and* $\cosh at = \frac{1}{2}\left(e^{at} + e^{-at}\right)$

$$F(s) = \frac{1}{2}\mathcal{L}\{f(e^{at})\} + \frac{1}{2}\mathcal{L}\{f(e^{-at})\}$$

$$F(s) = \frac{1}{2}\left(\frac{1}{s-a}\right) + \frac{1}{2}\left(\frac{1}{s-(-a)}\right) = \frac{1}{2}\left[\left(\frac{1}{s-a}\right) + \left(\frac{1}{s+a}\right)\right]$$

$$\therefore F(s)=\frac{1}{2}\left[\frac{(s+a)+(s-a)}{(s-a)(s+a)}\right]=\frac{s}{s^2-a^2}\ \ and\ \ s>a$$

5 $f(t)=\sinh at$ *and* $\sinh at=\frac{1}{2}\left(e^{at}-e^{-at}\right)$

$$F(s)=\frac{1}{2}\mathcal{L}\left\{f\left(e^{at}\right)\right\}-\frac{1}{2}\mathcal{L}\left\{f\left(e^{-at}\right)\right\}$$

$$F(s)=\frac{1}{2}\left(\frac{1}{s-a}\right)-\frac{1}{2}\left(\frac{1}{s-(-a)}\right)=\frac{1}{2}\left[\left(\frac{1}{s-a}\right)-\left(\frac{1}{s+a}\right)\right]$$

$$\therefore F(s)=\frac{1}{2}\left[\frac{(s-a)+(s+a)}{(s-a)(s+a)}\right]=\frac{a}{s^2-a^2}\ \ and\ \ s>a$$

6 $f(t)=\sin at$ *and* $\sin at=\frac{1}{2j}\left(e^{jat}-e^{-jat}\right)$

$$F(s)=\frac{1}{2j}\mathcal{L}\left\{f\left(e^{jat}\right)\right\}-\frac{1}{2j}\mathcal{L}\left\{f\left(e^{-jat}\right)\right\}$$

$$F(s)=\frac{1}{2j}\left(\frac{1}{s-a}\right)-\frac{1}{2j}\left(\frac{1}{s-(-a)}\right)=\frac{1}{2j}\left[\left(\frac{1}{s-a}\right)-\left(\frac{1}{s+a}\right)\right]$$

$$\therefore F(s)=\frac{1}{2j}\left[\frac{(s+ja)+(s-ja)}{(s-a)(s+a)}\right]=\frac{2ja}{2j}\left[\frac{1}{(s-a)(s+a)}\right]=\frac{a}{s^2+a^2}$$

$and\ \ s>a$

7 $f(t)=\cos at$ *and* $\cos at=\frac{1}{2}\left(e^{jat}+e^{-jat}\right)$

$$F(s)=\frac{1}{2}\mathcal{L}\left\{f\left(e^{jat}\right)\right\}+\frac{1}{2}\mathcal{L}\left\{f\left(e^{-jat}\right)\right\}$$

$$F(s)=\frac{1}{2}\left(\frac{1}{s-ja}\right)+\frac{1}{2}\left(\frac{1}{s-(-ja)}\right)=\frac{1}{2}\left[\left(\frac{1}{s-ja}\right)+\left(\frac{1}{s+ja}\right)\right]$$

$$\therefore F(s)=\frac{1}{2}\left[\frac{(s+ja)+(s-ja)}{(s-ja)(s+ja)}\right]=\frac{s}{s^2+a^2}\ \ and\ \ s>0$$

8 $f(t)=t\ \ ,\therefore\ f(t)=t=te^{-0t}$

$$F(s)=\frac{1}{(s+0)^2}=\frac{1}{s^2}$$

$$\therefore F(s)=\frac{1}{s^2}$$

4.1 INVERSE LAPLACE TRANSFORM

$f(t)=\mathcal{L}^{-1}\{F(s)\}$ *where* $\mathcal{L}^{-1}\{F(s)\}$ *is called the Inverse Laplace Transformation operator*

Examples

1 $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}=1$

2 $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\}=e^{at}$

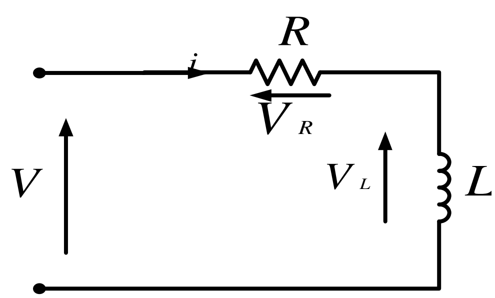
SUMMARY

For harmonic signals, $\frac{d}{dt}\xrightarrow{\text{Re}\,place} j\omega$, $\int dt \xrightarrow{\text{Re}\,place} \frac{1}{j\omega}$

For Laplace Transform, $\frac{d}{dt}\xrightarrow{\text{Re}place} s$, $\int dt \xrightarrow{\text{Re}\,place} \frac{1}{s}$

Example

An electrical circuit contains inductance L and resistance R connected to a constant voltage source E. If the current i is given by the differential equation $E-L\frac{di}{dt}=iR$,calculate the current in terms of time *t* at *t = 0 and i=0.*



$$E-L\frac{di}{dt}=iR$$

$$\mathcal{L}\{E\}=\mathcal{L}\left\{L\frac{di}{dt}\right\}+\mathcal{L}\{iR\}$$

$$E\,\mathcal{L}\{1\}=L\frac{d}{dt}\mathcal{L}\{i\}+R\,\mathcal{L}\{i\}$$

$$E\left(\frac{1}{s}\right)=[sL\,\mathcal{L}\{i\}-i_{(0)}]+R\,\mathcal{L}\{i\} \text{ but } i_{(0)}=0$$

$$\frac{E}{s}=sL\,\mathcal{L}\{i\}+R\,\mathcal{L}\{i\}$$

$$\frac{E}{s}=\mathcal{L}\{i\}[sL+R]$$

$$\mathcal{L}\{i\} = \frac{E}{s(sL+R)}$$

$$i = \mathcal{L}^{-1}\left\{\frac{E}{s(sL+R)}\right\}$$

Using Table

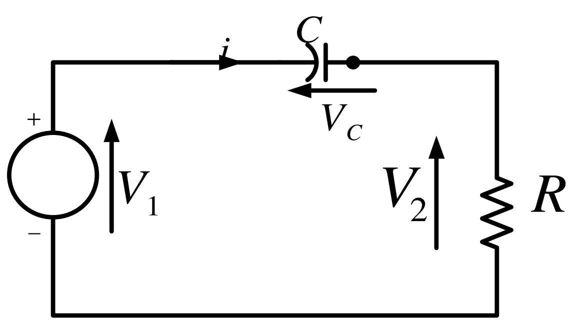
$$i = \mathcal{L}^{-1}\left\{\frac{E}{s(sL+R)}\right\} \Rightarrow i = \mathcal{L}^{-1}\left\{\frac{E}{sL\left(s+\frac{R}{L}\right)}\right\} \Rightarrow i = \frac{E}{L}\mathcal{L}^{-1}\left\{\frac{1}{(s+0)\left(s+\frac{R}{L}\right)}\right\}$$

$$\therefore i = \left(\frac{E}{L}\right)\left(\frac{1}{\frac{R}{L}-0}\right)\left(e^{0} - e^{-\frac{R}{L}t}\right) \Rightarrow i = \left(\frac{E}{R}\right)\left(1-e^{-\frac{R}{L}t}\right)$$

$$i = \left(\frac{E}{R}\right)\left(1-e^{-\frac{R}{L}t}\right)$$

Example

Determine the differential equation which relates the output voltage V_2 and the input voltage V_1. Using Laplace Transform technique, calculate the output voltage V_2. Assume the initial voltage across the capacitor $V_{C0} = 1$, $R = 1$, *C=1* and the input voltage $V_1 = 2e^{-t}$.



Solution

$$V_1 = \frac{1}{C}\int_0^t i\,dt + V_{C0} + iR \Rightarrow V_1 = \frac{1}{C}\int_0^t V_2\,dt + 1 + V_2$$

$$\frac{dV_1}{dt} = V_2 + \frac{dV_2}{dt} \Rightarrow sV_1(s) - V_1(0^+) = sV_2(s) - V_2(0^+) + V_2(s)$$

Laplace Transformation:

$$s\,\mathcal{L}\{V_1(s)\} - \mathcal{L}\{V_1(0^+)\} = s\,\mathcal{L}\{V_2(s)\} - \mathcal{L}\{V_2(0^+)\} + V_2(s)$$

$$\mathcal{L}\{V_1(s)\} = \mathcal{L}\{2e^{-t}\} = \frac{2}{2+1}$$

$$V_1(0^+) = \lim_{t\to 0} 2e^{-t} = 2$$

To find $V_2(0^+)$, limits are taken on both sides of the original voltage.

$$V_1(0^+) = \lim_{t\to 0} V_1(t) = \lim_{t\to 0}\left[\int_0^t V_2\,dt + V_{C0} + V_2(t)\right] = V_{C0} + V_2(0^+)$$

$$V_1(0^+) = V_{C0} + V_2(0^+) \Rightarrow V_2(0^+) = V_1(0^+) - V_{C0} \Rightarrow V_2(0^+) = 2 - 1 = 1$$

$$\therefore V_2(0^+) = 1$$

$$s\,\mathcal{L}\{V_1(s)\} - \mathcal{L}\{V_1(0^+)\} = s\,\mathcal{L}\{V_2(s)\} - \mathcal{L}\{V_2(0^+)\} + V_2(s)$$

$$s\left(\frac{2}{s+1}\right) - 2 = sV_2(s) - 1 + V_2(s)$$

$$s\left(\frac{2}{s+1}\right) - 1 = sV_2(s) + V_2(s) \Rightarrow \frac{s-1}{s+1} = V_2(s)(s+1) \Rightarrow V_2(s) = \frac{s-1}{(s+1)(s+1)} \Rightarrow V_2(s) = \frac{s-1}{(s+1)^2}$$

$$\therefore V_2(s) = \frac{s-1}{(s+1)^2}$$

Using Partial Fractions

$$\frac{s-1}{(s+1)^2} = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} = \frac{A + B(s+1)}{(s+1)^2} = \frac{(A+B) + sB}{(s+1)^2}$$

$$A + B = -1 \Rightarrow A = -1 - B$$

$$B = 1$$

$$\therefore A = -1 - 1 = -2$$

$$\frac{s-1}{(s+1)^2} = \frac{-2}{(s+1)^2} + \frac{1}{(s+1)} = -2\left[\frac{1}{(s+1)^2}\right] + \frac{1}{(s+1)}$$

Using Tables

$$-2\left[\frac{1}{(s+1)^2}\right] + \frac{1}{(s+1)} = -2te^{-t} + e^{-t}$$

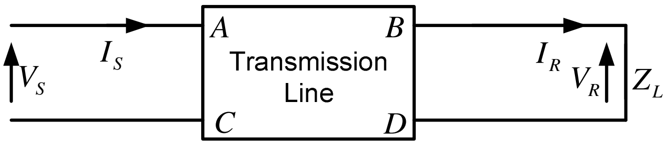
$$\therefore V_2(t) = e^{-t} - 2te^{-t}$$

USEFUL LAPLACE TRANSFORM PAIRS

$F(s)$	$f(t) \quad t>0$
$\frac{1}{s}$	1
$\frac{1}{s^2}$	t
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{s-a}$	e^{at}
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$, $n=1,2,3,\ldots.$
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$
$\frac{s}{(s+a)(s+b)}$	$\frac{1}{b-a}\left(ae^{-at}-be^{-bt}\right)$
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-a)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$
$\frac{s}{s^2+a^2}$	$\cos at$
$\frac{a}{s^2+a^2}$	$\sin at$
$\frac{a}{s^2-a^2}$	$\sinh at$
$\frac{s}{s^2-a^2}$	$\cosh at$
$\frac{1}{s(s+a)}$	$\frac{1}{a}\left(1-e^{-at}\right)$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left(1-\frac{be^{-at}}{b-a}+\frac{ae^{-bt}}{b-a}\right)$
$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2}\left(1-e^{-at}-ate^{-at}\right)$
$\frac{1}{s^2(s+a)}$	$\frac{1}{a^2}\left(at-1+e^{-at}\right)$
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$, $0!=1$

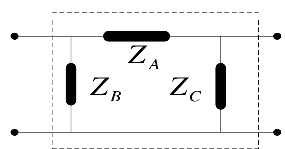
5.0 MULTI-PORT NETWORKS ANALYSIS

Two-Port Network

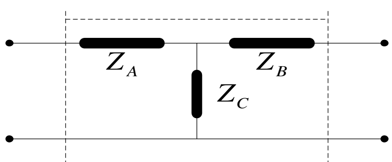


$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

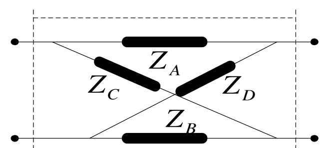
These are networks in which electrical energy is fed in at one pair of terminals and taken out at a second pair of terminals. Examples are transmission lines, telecommunication lines, wave filters transformers etc. The network between the input port and the output port is a transmission network for which a common relationship exists between the input and output voltage and current. The network may consist of passive elements (Resistors, Capacitors and Inductors) and active elements. The active elements contain sources of e.m.f. and pass out energy. The voltage source may can be a voltage controlled voltage source, a current controlled voltage source, voltage controlled current source or current controlled current source.



$\pi - Network$

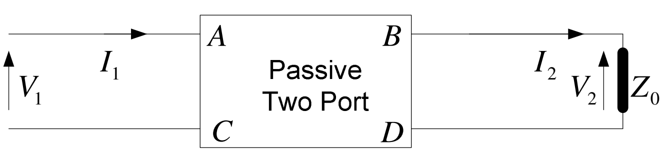


$T - Network$



$Lattice - Network$

CHARACTERISTIC IMPEDANCE



IMPEDANCE

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$
$$Z_{11} = \left|\frac{V_1}{I_1}\right|_{I_2=0}$$ *The out put is open circuited*

$$Z_{12} = \left|\frac{V_1}{I_2}\right|_{I_1=0}$$ *The input is open circuited*

$$Z_{21} = \left|\frac{V_2}{I_1}\right|_{I_2=0}$$ *The out put is open circuited*

$$Z_{22} = \left|\frac{V_2}{I_2}\right|_{I_1=0}$$ *The input is open circuited*

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} Z_{12} \\ Z_{21} Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z][I]$$

ADMITTANCE

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} Y_{12} \\ Y_{21} Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

HYBRID

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} h_{12} \\ h_{21} h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

If the output terminals of a two-port network are closed through an impedance Z_0, where is $Z_0 = \frac{V_2}{I_2}$ and if the input impedance $\frac{V_1}{I_1}$ is also Z_0, then the quantity Z_0 is the characteristic impedance.

Let $\begin{aligned} V_1 &= AV_2 + BI_2 \\ I_1 &= CV_2 + DI_2 \end{aligned}$

$$\frac{V_1}{I_1} = \frac{AV_2 + BI_2}{CV_2 + DI_2} = \frac{V_2\left(A + B\frac{I_2}{V_2}\right)}{I_2\left(C\frac{V_2}{I_2} + D\right)} = \left(\frac{V_2}{I_2}\right)\left(\frac{A + B\left(\frac{I_2}{V_2}\right)}{C\left(\frac{V_2}{I_2}\right) + D}\right)$$

but $Z_0 = \frac{V_2}{I_2}$

$$\therefore \frac{V_1}{I_1} = Z_0\left(\frac{A + B/Z_0}{CZ_0 + D}\right) = Z_0\left(\frac{A + B/Z_0}{D + CZ_0}\right)$$

For symmetrical two-port $A = D$ *and* $B/Z_0 = CZ_0$

$$\therefore Z_0 = \sqrt{B/C}$$

$$\therefore \frac{V_1}{I_1} = Z_0\left(\frac{A + CZ_0}{A + CZ_0}\right) = Z_0$$

OPEN CIRCUITED OUPUT $(I_2 = 0)$

$$V_1 = AV_2 + B0$$

$$I_1 = CV_2 + D0$$

$$\frac{V_1}{I_1} = \frac{AV_2}{CV_2} = \left(\frac{A}{C}\right)\left(\frac{V_2}{V_2}\right) = \frac{A}{C} = Z_{OC}$$ *and* Z_{OC} = *Open circuit impedance*

$$\therefore \frac{V_1}{I_1} = \frac{A}{C} = Z_{OC}$$

SHORT CIRCUITED OUPUT $(V_2 = 0)$

$$V_1 = A0 + BI_2 = BI_2$$

$$I_1 = C0 + DI_2 = DI_2$$

$$\frac{V_1}{I_1} = \frac{BI_2}{DI_2} = \left(\frac{B}{D}\right)\left(\frac{I_2}{I_2}\right) = \frac{B}{D} = \frac{B}{A} = Z_{SC}$$

For $A = D$ *and* Z_{SC} = *Short circuit impedance*

$$\therefore \frac{V_1}{I_1} = \frac{B}{A} = Z_{SC}$$

$$(Z_{OC})(Z_{SC}) = \left(\frac{V_1}{I_1}\right)^2 = \frac{BA}{AC} = \frac{B}{C}$$

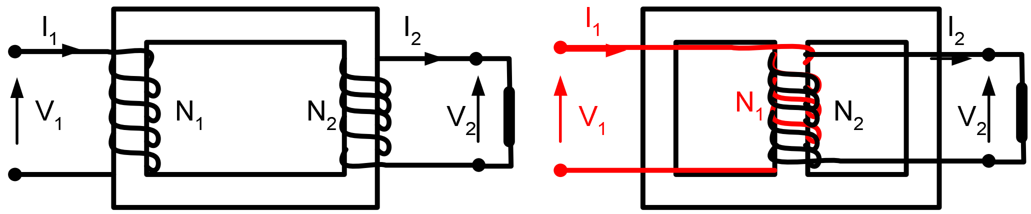
$$Z_0 = \frac{V_1}{I_1} = \sqrt{(Z_{OC})(Z_{SC})} = \sqrt{\frac{B}{C}}$$

The parameters ABCD are functions of frequency and Z_0 is a complex operator.

6.0 TRANSFORMERS

A transformer has four basic parts and these are:

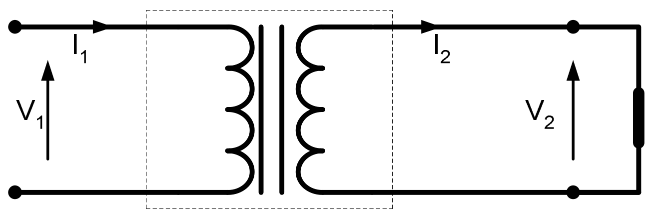
- Magnetic circuit
- Electric circuit
- Terminal tapings (connections and switches)
- A cooling system. Small transformers are air cooled and the method of cooling becomes more complicated in power transformers of high and extra high voltage. Oil and forced air is often used.



6.1 IDEAL TRANSFORMERS

An ideal transformer has no losses, no winding resistance and no reluctance. In addition there is no flux leakage.

EQUIVALENT CIRCUIT



V_1= primary voltage
I_1 = primary current
V_2 = secondary voltage
I_2 = secondary current
N_1 = primary winding
N_2 = secondary winding

6.2 TURNS RATIO OF AN IDEAL TRANSFORMER

$$V_1 = N_1(\frac{d\varphi}{dt}) \text{.........} \text{...}(1)$$

$$V_2 = N_2(\frac{d\varphi}{dt}) \text{...........}(2)$$

Divide equation (1) by equation (2)

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \text{..........} \text{...}(3)$$

For an ideal transformer, $F = S\phi$ *and* $S = 0$

$$\therefore \ F = 0 \text{..........} \quad \text{.....} \ (4)$$

$$F = N_1 I_1 - N_2 I_2 = 0$$

$$N_1 I_1 = N_2 I_2$$

$$\frac{N_1}{N_2} = \frac{I_2}{I_1} \text{..........} \quad \text{....} \ (5)$$

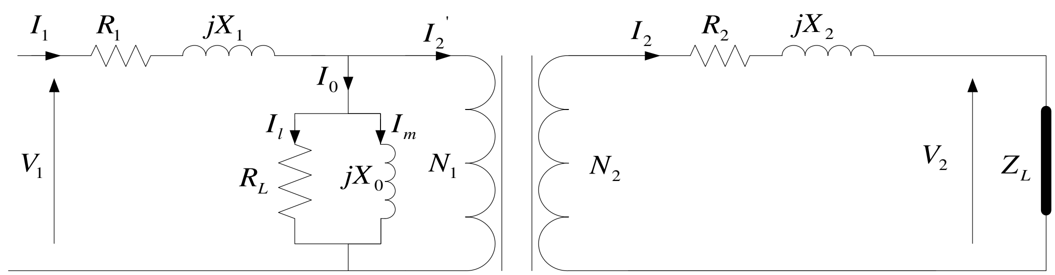
Using equations (3) and (5)

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} \quad \textit{Turns Ratio}$$

6.3 REAL TRANSFORMER

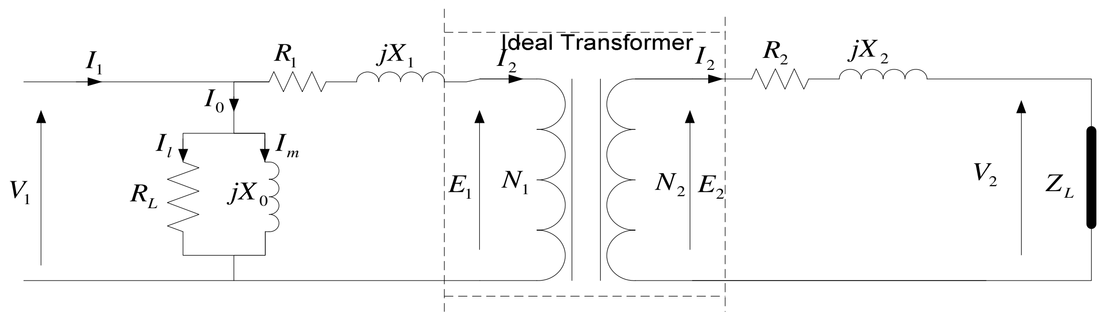
Real transformer = Ideal transformer + transformer losses

EQUIVALENT CIRCUIT



The current taken by the shunt arm of the equivalent circuit is small compared to the rated primary current. Further, the rated current causes only a small voltage drop in the series elements ($Z_1 = R_1 + jX_1$) so that the drop in these elements due to the current in the shunt elements is negligible. The shunt element

APPROXIMATE EQUIVALENT CIRCUIT



V_1= primary voltage
I_1 = primary current
E_1 = primary induced e.m.f.
V_2 = secondary voltage
I_2 = secondary current
E_2 = secondary induced e.m.f.

N_1 = primary winding
N_2 = secondary winding
I_0 = transformer no-load current (magnetizing current and iron losses)
I_l = current that accounts for iron losses
I_m= magnetizing current
R_L = resistance to account for iron losses
R_1 = primary winding resistance
R_2 =secondary winding resistance
X_1 = primary leakage reactance
X_2 = secondary leakage reactance
$Z_1 = R_1+jX_1$ = primary leakage impedance
$Z_2 = R_2+jX_2$ = secondary leakage impedance

6.4 TURNS RATIO OF A REAL TRANSFORMER

$$F = S\phi = I_1 N_1 - I_2 N_2 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$I_1 N_1 = I_2 N_2 + \phi S$$

$$I_1 = I_2 \frac{N_2}{N_1} + \frac{\phi S}{N_1} = I_2 \frac{N_2}{N_1} + I_m \ldots\ldots (2)$$

$I_2 >> I_m$ *Magnetizing current is very small and negligible compared to I_2.*

$$I_1 \approx I_2 \frac{N_2}{N_1}$$

$$\therefore \frac{N_1}{N_2} \approx \frac{I_2}{I_1} \ldots\ldots\ldots\ldots (3)$$

$\frac{E_1}{E_2} = \frac{N_1}{N_2}$ *For ideal transformer*

$E_1 \approx V_1$ *due to negligible voltage drop across Z_1.*

$E_2 \approx V_2$ *due to negligible voltage drop across Z_2.*

$$\therefore \frac{V_1}{V_2} \approx \frac{N_1}{N_2}$$

$\frac{V_1}{V_2} \approx \frac{N_1}{N_2} \approx \frac{I_2}{I_1}$ *Turns Ratio*

6.5 IMPEDANCE TRANSFORMATION (REFERRED QUANTITIES)

The primary quantity can be referred to the secondary and the secondary quantities can also be referred to the primary using the turns ratio equation.

$$\frac{V_1}{V_2} \approx \frac{N_1}{N_2} \approx \frac{I_2}{I_1}$$

$$Z_1 = \frac{V_1}{I_1} \Rightarrow V_1 = Z_1 I_1$$

$$Z_2 = \frac{V_2}{I_2} \Rightarrow V_2 = Z_2 I_2$$

$$\therefore \frac{Z_1 I_1}{Z_2 I_2} \approx \frac{N_1}{N_2} \approx \frac{I_2}{I_1} \Rightarrow \left(\frac{Z_1}{Z_2}\right)\left(\frac{I_1}{I_2}\right) \approx \frac{N_1}{N_2} \approx \frac{I_2}{I_1} \Rightarrow \left(\frac{Z_1}{Z_2}\right)\left(\frac{N_2}{N_1}\right) \approx \frac{N_1}{N_2} \approx \frac{I_2}{I_1}$$

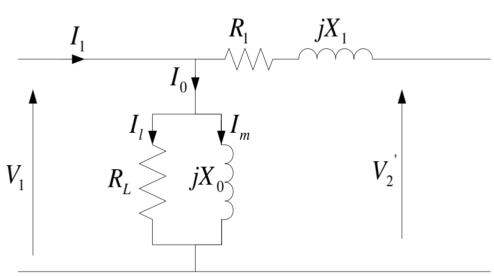
$$\left(\frac{Z_1}{Z_2}\right) \approx \left(\frac{N_1}{N_2}\right)^2 \approx \left(\frac{N_1}{N_2}\right)\left(\frac{I_1}{I_2}\right)$$

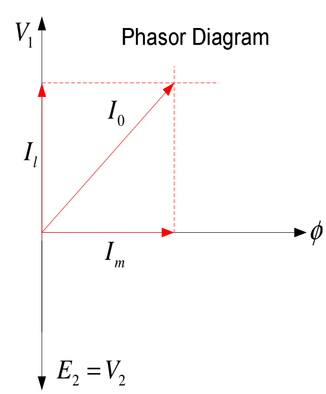
$$\therefore Z_1 \approx Z_2\left(\frac{N_1}{N_2}\right)^2 \approx n^2 Z_2$$

$$Z_2 \approx Z_1\left(\frac{N_2}{N_1}\right)^2 \approx \frac{1}{n^2} Z_1$$

6.6 PHASOR DIAGRAMS AND REFERRED QUANTITIES

NO LOAD TRANSFORMER





$$I_2 = I'_2 = 0$$

$$I_1 = I_0$$

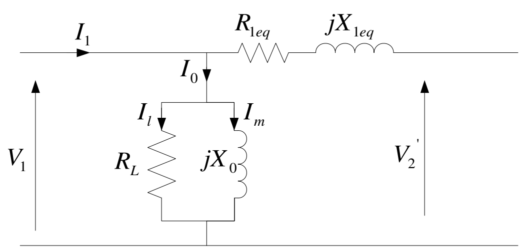
$$E_2 = V_2$$

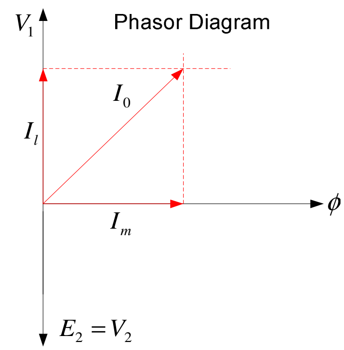
$$E_1 = V_1 = V_2'$$

$$\frac{V_2'}{V_2} = \frac{N_1}{N_2}$$

$$V_2' = \frac{N_1}{N_2} V_2 = nV_2 = E_1$$

EQUIVALENT CIRCUIT REFERRED TO THE PRIMARY

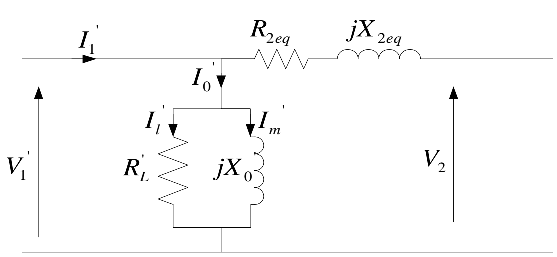


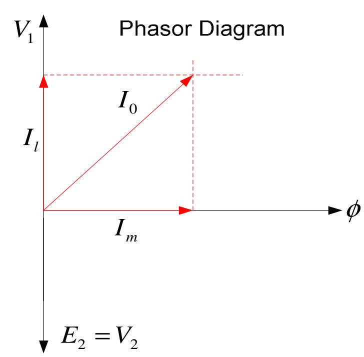


$$R_{1eq} = R_1 + R_2^{'} \text{ where } R_2^{'} = R_2\left(\frac{N_1}{N_2}\right)^2 = n^2 R_2$$

$$X_{1eq} = X_1 + X_2^{'} \text{ where } X_2^{'} = X_2\left(\frac{N_1}{N_2}\right)^2 = n^2 X_2$$

EQUIVALENT CIRCUIT REFERRED TO THE SECONDARY





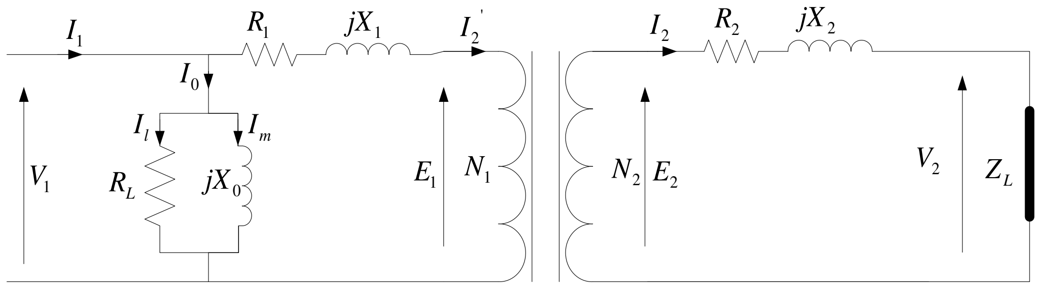
$$R_{2eq} = R_2 + R_1^{'} \text{ where } R_1^{'} = R_1\left(\frac{N_2}{N_1}\right)^2 = \frac{1}{n^2} R_1$$

$$X_{2eq} = X_2 + X_1^{'} \text{ where } X_1^{'} = X_1\left(\frac{N_2}{N_1}\right)^2 = \frac{1}{n^2} X_1$$

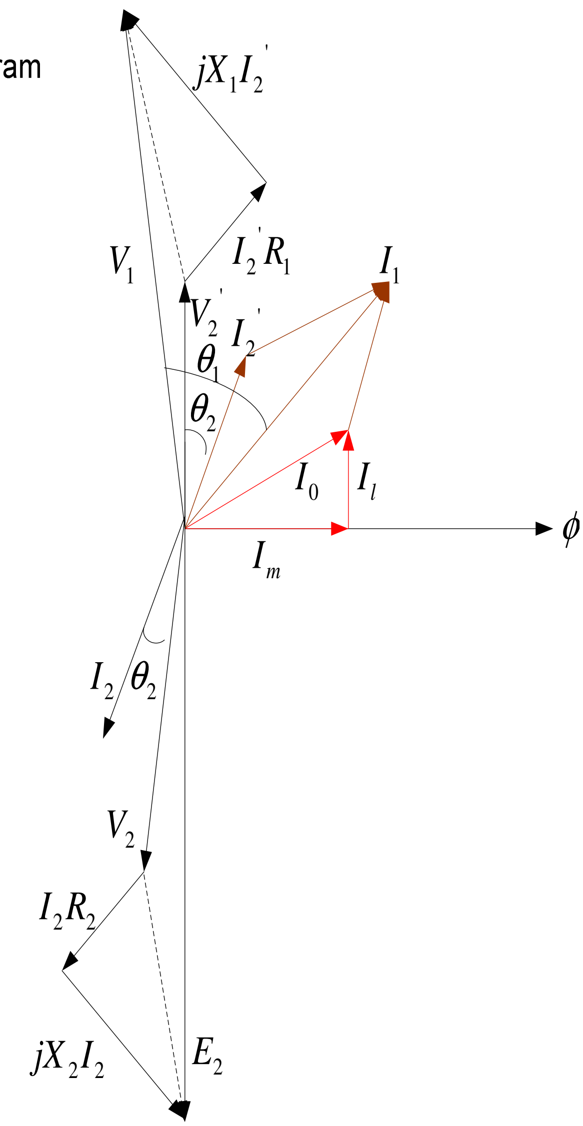
$$R_0^{'} = R_0\left(\frac{N_2}{N_1}\right)^2 = \frac{1}{n^2} R_0$$

$$X_0^{'} = X_0\left(\frac{N_2}{N_1}\right)^2 = \frac{1}{n^2} X_0$$

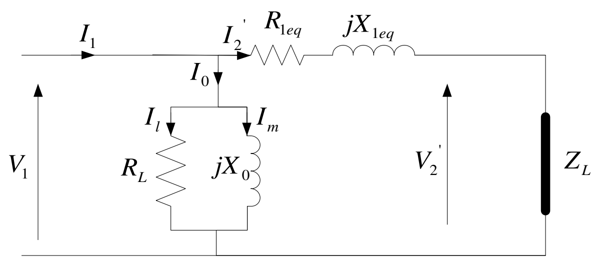
ON LOAD TRANSFORMER ($I_2 \neq 0$)

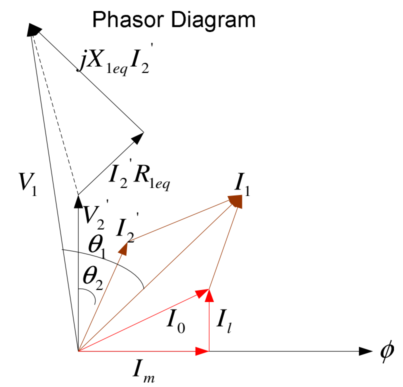


Phasor Diagram

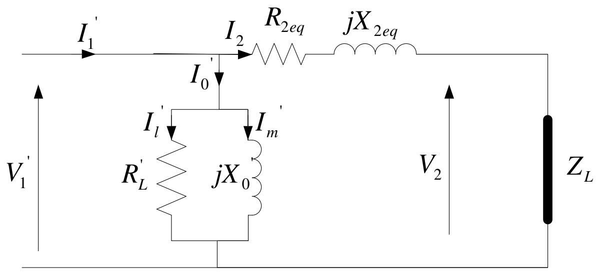


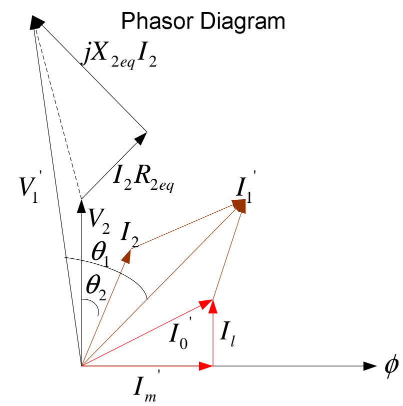
EQUIVALENT CIRCUIT REFERRED TO THE PRIMARY





EQUIVALENT CIRCUIT REFERRED TO THE SECONDARY





6.7 TRANSFORMER EFFICIENCY

Efficiency $\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{P_{in} - P_{losses}}{P_{in}} = 1 - \dfrac{P_{losses}}{P_{in}}$

Efficiency is always less than one for real transformers.

P_{losses} = Iron losses + Copper losses = P_{Fe} + P_{cu}

$P_{Fe} = \dfrac{V_1^{\,2}}{R_L}$ *is a constant for constant supply voltage V_1.*

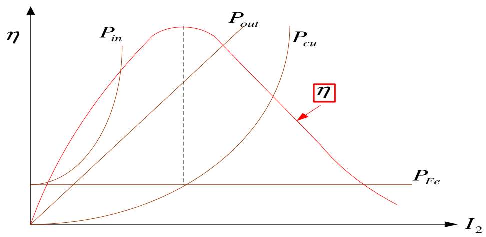
$P_{cu} = \left(I_2^{'}\right)^2 R_{1eq}$ *is proportional to the load current.*

$P_{out} = V_2 I_2 \cos\theta_2$

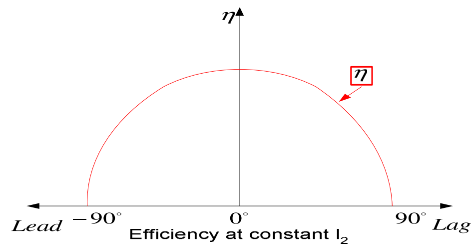
$P_{in} = P_{out} + P_{losses} = V_2 I_2 \cos\theta_2 + P_{Fe} + P_{cu}$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_{Fe} + P_{cu}} = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_{Fe} + \left(I_2^{'}\right)^2 R_{1eq}} = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_{Fe} + \left(I_2\right)^2 R_{2eq}}$$

Transformer efficiency depends on load current I_2 and load phase angle θ_2.



Efficiency at constant pf



Efficiency at constant I_2

MAXIMUM EFFICIENCY

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_{Fe} + \left(I_2^{'}\right)^2 R_{1eq}} = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_{Fe} + \left(I_2\right)^2 R_{2eq}} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$R_{2eq} = R_{1eq}\left(\frac{N_2}{N_1}\right)^2$$

$$\frac{I_2}{I_2^{'}} = \frac{N_1}{N_2}$$

$$\frac{I_2^{2}}{I_2^{'2}} = \left(\frac{N_1}{N_2}\right)^2 = \frac{R_{1eq}}{R_{2eq}} \Rightarrow I_2^{2} R_{2eq} = I_2^{'2} R_{1eq}$$

Divide equation (1) by I_2:

$$\eta = \frac{V_2 \cos\theta_2}{V_2 \cos\theta_2 + \frac{P_{Fe}}{I_2} + I_2 R_{2eq}}$$

$$\eta_{\max} \Rightarrow \frac{d}{dI_2}\left(\frac{P_{Fe}}{I_2} + I_2 R_{2eq}\right) = 0 = -\frac{P_{Fe}}{I_2^{2}} + R_{2eq}$$

$$\therefore P_{Fe} = I_2^{2} R_{2eq} \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ .(2)$$

$$But\ ; P_{cu} = I_2^{2} R_{2eq}$$

$$\therefore P_{Fe} = P_{cu}$$

At maximum efficiency, the copper loss has such a value that it is equal to iron loss.

Divide equation (1) by I_2:

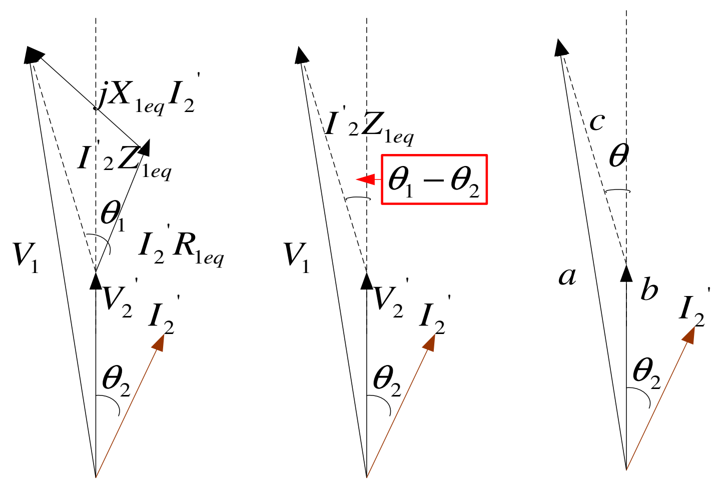
$$\eta = \frac{V_2 I_2}{V_2 I_2 + \frac{P_{Fe} + I_2^{2} R_{2eq}}{\cos\theta_2}} \ldots\ldots\ldots\ldots(3)$$

Maximum efficiency occurs when the power factor $(\cos\theta_2)$ is a maximum. i.e. $(\cos\theta_2) = 1.0$

6.8 TRANSFORMER REGULATION

The voltage regulation of a transformer is defined as the variation of the secondary voltage between no-load and full-load expressed as a percentage on no-load voltage assuming a constant primary supply voltage. It is a fractional change of the output voltage for a given load current at a constant supply voltage.

$$\text{Regulation} = \frac{\left|V_{2(NL)}\right| - \left|V_{2(L)}\right|}{\left|V_{2(NL)}\right|} = \frac{\left|V'_{2(NL)}\right| - \left|V'_{2(L)}\right|}{\left|V'_{2(NL)}\right|} = \frac{\left|V_1\right| - \left|V'_{2(L)}\right|}{\left|V_1\right|}$$



$$a^2 = b^2 + c^2 + 2bc\cos\theta$$ *(Cosine rule)*

Dividing by b^2;

$$\left(\frac{a}{b}\right)^2 = 1 + \left(\frac{c}{b}\right)^2 + 2\left(\frac{c}{b}\right)\cos\theta$$

For c<<b, then although $\left(\frac{c}{b}\right)$ is retained, $\left(\frac{c}{b}\right)^2$ will be negligible.

$$\therefore \left(\frac{a}{b}\right)^2 = 1 + 2\left(\frac{c}{b}\right)\cos\theta$$

$$\left(\frac{a}{b}\right) = \left\{1 + 2\left(\frac{c}{b}\right)\cos\theta\right\}^{\frac{1}{2}}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots + \binom{n}{r}x^r + \ldots\ldots$$

Valid for all $|x| < 1$

Let $x = 2\left(\frac{c}{b}\right)\cos\theta$

$$\left(1+2\left\{\frac{c}{b}\right\}\cos\theta\right)^{\frac{1}{2}} = 1+\frac{c}{b}\cos\theta + \textit{higher powers of}\left(\frac{c}{b}\right).$$

Higher powers of $\left(\frac{c}{b}\right)$ *can be neglected.*

$$\frac{a}{b} = \left(1+2\frac{c}{b}\cos\theta\right)^{\frac{1}{2}} \approx 1+\frac{c}{b}\cos\theta \Rightarrow \frac{a}{b} = 1+\frac{c}{b}\cos\theta \Rightarrow 1 = \frac{b}{a}+\frac{c}{a}\cos\theta$$

$$1-\frac{b}{a} = \frac{c}{a}\cos\theta \Rightarrow \frac{a-b}{a} = \frac{c}{a}\cos\theta$$

$$\therefore \text{Re}\,gulation = \frac{a-b}{a} = \frac{c}{a}\cos\theta = \frac{|V_1|-|V_2^{'}|}{|V_1|} = \frac{I_2^{'}Z_{1eq}}{V_1}\cos(\theta_1-\theta_2) = \frac{I_2 Z_{2eq}}{V_2}\cos(\theta_1-\theta_2)$$

$$\text{Re}\,gulation = \frac{I_2^{'}Z_{1eq}}{V_1}\cos(\theta_1-\theta_2) = \frac{I_2 Z_{2eq}}{V_2}\cos(\theta_1-\theta_2) \quad \textit{for lagging } \theta_2$$

$$\text{Re}\,gulation = \frac{I_2^{'}Z_{1eq}}{V_1}\cos(\theta_1+\theta_2) = \frac{I_2 Z_{2eq}}{V_2}\cos(\theta_1+\theta_2) \quad \textit{for leading } \theta_2$$

$$\cos(\theta_1 \mp \theta_2) = \cos\theta_1\cos\theta_2 \pm \sin\theta_1\sin\theta_2$$

$$Z_{1eq}\cos(\theta_1 \mp \theta_2) = Z_{1eq}\cos\theta_1\cos\theta_2 \pm \sin\theta_1\sin\theta_2 = R_{1eq}\cos\theta_2 \pm X_{1eq}\sin\theta_2$$

$$\text{Re}\,gulation = \frac{I_2^{'}Z_{1eq}}{V_1}\cos(\theta_1-\theta_2) = \frac{I_2^{'}}{V_1}\left(R_{1eq}\cos\theta_2 + X_{1eq}\sin\theta_2\right)$$

for lagging θ_2

$$\text{Re}\,gulation = \frac{I_2 Z_{2eq}}{V_2}\cos(\theta_1-\theta_2) = \frac{I_2}{V_2}\left(R_{2eq}\cos\theta_2 + X_{2eq}\sin\theta_2\right)$$

for lagging θ_2

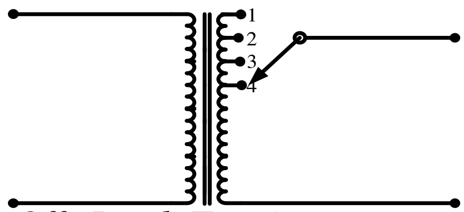
$$\text{Re}\,gulation = \frac{I_2^{'}Z_{1eq}}{V_1}\cos(\theta_1+\theta_2) = \frac{I_2^{'}}{V_1}\left(R_{1eq}\cos\theta_2 - X_{1eq}\sin\theta_2\right)$$

for leading θ_2

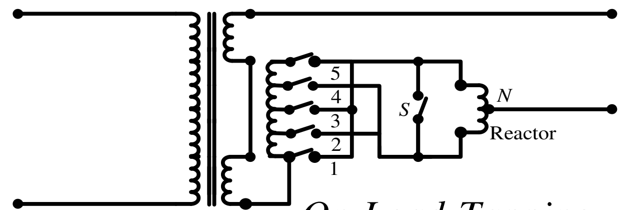
$$\text{Re}\,gulation = \frac{I_2 Z_{2eq}}{V_2}\cos(\theta_1+\theta_2) = \frac{I_2}{V_2}\left(R_{2eq}\cos\theta_2 - X_{2eq}\sin\theta_2\right)$$

for leading θ_2

Tap Changing Transformers



Off Load Tapping

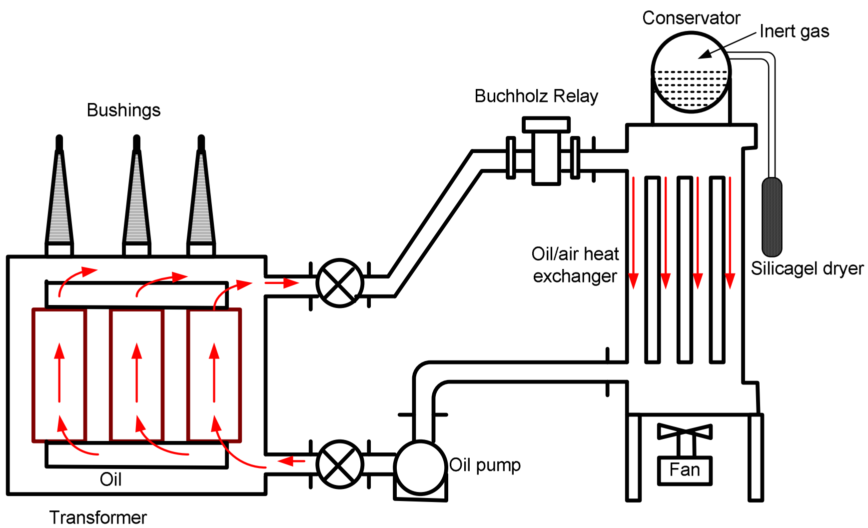


On Load Tapping

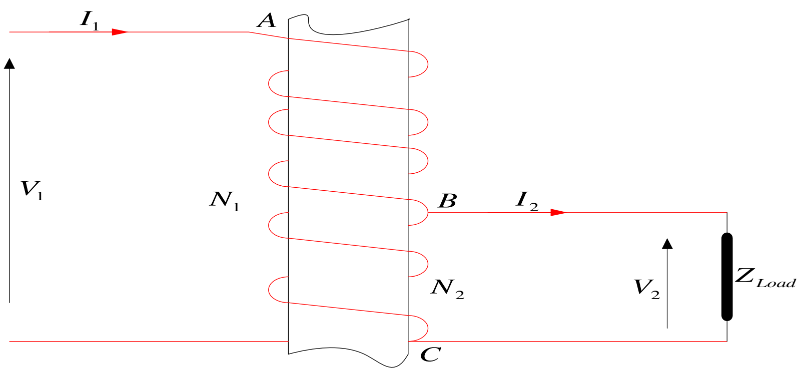
Tap changing transformers is one of the methods used to control voltages in a network. The method makes use of a turns ratio change which can be done either off load or on load.
In the case of the off load tapping, the variable tap can be on the primary or secondary side of the transformer. Varying the position of the tap alters the output secondary voltage. Operating of an off load tap changer results in an open circuit and disruption of supply. If an attempt is made to carry out tap changing on load, excessive arcing would result.
For on load tapping, all the windings are in circuit when tap changing is started. The short circuiting switch ‘s’ allows half of the total current to flow through each half of the reactor and since the current in each half of the reactor will be in opposition, no resultant flux will be set up in the reactor and there will be no inductive voltage across it. Assuming the transformer is on tap 5, if it is desired to alter the tap position on load to position 4, the reactor short circuiting switch is opened. The load current will flow through one half of the reactor coil only so that there will be a voltage drop across the reactor. When switch 4 is closed, the coil between the taps 4 and 5 will be connected through the whole reactor winding. A circulating current will flow through this local circuit, but its value will be limited by the reactor. Switch 5 is then opened and the reactor short circuiting switch closed thus completing the operation. The tap coils are physically placed in the centre of the transformer limb to avoid unbalanced axial forces acting on the coils. Electrically the taps are at one end of the winding.

Transformer Cooling

The cooling problem in electric apparatus increases in difficult with the increased size of the equipment. A cooling system is required as a result of heat generated by the losses. Cooling ensures that the life of a machine is not shortened by overheating. The operating temperature is closely associated with its life expectancy because deterioration of insulation is a function of both time and temperature. $Life = Ae^{B/T}$ where A and B are constants and T absolute temperature. Deterioration is a chemical phenomenon involving slow oxidation and brittle hardening leading to loss of mechanical durability and dielectric strength. Cooling radiators or ducts for motors must be provided to ensure effective removal of heat arising from losses. Small transformers are air cooled. High voltage (HV) and extra high voltage (EHV) transformers with higher rating, transformer oil is used to remove the heat from the windings. Transformer oil has good viscosity, electric strength with a flash point higher than 160˚C. Oil vapour ignites spontaneously while oil will ignite and burn at about 200˚C which is approximately 25% above the flash point. The silica gel removes moisture from the oil and prevents ingress of moisture by absorbing the moisture.

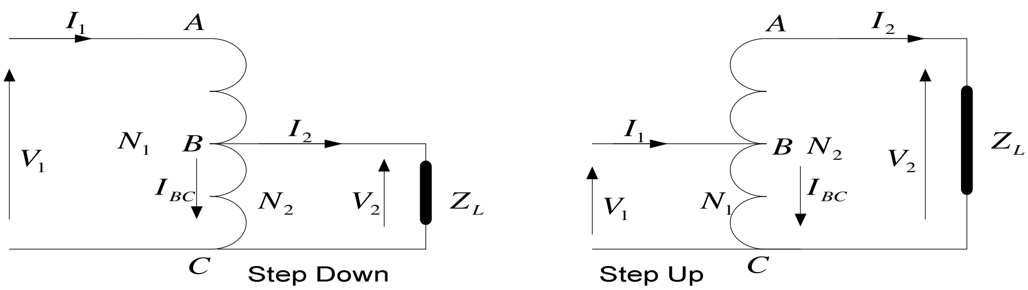


6.9 AUTOTRANSFORMERS



V_1 = primary voltage
V_2 = secondary voltage
I_1 = primary current
I_2 = secondary current
N_1= (A-C) = primary winding
N_2 = (B-C) = secondary winding

An autotransformer is a transformer connected in a special way in which the winding A-B is provided with extra insulation. Its performance is governed by the same fundamental principles already discussed for a two winding transformer.



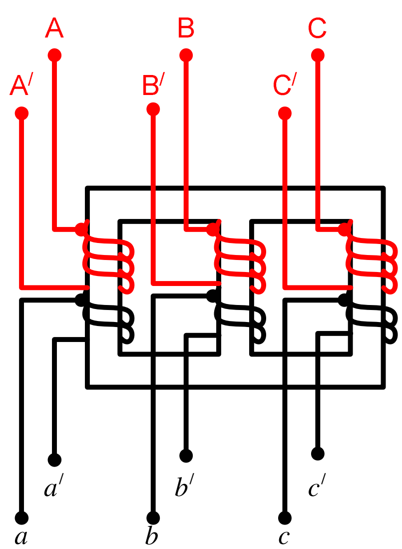
AUTOTRANSFORMERS ADVANTAGES

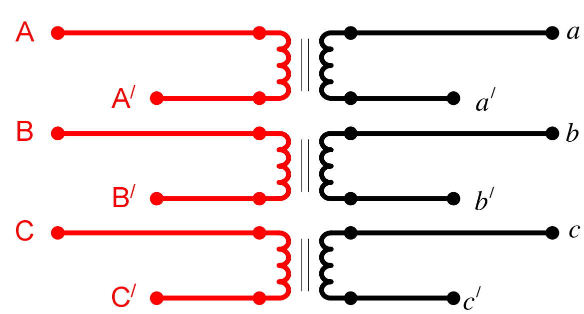
- Savings in winding material since the secondary winding is merged into the primary winding.
- Lower leakage reactance, lower losses and small exciting current.
- Costs less than a two winding transformer of the same rating.

AUTOTRANSFORMERS DISADVANTAGES

- The direct connection between the primary and the secondary winding in a step down autotransformer pause a danger in the event on an open circuit between B and C. The high voltage would be applied to the secondary.
- The short circuit current is much higher compared to the two winding transformer.

6.10THREE-PHASE TRANSFORMERS





The transformer windings can be connected in star/delta, delta/star, star/star or delta/delta depending on the connections of A', B', C' and a', b', c'.

Transformer Connections

Star/Star

This is an economical connection for small high voltage transformers. The turns per phase insulation is minimised due to the neutral points on both sides which also provides a neutral connection. It has the disadvantage of primary current being unbalanced for unbalanced secondary load.

Delta/Delta

This connection is suitable large low voltage transformers. Large load unbalance can be tolerated. It is very useful for high current low voltage transformers and can supply large unbalanced loads without disturbing the magnetic equilibrium. There is however no available star point.

Star/Delta

This is used for substation transformers supplied from the grid. The star point serves a mixed 1-phase and 3-phase loads, and a delta winding carries triplen currents and so stabilizes the star point.

Delta/Zigzag or Star/Zigzag

This is suitable for smaller powers with large out of balance neutral currents. The Zigzag winding establishes magnetic equilibrium.

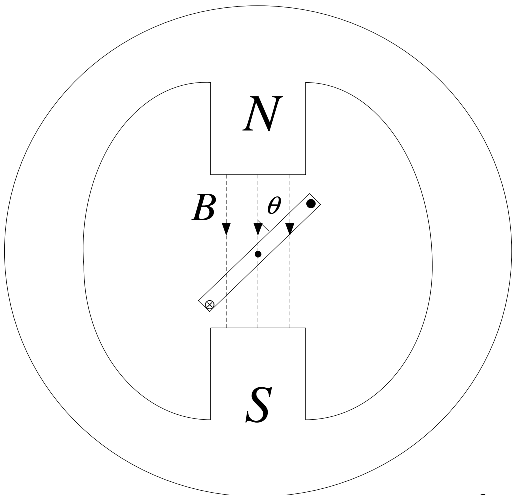
6.11 PARALLEL TRANSFORMER CONNECTION

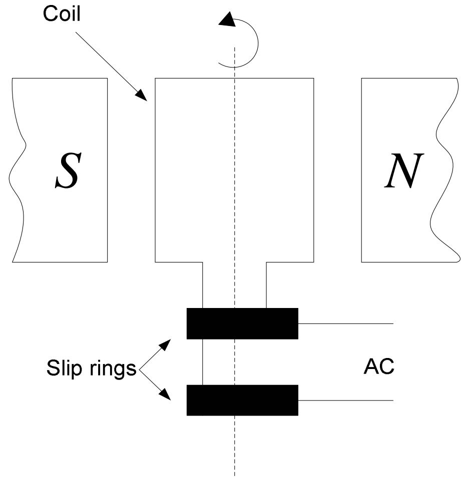
Parallel operation of transformers is necessary in order to increase the capacity or the total substation MVA. Three-phase transformers can be operated in parallel under the following conditions:

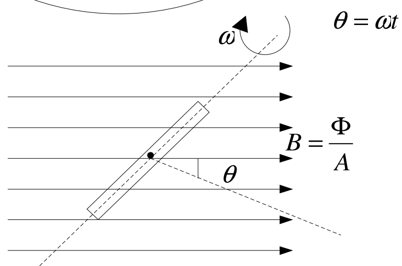
(a) The secondary must have the same phase sequence
(b) All corresponding secondary line voltages must be in phase
(c) The secondary must give the same magnitude of line voltage
(d) The transformer impedance must be the same or nearly the same

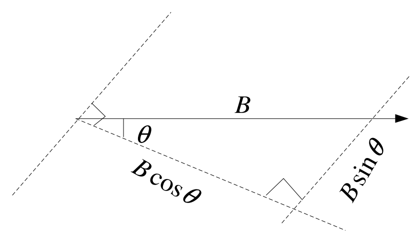
If conditions (a), (b) and (c) are not complied with, the secondary windings will simply short circuit one another resulting in serious damage. If condition (d) is not complied with, the transformer will not share the total load in proportion to their ratings. The transformer with the lower reactance will be overloaded before the total output reaches the sum of the individual rating.

7.0 ALTERNATING CURRENT (AC)









The rotating coil is rotated about a horizontal axis in its own plane at right angles to a uniform magnetic field of flux density B.

$$\therefore \Phi = BA\cos\theta$$

By Faraday's law;

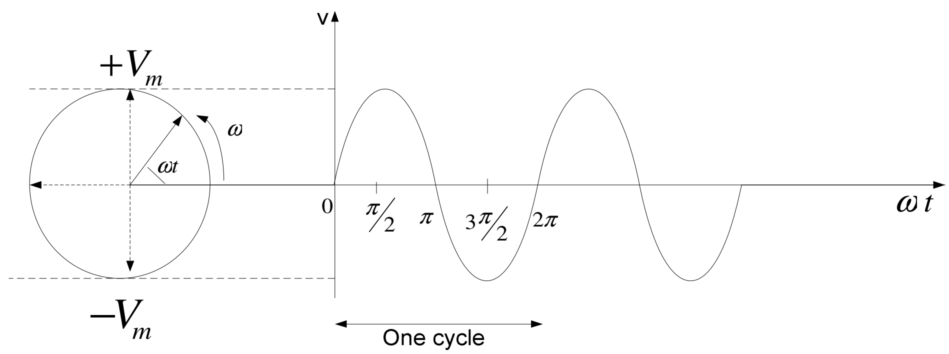
$$v = \quad N\frac{d\Phi}{dt} = \quad N\frac{d}{dt}(BA\cos\omega t)$$

$$v = \quad NBA\frac{d}{dt}(\cos\omega t) = \quad \omega NBA(\quad \sin\omega t) = \omega NBA(\sin\omega t)$$

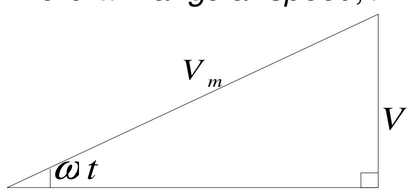
$$for\,\theta = 90°, \Rightarrow V_m = \omega NBA$$

$$\therefore v = V_m \sin \omega t$$

$$similarly \;\; ; i = I_m \sin \omega t$$



$\theta = \omega t$ where $\omega =$ angular speed, $\theta =$ displacement angle, t= time (s)

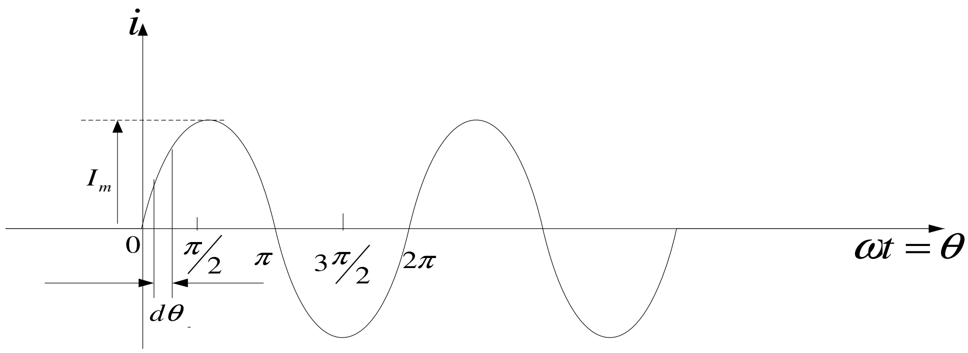


$$\sin \omega t = \frac{V}{V_m} \Rightarrow \therefore V = V_m \sin \omega t$$

Frequency (f) = Number of complete cycles per second (Hz)

Period T = $\frac{1}{f}$ = time for one cycle.

AVERAGE VALUES



$$i = I_m \sin \omega t = I_m \sin \theta$$

For a very small interval $d\theta$, the area of the shaded strip is $id\theta$. The total area enclosed by the current wave over half cycle can be calculated by integration $\int_0^\pi id\theta = \int_0^\pi I_m \sin \theta d\theta = I_m \int_0^\pi \sin \theta d\theta$.

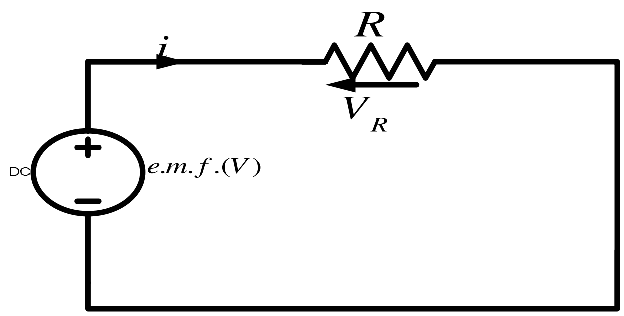
The average current value over half cycle is;

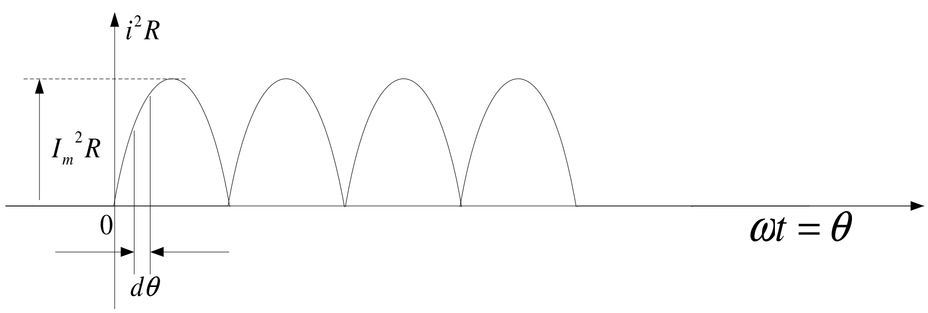
$$I_{av} = \frac{I_m \int_0^\pi \sin \theta d\theta}{\pi} = \frac{2I_m}{\pi} = 0.6366 I_m$$

$$similary\ V_{av} = \frac{V_m\int_0^{\pi}\sin\theta d\theta}{\pi} = \frac{2V_m}{\pi} = 0.6366V_m$$

RMS VALUES

The root mean square (rms) or the effective value is more useful than the average value for telling us the heating effect of an alternating current or voltage.





The instantaneous heating effect = i^2R

During interval $d\theta$ *, heat generated* $= i^2Rd\theta$ *represented by the area of the shaded strip.*

The average heating effect during the first half cycle is $\frac{\int_0^{\pi} i^2Rd\theta}{\pi}$.

$$\frac{\int_0^{\pi} i^2Rd\theta}{\pi} = \frac{\int_0^{\pi} I_m^{\ 2}R\sin^2\theta d\theta}{\pi} = \frac{I_m^{\ 2}R\int_0^{\pi}\sin^2\theta d\theta}{\pi} = \left(\frac{I_m^{\ 2}R}{2\pi}\right)\int_0^{\pi}(1-\cos 2\theta)d\theta = \frac{I_m^{\ 2}R}{2}$$

If *the current I is the value of the direct current through the same resistance to produce the same heating effect,* $I^2R = \frac{1}{2}I_m^{\ 2}R$.

$\therefore\ I = \dfrac{I_m}{\sqrt{2}} = 0.7071\,I_m$ *where I is the rms value i.e.* $I_{rms} = \dfrac{I_m}{\sqrt{2}} = 0.7071 I_m$.

Similarly $V_{rms} = \dfrac{V_m}{\sqrt{2}} = 0.7071 V_m$

FORM FACTOR

Current form factor = $\dfrac{I_{rms}}{I_{av}} = \dfrac{I_m/\sqrt{2}}{2I_m/\pi} = 1.111$

Voltage form factor= $\dfrac{V_{rms}}{V_{av}} = \dfrac{V_m/\sqrt{2}}{2V_m/\pi} = 1.111$

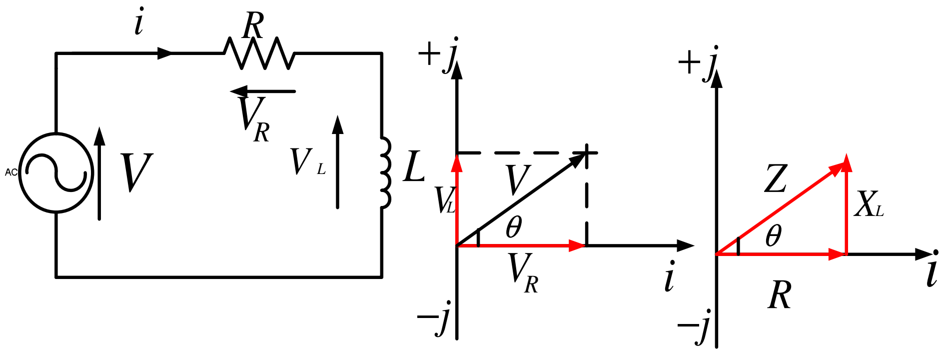
PEAK FACTOR (CREST FACTOR)

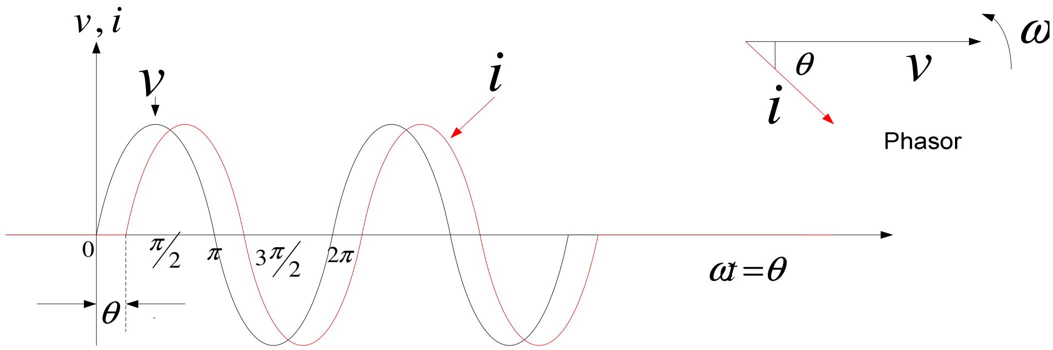
Current peak factor = $\dfrac{I_m}{I_{rms}} = \dfrac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$

Voltage peak factor = $\dfrac{V_m}{V_{rms}} = \dfrac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = 1.414$

7.1 Alternating Current Circuits

RL Series Connection





$$\overline{V} = \overline{V_R} + \overline{V_L} \text{ and } V_L = jiX_L \text{ and } V_R = iR$$

$$\therefore V = iR + jiX_L = i(R + jX_L)$$

$$Z = \frac{V}{i} = R + jX_L = \sqrt{R^2 + X_L{}^2}$$

$$\tan\theta = \frac{X_L}{R} \Rightarrow \theta = \tan^{-1}\frac{X_L}{R}$$

Z is the circuit impedance and θ is the phase angle.

$$Y = \frac{1}{Z} = \frac{1}{R + jX_L} = \left(\frac{1}{R + jX_L}\right)\left(\frac{R - jX_L}{R - jX_L}\right) = \frac{R - jX_L}{R^2 + X_L^2} = \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2}$$

$$Y = \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2} = G - jB_L$$

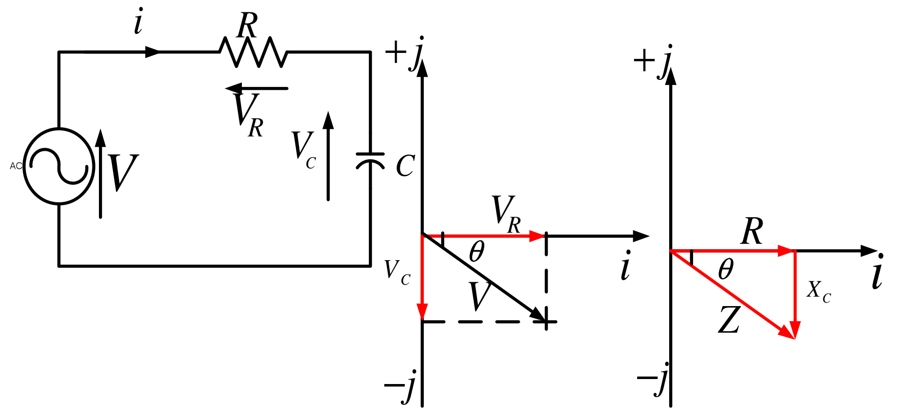
$$\therefore Y = G - jB_L$$

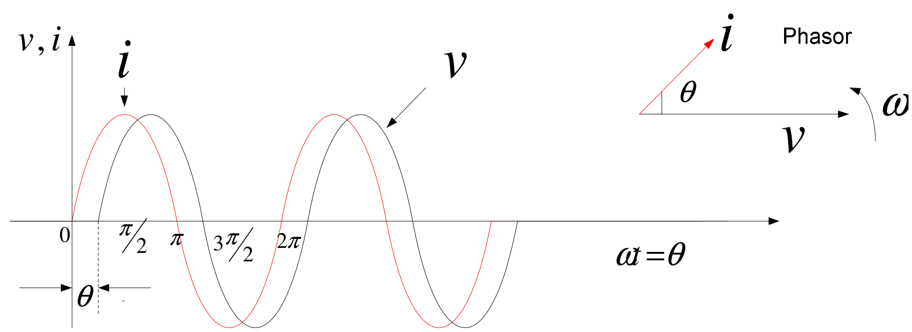
Y = Admittance

G = Conductance

B_L = Susceptance

RC SERIES CONNECTION





$$\overline{V} = \overline{V_R} - \overline{V_C} \text{ and } V_L = -jiX_C \text{ and } V_R = iR$$

$$\therefore V = iR - jiX_C = i(R - jX_C)$$

$$Z = \frac{V}{i} = R - jX_C = \sqrt{R^2 + X_C{}^2}$$

$$\tan\theta = \frac{-X_C}{R}$$

$$\theta = \tan^{-1}\frac{-X_C}{R}$$

Z is the circuit impedance and θ is the phase angle.

$$Y = \frac{1}{Z} = \frac{1}{R - jX_C} = \left(\frac{1}{R - jX_C}\right)\left(\frac{R + jX_C}{R + jX_C}\right) = \frac{R + jX_C}{R^2 + X_C^2} = \frac{R}{R^2 + X_C^2} + j\frac{X_C}{R^2 + X_C^2}$$

$$Y = \frac{R}{R^2 + X_C^2} + j\frac{X_L}{R^2 + X_C^2} = G + jB_C$$

$$\therefore Y = G + jB_C$$

Y = Admittance
G = Conductance
B_C = Susceptance

RLC SERIS CONNECTION

(a) $|V_C| > |V_L|$

$\overline{V} = \overline{V_R} + \overline{V_L} - \overline{V_C}$ and $V_L = -jiX_C, V_R = iR \; and \; V_L = jiX_L$

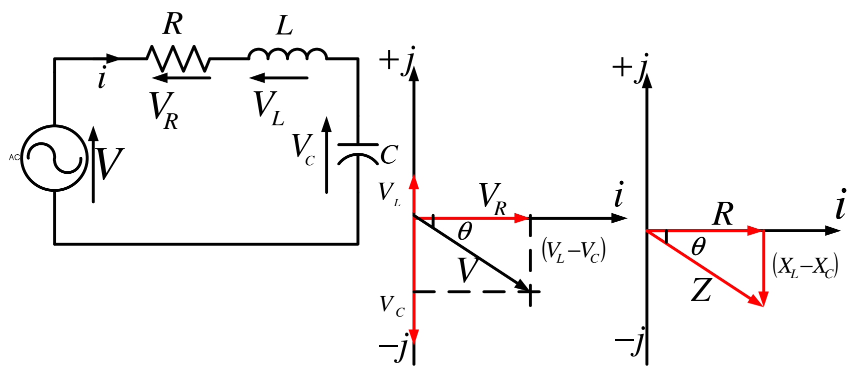
$$\therefore V = iR - jiX_C + jiX_L = i(R - jiX_C + jX_L) = i[R - j(X_C - X_L)]$$

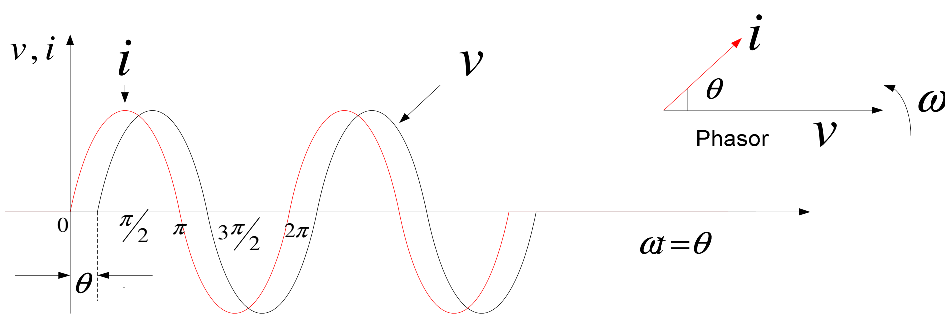
$$Z = \frac{V}{i} = [R - j(X_C - X_L)] = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\tan\theta = \frac{-(X_C - X_L)}{R}$$

$$\theta = \tan^{-1}\frac{-(X_C - X_L)}{R}$$

Z is the circuit impedance and θ is the phase angle.





(b) $|V_L| > |V_C|$

$\overline{V} = \overline{V_R} + \overline{V_L} - \overline{V_C}$ and $V_L = -jiX_C, V_R = iR \; and \; V_L = jiX_L$

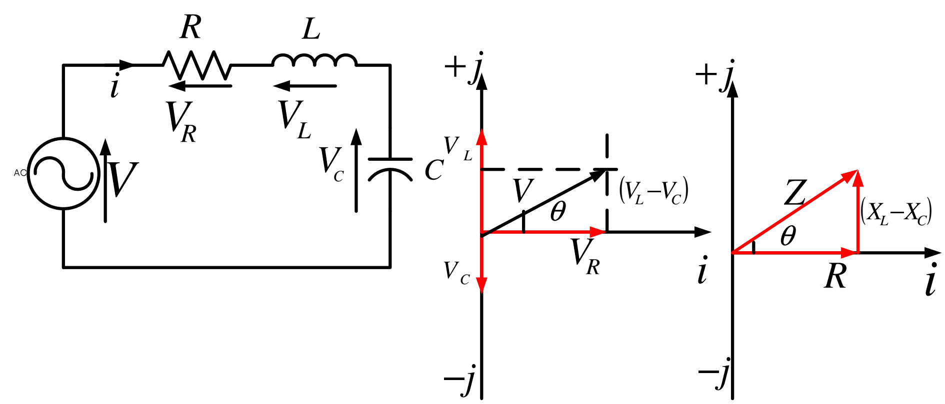
$$\therefore V = iR + jX_L - jiX_C = i(R + jX_L - jX_C) = i[R + j(X_L - X_C)]$$

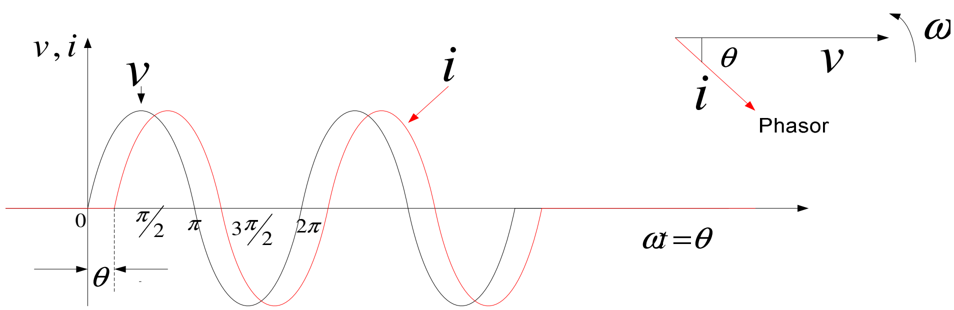
$$Z = \frac{V}{i} = [R + j(X_L - X_C)] = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\theta = \frac{X_L - X_C}{R}$$

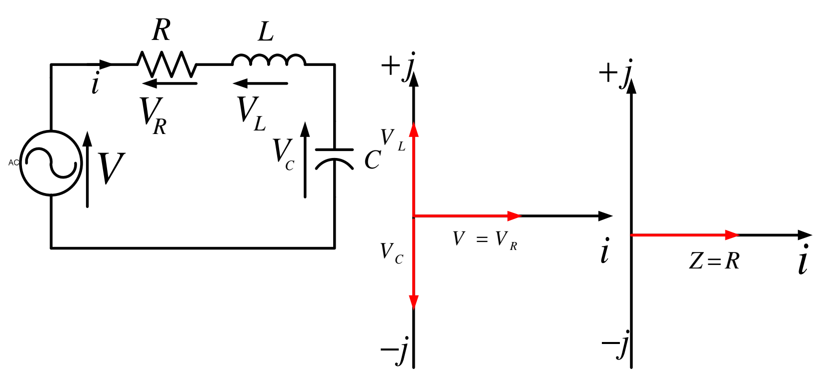
$$\theta = \tan^{-1}\frac{X_L - X_C}{R}$$

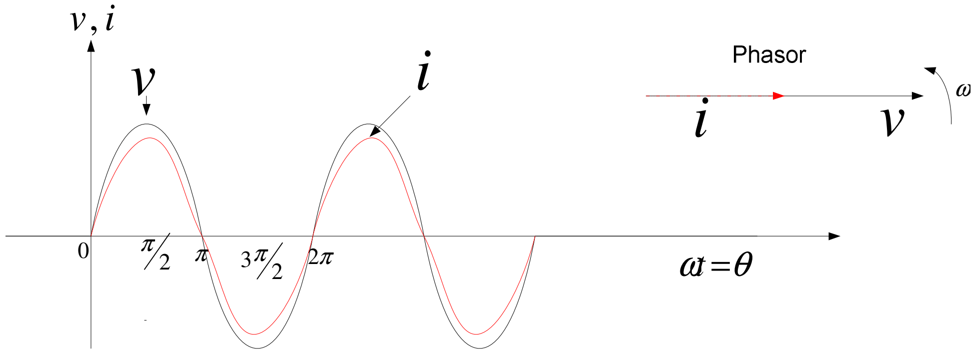
Z is the circuit impedance and θ is the phase angle.





(c) $|V_L| = |V_C|$





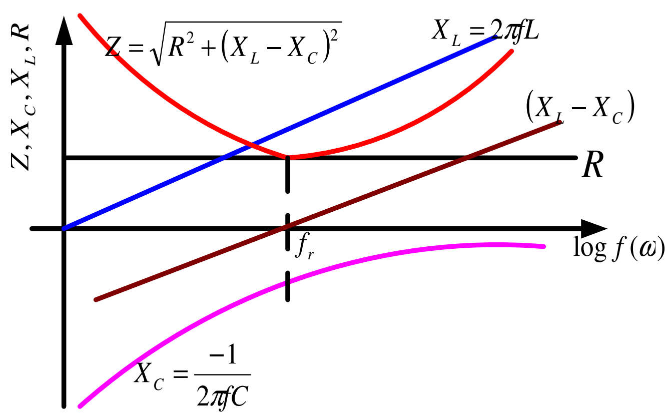
$$\overline{V} = \overline{V_R} + \overline{V_L} - \overline{V_C} \text{ and } V_L = -jiX_C, V_R = iR \text{ and } V_L = jX_L$$

$$\therefore V = iR + jX_L - jiX_C = i(R + jX_L - jX_C) = i[R + j(X_L - X_C)] = i[R + j0] = iR$$

$$Z = \frac{V}{i} = R$$

$$\tan\theta = \frac{X_L - X_C}{R} = \frac{0}{R} = 0 \Rightarrow \theta = \tan^{-1} 0 = 0^\circ$$

Z is equal R and θ is zero. At resonant frequency f_r, the impedance Z of the series circuit reaches its minimum value when $X_C = X_L$ and $X_L - X_C = 0$. When this occurs, the circuit is said to be resonant.

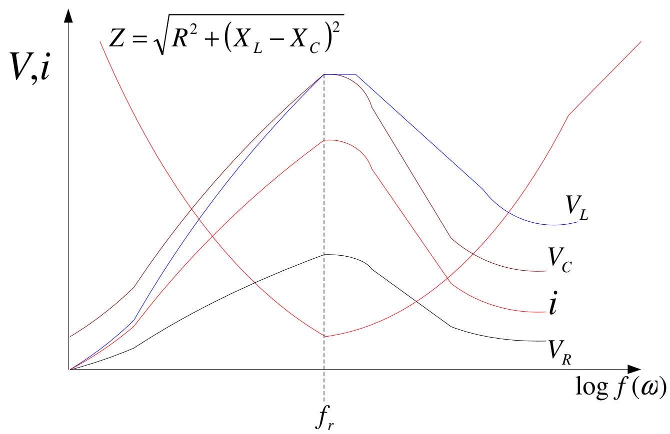


$$X_L \quad X_C = \omega L \quad \frac{1}{\omega C} = 2\pi f_r L \quad \frac{1}{2\pi f_r C} = 0$$

$$\therefore 2\pi f_r L = \frac{1}{2\pi f_r C} \Rightarrow f_r^{\,2} = \frac{1}{4\pi^2 LC} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

At resonance, $V_L = V_C$ and $V = V_R$. At this point, the values of V_L and V_C can be several times greater than V_R depending on the values of R, L and C. Voltage magnification also known as the Q-factor at resonance is $\frac{V_L or V_C}{V_R}$. Q-factor is a measure of voltage magnification in series circuits.

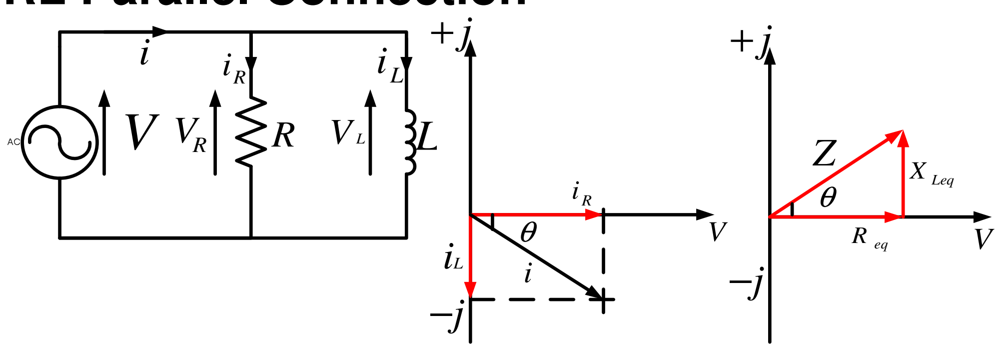
$$Q = \frac{2\pi f_r Li}{iR} = \frac{2\pi f_r L}{R} = \frac{2\pi\left(\frac{1}{2\pi\sqrt{LC}}\right)L}{R} = \frac{L}{R\sqrt{LC}} = \left(\frac{1}{R}\right)\sqrt{\frac{L^2}{LC}} = \left(\frac{1}{R}\right)\sqrt{\frac{L}{C}}$$

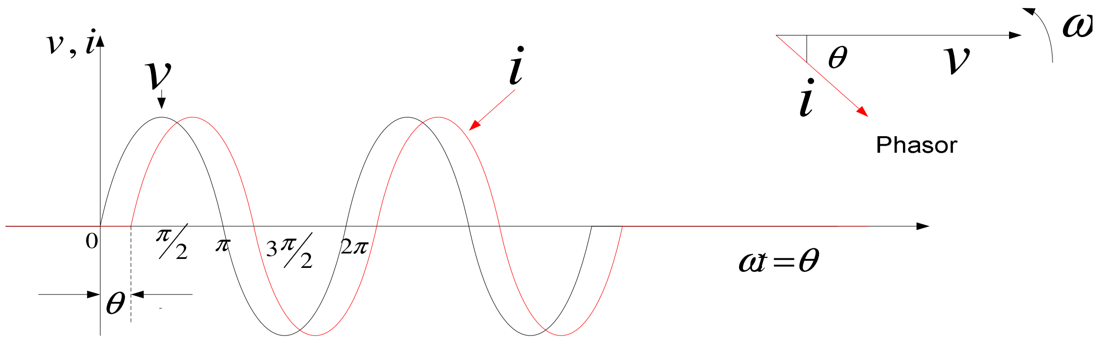


A series resonant circuit is often referred to as an acceptor circuit due to its maximum current at resonance. The frequency of the applied voltage is the same as the natural frequency of oscillation of the circuit. The energy is maintained in oscillation between *L* and *C* i.e. $\left(\frac{1}{2}\right)LI_m^{\ 2} = \left(\frac{1}{2}\right)CV_m^{\ 2}$. The power taken from the supply is just enough to supply losses in the resistance R. Therefore the Q-factor can be redefined as follows:

$$Q = \frac{(W_L or W_C)2\pi}{W_R / cycle} = \frac{\left[\left(\frac{1}{2}\right)LI_m^{\ 2}\right]2\pi}{\left(\frac{I_m}{\sqrt{2}}\right)^2\left(\frac{R}{f_r}\right)} = \frac{2\pi f_r L}{R} = \frac{\omega L}{R}.$$

RL Parallel Connection





$$\bar{i} = \bar{i}_R - \bar{i}_L$$

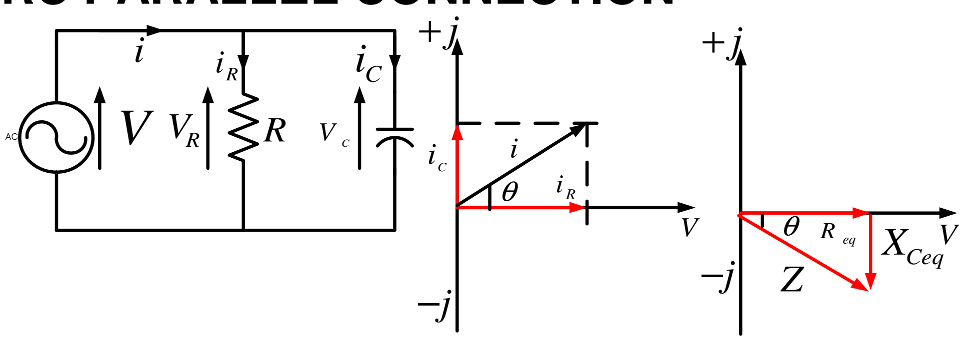
$$i = i_R - j i_L$$

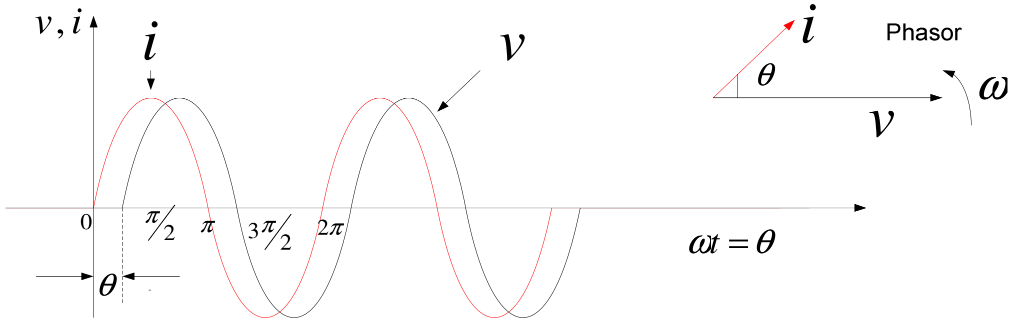
$$i = \frac{V}{R} - j\frac{V}{X_L} = V\left(\frac{1}{R} - j\frac{1}{X_L}\right)$$

$$Y = \frac{i}{V} = \frac{1}{Z} = \left(\frac{1}{R} - j\frac{1}{X_L}\right) = \left(\frac{1}{R} - j\frac{1}{\omega L}\right)$$

$$\therefore Z = \frac{1}{Y} = \frac{1}{\left(\frac{1}{R} - j\frac{1}{\omega L}\right)} = \left(\frac{R\omega^2 L^2}{R^2 + \omega^2 L^2} + j\frac{\omega L R^2}{R^2 + \omega^2 L^2}\right)$$

RC PARALLEL CONNECTION





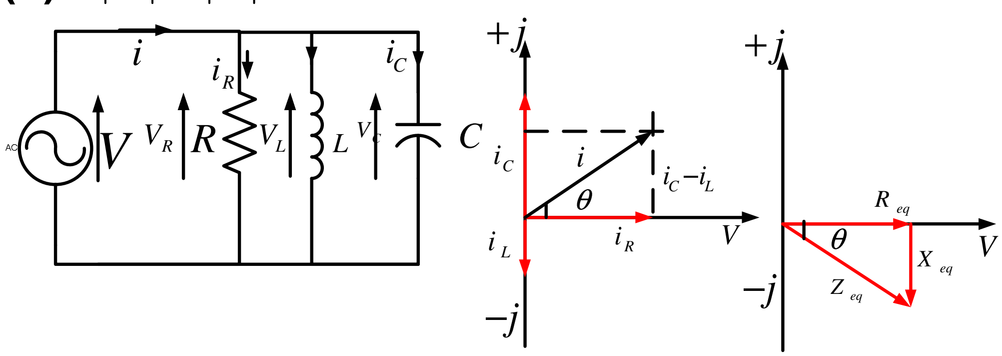
$$\bar{i} = \bar{i}_R + \bar{i}_C \Rightarrow i = i_R + j i_C = \frac{V}{R} + j\frac{V}{X_C} = V\left(\frac{1}{R} + j\frac{1}{X_C}\right)$$

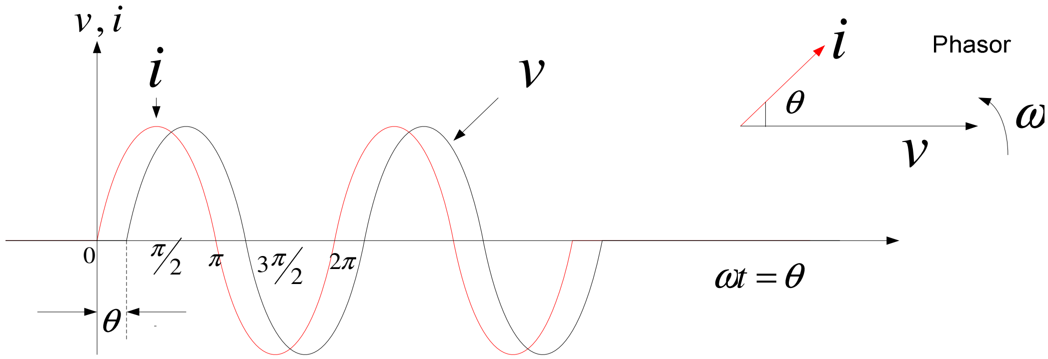
$$Y = \frac{i}{V} = \frac{1}{Z} = \left(\frac{1}{R} + j\frac{1}{X_C}\right) = \left(\frac{1}{R} + j\frac{1}{\frac{1}{\omega C}}\right) = \left(\frac{1}{R} + j\omega C\right)$$

$$\therefore Z = \frac{1}{Y} = \frac{1}{\left(\frac{1}{R} + j\omega C\right)} = \left(\frac{R}{1+\omega^2 C^2 R^2} - j\frac{\omega C R^2}{1+\omega^2 C^2 R^2}\right)$$

RLC PARALLEL CONNECTION

(a) $|i_C| > |i_L|$





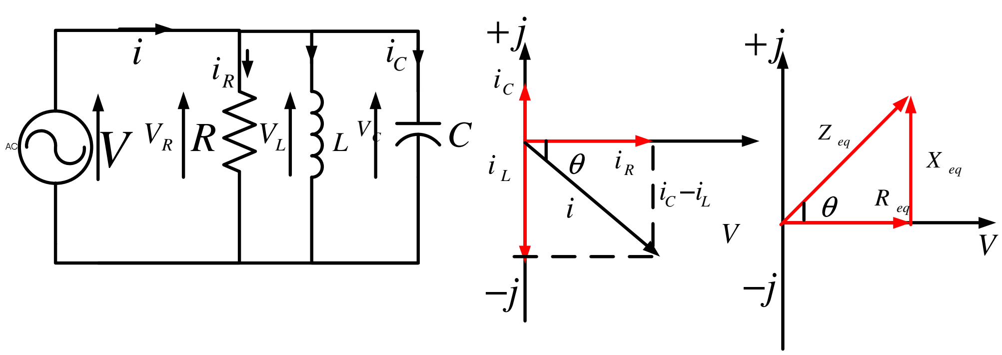
$$\bar{i} = \bar{i}_R + \bar{i}_C - \bar{i}_L$$

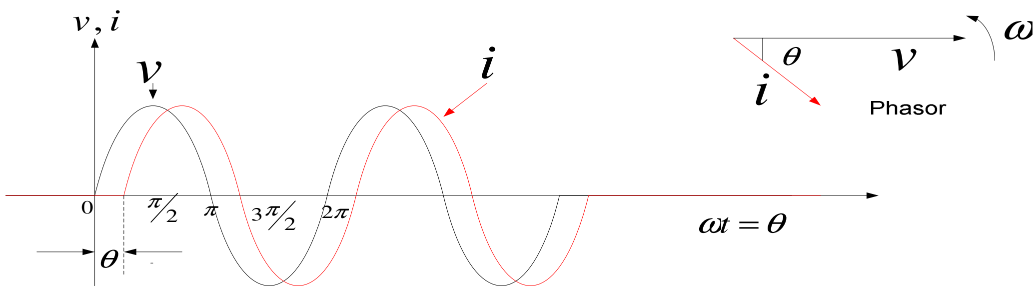
$$i = i_R + ji_C - ji_L = \frac{V}{R} + j\frac{V}{X_C} - j\frac{V}{X_L} = V\left(\frac{1}{R} + j\frac{1}{X_C} - j\frac{1}{X_L}\right) = V\left(\frac{1}{R} + j\left\{\frac{1}{X_C} - \frac{1}{X_L}\right\}\right)$$

$$Y = \frac{i}{V} = \frac{1}{Z} = \left(\frac{1}{R} + j\left\{\frac{1}{X_C} - \frac{1}{X_L}\right\}\right) = \left(\frac{1}{R} + j\left\{\frac{1}{\frac{1}{\omega C}} - \frac{1}{\omega L}\right\}\right) = \left(\frac{1}{R} + j\left\{\omega C - \frac{1}{\omega L}\right\}\right)$$

$$\therefore Z = \frac{1}{Y} = \frac{1}{\left(\frac{1}{R} + j\left\{\omega C - \frac{1}{\omega L}\right\}\right)} = \left(\frac{R\omega^2 L^2}{\omega^2 L^2 + R^2\left(\omega^2 LC - 1\right)^2} - j\frac{R^2 \omega L\left(\omega^2 LC - 1\right)}{\omega^2 L^2 + R^2\left(\omega^2 LC - 1\right)^2}\right)$$

(b) $|i_L| > |i_C|$





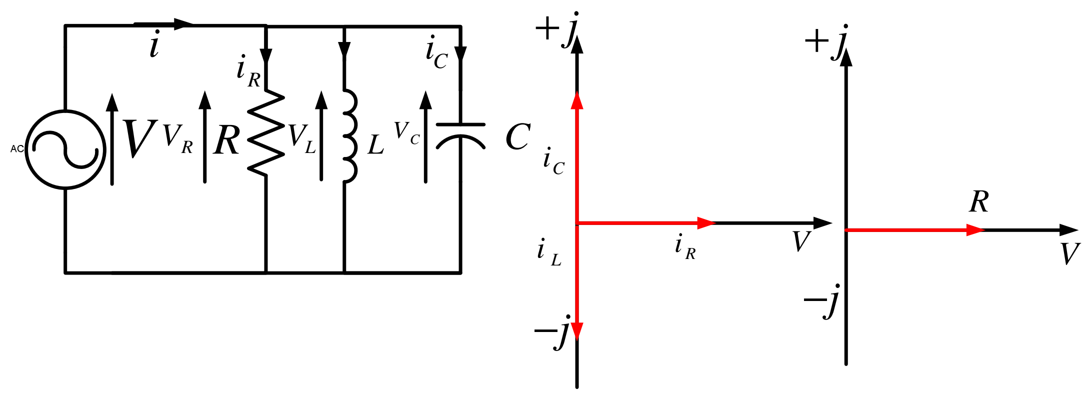
$$\bar{i} = \overline{i_R} - \overline{i_L} + \overline{i_C}$$

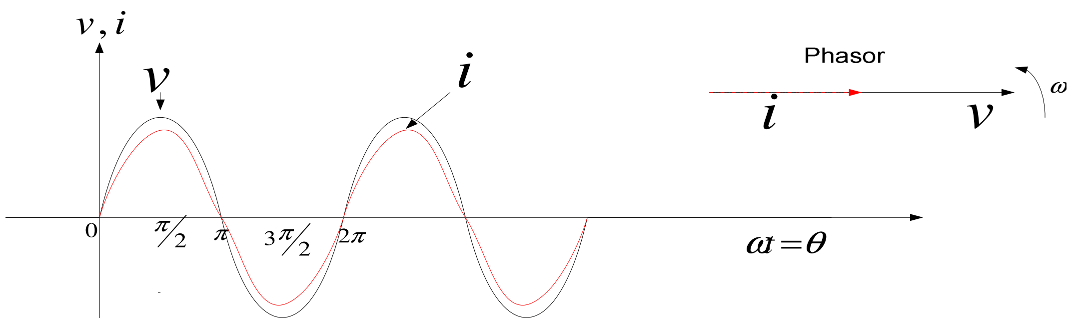
$$i = i_R - ji_L + ji_L = \frac{V}{R} - j\frac{V}{X_L} + j\frac{V}{X_C} = V\left(\frac{1}{R} - j\frac{1}{X_L} - j\frac{1}{X_C}\right) = V\left(\frac{1}{R} - j\left\{\frac{1}{X_L} - \frac{1}{X_C}\right\}\right)$$

$$Y = \frac{i}{V} = \frac{1}{Z} = \left(\frac{1}{R} - j\left\{\frac{1}{X_L} - \frac{1}{X_C}\right\}\right) = \left(\frac{1}{R} - j\left\{\frac{1}{\omega L} - \frac{1}{\frac{1}{\omega C}}\right\}\right) = \left(\frac{1}{R} - j\left\{\frac{1}{\omega L} - \omega C\right\}\right)$$

$$\therefore Z = \frac{1}{Y} = \frac{1}{\left(\frac{1}{R} - j\left\{\frac{1}{\omega L} - \omega C\right\}\right)} = \left(\frac{R\omega^2 L^2}{\omega^2 L^2 + R^2\left(1-\omega^2 LC\right)^2} + j\frac{R^2\omega L\left(1-\omega^2 LC\right)}{\omega^2 L^2 + R^2\left(1-\omega^2 LC\right)^2}\right)$$

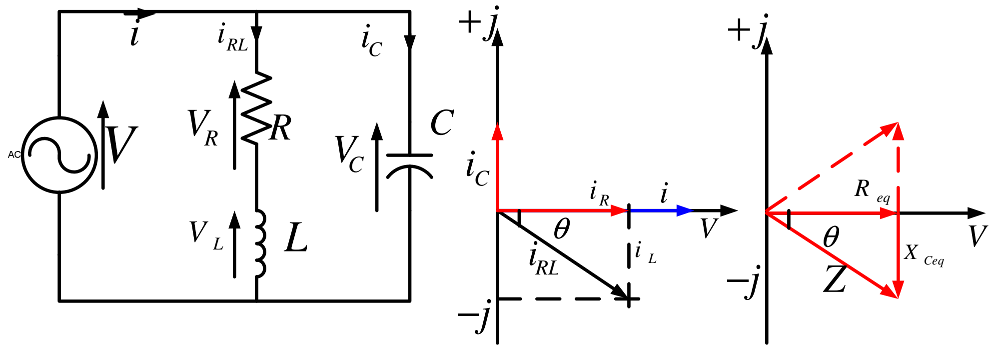
(c) $|i_L| = |i_C|$





$$\bar{i}=\bar{i}_R-\bar{i}_L+\bar{i}_C=\bar{i}_R\Rightarrow i=i_R=\frac{V}{R}=V\left(\frac{1}{R}\right)\Rightarrow Y=\frac{i}{V}=\frac{1}{Z}=\left(\frac{1}{R}\right)\Rightarrow\therefore Z=\frac{1}{Y}=\frac{1}{\left(\frac{1}{R}\right)}=R$$

In parallel connections, inductances are not pure and contain internal resistance in series with the coil.



$$i=i_R=i_{RL}\cos\theta \; and \; i_C=i_L=i_{RL}\sin\theta$$

At resonance, $i_C=i_L=i_{RL}\sin\theta$ and $i_{RL}=\frac{V}{R+jX_L}=\frac{V}{\sqrt{R^2+\omega^2L^2}}$,

$$\sin\theta=\frac{\omega L}{\sqrt{R^2+\omega^2L^2}}.$$

$$i_C=i_{RL}\sin\theta=\left(\frac{V}{\sqrt{R^2+\omega^2L^2}}\right)\left(\frac{\omega L}{\sqrt{R^2+\omega^2L^2}}\right)=\frac{\omega LV}{R^2+\omega^2L^2}$$

$$i_C=\frac{V}{\frac{1}{\omega C}}=\omega CV\Rightarrow\therefore\omega CV=\frac{\omega LV}{R^2+\omega^2L^2}\Rightarrow C=\frac{L}{R^2+\omega^2L^2}$$

$$\frac{L}{C}=R^2+\omega^2L^2\Rightarrow\omega^2=\frac{1}{LC}-\frac{R^2}{L^2}\Rightarrow\omega=2\pi f_r=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

$$\therefore f_r=\left(\frac{1}{2\pi}\right)\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

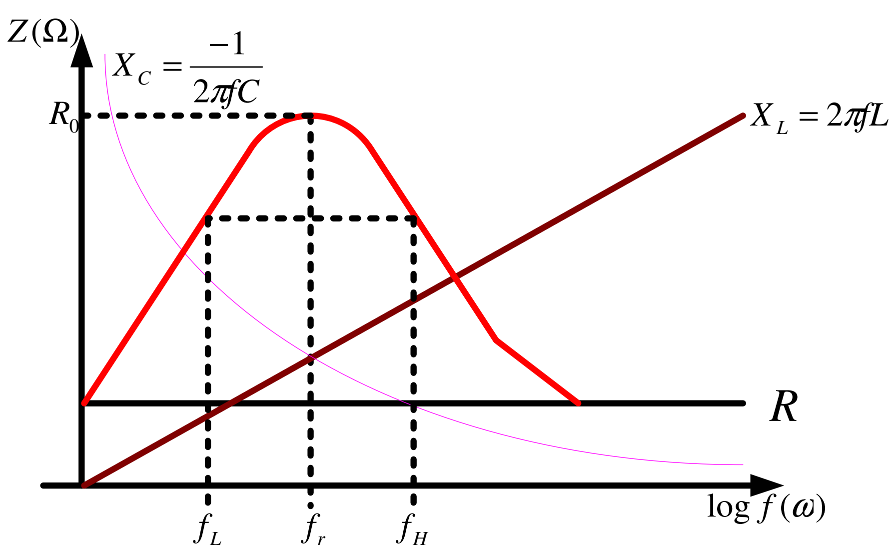
If the internal resistance R is negligible $(R\approx 0)$, then $\frac{R^2}{L^2}\approx 0$ and the resonant frequency becomes $f_r=\left(\frac{1}{2\pi}\right)\sqrt{\frac{1}{LC}}=\frac{1}{2\pi\sqrt{LC}}$. The current circulating in L and C at resonance can be many times the current from the source. Therefore the Q-factor (current magnification) becomes:

$$Q = \frac{i_C}{i} = \frac{i_{RL} \sin\theta}{i_{RL} \cos\theta} = \tan\theta = \frac{\omega L}{R}.$$

The Q-factor is a measure of current magnification in parallel circuits. The resultant current in a resonant parallel circuit is in phase with the supply voltage. The impedance at resonant frequency is $Z = \frac{V}{i} = \frac{V}{\frac{i_C}{\tan\theta}} = \frac{V \tan\theta}{i_C}$.

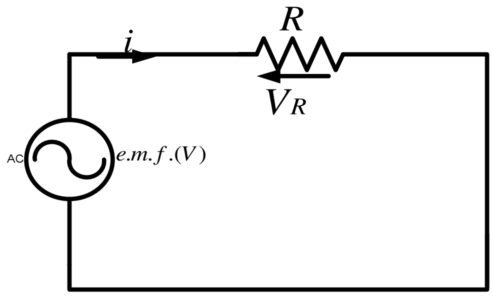
But $Z = \frac{V}{i} = \frac{V\tan\theta}{i_C} = \frac{V\tan\theta}{\omega CV} = \frac{V\frac{\omega L}{R}}{\omega CV} = \frac{L}{RC}$.

The impedance is equivalent to a non-reactive resistance of $\frac{L}{RC}(\Omega)$ known as the dynamic resistance R_0.

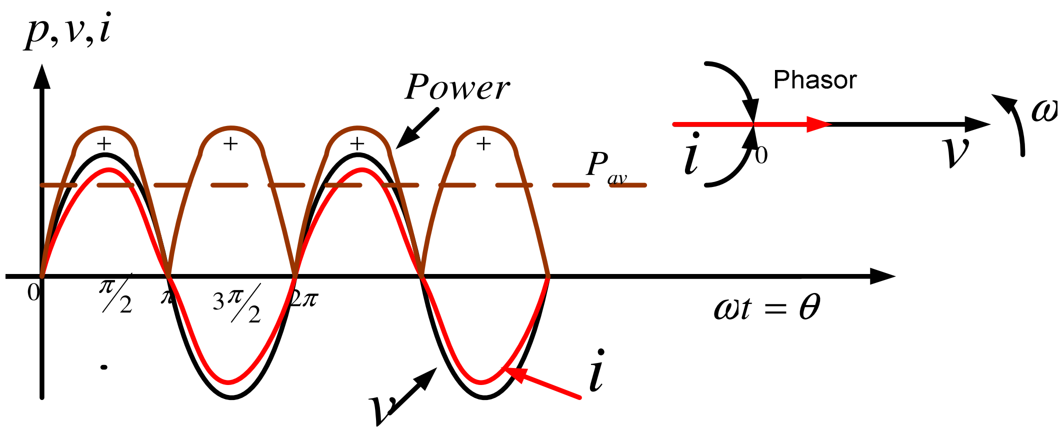


8.0 POWER IN AC CIRCUITS

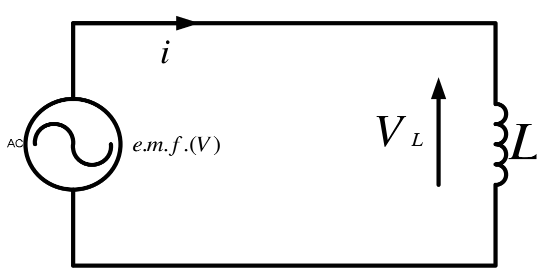
PURE RESISTANCE



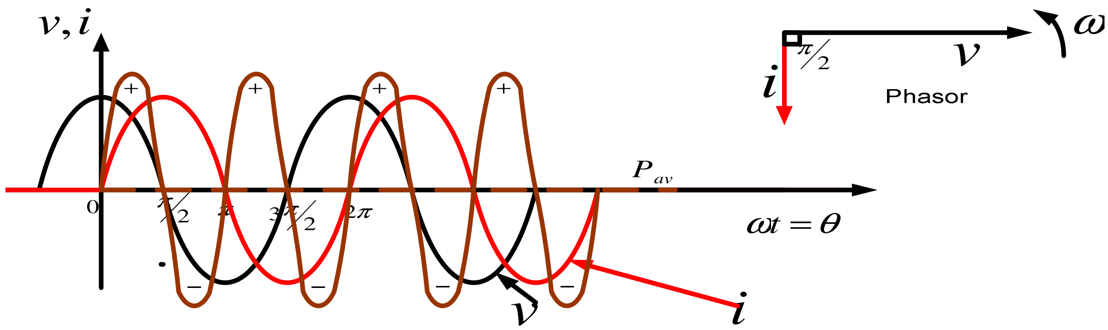
The power consumed by a pure resistance where the voltage and current are in phase is $P = vi\cos\theta = vi\cos 0 = vi.$ It is the product of current in phase with the voltage and current.



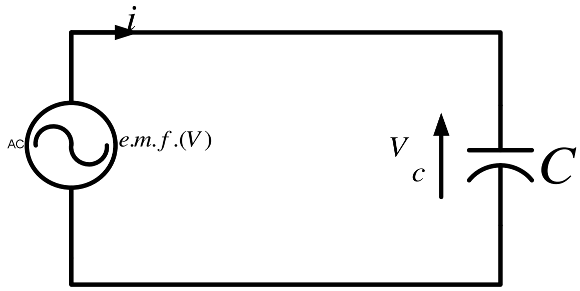
PURE INDUCTANCE



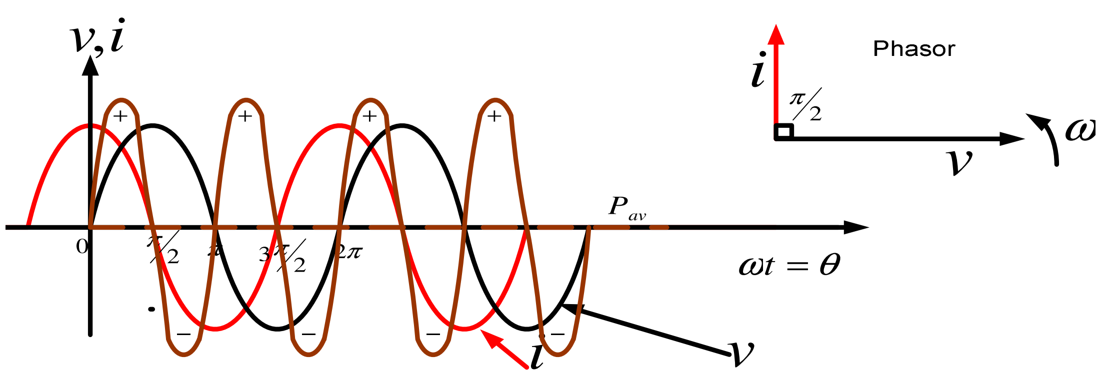
The power consumed by a pure inductance where the voltage and current are in out of phase by $\pi/2$ is $P = vi\cos\theta = vi\cos\pi/2 = 0.$ The energy from supply in the first quarter (+) builds up the magnetic flux and is stored in the fields established. In the second quarter, the energy is returned to the supply when the current is reduced and the magnetic field allowed to collapse. Consequently the net energy absorbed by the coil during a complete cycle is zero; giving zero average power over a complete cycle.



PURE CAPACITANCE

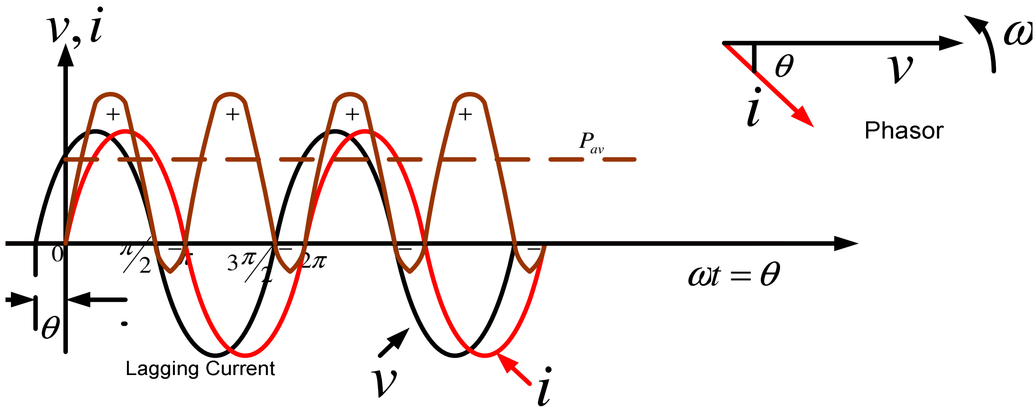


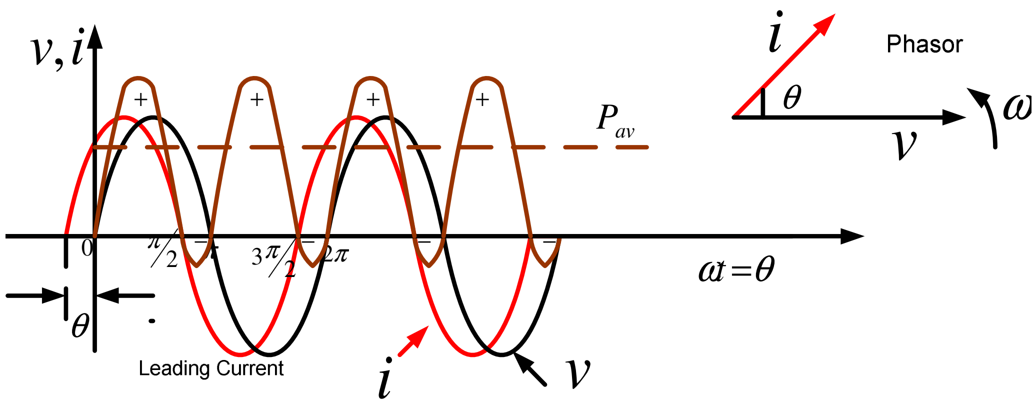
The power consumed by a pure capacitance where the voltage and current are in out of phase by $\pi/2$ is $P = vi\cos\theta = vi\cos\pi/2 = 0.$ The energy from supply in the first quarter (+) builds up the electric flux and is stored in the electric fields established. In the second quarter, the energy is returned to the supply when the current is reduced and the electric field allowed to collapse. Consequently the net energy absorbed by the capacitor during a complete cycle is zero; giving zero average power over a complete cycle.



COMBINATION OF RESISTANCE AND REACTANCE

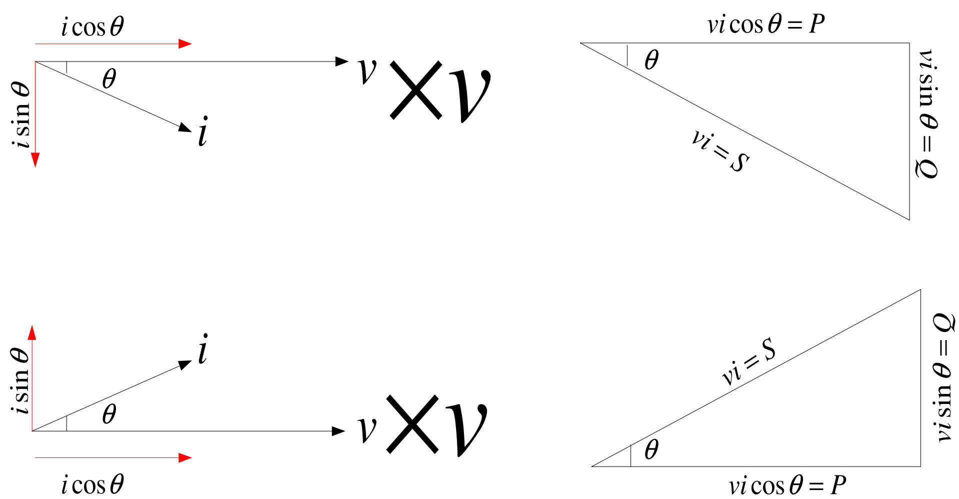
Power consumption by a combination of resistance and reactance causes a phase displacement of θ which lies between 0 and $\pi/2$.





The power consumed by a combination of resistance and reactance where the voltage and current are in out of phase by θ is $P = vi\cos\theta$ The product of current and voltage is only an apparent power because the current and voltage are not in phase. The positive power (+) is larger than the (-) giving an average output power of P_{av}.

POWER TRIANGLE

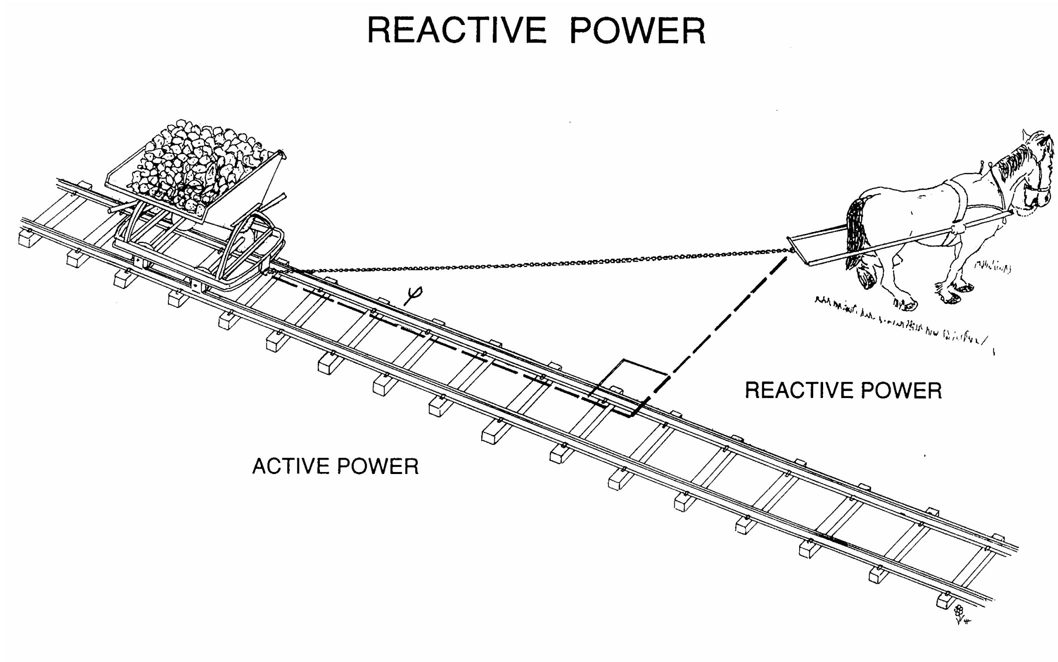


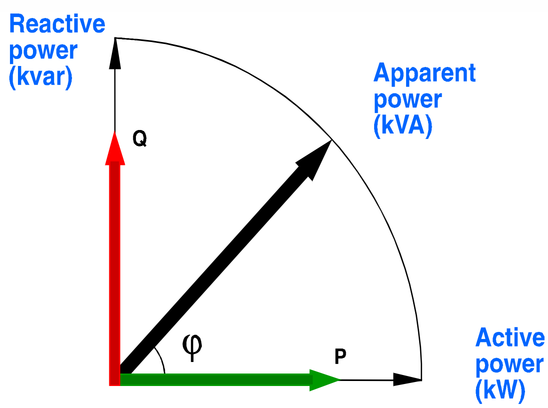
$$Apparent \;\; power \;\; S = vi(VA)$$

$$\text{Real} \;\; power \;\; P = vi\cos\theta(W)$$

$$\text{Reactive} \;\; power \;\; or \;\; imaginary \;\; Q = vi\sin\theta(VAr)$$

$$Power \;\; factor, \; pf = \frac{P}{S} = \frac{vi\cos\theta}{vi} = \cos\theta$$

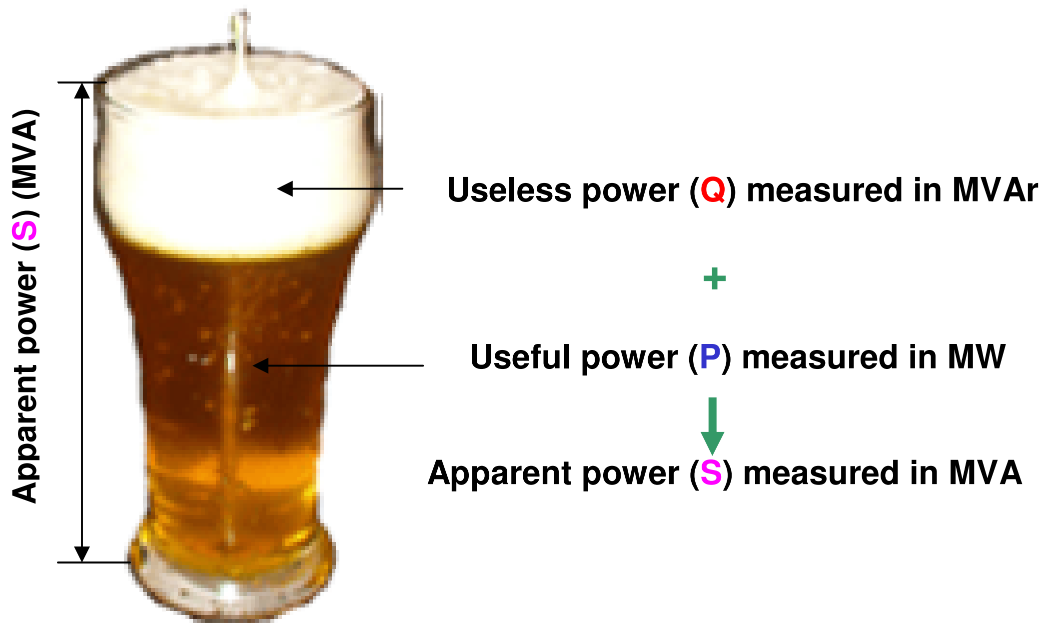




...compete over space in wires and apparatuses.

Interested In:

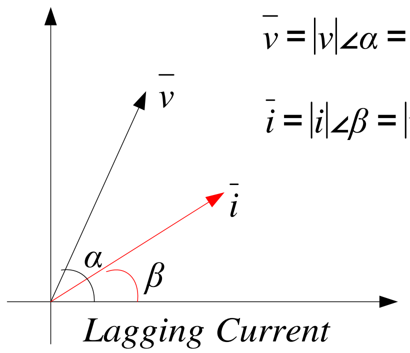
Amount of power moving through the system



8.1 POWER FACTOR IMPROVEMENT

Power factor improvement also known as power factor correction refers to the process of improving the power factor such that $\cos\theta \approx 1$. The process reduces reactive power and the current losses due to i^2R losses in conductors by reducing the current that is associated with reactive power. Inductive loads can have their power factor improved by adding a capacitor in parallel and capacitive loads can have their power factor improved by adding an inductor in parallel. An over excited synchronous motor can also be used in place of a capacitor and an under excited synchronous motor can be used in place of an inductor.

8.2 COMPLEX POWER (POWER WITH j NOTATION)



Lagging Current

$\bar{v} = |v|\angle\alpha = |v|(\cos\alpha + j\sin\alpha)$

$\bar{i} = |i|\angle\beta = |v|(\cos\beta + j\sin\beta)$

$$P = |v||i|\cos(\alpha - \beta) = |v||i|(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$$Q = |v||i|\sin(\alpha - \beta) = |v||i|(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$$

But multiplying $\bar{v}$ and $\bar{i}$, the following product is obtained;

$$S = \overline{v}\overline{i}\angle(\alpha + \beta) = |v||i|\{\cos(\alpha + \beta) + j\sin(\alpha + \beta)\}$$

$$\therefore P = |v||i|\cos(\alpha + \beta)$$

$$Q = |v||i|\sin(\alpha + \beta)$$

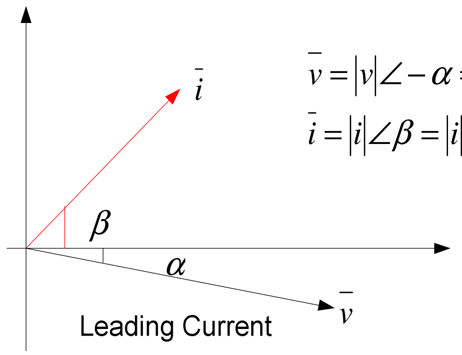
The values of power P and reactive power Q obtained differ from the correct values for both the real and quadrature components and in polar form, the angle is $(\alpha + \beta)$ instead of the correct value of $(\alpha - \beta)$. The correct expressions are obtained by using the conjugate of $\bar{i}$ denoted $\bar{i}^*$ in the multiplication. $\therefore\ \bar{i}^* = |i|\angle - \beta$.

$$S = vi \quad = |v||i|\angle(\alpha \quad \beta) = |v||i|\{\cos(\alpha \quad \beta) + j\sin(\alpha \quad \beta)\} = P + jQ$$

if $v = |v|\angle 0°$ and $i = |i|\angle - \beta$, then

$$S = vi^* = |v||i|\angle(+\beta) = |v||i|(\cos\beta + j\sin\beta) = P + jQ$$

The sign of the reactive power is positive for lagging current.



$$\bar{v} = |v|\angle -\alpha = |v|(\cos\alpha - j\sin\alpha)$$
$$\bar{i} = |i|\angle\beta = |i|(\cos\beta + j\sin\beta)$$

$$P = |v||i|\cos(-\alpha-\beta) = |v||i|\cos\{-(\alpha+\beta)\} = |v||i|(\cos\beta\cos\alpha + \sin\beta\sin\alpha)$$
$$Q = |v||i|\sin(-\alpha-\beta) = |v||i|\sin\{-(\alpha+\beta)\} = |v||i|(-\sin\beta\cos\alpha - \cos\beta\sin\alpha)$$

But multiplying $\bar{v}$ and $\bar{i}$, the following product is obtained;

$$S = \bar{v}\bar{i}\angle(-\alpha+\beta) = \bar{v}\bar{i}\angle\{-(\alpha-\beta)\} = |v||i|\{\cos[-(\alpha-\beta)] - j\sin[-(\alpha-\beta)]\}$$
$$\therefore P = |v||i|\cos[-(\alpha-\beta)]$$
$$Q = -|v||i|\sin[-(\alpha-\beta)]$$

The values of power P and reactive power Q obtained differ from the correct values for both the real and quadrature components and in polar form, the angle is $\{-(\alpha-\beta)\}$ instead of the correct value of $\{-(\alpha+\beta)\}$. The correct expressions are obtained by using the conjugate of $\bar{i}$ denoted $\bar{i}^*$ in the multiplication. $\therefore \bar{i}^* = |i|\angle -\beta$.

$$S = vi^* = |v||i|\angle(-\alpha-\beta) = |v||i|\angle\{-(\alpha+\beta)\} = |v||i|\{\cos(\alpha+\beta) - j\sin(\alpha+\beta)\} = P - jQ$$

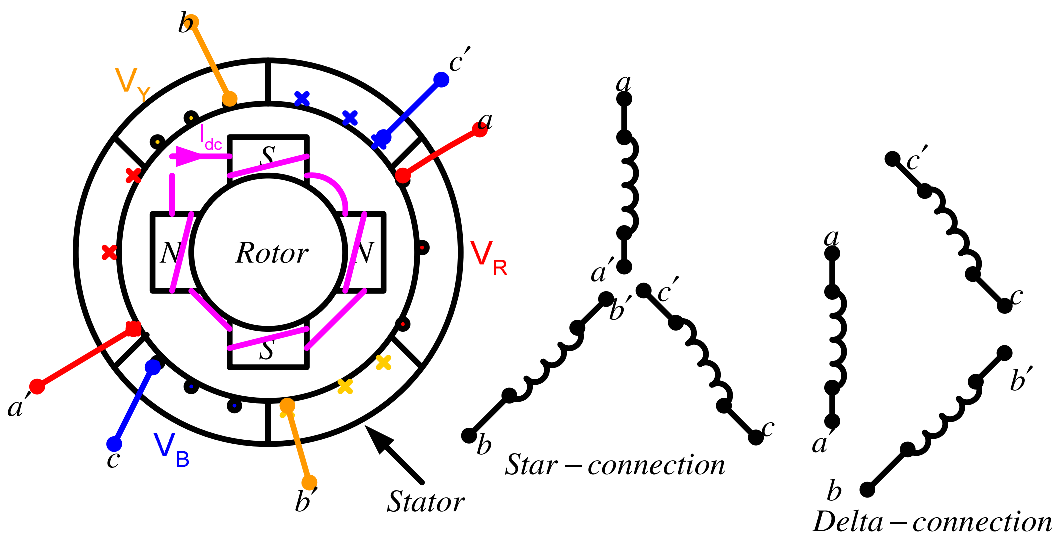
if $v = |v|\angle 0°$ and $i = |i|\angle\beta$, then

$$S = vi^* = |v||i|\angle(-\beta) = |v||i|(\cos\beta - j\sin\beta) = P - jQ$$

The sign of the reactive power is negative for leading current.

9.0 THREE-PHASE SUPPLY

THREE-PHASE ALTERNATOR



The dc current supplies excitation or magnetizing current to form the N-S poles on the rotor. For high sped and high efficiency alternators, the generated voltage is obtained from stator windings and the dc supply is through the slip rings on the rotor. When the field on the rotor is rotated, voltages are generated in three phases in accordance with Faraday's law:

$$V = N\frac{\partial\phi}{\partial t} + Nu\frac{\partial\phi}{\partial x}$$

Where $N\frac{\partial\phi}{\partial t}$ = *flux linking*

$Nu\frac{\partial\phi}{\partial x}$ = *flux cutting*

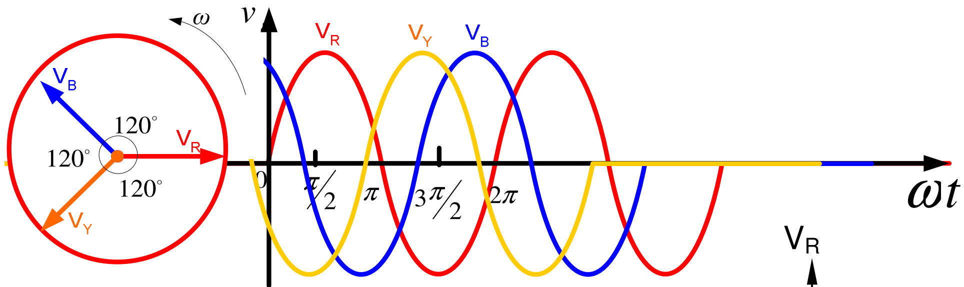
u = *linear speed*

∂x = *linear displacement*

ϕ = *flux*

N = *Number of turns*

For sinusoidally distributed flux over the poles, the flux linking any phase varies sinusoidally with time. Sinusoidal voltages are therefore induced in three phases. The three waves will be displaced by 120 electrical degrees in time as a result of the three phases being displaced 120° in space. For balanced three-phase supply, the voltages in the three phases are the same and equal in magnitude but spaced 120°. For balanced loads, the impedances and phase angles in each phase are the same and equal. The system is said to be balanced if the supply and the loads are balanced.



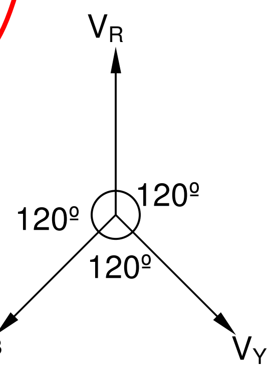
$\omega =$ Angular Speed

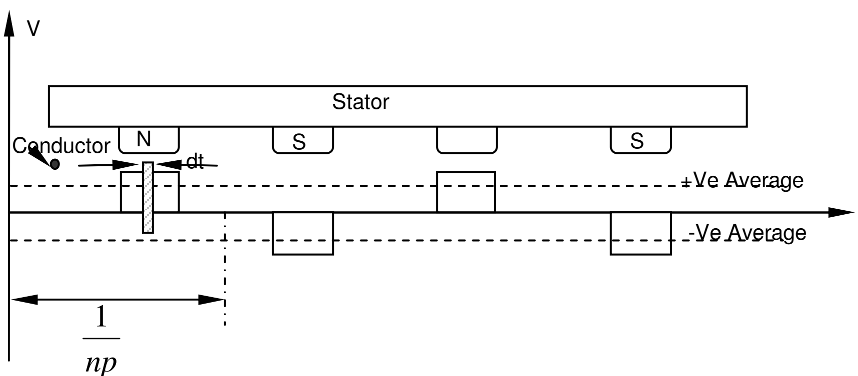
$\omega t = \theta =$ Angular displacement

$V_R = V_m \sin \omega t$ *Taken as reference*

$V_Y = V_m \sin\left(\omega t - 120^\circ\right)$

$V_B = V_m \sin\left(\omega t - 240^\circ\right) = V_m \sin\left(\omega t + 120^\circ\right)$





$$v = Blu \Rightarrow V_{av} = \frac{\int Bludt}{\int dt} = \frac{\int Bludt}{\frac{1}{np}} = \frac{\int Bldx}{\frac{1}{np}} = \frac{\phi}{\frac{1}{np}} = \phi np$$

$V_{av} = \varphi np \;\; for \;\; one \;\; conductor$

If the whole armature has Z conductors in series, then the average induced voltage for the whole machine is $V_{av} = Z_s n\varphi pk$ where k is the winding factor.

$$\frac{V_{rms}}{V_{av}} = \frac{V_m / \sqrt{2}}{2V_m / \pi} = 1.111$$

$V_{rms} = 1.111 V_{av} = 1.11 Z_s pn\varphi k$ *This is the output of the ac generator or alternator. A conductor passing one pole gives a half cycle. In a p pole machine running at n rev/sec* $f = \frac{np}{2}$.

The alternator output voltage is proportional to the excitation current. The voltage increases with an increased field current or excitation current.

$$V = 2.22\,\varphi f Z_s k$$

Where V = Output voltage

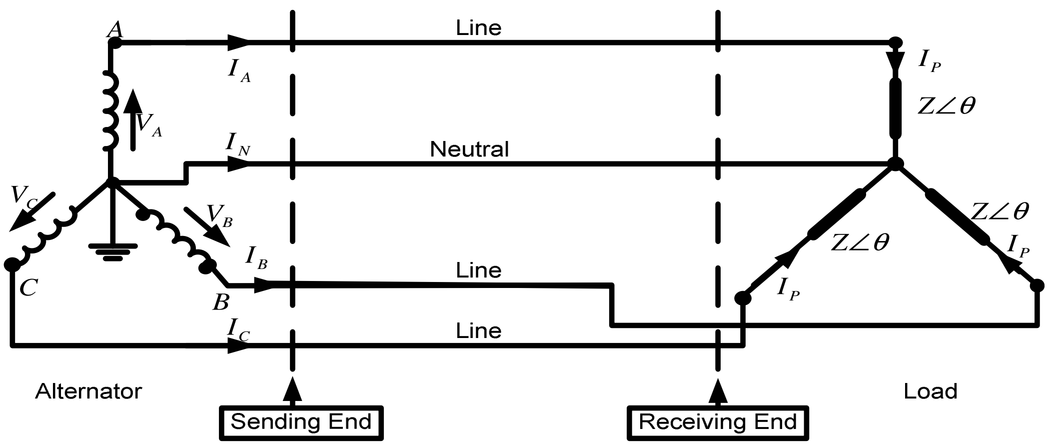
ϕ = Magnetic flux due to current excitation

f = Frequency

Z_s = Conductors in series

k = Distribution factor of the winding

STAR/STAR BALANCED LOAD.



For calculation purposes, choose V_A as the referrence.

$$V_A = |v|\angle 0^\circ,$$

$$V_B = |v|\angle -120^\circ,$$

$$V_C = |v|\angle -240^\circ = |v|\angle 120^\circ$$

$$I_A = \frac{V_A\angle 0^\circ}{Z\angle\theta} = \frac{v}{Z}\angle -\theta$$

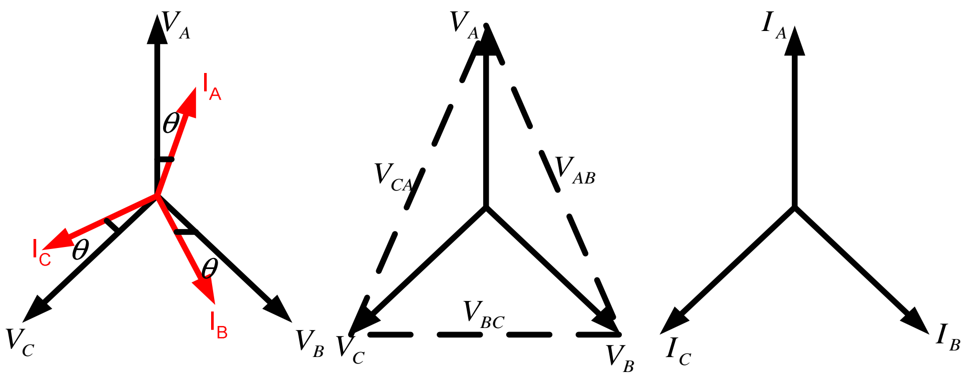
$$I_B = \frac{V_B\angle -120^\circ}{Z\angle\theta} = \frac{v}{Z}\angle -\theta -120^\circ$$

$$I_C = \frac{V_C\angle -240^\circ}{Z\angle\theta} = \frac{v}{Z}\angle -\theta - 240^\circ = \frac{v}{Z}\angle 120^\circ - \theta$$

$$I_N = I_A + I_B + I_C$$

$$For\ I_A = I_B = I_C,$$

$$I_N = 0$$



$$V_A = V_B = V_C = V_P = \text{phase voltage}$$

$$V_{AB} = V_{BC} = V_{CA} = V_L = \text{line voltage}$$

$$I_A = I_B = I_C = I_P = I_L$$

$$\therefore I_L = I_P$$

$$I_P = \text{phase current}$$

$$I_L = \text{line current}$$

Line voltage is voltage between any two lines while a phase voltage refers to voltage between any line and neutral.

$$V_L^2 = V_P^2 + V_P^2 - 2V_P V_P \cos 120^\circ = 2V_P^2 + V_P^2 = 3V_P^2$$

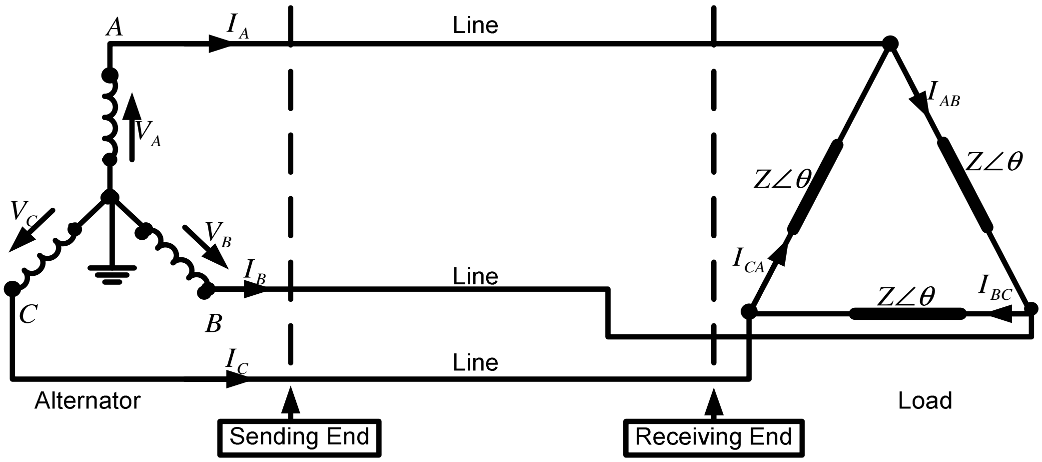
$$\therefore V_L = \sqrt{3} V_P$$

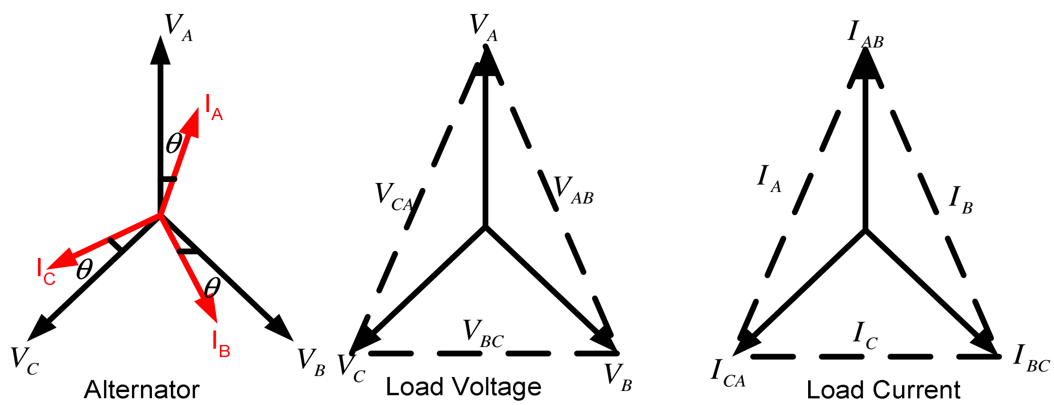
$$\text{Power in one phase} = V_P I_P \cos\theta$$

$$\text{Power in three phases} = 3V_P I_P \cos\theta = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta = \sqrt{3} V_L I_L \cos\theta$$

$$\therefore P = \sqrt{3} V_L I_L \cos\theta$$

STAR/DELTA BALANCED LOAD





$$V_A = V_B = V_C = V_P = \text{phase voltage}$$

$$V_{AB} = V_{BC} = V_{CA} = V_L = \text{line voltage}$$

$$I_A = I_B = I_C = I_L$$

$$I_{AB} = I_{BC} = I_{CA} = I_P$$

$$I_P = \text{phase current}$$

$$I_L = \text{line current}$$

$$I_L^{\ 2} = I_P^{\ 2} + I_P^{\ 2} - 2I_P I_P \cos 120^\circ = 2I_P^{\ 2} + I_P^{\ 2} = 3I_P^{\ 2}$$

$$\therefore I_L = \sqrt{3} I_P$$

$$I_A = I_{AB} - I_{CA}$$

$$I_B = I_{BC} - I_{AB}$$

$$I_C = I_{CA} - I_{BC}$$

$$Power\ in\ one\ phase = V_P I_P \cos\theta$$

$$Power\ in\ three\ phases = 3V_P I_P \cos\theta = 3V_L(\frac{I_L}{\sqrt{3}})\cos\theta = \sqrt{3}V_L I_L \cos\theta$$

$$\therefore P = \sqrt{3}V_L I_L \cos\theta$$

$$Apparent\ power\ S = \sqrt{3}V_L I_L (VA)$$

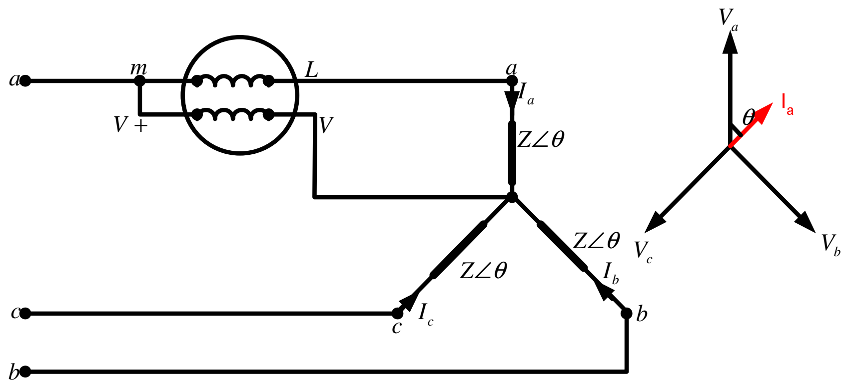
$$\text{Real}\ power\ P = \sqrt{3}V_L I_L \cos\theta (W)$$

$$\text{Reactive}\ power\ or\ imaginary\ Q = \sqrt{3}V_L I_L \sin\theta (VAr)$$

$$Power\ factor,\ pf = \frac{P}{S} = \frac{\sqrt{3}V_L I_L \cos\theta}{\sqrt{3}V_L I_L} = \cos\theta$$

9.1 POWER MEASUREMENT IN THREE-PHSAES

ONE WATTMETER METHOD

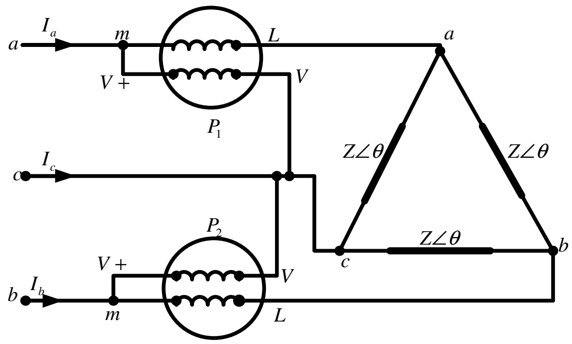


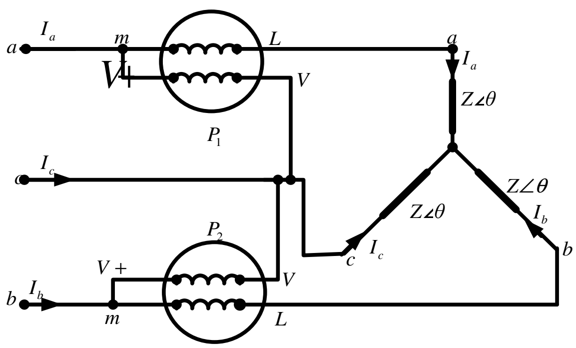
Power in one phase = $V_a I_a \cos\theta$ *(wattmeter reading)*

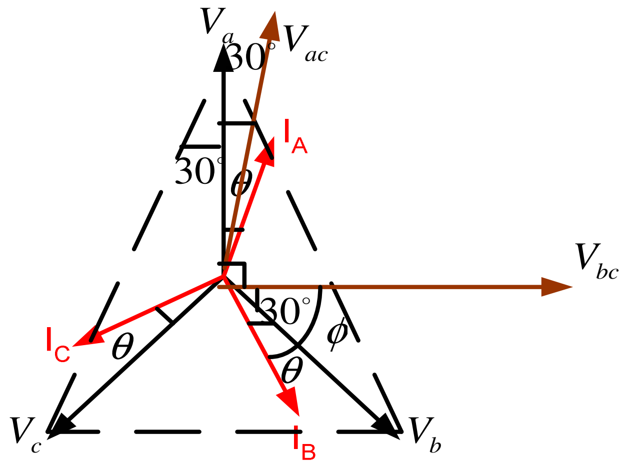
Total power = $3V_a I_a \cos\theta$ *(3 x Wattmeter reading)*

The one wattmeter method is used to measure power in a star connected balanced load with an accessible neutral point. The current coil is connected in one phase and the voltage coil is connected between the phase and the neutral point.

TWO WATTMETER METHOD







Total power is obtained by taking the sum of the two wattmeter readings.

Instantaneous power in P_1 wattmeter $P_1 = i_a V_{ac} = i_a(V_a - V_c)$

Average power in P_1 wattmeter $P_1 = V_{ac} i_a \cos(\theta - 30^\circ)$

Average power in P_2 wattmeter $P_2 = V_{bc} i_b \cos\phi = V_{bc} i_b \cos(\theta + 30^\circ)$

$$P_{total} = P_1 + P_2 = V_{ac} i_a \cos(\theta - 30^\circ) + V_{bc} i_b \cos(\theta + 30^\circ) = V_L I_L \cos(\theta - 30^\circ) + V_L I_L \cos(\theta - 30^\circ)$$

$$Note: \cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$P_{total} = V_L I_L \{\cos(\theta - 30^\circ) + \cos(\theta + 30^\circ)\} = V_L I_L \left\{2\cos\frac{1}{2}(\theta - 30^\circ + \theta + 30^\circ)\cos\frac{1}{2}(\theta - 30^\circ - (\theta + 30^\circ))\right\}$$

$$\therefore P_{total} = 2\cos\theta\cos(-30^\circ) = V_L I_L 2\cos\theta\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3} V_L I_L \cos\theta$$

The two wattmeter method measures three-phase power for balanced and unbalanced system.

REACTIVE POWER MEASUREMENT

The reactive power Q is the difference between the two wattmeter readings.

$$Q = P_1 - P_2 = V_L I_L \{\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)\}$$

$$Note: \cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$Q = V_L I_L \left\{-2\sin\frac{1}{2}(\theta - 30^\circ + \theta + 30^\circ)\sin\frac{1}{2}[(\theta - 30^\circ) - (\theta + 30^\circ)]\right\} = -2V_L I_L \sin\theta\left(-\frac{1}{2}\right) = V_L I_L \sin\theta$$

$$\therefore Q = V_L I_L \sin\theta$$

The total reactive power is obtained by multiplying the difference $\left(P_1 - P_2\right)$ by $\sqrt{3}$.

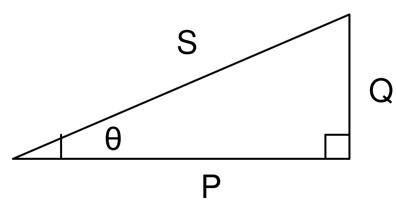
$$Q_{total} = \sqrt{3} V_L I_L \sin\theta$$

POWER FACTOR MEASUREMENT

Power factor $pf = \cos\theta$

$$P_1 + P_2 = \sqrt{3} V_L I_L \cos\theta$$

$$\therefore \cos\theta = \frac{P_1 - P_2}{\sqrt{3} V_L I_L}$$

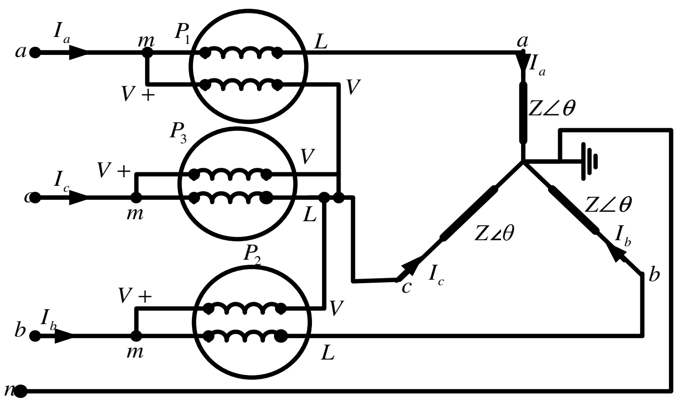


$$\frac{Q}{P} = \frac{P_1 - P_2}{P_1 + P_2} = \frac{V_L I_L \sin\theta}{\sqrt{3} V_L I_L \cos\theta} = \frac{1}{\sqrt{3}} \tan\theta$$

$$\therefore \tan\theta = \frac{\sqrt{3}\left(P_1 - P_2\right)}{P_1 + P_2}$$

$$\therefore pf = \cos\theta = \cos\left\{\tan^{-1}\left(\frac{\sqrt{3}\left(P_1 - P\right)}{P_1 + P_2}\right)\right\}$$

THREE WATTMETER METHOD



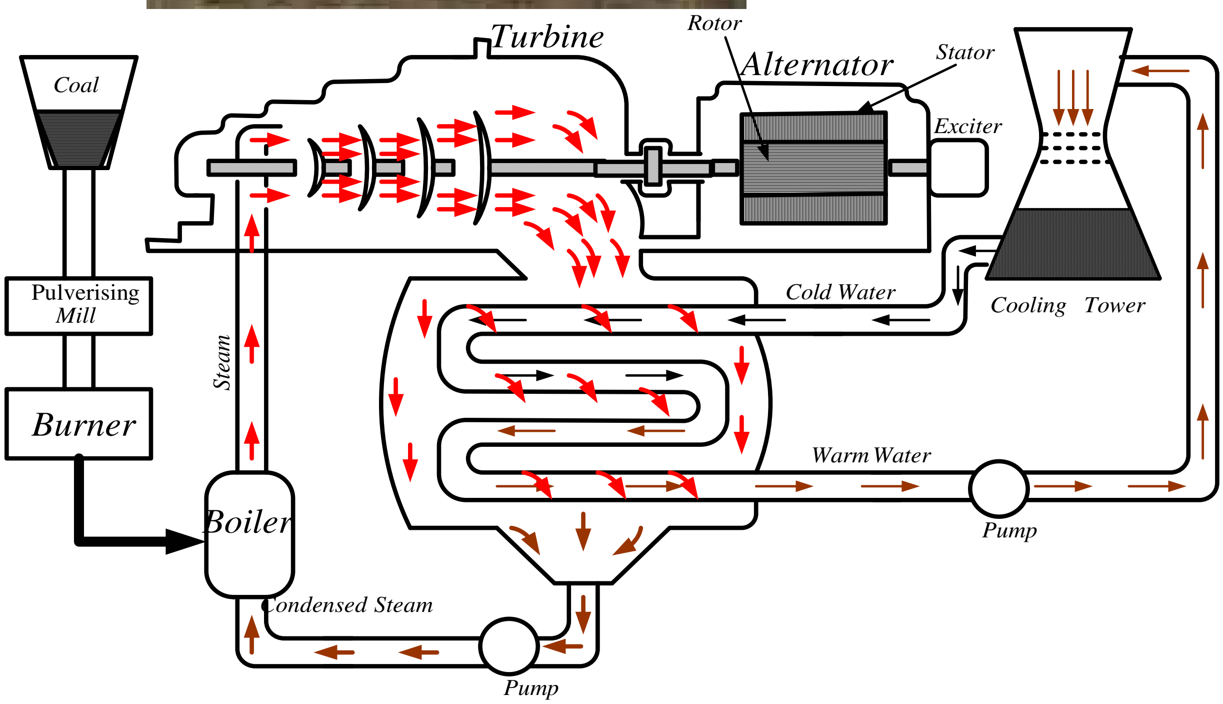
$$P_{total} = P_1 + P_2 + P_3$$

BLONDEL'S THEOREM

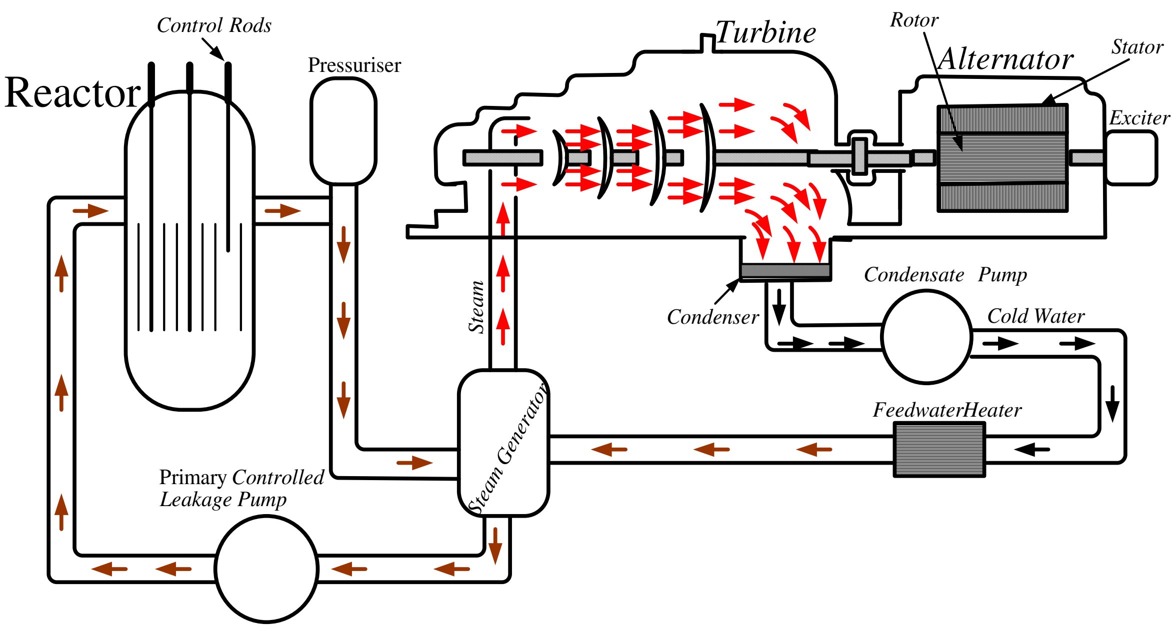
The theorem states that the minimum number of wattmeters required to measure the power in a polyphase system is one less than the number of wires carrying current in the system. For a three-phase four wire system, three wattmeters are required. For a three-phase three wire system, only two wattmeters are required.

10.0SOURCES OF ELECTRICAL ENERGY

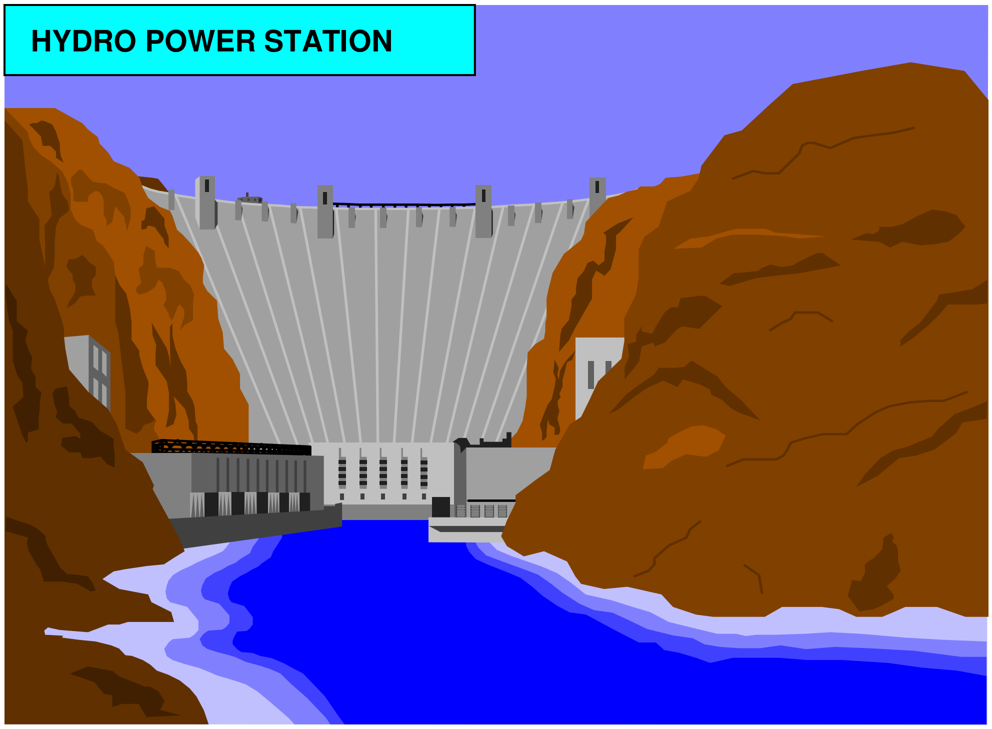
THERMAL POWER STATION

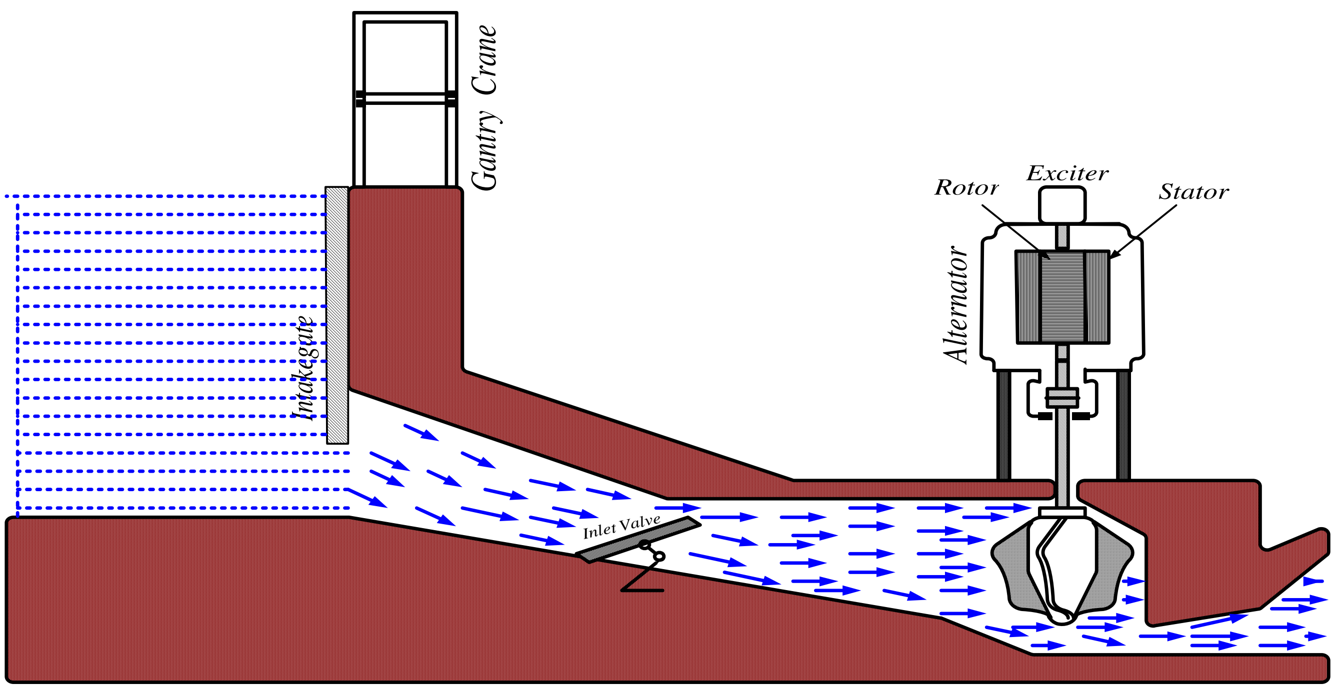


The coal is conveyed to a mill and crushed into fine powder i.e. pulverised. The pulverised fuel is blown into the boiler where it mixes with a supply of air for combustion. The combustion of coal produces steam at high temperatures and pressure which passed on to steam turbines and rotation occurs.



The basic reactor consists of the fuel in the form of rods or pellets of uranium dioxide in bundles of stainless steel tubes. Steam is produced at high temperatures and pressure which is passed on to steam turbines and rotation occurs. The steam production efficiency in nuclear power plants much higher than in coal fired power plants.

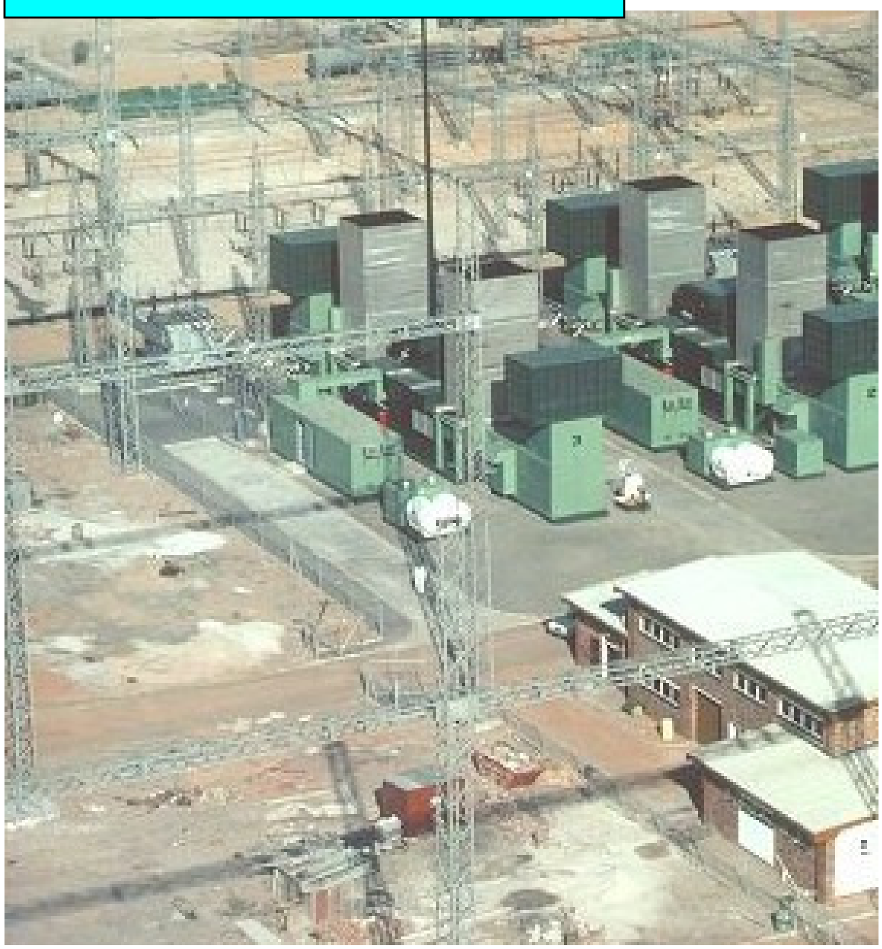


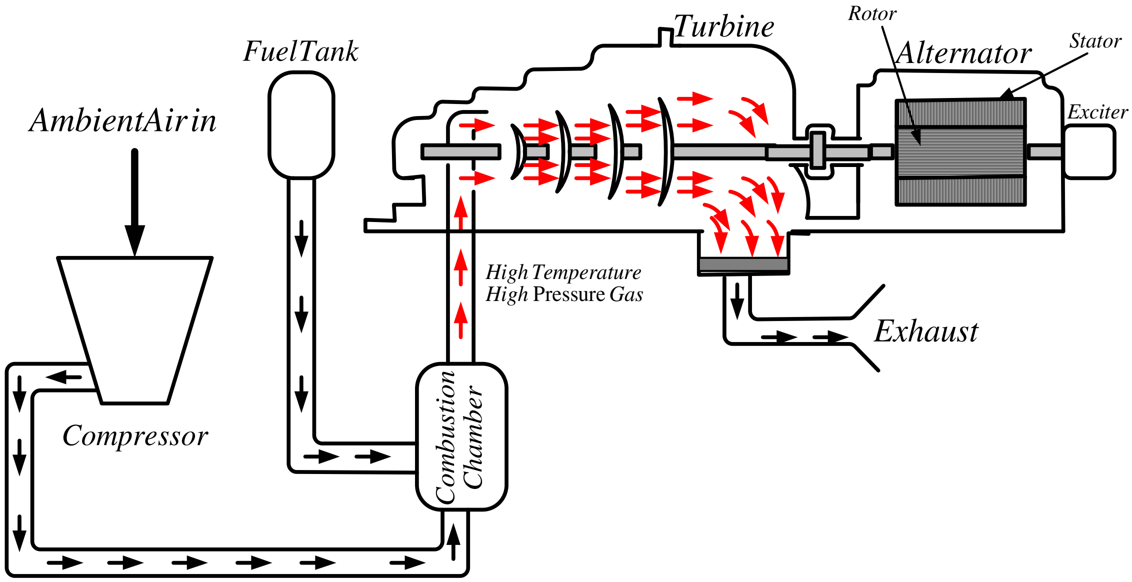


The hydro-electric station is the oldest form of energy conversion and the source of energy is free of cost. This attractive feature is offset by the very high capital cost of civil engineering construction works. Unfortunately, the geographical conditions necessary for hydro-generation are not commonly found, especially in Britain. An alternative to the conventional hydro-electric power stations is the pumped storage stations.

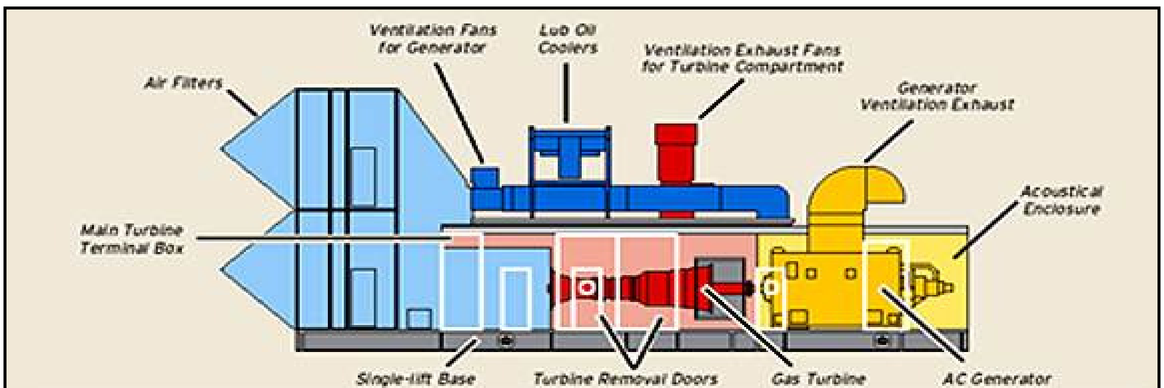
PUMP STORAGE SCHEME

GAS TURBINE POWER STATION





The gas turbine alternator (GTA) is less economical to operate for normal running. Its main advantage lies in the ability to start and take up load very quickly. Hence the GTA comes into use as a method for dealing with the peaks of the system load. A further use for the GTA is as a synchronous compensator to assist with maintaining voltage levels. Even on economic grounds it is better to meet peak loads by starting up GTAs from cold in the order of two minutes than running spare steam plant continuously.
The installation consists of a turbine, a combustion chamber and a compressor. The compressed air is delivered to the combustion chamber where continuous combustion of injected fuel oil is maintained; the hot gases then drive the turbine.

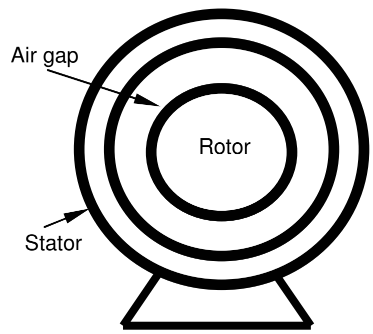


Base Plate Length	71' 0"	(21.6m)	Overall Length	78' 6"	(23.9 m)
Base Plate Width	13' 7"	(4.2 m)	Overall Width*	21' 2"	(6.2 m)
Enclosure Height	13' 6"	(4.1 m)	Overall Height*	34' 0"	(10.4 m)
* Includes air filter			Base Plate Foundation Load*	420,000 lb	(190,900 kg)

	Power kW	Heat Rate Btu/kWh LHV	Heat Rate kJ/kWh LHV	No. Shafts	Pressure Ratio	Shaft Speed rpm	Exhaust Flow lb/s	Exhaust Flow kg/s	Exhaust Temp. °F	Exhaust Temp. °C
LM2500 60 Hz										
LM2500PE	22719	9311	9789	2	19.1	3600	153	69.4	992	533
LM2500 50 Hz										
LM2500PE	21719	9653	10141	2	18	3000	154	69.8	1000	538

11.0 THREE-PHASE INDUCTION MOTORS

Induction motors have two major mechanical members; the stationary member called the stator and the rotating member known as the rotor. The two members are separated by a small air gap clearance to avoid mechanical rubbing. The three-phase supply is connected to the stator. The three-phase supply sets a synchronously rotating magnetic field in the air gap and field induces currents in the rotor provided the rotor speed is less than the synchronous speed of the rotating air gap magnetic field. The stator can therefore be referred to as the primary circuit and the rotor as the secondary circuit.



The rotating air gap magnetic field induces a voltage in the rotor by transformer action. Since the rotor windings are short circuited, the induced voltage in the rotor produces rotor currents which cause rotor rotation.

THREE-PHASE INDUCTION MOTOR/THREE-PHASE TRANSFORMER ANALOGY

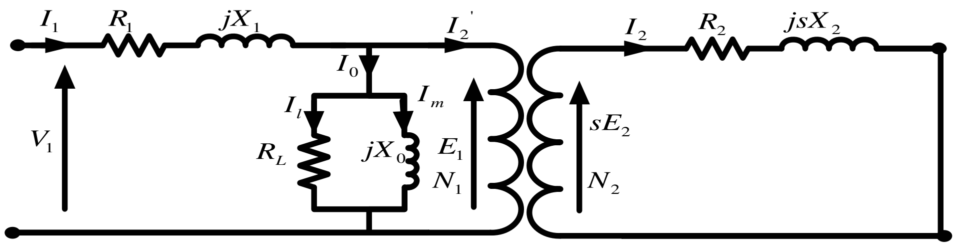
Similarities

- Both have two coupled windings (primary and secondary)
- Both have similar losses (copper and iron losses)

Differences

- Induced voltage in the motor is speed dependant
- Poor coupling in the motor air gap causes higher losses compared to transformer losses
- Motor secondary reactance is speed dependant

EQUIVALENT CIRCUIT



Equivalent Circuit of a 3-phase induction motor

V_1= Stator supply voltage
I_1 = Stator supply current
E_1 = primary induced e.m.f.
I_2 = Rotor current

sE_2 = Rotor induced e.m.f. where $s = \frac{n_s - n}{n_s}$

s = Slip
n_s *= Synchronous speed*
n *= Rotor speed*
N_1 = Stator winding
N_2 = Rotor winding
I_0 = Motor no-load current (magnetizing current and iron losses)
I_l = current that accounts for iron losses
I_m= magnetizing current
R_L = resistance to account for iron losses
R_1 =Stator winding resistance
R_2 =Rotor winding resistance
X_1 = Stator leakage reactance
X_2 = Rotor leakage reactance
$Z_1 = R_1+jsX_1$ = Stator leakage impedance
$Z_2 = R_2+jsX_2$ = Rotor leakage impedance

SLIP (s)
When a stator winding is excited from a balanced three-phase supply, the air gap magnetic field set up travels at synchronous speed. The rotor speed is slightly less than the synchronous speed so that the rotor runs with a per unit slip 's' defined as $s = \frac{n_s - n}{n_s}$ and $n_s = \frac{f}{p}$.

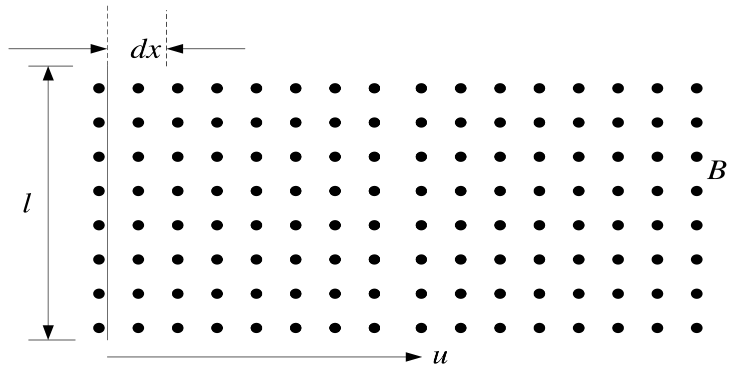
f = *supply frequency and* p = *number of pole pairs*

$$s = \frac{n_s - n}{n_s} \Rightarrow sn_s = n_s - n \Rightarrow n = (n_s - sn_s) \Rightarrow n = n_s(1-s)$$

The frequency of the rotor e.m.f. and current is proportional to the difference in speed between the rotating field and the rotor $\left(f_{rotor} = p(n_s - n)\right)$.

$$\therefore \frac{f_{rotor}}{p} = (n_s - n) \Rightarrow \frac{f_{rotor}}{p} = sn_s \Rightarrow f_{rotor} = spn_s \Rightarrow f_{rotor} = sf$$

PRINCIPLE OF OPERATION



B = Air gap magnetic flux density
l = Conductor length
u = Conductor linear speed
dx = Conductor linear displacement

A rotor conductor with length 'l' moving at a linear speed 'u' in the magnetic flux density B, the conductor 'l', linear speed 'u' and the flux density B will be mutually perpendicular.

By Faraday's law $v = N\frac{\partial\phi}{\partial t} + Nu\frac{\partial\phi}{\partial x}$

where N *= Number of turns*
v *= Induced voltage*
$N\frac{\partial\phi}{\partial t}$ *= Stator-rotor flux linking (Transformer action)*
$Nu\frac{\partial\phi}{\partial x}$ *= Rotor flux cutting*

For N=1, induced voltage by flux cutting $v = u\frac{d\phi}{dt}$ *and* $d\phi = Bldx$

$$\therefore v = uBl\frac{dx}{dx} = Blu \Rightarrow v = Blu$$

The rotor conductor carries a current 'i' produced as a result of transformer action between the stator and the rotor windings. The conductor experiences a mechanical force when a voltage is induced due to flux cutting.

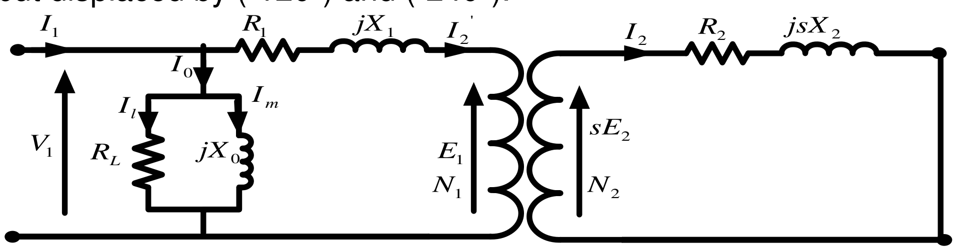
Instantaneous power $p = vi = F_m u \Rightarrow F_m = \frac{vi}{u} = Bli$

F_m is the mechanical force.

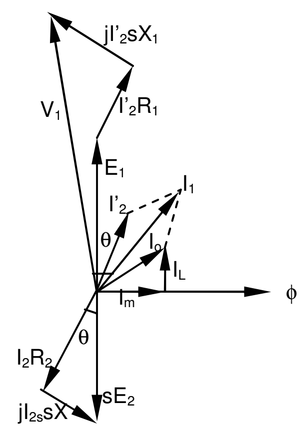
The rotor conductor experiences a force F_m and the rotor rotates. The rotation reduces the rate of change of flux and so both the magnitude and frequency of the induced voltage are reduced in proportion i.e. sf and sE_2.

ELECTRCAL/MECHANICAL CONVERSION OF POWER

A balanced three-phase supply is assumed and therefore it is adequate to consider the reference phase only. The other two phases being the same but displaced by (-120º) and (-240º).



Approximate Equivalent Circuit of a 3-phase induction motor



Phasor Diagram

The input power to the stator, $P_{stator} = V_1 I_1 \cos\theta_1$

The power losses in the stator are, $P_{SL} = I_2^{/2} R_1 + I_L^2 R_L$

where $I_2^{/2} R_1$ *= Copper losses and* $I_L^2 R_L$ *= Iron losses.*

The power losses in the rotor, $P_{RL} = I_2^2 R_2$

Available air gap power $P_g = E_1 I'_2 \cos\theta_2 = sE_2 I_2 \cos\theta_2$

Where $E_1 I'_2 = SE_2 I_2$ *from turns ratio. i.e.* $\frac{E_1}{sE_2} = \frac{N_1}{N_2} = \frac{I_2}{I_2'}$

For $N_1 = N_2; \Rightarrow E_1 = sE_2 \Rightarrow I_2' = I_2$

GROSS MECHANICAL POWER OUTPUT (P_m)

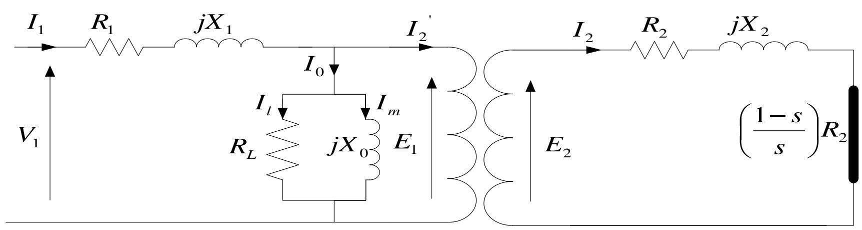
At starting, $s = 1$ hence $P_g = E_2 I_2 \cos\theta_2$. When the rotor begins to rotate, $P_g = sE_2 I_2 \cos\theta_2$. From stand still to rotation, there will be a loss of $E_2 I_2 \cos\theta_2 - sE_2 I_2 \cos\theta_2 = (1-s)E_2 I_2 \cos\theta_2$.

Using the equivalent circuit,

$$I_2 = \frac{sE_2}{R_2 + jsX_2} = \frac{E_2}{\frac{R_2}{s} + jX_2}$$

$$\frac{R_2}{s} = \frac{R_2}{s} - R_2 + R_2 = \left(\frac{R_2}{s} - R_2\right) + R_2 = \left(\frac{R_2 - sR_2}{s}\right) + R_2 = R_2\left(\frac{1-s}{s}\right) + R_2$$

$$\therefore \frac{R_2}{s} = R_2\left(\frac{1-s}{s}\right) + R_2$$



Equivalent Circuit of a 3-phase induction motor

$\left(\frac{1-s}{s}\right)R_2$ is a fictitious resistance which accounts for power loss $(1-s)E_2 I_2 \cos\theta_2$. Hence the mechanical output power produced when the rotor begins to rotate is:

$$P_m = I_2^{\,2} R_2\left(\frac{1-s}{s}\right) \Rightarrow P_m = (1-s)E_2 I_2 \cos\theta_2 = (1-s)P_g$$

. $P_g = E_2 I_2 \cos\theta_2$ *at starting.*

POWER FLOW AND LOSSES

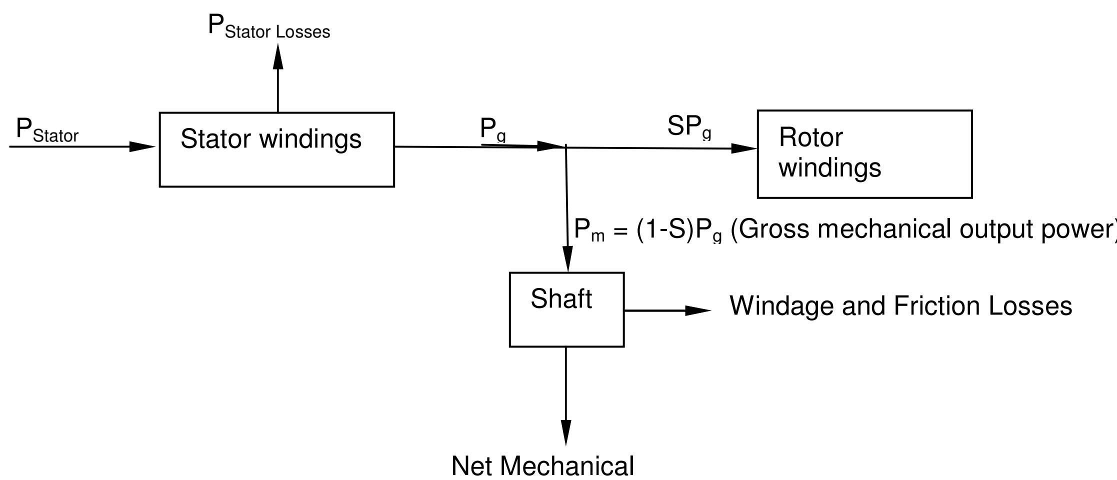
$$P_g = P_{stator} - P_{statorlosses}$$

$$P_g = P_{Rotorlosses} + P_m = I_2^{\,2} R_2 + I_2^{\,2} R_2 \left(\frac{1-s}{s} \right) = \frac{I_2^{\,2} R_2}{s}$$

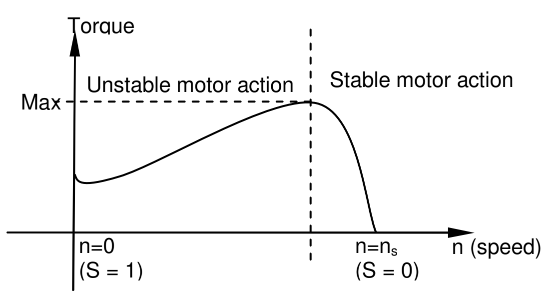
$$P_{Rotorlosses} = sP_g$$

$$P_m = I_2^{\,2} R_2 \left(\frac{1-s}{s} \right) = \left(\frac{1-s}{s} \right) P_{Rotorlosse} = sP_g \left(\frac{1-s}{s} \right) - (1-s) P_g$$

$$P_g = sP_g + (1-s) P_g$$



TORQUE/SPEED CHARACTERISTICS



TESTS FOR EQUIVALENT CIRCUIT PARAMETERS

No-load tests (S = 0 and n = n_s)

The induction motor is allowed to run without load. Small current is induced to overcome windage and friction losses. Hence the slip 'S' is small and approximately equal to zero. The on no-load consists mainly of core losses (iron losses) hence the readings or data obtained gives the value of the shunt circuit. Sometimes it is necessary to separate windage and friction losses. This test is similar to open circuit test for power transformers.

Locked rotor tests (S = 1 and n = 0)

With a locked rotor, the stator supply is adjusted to reach the motor rated current. The test is equivalent to short circuit test for power transformers. The shunt losses can be ignored and the readings or data obtained gives the value of the copper losses.

Resistance test

The stator and rotor resistance can be measured directly using a suitable ohmmeter.

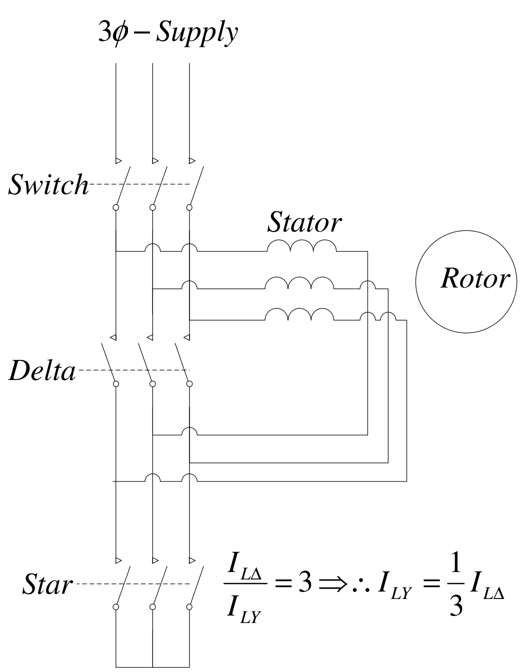
STARTING OF INDUCTION MOTORS

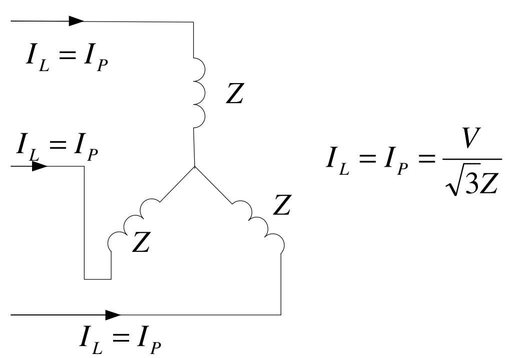
Direct On Line

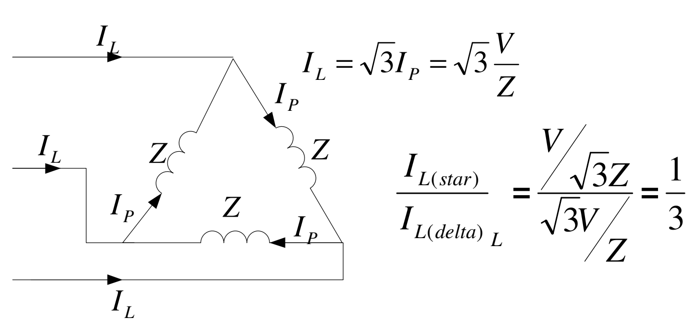
This method is used only for small motors of 2.0kW or less. The motor normally takes five to six times the full-load current at low power factor.

Star-Delta Starting

The motor is started in star and then switched to delta connection when the speed is within 10% of its full-load speed. The method is cheap and the starting current and starting torque are reduced by $\frac{1}{3}$.

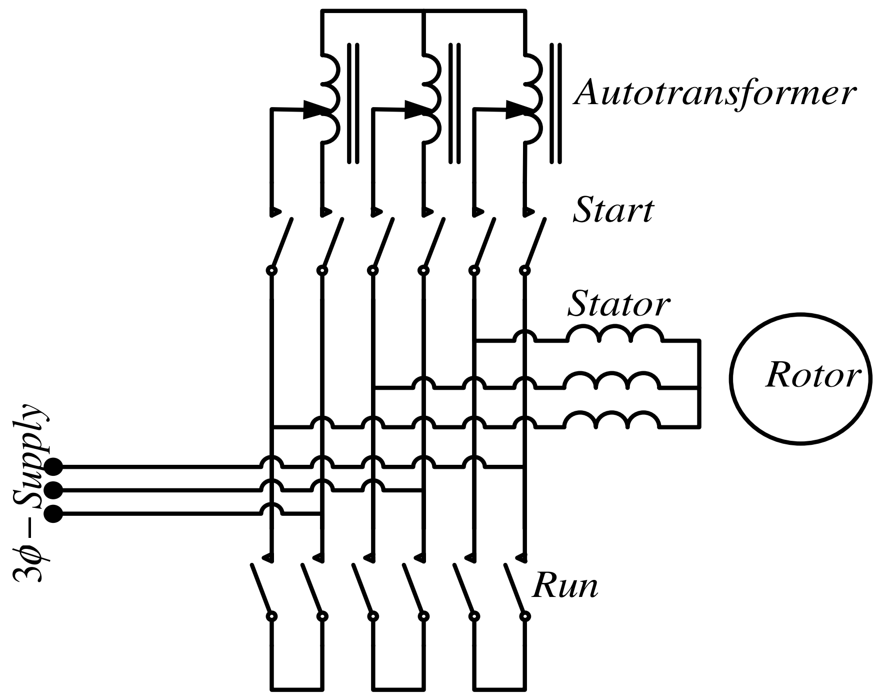






Autotransformer Starting

The induction motor is started using an auto transformer and then switched directly to the mains when the motor has run up to almost its rated speed. The autotransformer reduces the starting voltage between 40% to 75% of the line voltage. The reduced voltage reduces the starting current and torque.

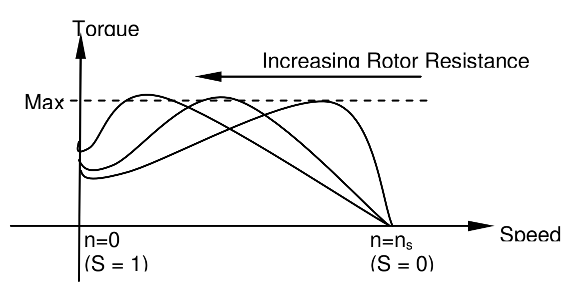


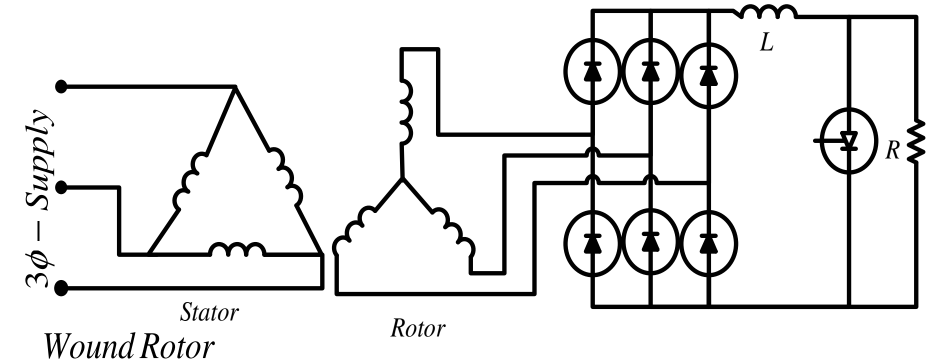
Autotransformer Starting

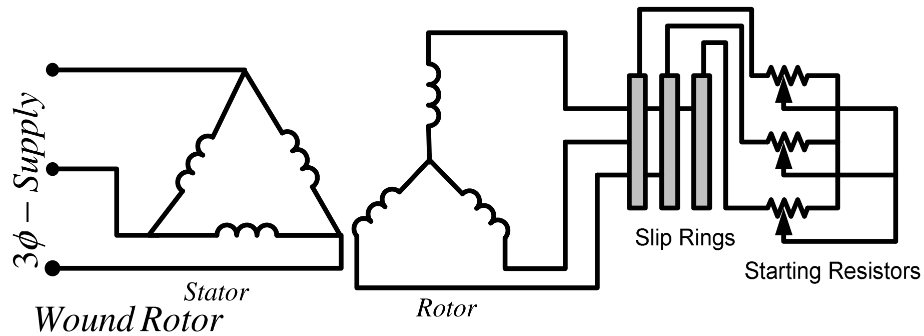
Wound Rotor Machine Starting

The slip ring induction motors are started by means of external resistors connected through the slip rings to the rotor circuit. The motor is started with all the resistors in circuit giving a high starting torque. As the motor runs up to its rated speed, the external rotor resistance is reduced until the motor attains full speed and runs with no external resistance at full speed.

SLIP/TORQUE CURVES FOR ROTOR RESISTANCE STARTING

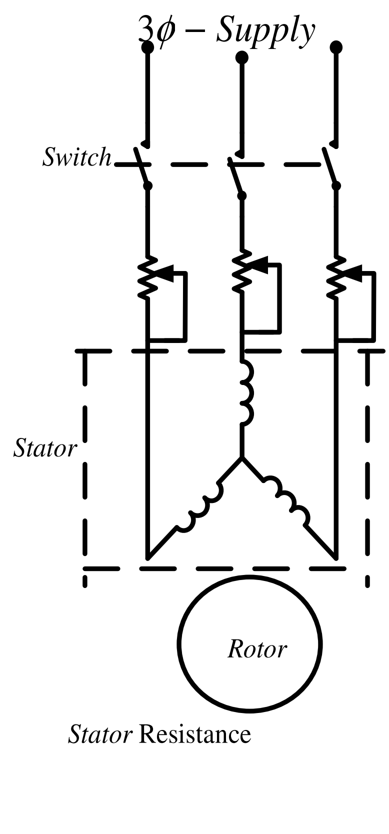






Stator resistance Starting

The stator resistors are used to limit the starting current in the stator and also limits the initial starting torque. The shock to the motor is thus reduced. The method has the main disadvantages of I^2R losses as heat on starting.

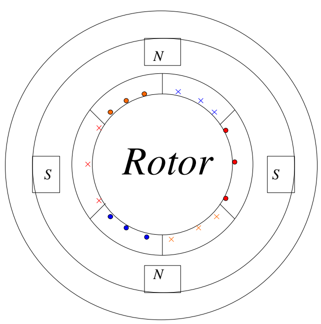


12.0 SYNCHRONOUS MACHINES

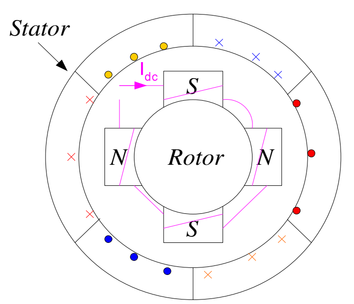
A synchronous machine is an a.c. machine in which the rotor moves at a constant speed which is proportional to the frequency of the current in the armature winding. For the synchronous motor, the rotor speed remains constant irrespective of the load, provided the supply frequency remains constant. For a generator, the rotor speed must be maintained at a constant in order to produce a constant frequency on the output. The field of a synchronous machine is a steady one. For small machine, the field is produced by permanent magnets and for large machines the field is excited by a d.c. current.

Armature on the Rotor

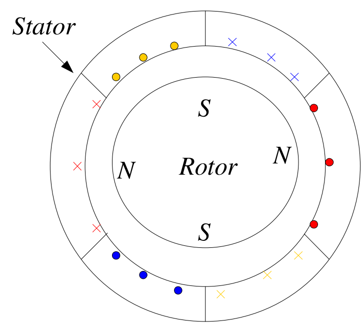
The stator carries a salient pole field winding excited by a direct current. The output a.c. voltage is obtained through the brush riggings on the rotor. The difficulties of passing relatively large current at high voltage across the moving contacts have made the stator wound armature the choice for large machines.



Armature On Rotor



Salient Pole Rotor



Cylindrica l Rotor

Salient pole rotor

The salient pole machine has concentrated field windings and its speed is limited to take into account centrifugal forces. Salient pole alternators are generally used where a prime mover is a water turbine or a reciprocating engine of about1,500 rev/min

Cylindrical rotor

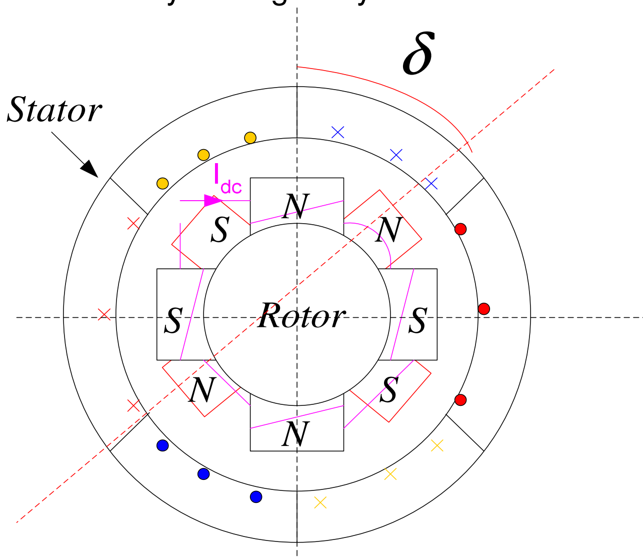
The cylindrical rotor has a distributed winding in the rotor slots and this makes them suitable for high speed operations. They are generally used in high speed machine operations of about 3000 revolutions per minute.

TORQUE

At synchronous speed, the rotating magnetic field on the rotor established by d.c. supply on the rotor travels at the same speed as the field created by the armature current on the stator thereby creating a steady torque. At starting, the synchronous motor produces an alternating torque on the rotor and the net torque is zero.

LOAD ANGLE

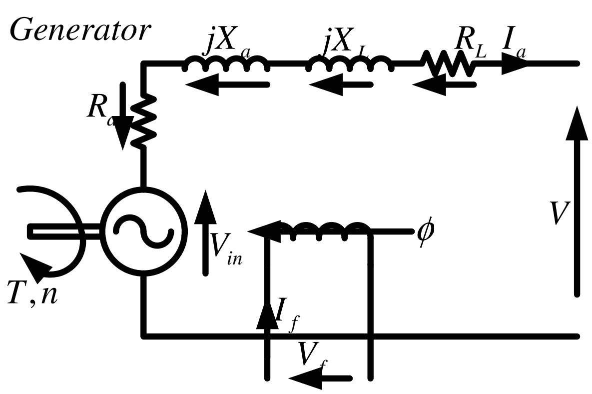
For no-loads, the rotor poles axis and the armature are coincidental. On loads, the rotor is retarded by an angle say ‘δ’ known as the load angle.

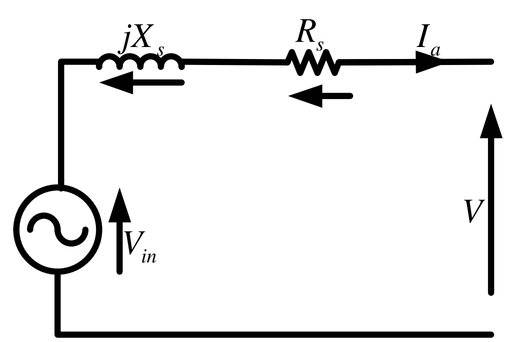


ARMATURE REACTION

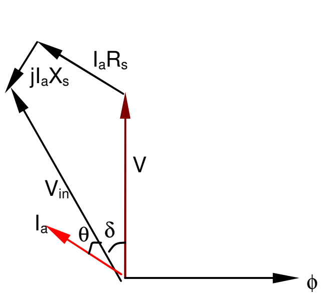
Maximum voltage will be induced when the centre lines on the rotor magnet and the armature coincide. The armature reaction has the effect on the operation of a synchronous machine with respect to both the power factor and at which it operates and the amount of excitation that it needs. At unity power factor, the current in the stator produces an m.m.f. which interacts with that of the rotor resulting into a distortion of flux across the pole faces. This weakens the flux. The m.m.f. arising from the current flowing in the stator is known as the armature reaction.

SYNCHRONOUS MACHINE EQUIVALENT CIRCUITS AND PHASOR DIAGRAMS

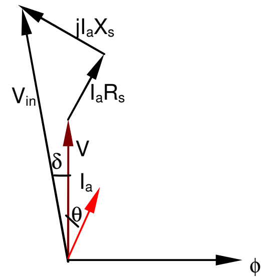




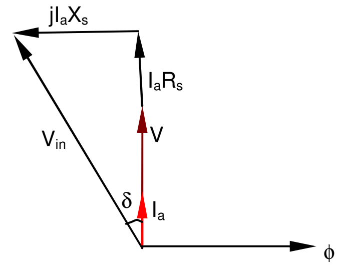
$$V_{in} = V + I_a(R_s + jX_s)$$



Phasor Diagram at Leading pf

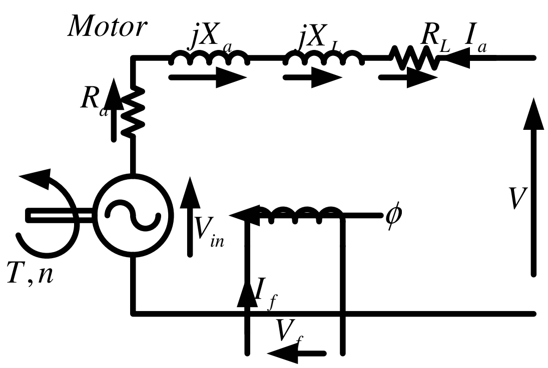


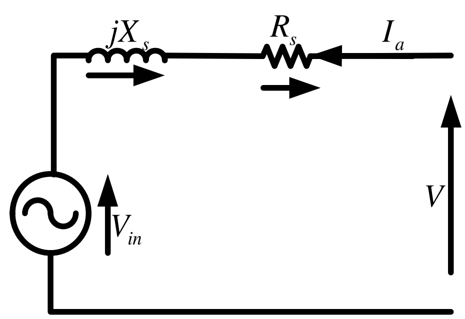
Phasor Diagram at Lagging pf



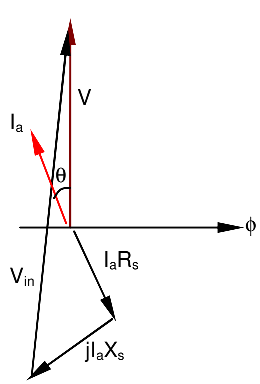
Phasor Diagram at Unity pf

V_{in} = Induced voltage in the armature
V = Armature output voltage
V_f = Excitation voltage or field supply voltage
I_a = Armature current
I_f = Field current or excitation current
$R_s = R_L + R_a$ = Synchronous Resistance
R_a = Armature Resistance
R_L = Resistance which accounts for I^2R losses in the armature
$X_s = X_L + X_a$ = Synchronous Reactance
X_a = Armature Reactance
X_L = Reactance which accounts for leakage inductance
$Z_s = R_s + jX_s$ = Synchronous Impedance

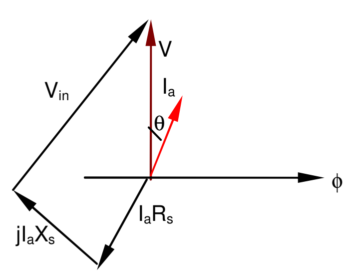




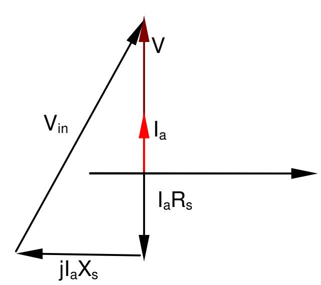
$$V = V_{in} + I_a\left(R_s + jX_s\right)$$



Phasor Diagram at Leading pf



Phasor Diagram at Lagging pf



Phasor Diagram at Unity pf

ALTERNATOR VOLTAGE REGULATION

The voltage regulation of an alternator is a fractional change of voltage associated with some stated change of load. It is a rise in terminal voltage when a given load is thrown off.

$$\text{Regulation}=\frac{|V_{in}| \quad |V|}{|V|}$$

SYNCHRONOUS MACHINE OUTPUT VOLTAGE

The output voltage is proportional to the excitation current. The voltage increases with an increased field current or excitation current.

$$V = 2.22\phi f Z_s K \text{ and } Toque \;=\; \frac{Power}{2\pi n_s}.$$

Where V = Output voltage

ϕ = Magnetic flux due to current excitation

f = Frequency

Z_s = Conductors in series

K = Distribution factor of the winding

STARTING OF SYNCHRONOUS MOTORS

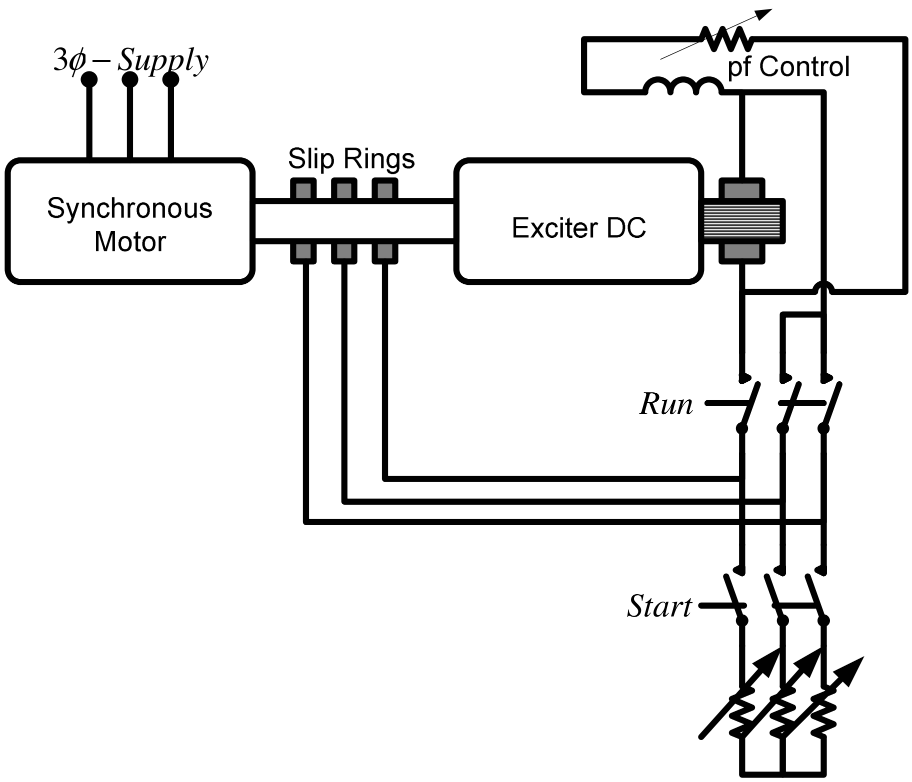
The synchronous motor has no starting torque at constant frequency.

Variable Frequency Method

The stator supply frequency is reduced to reduce the speed of the stator magnetic field to allow low enough value that the rotor can accelerate and lock in during one half cycle of the magnetic field's rotation. The machine is then capable of developing full load torque through out the speed range as the frequency is increased to rated value.

Synchronous Induction Motor Action

The synchronous motor is accelerated from stand still by an induction motor type action and then switched to an exciter. The motor locks in and reaches synchronous speed.

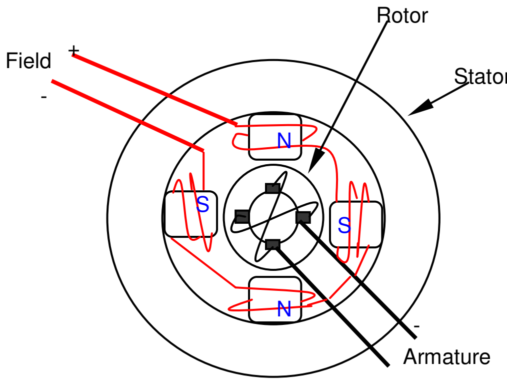


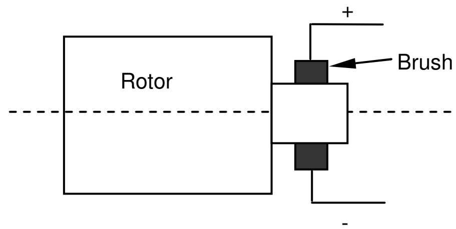
SYNCHRONISING

The method of connecting an incoming alternator to a live busbar is called synchronising. In modern power stations, alternators are synchronised automatically and synchronising circuits allow synchronisation under the following conditions:

- The frequency of the induced voltage in the on coming machine must equal the frequency of the live busbar voltage
- The induced voltage in the on coming machine must equal the live busbar voltage in magnitude and phase.
- The phase sequence of the live busbar voltage and the induced voltage in the on coming machine must be the same.

13.0 DIRECT CURRENT (DC) MACHINES





13.1 DC Generator

The rotor is connected to a prime mover and the excitation provided by a dc source on the stator to form the N-S poles. If a single conductor on the rotor (armature) is considered and imagined stretched out, the output through the commutators (brushes) will be a rippled positive dc for all North poles and rippled negative for all South poles.

Armature Winding

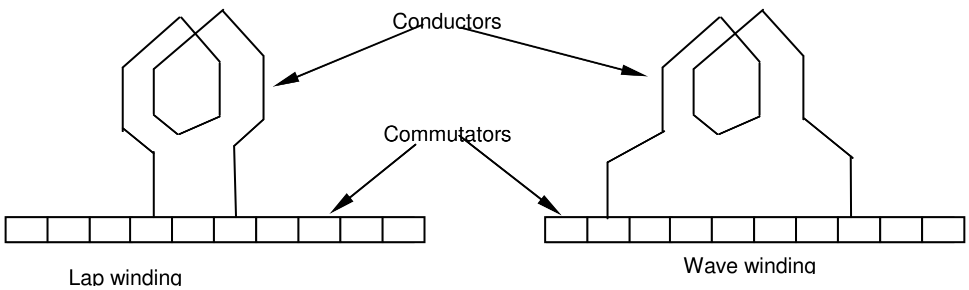
The armature windings can be divided into two groups depending upon the manner in which the wires are joined to the commutator.

Lap winding

The two ends of any one coil or winding in the rotor slots go round North to South without taking the same route. The number of brushes equals the number of poles ‘p’. The brushes with similar polarities are joined together and the number of paths in parallel ‘a’ equals the number of poles ‘p’. $a = p$.

Wave Winding

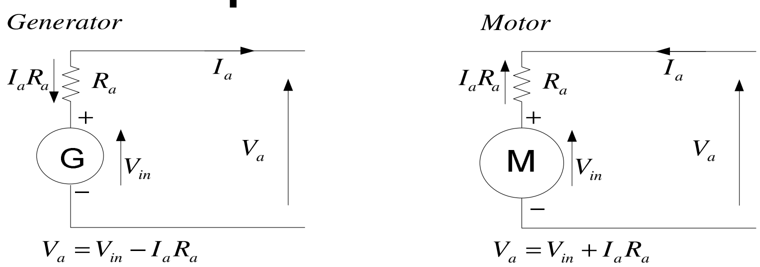
The two ends of each coil are bent in opposite direction and taken to segments some distance apart. The number of brushes may just equal to or sometimes to the number of poles ‘p’. However in all cases, the number of parallel paths ‘a’ equals to two. *a = 2.*



13.2 Induced Voltage in DC Machines

$V_{av} = \phi np$ for one conductor on the armature. If the whole armature has Z conductors in series, then the number of conductors in series is $\frac{Z}{a}$. Therefore the average induced voltage for the whole machine is $V_{av} = \frac{pZ}{a} n\phi$. i.e. $V_{in} = \frac{pZ}{a} n\phi$.

Circuit Representation Of Dc Machines



Real machine = Ideal machine + Losses

13.3 Losses

Mechanical Losses

- Windage losses: caused by friction between moving parts of the machine and air inside the motor casing. The losses vary with the cube of the speed of rotation of the machine.
- Friction losses: due to friction of the bearings in the machine

Core losses Or Iron Losses

The losses are due to hysteresis losses and eddy currents.

Stray Losses (Miscellaneous Losses)

The losses that cannot be accounted for.

Electrical Losses

- *Armature losses* = $I_a^2 R_a$ Where I_a = *armature current*
- *Field losses* = $I_f^2 R_f$

R_a = *armature resistance*

I_f = *field current*

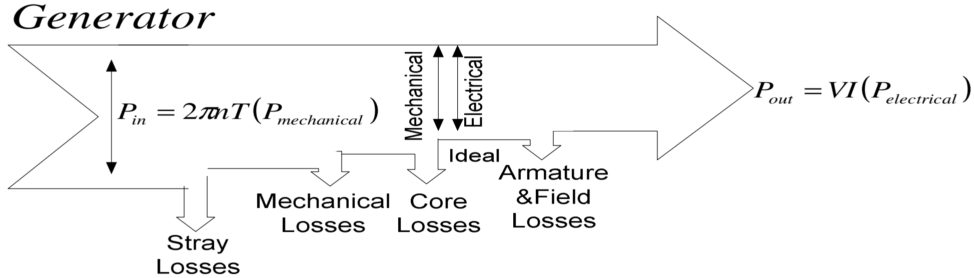
R_f = *field resistance*

Brush Losses

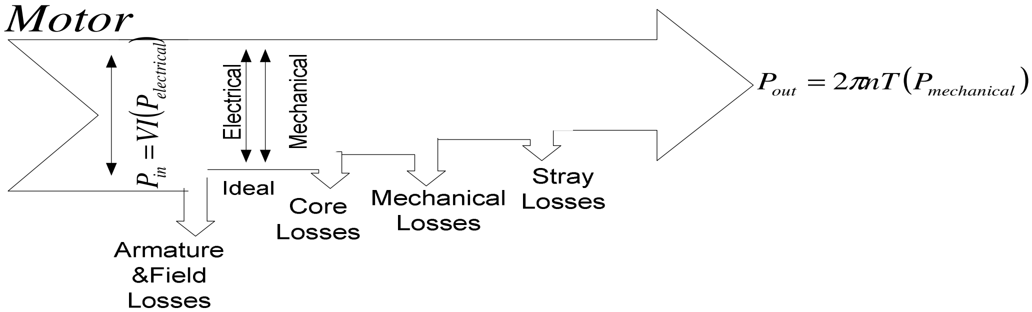
This is due to the drop in power lost across the contact potential and brushes of the machine.

13.4 POWER FLOW DIAGRAM

Generator



Motor



TORQUE

An ideal machine in which all losses are neglected will have all the mechanical energy converted to electrical for a generator and all the electrical energy converted to mechanical energy for a motor.

By conservation of energy; $2\pi nT = V_{in} I_a$

Where V_{in} = *Induced voltage*

I_a = *Armature current*

T = *Torque*

$2\pi nT$ = *Angular speed in rad/sec*

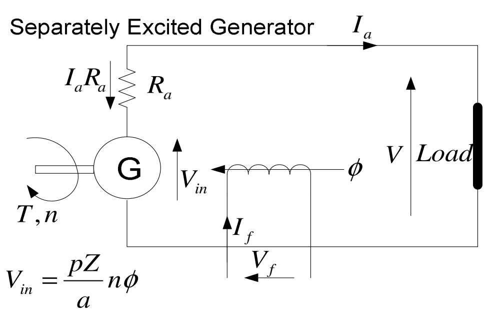
n = *Angular speed in rev/sec*

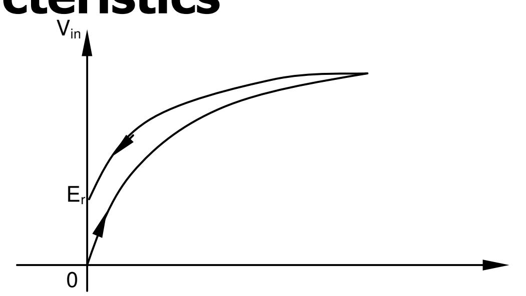
Also

$$V_{in} = \frac{pZ}{a} n\phi \Rightarrow 2\pi nT = V_{in} I_a \Rightarrow 2\pi nT = \frac{pZ}{a} n\phi I_a$$

$$\therefore T = \left(\frac{1}{2\pi}\right)\frac{pZ}{a}\phi I_a$$

13.5 DC Generator Characteristics



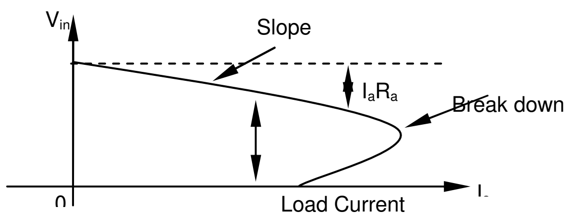


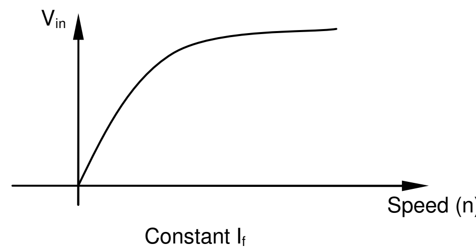
Open Circuit Voltage: (I_a = 0 and n = Constant

Open Circuit Voltage

The armature is driven at constant speed on no load and the excitation current I_f increased from zero up to the maximum possible value and then reduced to zero. The resulting difference in curves is due to hysteresis and residual magnetism E_r in the poles.

Load Test (Generator on Load)



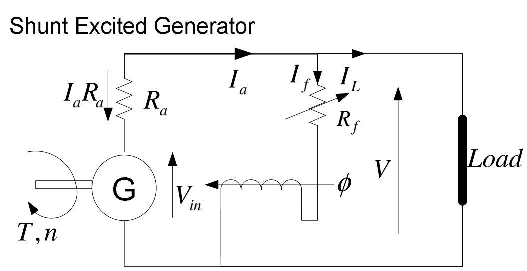


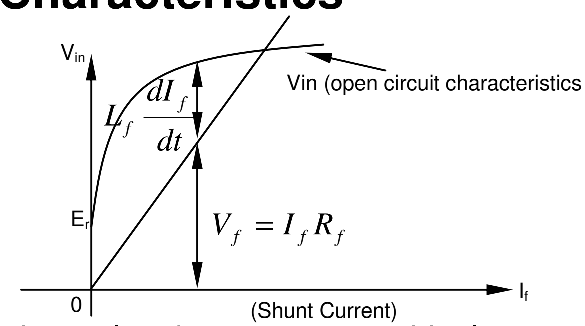
$$V_a = V_{in} - I_a R_a$$
$$Slope = \frac{dV_a}{dI_a} = 0 - R_a$$

For constant speed and constant excitation current If, $V_a = V_{in} - I_a R_a$; and $Slope = \frac{dV_a}{dI_a} = 0 - R_a$.

Increasing the load current I_a increases copper losses and voltage droop. Another factor that may cause a decrease in the terminal voltage is the decrease of flux as a result of demagnetising ampere-turns of the armature and magnetic saturation in the armature teeth due to flux distortion.

Shunt Excited Generator Characteristics

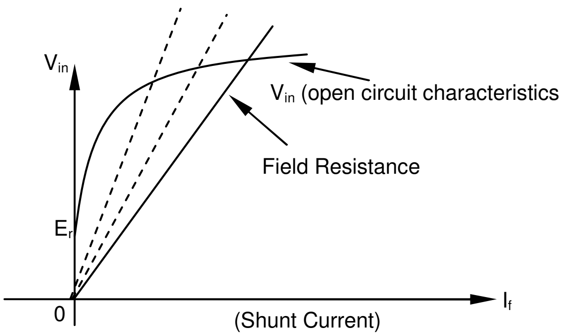


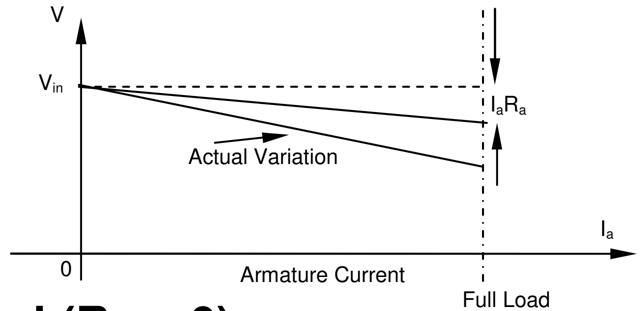


A shunt generator will excite only if the poles have some residual magnetism.

At start, $V_a = E_r = L_f \frac{dI_f}{dt} = V_{in}$ *where* $R_a \approx 0$.

Since $\frac{dI_f}{dt}$ is positive, the field current is increasing and also V_a increases. The E_r voltage which is V_{in}, circulates a shunt current until e.m.f. reaches maximum. $V_{in} = L_f \frac{dI_f}{dt} + I_f R_f$ and $L_f \frac{dI_f}{dt} = V_{in} - I_f R_f$ When I_f increases such that open circuit characteristic and the field resistance line intersect, then $L_f \frac{dI_f}{dt} = 0$. This happens when I_f becomes a constant giving $\frac{dI_f}{dt} = 0$. There will be no further increase in I_f. The $L_f \frac{dI_f}{dt}$ build up ceases and the point of intersection determines the no-load voltage of the shunt generator. The point of intersection can be adjusted using the field resistance.

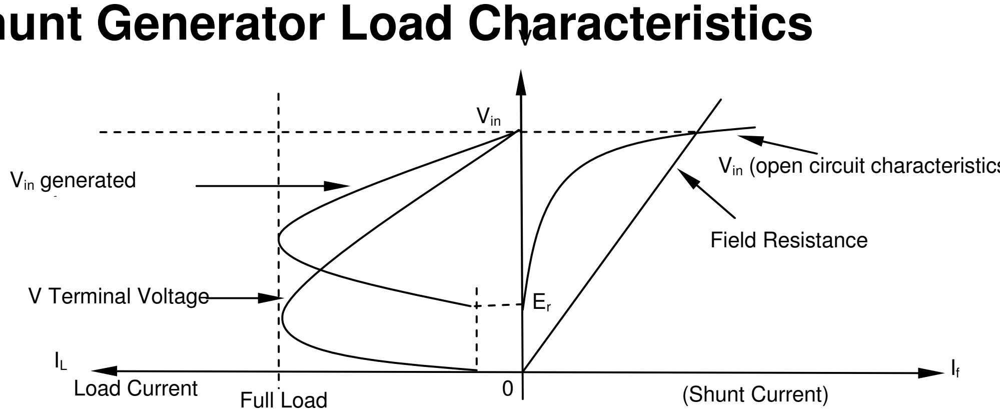




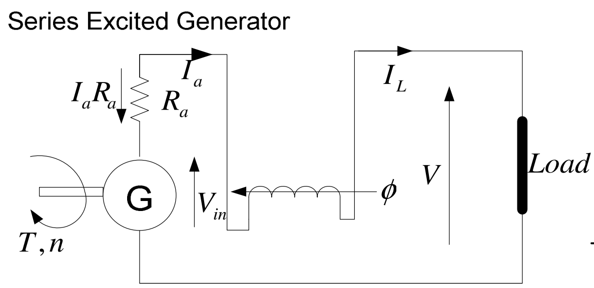
Shunt Generator On Load (Ra ≠ 0)

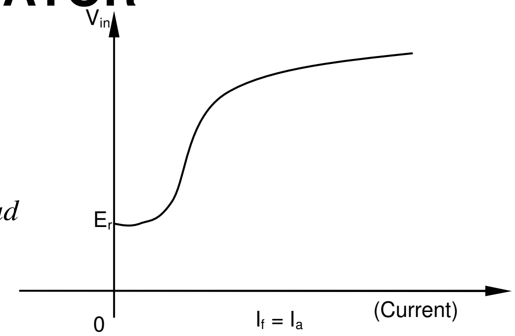
$V_a = V_{in} - I_a R_a$; an increase in I_a leads to higher $I_a R_a$ drop and this decreases the terminal voltage. $I_f = \frac{V_a}{R_f}$. The decrease in V_a reduces I_f and in turn reduces the generated voltage V_{in} since $V_{in} = \frac{pZ}{a} n\phi$ and $\phi \propto I_f$. Reduced V_{in} reduces V_a even further. Hence voltage regulation is generally poor for most applications. $\text{Re}\ gulation = \frac{|V_{in}| - |V_a|}{|V_a|}$

Shunt Generator Load Characteristics



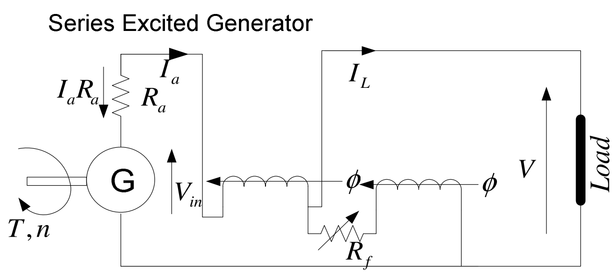
SERIES EXCITED GENERATOR

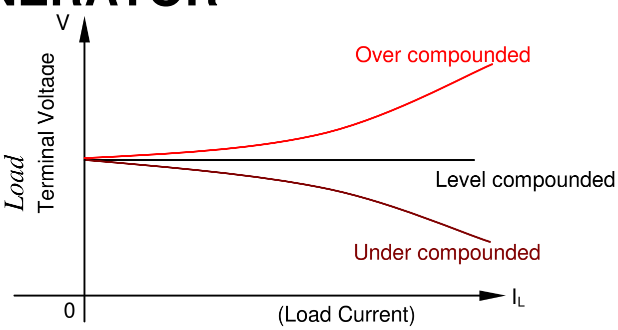




The armature current passes through the field winding, hence the winding consists of comparatively few turns of thick wires or strips. The series wound generator is quite unstable when voltage is to be maintained or even approximate constant over a wide range of load current.

COMPOUND WOUND GENERATOR





Load Characteristics of Compounded

Cumulative/Differential Compound Winding

The machine has a better regulation since load current passes through the series field. An increase in armature current I_a straightens the pole m.m.f. and hence increases the induced voltage V_{in}. The flux formed by the series and shunt field winding are cumulative if they add up or differential if they subtract.

Over Compounded Generator

If the increase in V_{in} is such that the terminal voltage V_a at full-load is greater than the no-load voltage (negative regulation), the generator is said to be over compounded. The increase in series turns increases the terminal voltage.

Under Compounded Generator

If the increase in V_{in} is not sufficient to overcome the IR drops, then positive regulation occurs and the machine is said to be under compounded.

Level or Flat Compounded

The increase in V_{in} exactly compensates for IR drop so that the terminal voltage V_a on full-load is the same as the no-load value. Such a machine is said to be level or flat compounded.

13.6 DC MOTORS

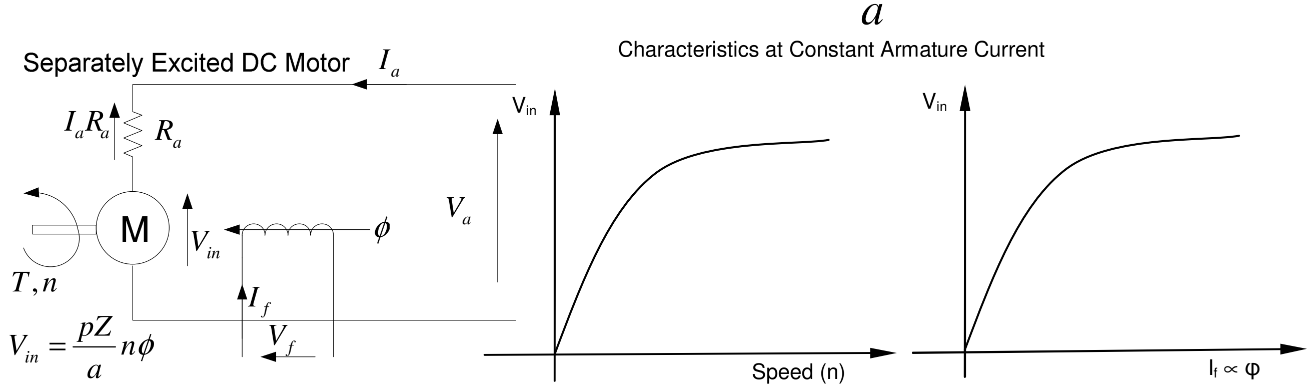
Toque in DC Motors

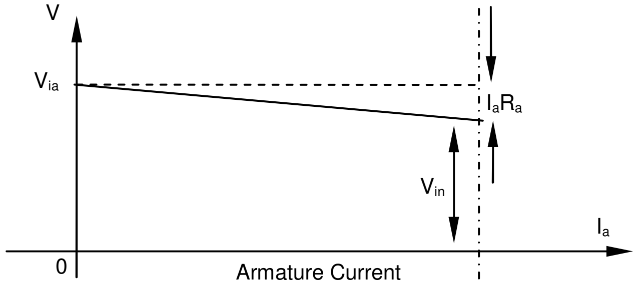
The field current in the stator produces a magnetic field which is proportional to the field current and the number of turns in the field winding. The armature current from the supply flows through the brush riggings on the rotor to the armature winding. This current also produces a magnetic field. The interaction between the 'field' magnetic field and the armature magnetic field, creates a twisting or rotating force called torque. When the armature of the motor starts to run, the armature windings cut the field of the magnetic field set up by the field current and the voltage is induced in the armature which is proportional the speed and the magnetic field. i.e. $V_{in} = \frac{pZ}{a} n\phi$. Reversing the one of these fields, reverses the motor torque which initially brakes and then the motor accelerates in the opposite direction. Reversing of the motor is achieved by reversing the direction of either the armature current or the field current.

Separately Excited DC Motor

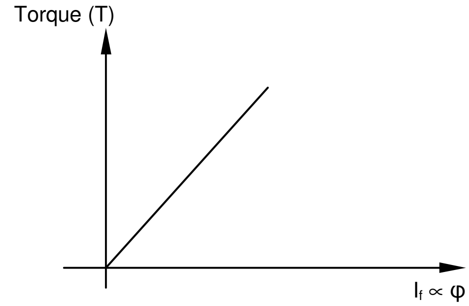
At constant armature current, the field current can be varied to vary the torque. Increasing the field current increases the torque which in turn reduces the motor speed.

$$V_{in} = V_a - I_a R_a \Rightarrow \frac{pZ}{a} n\phi = V_a - I_a R_a \Rightarrow n = \frac{V_a - I_a R}{\frac{pZ}{a}\phi}$$

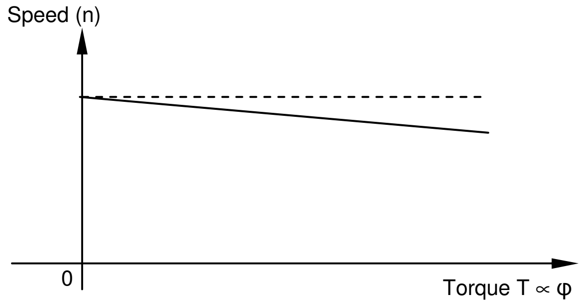


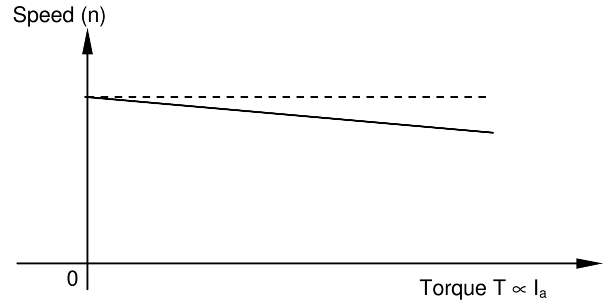


$$V_a = V_{in} + I_aR_a$$



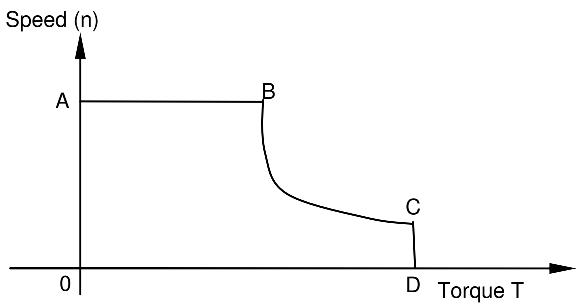
$$T = \left(\frac{1}{2\pi}\right)\frac{pZ}{a}\phi I_a$$

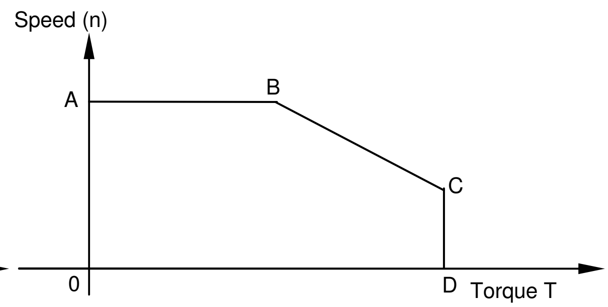




Controlled Armature and Field Current Supply

When a control system is added to the armature, the armature voltage can be varied. The field voltage or current can also be varied using a suitable control system such as thyristors (SCRs). Using the armature and field controls, the motion characteristic can have any shape.





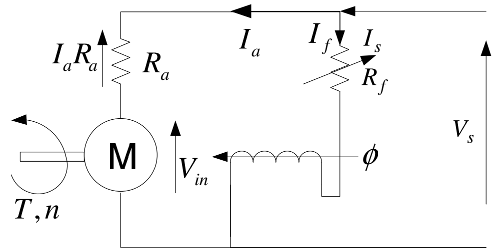
AB = Constant speed with increasing torque. The control is the armature current at constant field supply current. $T = \left(\frac{1}{2\pi}\right)\frac{pZ}{a}\phi I_a$. Increasing the armature current increases the torque.

BC = Variable speed and torque. The controls are both the armature and field currents. $T = \left(\frac{1}{2\pi}\right)\frac{pZ}{a}\phi I_a$ (Vary I_a to vary torque).

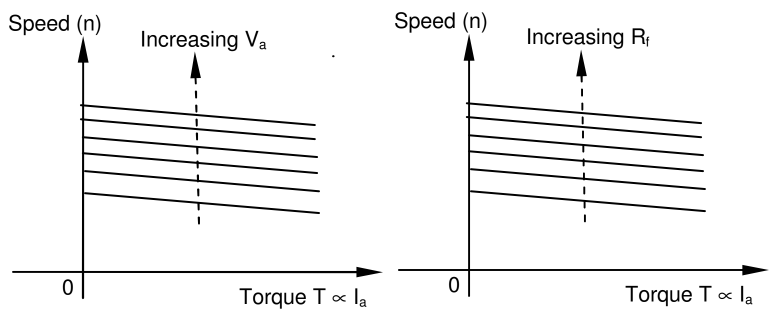
$n = \dfrac{V_a - I_aR}{\frac{pZ}{a}\phi}$ (Vary I_f to vary speed).

CD = Constant torque with decreasing speed. The control is the field current at constant armature supply current. Field current variation may affect the torque. Increasing the armature current and reducing the field current provides a constant torque until brakes are applied.

DC SHUNT MOTOR



Controlled Armature and Field Supplies



$$V_a = V_s = \frac{pZ}{a} n\phi + I_a R_a$$

Constant Supply Voltage

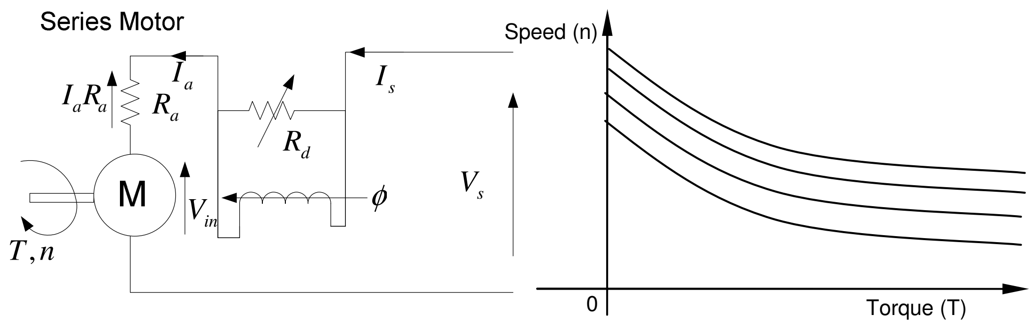
The field of a shunt motor is in parallel with the armature. For a constant supply voltage V_s, the field current will be a constant on a constant field resistance. $I_f = \frac{V_a}{R_f}$ = *Constant at Constant V_s.*

At constant supply voltage and constant field resistance, the speed will be a constant. Hence shunt motors are classified as constant speed motors.

Torque-Speed Curves

The family of torque-speed curves are flexible with the use of field resistance. The disadvantage is that as the field current decreases, the flux also decreases and the available torque is reduced. It is preferred to vary the armature voltage or supply voltage which although expensive, the flux is maintained by simultaneously adjusting the field resistance to give a constant field supply.

DC SERIES MOTOR

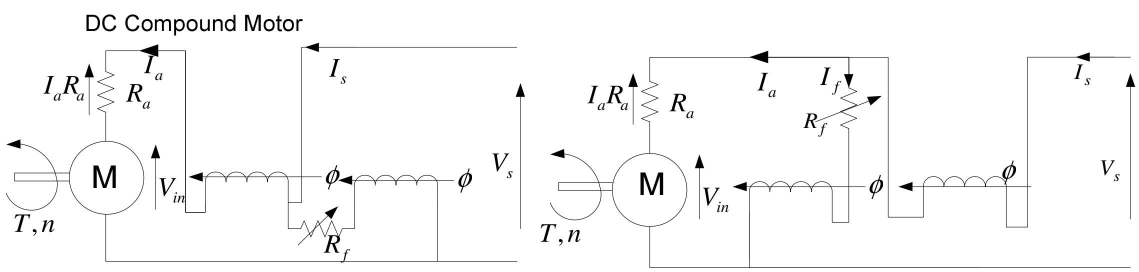


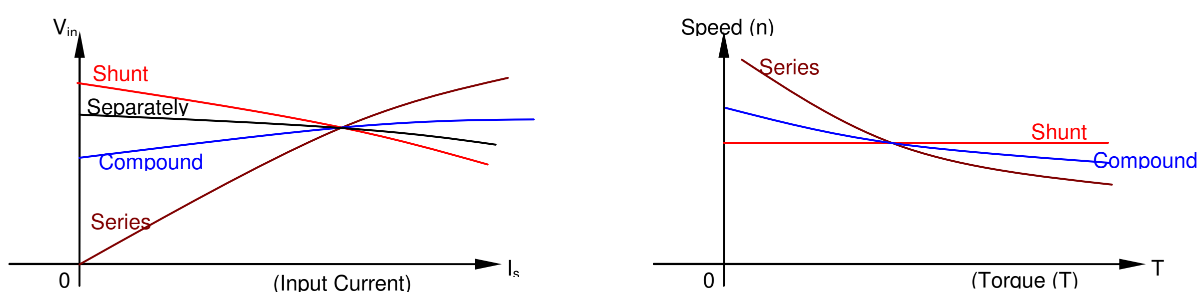
The field winding carries full-load current. Therefore the field conductor is larger than the armature conductor;

$$R_f << R_a \Rightarrow \therefore V_s = I_a\left(R_f + R_a\right) + V_{in} = I_a R_a + \frac{pZ}{a} n\phi \, .$$

Sometimes a diverter resistor R_d in parallel with the field circuit is employed so that a family of curves can be obtained. The supply voltage is varied for controls and the field current maintained using a diverter resistor. A series motor must always be connected to a mechanical load before starting or else a dangerous high speed will develop.

DC CMPOUND MOTORS





Load Characteristics of Compounded Motor

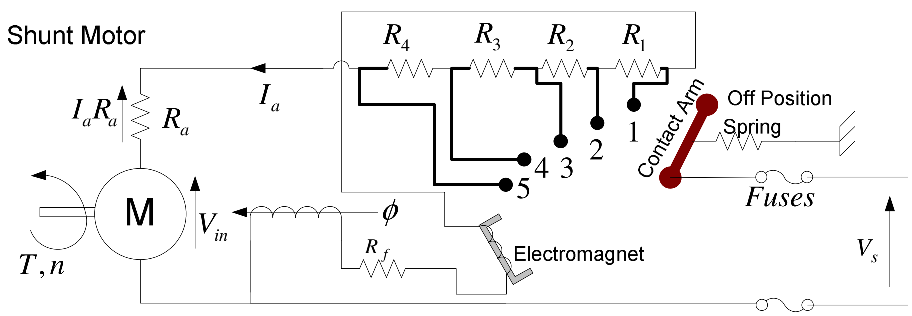
The field can either be cumulative or differential. In a cumulative circuit, the m.m.f. of the series field adds to that of the shunt field. In the differential connection, the m.m.f. series field opposes that of the shunt field. The speed-load characteristic is intermediate between those of a shunt and series motor, but retains to a considerable degree the advantages of a shunt motor.

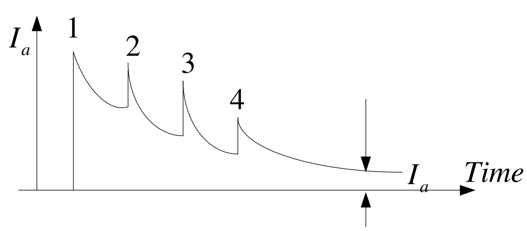
13.7 STARTING OF DC MOTORS

At starting, the induced voltage V_{in} is zero such that the supply voltage would be limited only by the armature resistance. A very high dangerous current can flow since the armature resistance is very small. $V_s = V_{in} + I_a R_a$, *at start,* $V_{in} = 0$

$$V_s = +I_a R_a \Rightarrow I_a = \frac{V_s}{R_a} = \frac{V_s}{0} \approx \infty .$$

Except for very small motors, a variable resistor in series with the armature which is reduced as the armature accelerates is necessary to limit the starting current. If the starting current is unlimited, the motor can be subjected to severe mechanical shock and cause the supply fuses to blow.





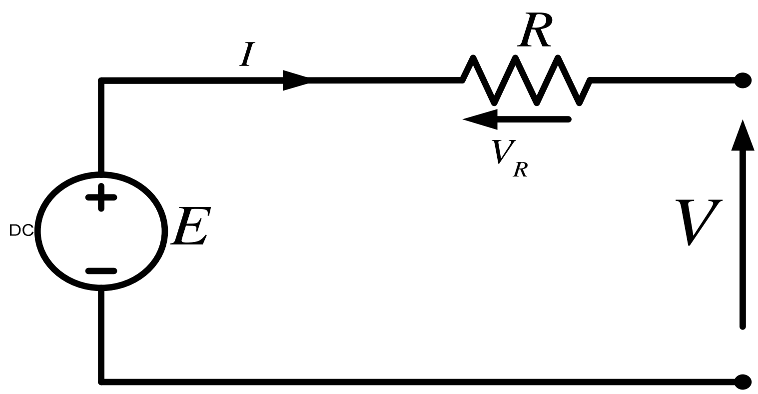
As the armature accelerates, the generated e.m.f. opposes the supply voltage and the armature current decreases.

$$V_a = V_{in} + I_a (R_a + R_1 + R_2 + R_3 + R_4) \text{ and } I_a = \frac{V_a - V_{in}}{(R_a + R_1 + R_2 + R_3 + R_4)}.$$

When the armature current falls to the required value, R_1 is shorted out to reduce the resistance and maintain the armature current. This operation is repeated until all the resistors are shorted out of the armature circuit.

14.0 ELECTRIC POWER FLOW

DC Circuits

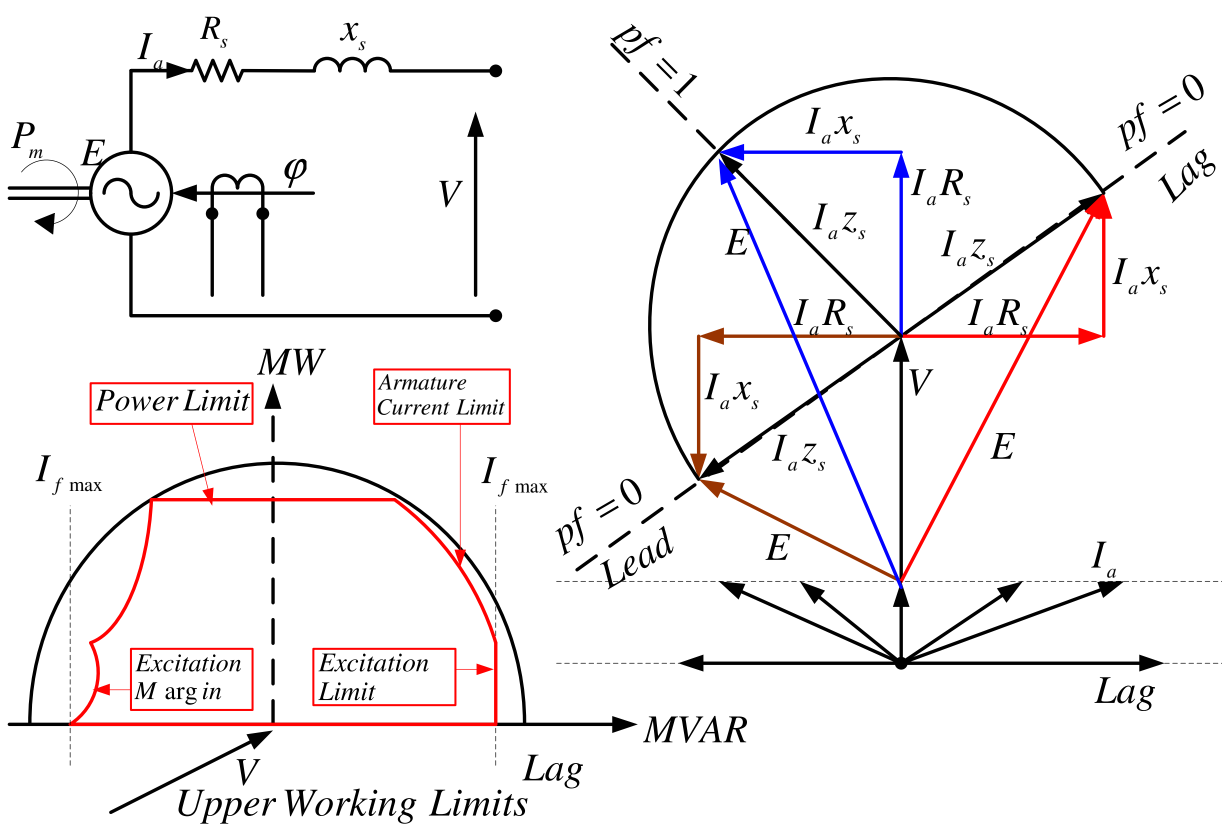


$$P = I^2 R = \left(\frac{E - V}{R}\right)^2 R = \left(\frac{\Delta V}{R}\right)^2 R = \frac{\Delta V^2}{R} = \Delta VI$$

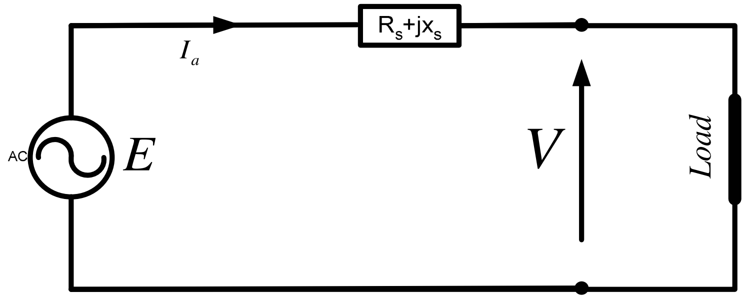
$$\therefore P = \Delta VI$$

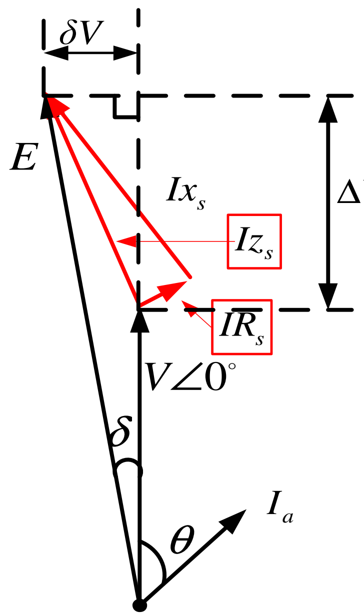
The effect of voltage difference in a dc circuit leads to power transfer.

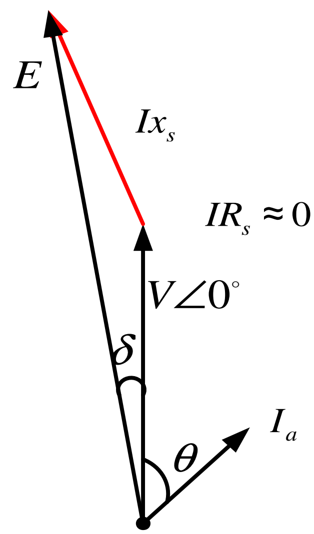
AC Circuits



Voltage Difference (ΔV)







$$E^2 = (V + \Delta V)^2 + (\delta V)^2$$

$$\Delta V = I_a R_s \cos\theta + I_a x_s \sin\theta$$

$$\delta V = I_a x_s \cos\theta - I_a R_s \sin\theta$$

$$E^2 = (V + I_a R_s \cos\theta + I_a x_s \sin\theta)^2 + (I_a x_s \cos\theta - I_a R_s \sin\theta)^2$$

$$P = VI_a \cos\theta \Rightarrow \frac{P}{V} = I_a \cos\theta$$

$$\therefore I_a R_s \cos\theta = \frac{R_s P}{V}$$

$$Q = VI_a \sin\theta \Rightarrow \frac{Q}{V} = I_a \sin\theta$$

$$\therefore I_a x_s \sin\theta = \frac{x_s Q}{V}$$

$$E^2 = (V + \frac{R_s P}{V} + \frac{x_s Q}{V})^2 + (\frac{x_s P}{V} - \frac{R_s Q}{V})^2$$

$$\Delta V = \frac{R_s P}{V} + \frac{x_s Q}{V} = \frac{R_s P + x_s Q}{V}$$

$$\therefore \Delta V = \frac{R_s P + x_s Q}{V}$$

$$|E| - |V| = \Delta V = \frac{R_s P + x_s Q}{V} \approx \frac{x_s Q}{V}; \; R_s \approx 0$$

$$\therefore |E| - |V| = \Delta V \approx \frac{x_s Q}{V}$$

$$|E| - |V| = \Delta V \approx \frac{x_s Q}{V} = \frac{Q}{I_a}$$

$$\therefore Q = \Delta V I_a$$

In ac circuits, the voltage difference usually leads to transfer of reactive power. This is the general effect in power systems due to the normally reactive behaviour of the lines.

Phase Angle Difference (∂V)

$$Z_s = R_s + jx_s, R_s \approx 0 \Rightarrow Z_s = R_s$$

$R_s << x_s$ for most practical transmission lines

$$|E|\angle\delta = |V|\angle 0^\circ + jx_s I_a \angle -\theta = |V|\angle 0^\circ + x_s I_a \angle 90^\circ - \theta$$

$$|E| = |V|\angle -\delta + |I_a||x_s|\angle 90^\circ - \theta - \delta$$

$$|I_a||x_s|\angle 90^\circ - \theta - \delta = |E|\angle 0^\circ - |V|\angle -\delta$$

$$|I_a|\angle -\theta - \delta = \frac{|E|\angle -90^\circ}{|x_s|} - \frac{|V|\angle -\delta - 90^\circ}{|x_s|}$$

$$S = EI^* = P + iQ = |E||I_a|\angle\theta + \delta = \frac{|E||E|\angle 90^\circ}{|x_s|} - \frac{|E||V|\angle\delta + 90^\circ}{|x_s|}$$

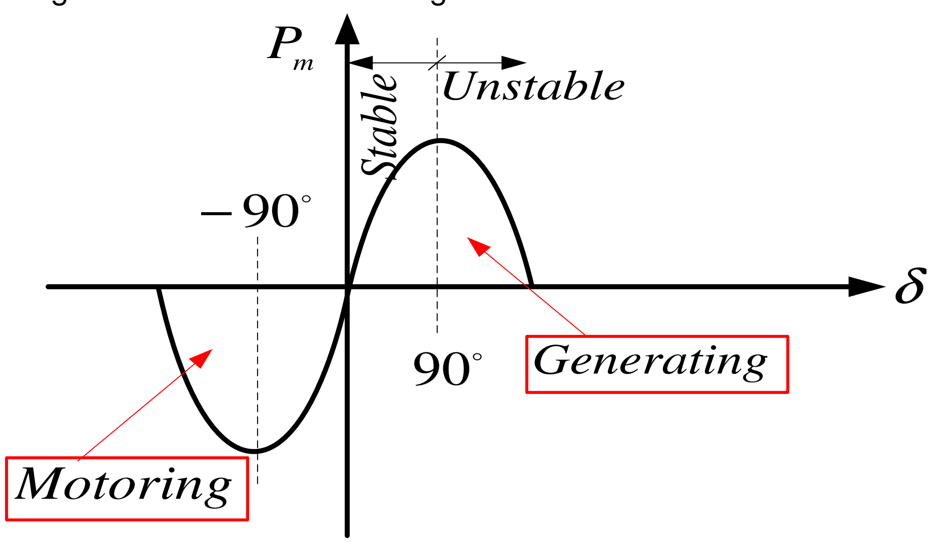
$$P=|E||I_a|\cos(\theta+\delta)=\frac{|E||E|}{|x_s|}\cos 90^\circ-\frac{|E||V|}{|x_s|}\cos(\delta+90^\circ)$$

$$P=\frac{|E||E|}{|x_s|}\cos 90^\circ-\frac{|E||V|}{|x_s|}\cos(\delta+90^\circ)=0-\frac{|E||V|}{|x_s|}\{(\cos\delta\cos 90^\circ)-\sin\delta\sin 90^\circ\}$$

$$P=-\frac{|E||V|}{|x_s|}\{(0)-\sin\delta\}=\frac{|E||V|}{|x_s|}\sin\delta$$

$$\therefore P=\frac{|E||V|}{|x_s|}\sin\delta$$

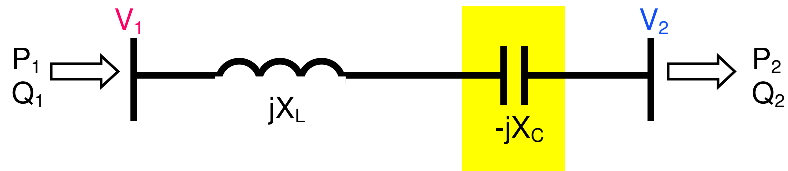
The difference in angle or the effect of phase difference results in power transfer. The angle δ is an electrical angle between the excitation voltage E and the busbar voltage V.

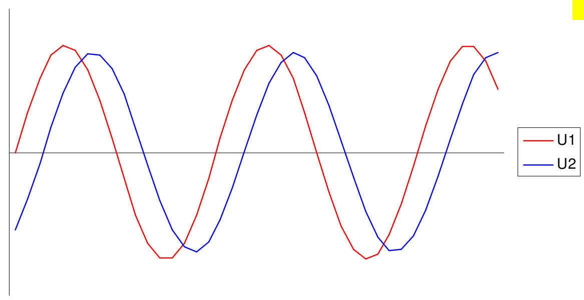


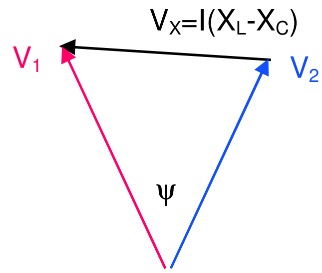
15.0 VOLTAGE VARIATION AND CONTROL

15.1 Series Capacitors

Series Capacitor basic theory - Transfer capability

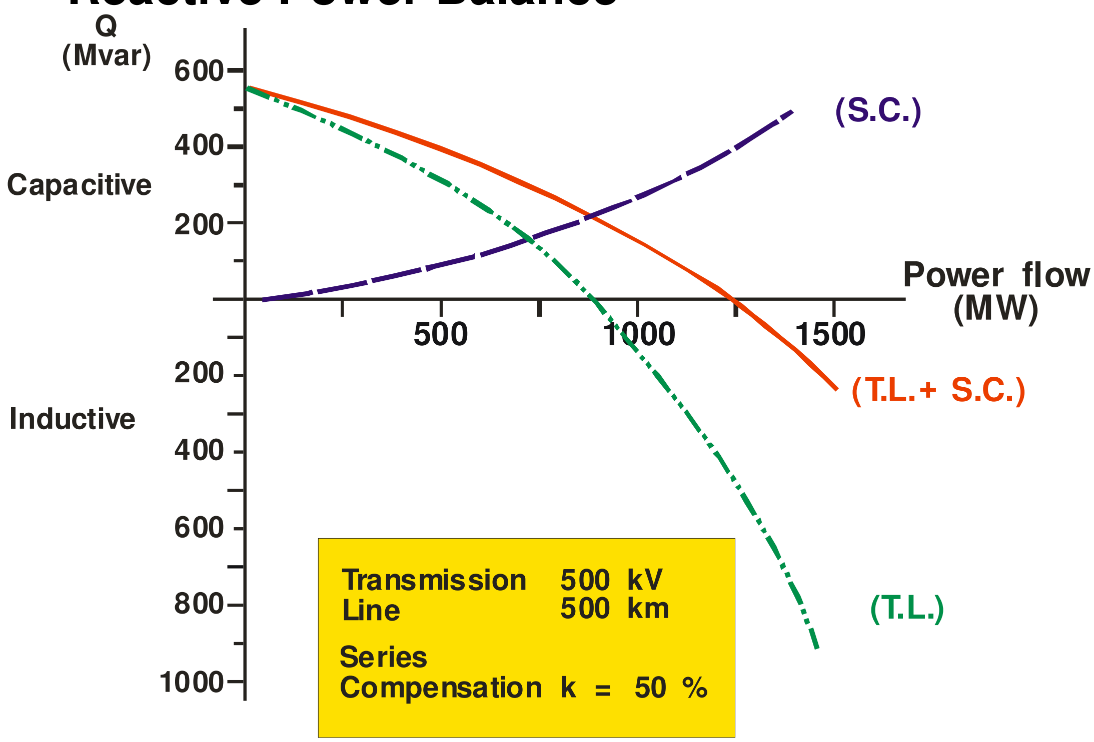






$$P = \frac{V_1 \cdot V_2}{X_L - X_C} \cdot \sin \Psi$$

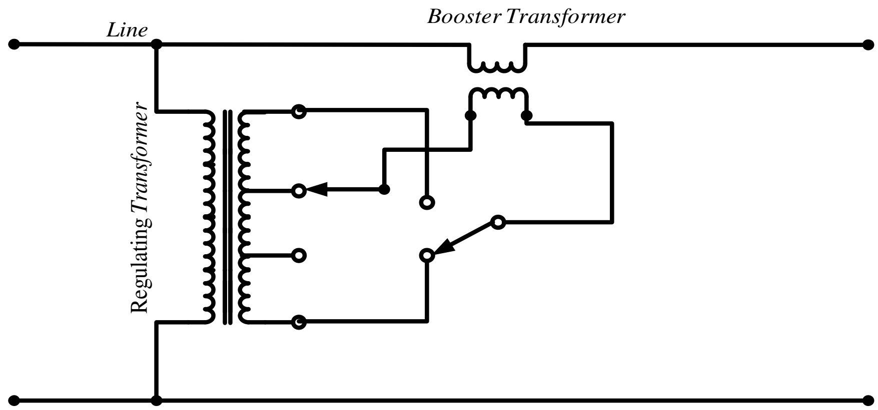
Reactive Power Balance



15.2 Synchronous Compensators

Synchronous compensators are synchronous alternators with a prime mover disengaged. The machine supplies vars when over excited during peal load conditions and it consumes vars when under excited during light load conditions.

15.3 Booster Transformers

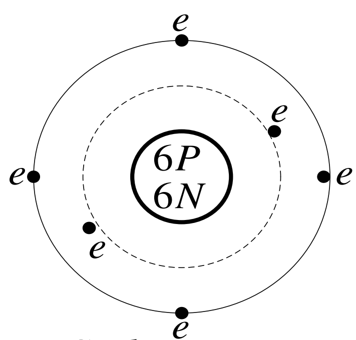


The taps can be an off load tapping or an on load tapping depending on the requirements. The tapping can be made by means of a motor operated controller and arrangements are made to reverse the connections to the primaries of the regulating transformers so that both buck and boost can be obtained. The disadvantages of the booster transformers are that it very expensive and less efficiency due to losses.

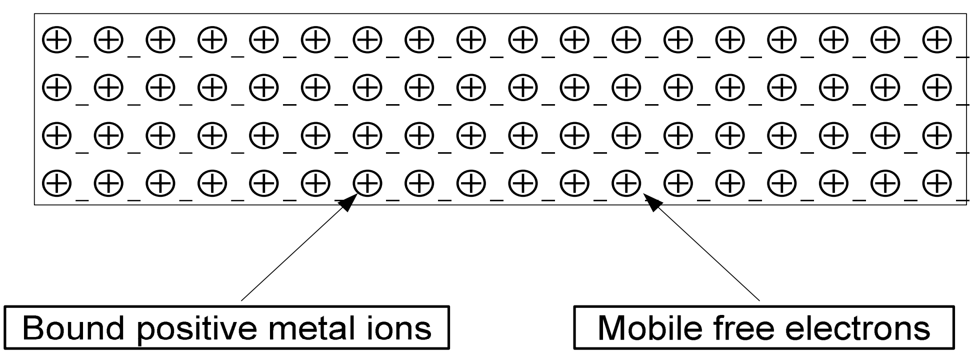
16.0 CONDUCTORS AND SEMICONDUCTORS

METALS

An atom keeps the electrons to its shell due to $Electrical\ attraction = \left(\frac{1}{4\pi\varepsilon}\right)\left(\frac{e^2}{r^2}\right)$. A solid metal is electrically neutral, however due to free movement of electrons in the atoms' outer shell, when an atom gives up one electron, a bound metal ion of positive charge is produced. There are approximately 10^{23} free electrons in a metal per cm^3. The free electrons have a random movement resulting from temperature changes.

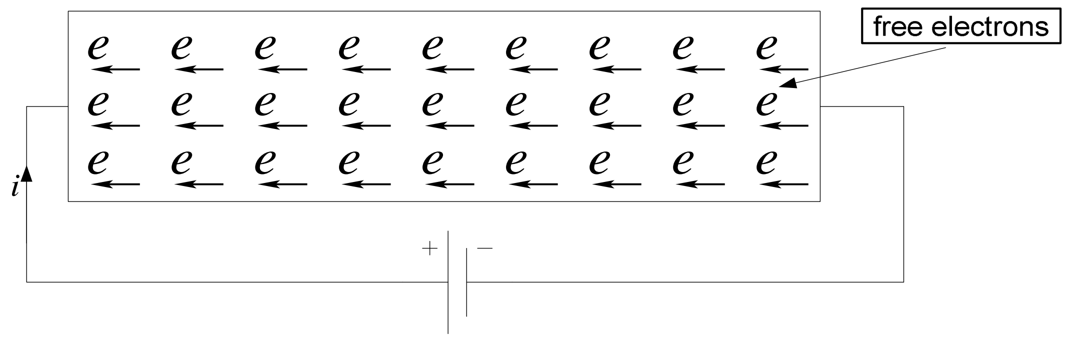


Carbon atom



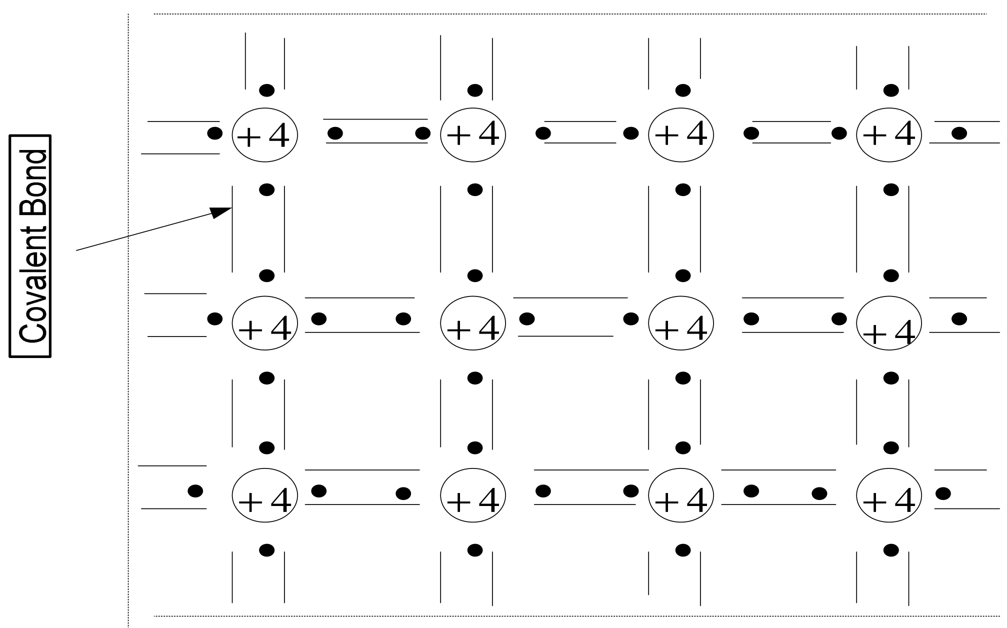
MOVEMENT OF ELECTRONS

When a battery is connected to the metal, the positive side of the battery attracts the negatively charged free electrons and the negative side of the battery repels the electrons. This results in a flow of a stream of electrons from the negative to positive side of the battery terminal. This means that the current flows from the negative to the positive side of the battery. However, by conversion, the current flow is from the positive to the negative terminal. In metals, electric current conduction is mainly due to 'free electrons' while in liquids it is due to ions.



SEMI-CONDUCTORS

A semi conductor is a material whose conductivity lies between that of a good conductor and a good insulator. Semi conductors have a crystalline structure in which atoms are arranged in a symmetrical periodic array. The covalent bonds in the crystalline structure serve to keep the atoms together in a crystal formation.



INTRINSIC SEMI-CONDUCTORS

A crystal is regarded as pure when it has no impurities and such a crystal is referred to as an intrinsic semi conductor. Intrinsic conductivity is temperature dependant and is as a result of movement of both the electrons and the holes which free themselves from the parent atom. A hole is a vacancy created when an electron is removed. Conductivity takes place when covalent bonds are broken due to mainly external energy such as temperature or battery source.

$$\sigma = q(\mu_e n + \mu_p p)$$

where μ_e = *electron mobility constant*

μ_p = *hole mobility constant*

p = *number of free holes*

n = *number of free electrons*

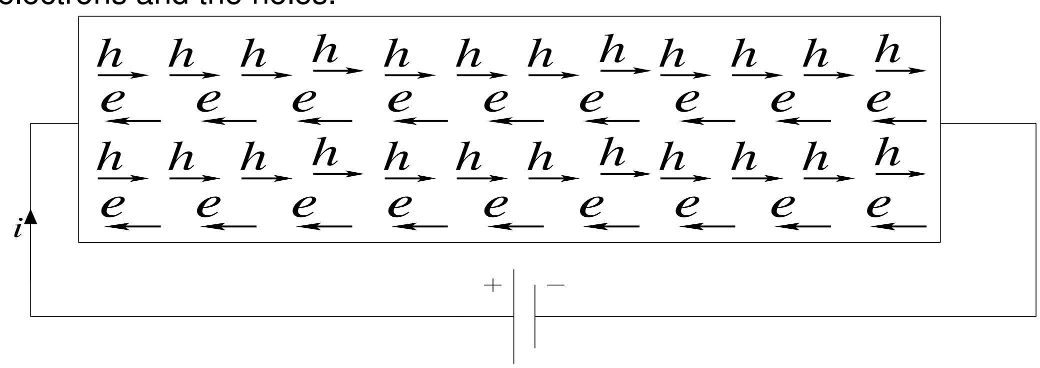
σ = *conductivity* = $\frac{J}{E}, (J = \frac{I}{A}, \quad E = \frac{V}{l})$

q = *charge*

The number of holes and electrons is equal in an intrinsic semi conductor.

INTRINSIC ELECTRIC CONDUCTION

When a battery is applied across an intrinsic semi conductor, the covalent bonds are broken and the electrons are attracted towards the positive terminal of the battery. The holes are attracted towards the negative terminal of the battery. Current flows due to the movement of both the electrons and the holes.

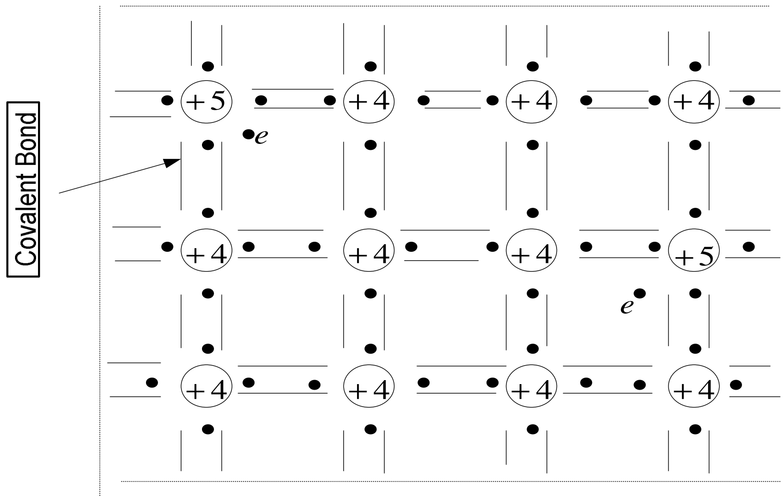


EXTRINSIC SEMI-CONDUCTORS

A crystal that is impure or has been doped with other atoms is known as extrinsic semi conductor. There are two types of extrinsic semi conductors, the n-type and the p-type.

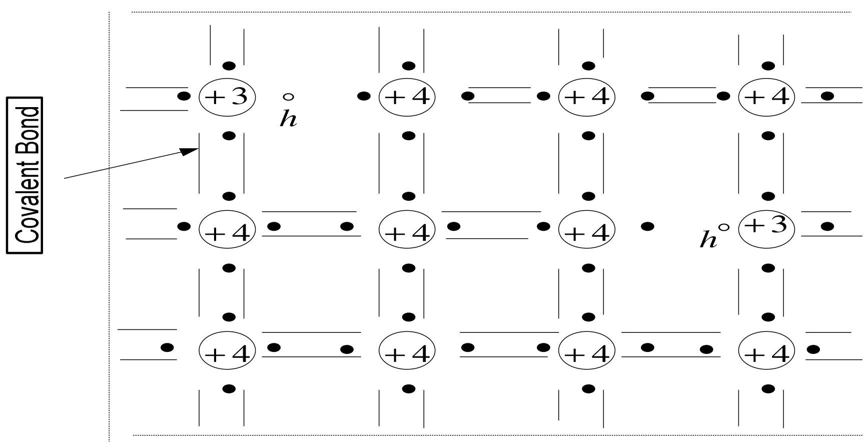
N-TYPE SEMI-CONDUCTORS

When a five valence atom (phosphorus, arsenic or antimony) is introduced into a pure four valence (germanium or silicon), a covalent bond is formed leaving excess electrons. The substitution of a pentavalent atom for tetravalent atom provides a free electron 'e'. In the n-type semi conductor, a pentavalent atom is termed the donor because it donates a free electron. A crystal doped with such impurities will have excess electrons available for electric conduction.



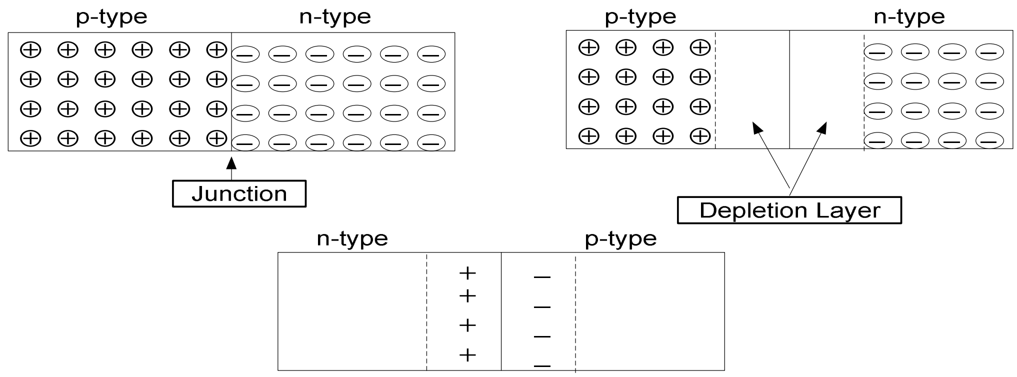
P-TYPE SEMI-CONDUCTOR

When a three valence atom (indium, boron or aluminium) is introduced into a pure tetravalent valence (germanium or silicon), a covalent bond is formed leaving an incomplete valence bond thereby creating a vacancy called hole 'h'. The trivalent atom will accept the electrons from adjacent tetravalent atoms. The hole can attract nearby electrons. The substitution of a trivalent atom for tetravalent atom provides a positively charged hole. In the p-type semi conductor, a trivalent atom is termed the acceptor because it accepts a free electron. A crystal doped with such impurities will have excess holes available for electric conduction.

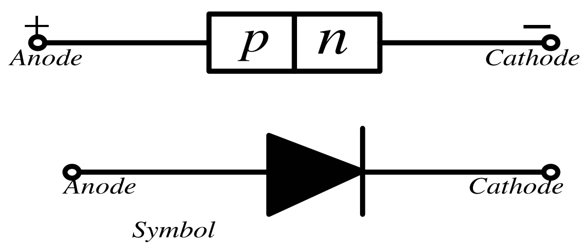


16.1 JUNCTION DIODE (P-N JUNCTION)

When a piece of 'p' and 'n' type of semi conductors are joined together, in the region of the junction, the electrons in the n-type move over to neutralize the holes in the p-type. This leaves a small region in the vicinity of the junction without any charge carriers. The movement of holes from p-region leaves a negatively charge acceptor atoms and the movement of electrons from n-region leaves a positively charge donor atoms. A net negative charge on the p-side and a net positive charge n-side will develop. This space charged region is known as the depletion layer. The static charge in the depletion layer forms a potential barrier which stops any farther movement of holes or electrons across the region.



DIODE CIRCUIT SYMBOL



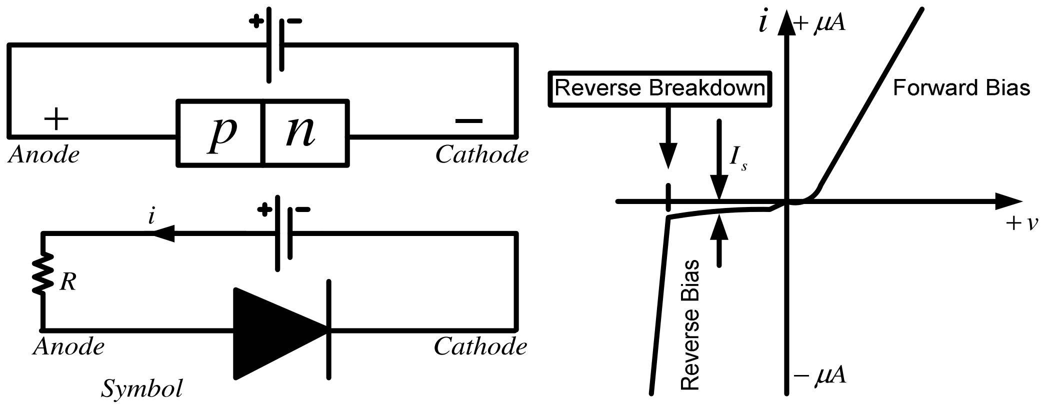
For germanium junction diode in which surface leakage is negligible;

$$i = I_s\left(e^{\frac{qv}{KT}} - 1\right)$$

I_s = Saturation current with negative bias or reverse saturation current
q = Electronic charge = 1.6 x 10^{-19} C
v = Potential difference in volts across the junction
K= Boltzman's constant = 1.6 x 10^{-23} J/K
T = Thermodynamic temperature = 273.15 +tºC

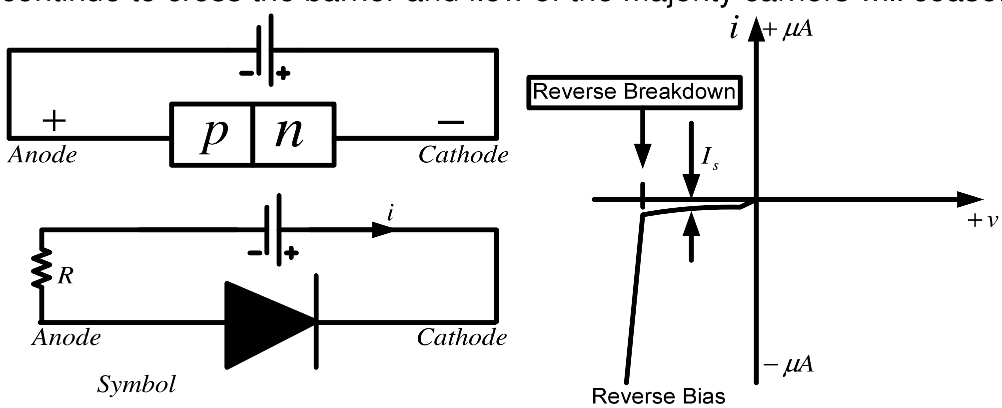
FORWARD BIAS

The holes in the p-region are repelled from the positive battery terminal and the electrons in the n-region are repelled from the negative terminal. The depletion layer is reduced and the potential barrier lowered. Electrons and holes move freely across the junction and a high current flows.



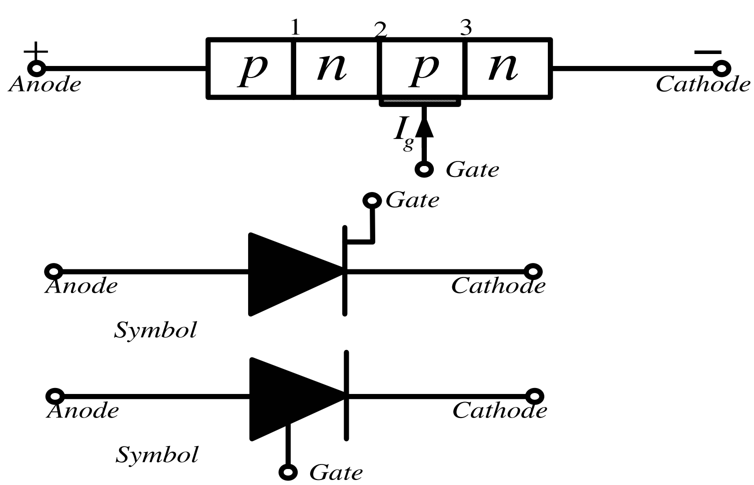
REVERSE BIAS

The n-region is made positive with respect to the p-region. The free electrons in the n-region are attracted to the positive terminal of the battery and the holes in the p-region are attracted towards the negative terminal of the battery. The depletion layer widens and the potential barrier increases. Only a few minority carriers generated by temperature will continue to cross the barrier and flow of the majority carriers will cease.



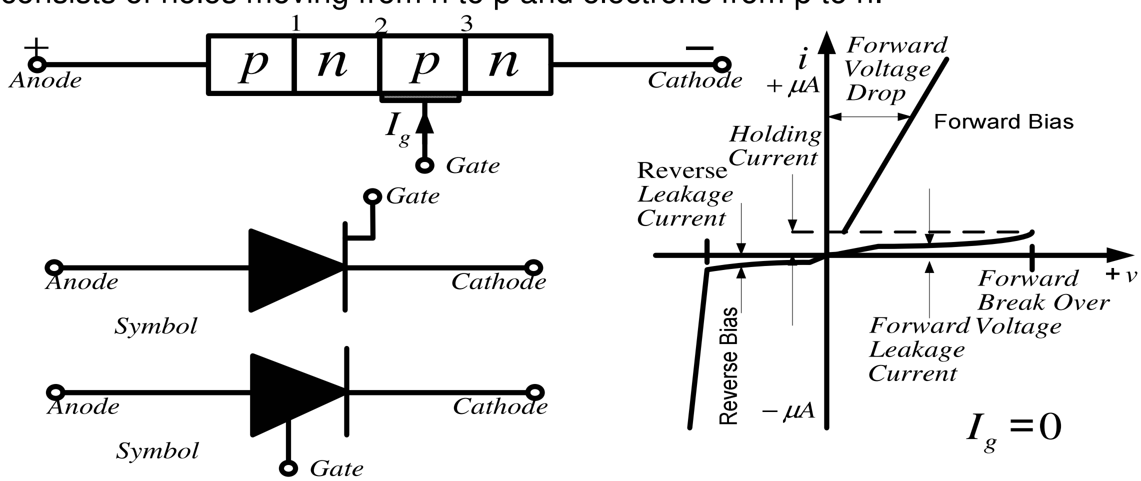
16.2 THYRISTOR (SCR)

Thyristors also known as Silicon Controlled Rectifiers (SCR) are a four layer semi-conductor junction device.



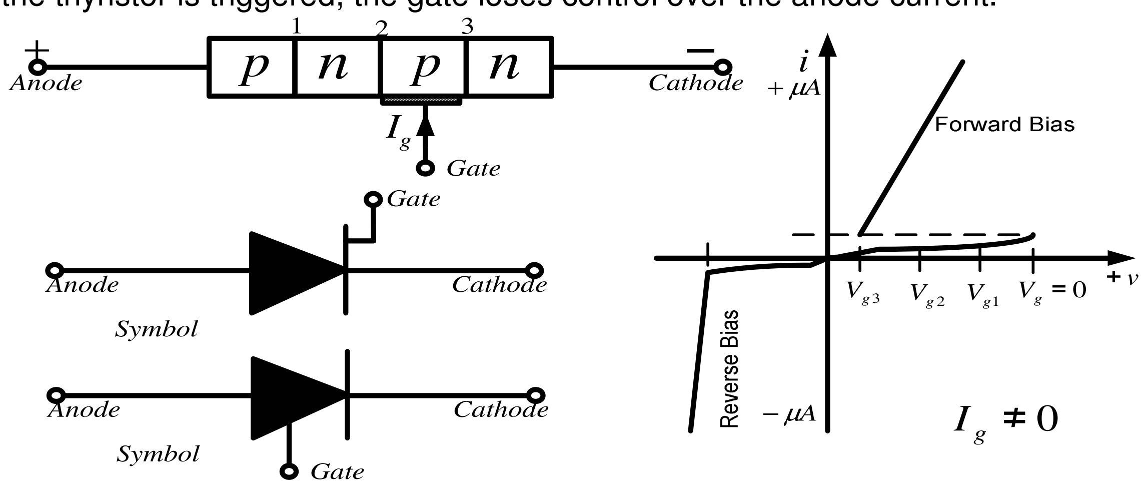
Open Circuit Gate ($I_g = 0$)

The anode is made positive with respect to the cathode. Junctions 1 and 3 will be forward biased and junctions 2 will be reverse biased. Junction 2 acts as reverse biased diode junction and the reverse bias current consists of holes moving from n to p and electrons from p to n.



Gate Control (Gate Triggered or Fired)

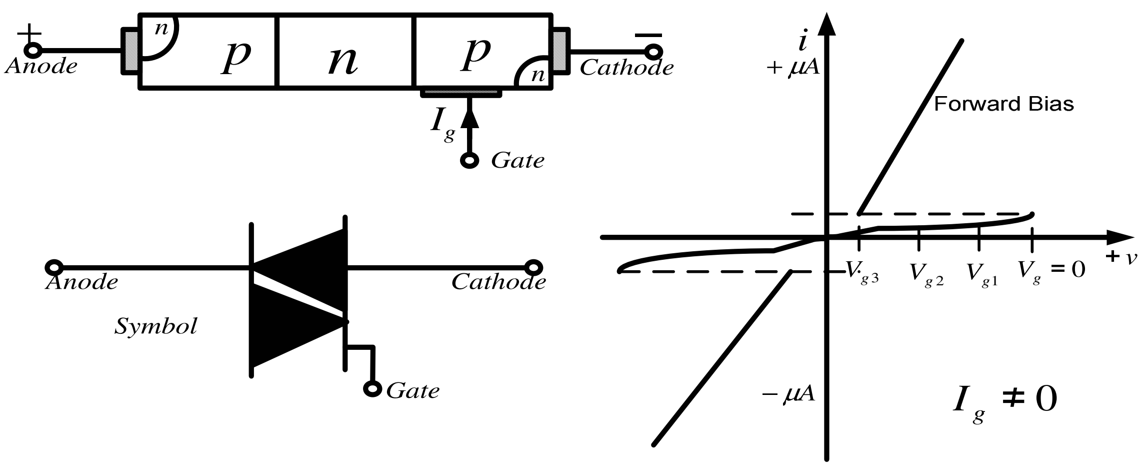
When the current is fed into the gate connection, the current across junction 3 is increased. Increasing the gate current or gate volyage reduces the break over voltage and the thyristor behaves like a junction diode. Once the gate has triggered the thyristor, the thyristor will continue conducting and will only stop conducting when the anode is reversed in direction or falls to almost zero or below the holding current value. Once the thyristor is triggered, the gate loses control over the anode current.



16.3 Triacs

The triac is equivalent to two thyristors connected together in inverse parallel. When a single thyristor is used to control power in an ac circuit, either by off-on switching or phase control, other devices, such as diodes in a bridge arrangement, must be connected around the thyristor to accommodate both directions of current flow. On the other hand, the diodes can be eliminated f a second thyristor is connected in inverse parallel across the first one.

The triac was developed to satisfy the need for ac power control with a single device. The triac is basically a five layer silicon semi-conductor device of the form ***n-p-n-p-n***. The n regions at the extreme ends are shorted to the adjacent p region, by the contact metallization. The overall result of these shorts is to minimise the effect of an n region, which is positively biased. As a result, the triac looks like a ***p-n-p-n*** configuration independent of the polarity of the bias voltage and thus performs functionally as though two thyristors were connected in inverse parallel.



16.4 Power Electronics

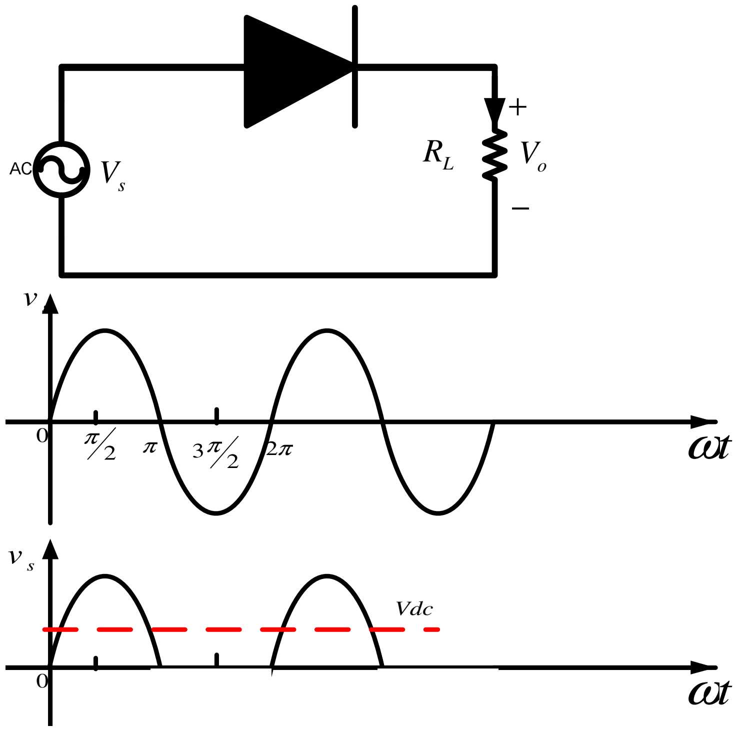
Power electronics deals with the applications of electronic devices and associated components to the conversion, control and conditioning of electric power. The primary characteristics of electric power which are subject to control is the ac and dc voltage and current.

Rectifiers (AC-DC Converters)

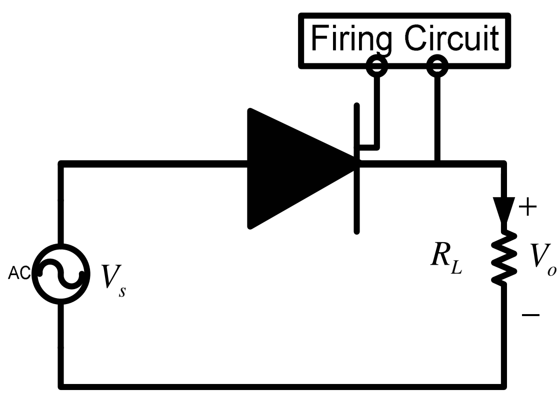
A rectifier is assembled from diodes and/or thyristors which supply direct current (dc) from an ac supply line.

SINGLE PHASE HALF WAVE RECTIFIER

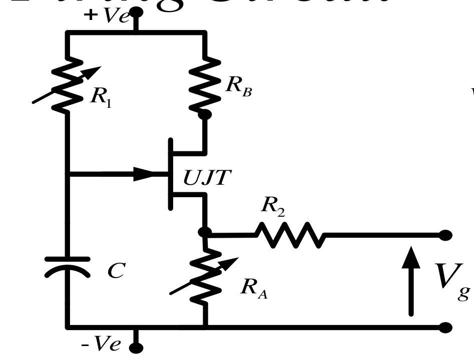
(a) Diode Rectifier

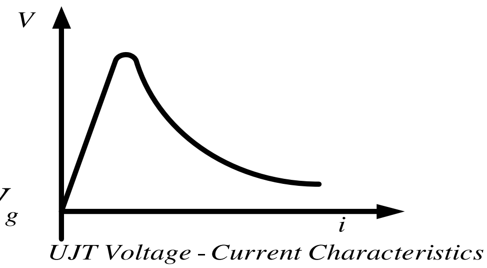


(b) Thyristor Rectifier

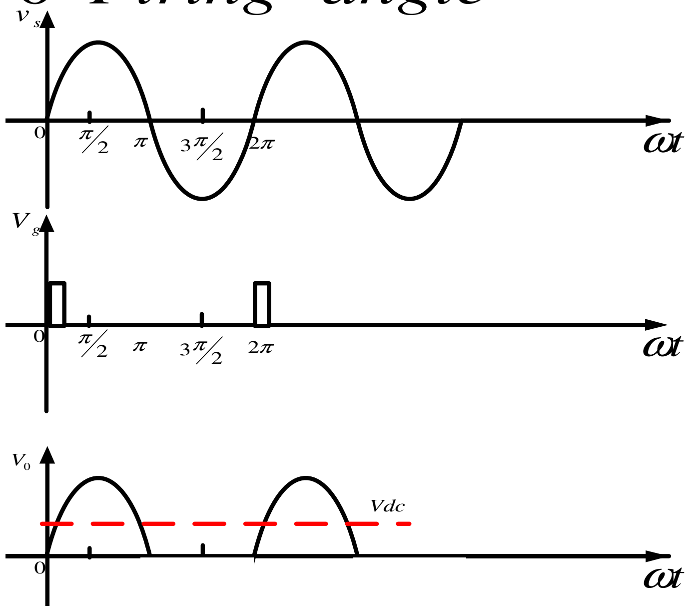


Firing Circuit

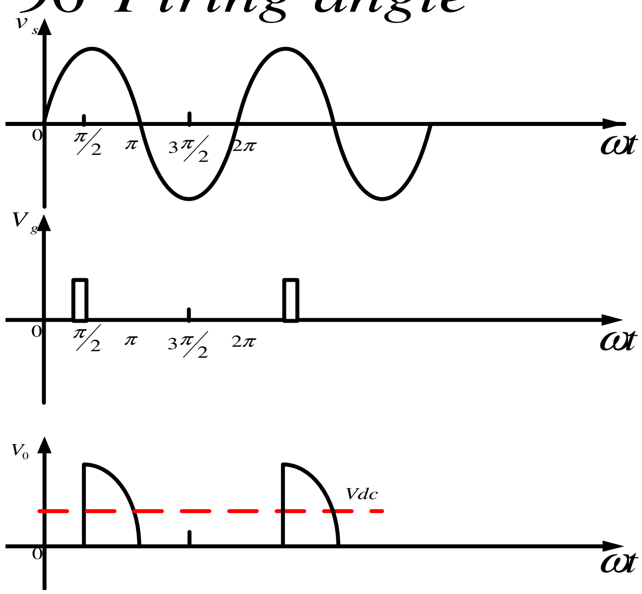




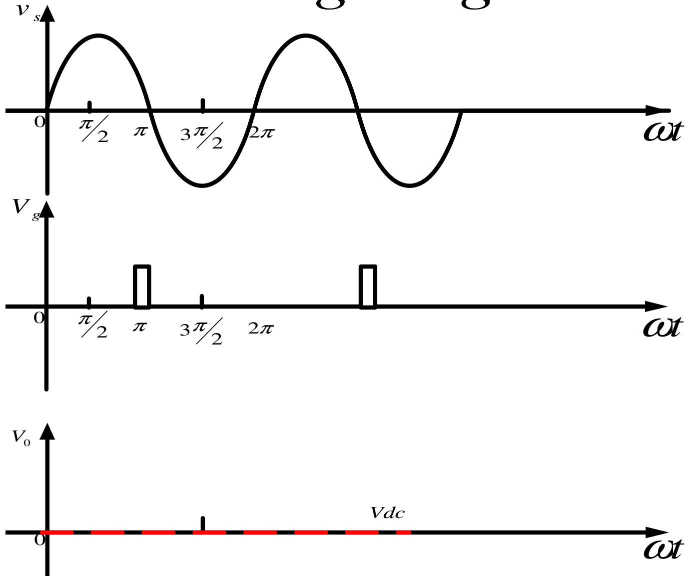
0° *Firing angle*



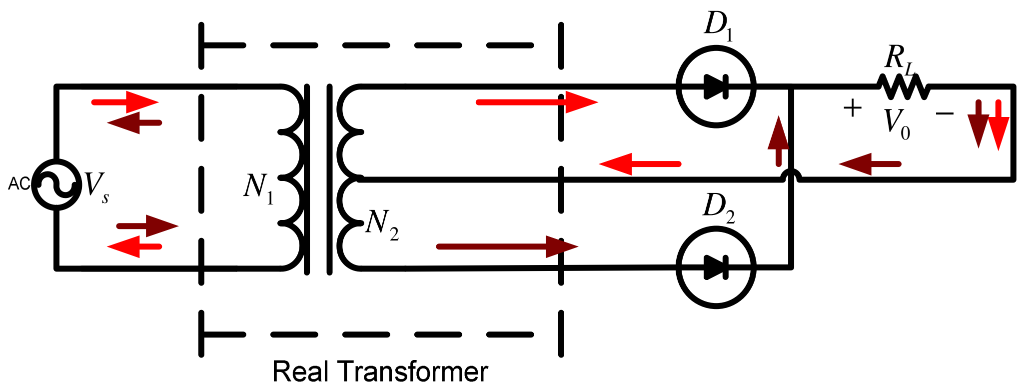
90° *Firing angle*

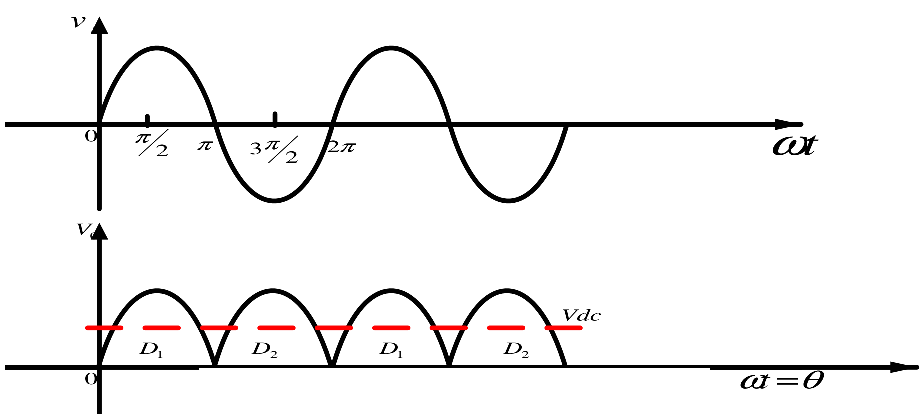


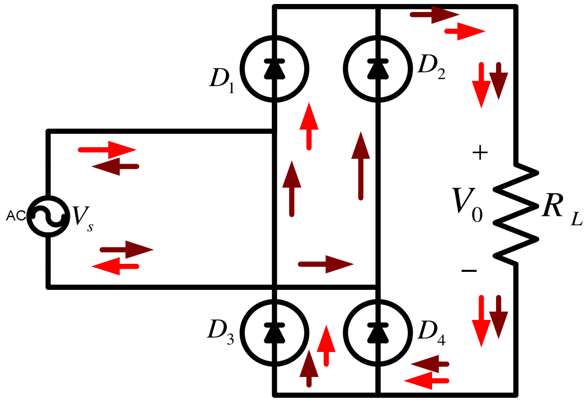
180° *Firing angle*

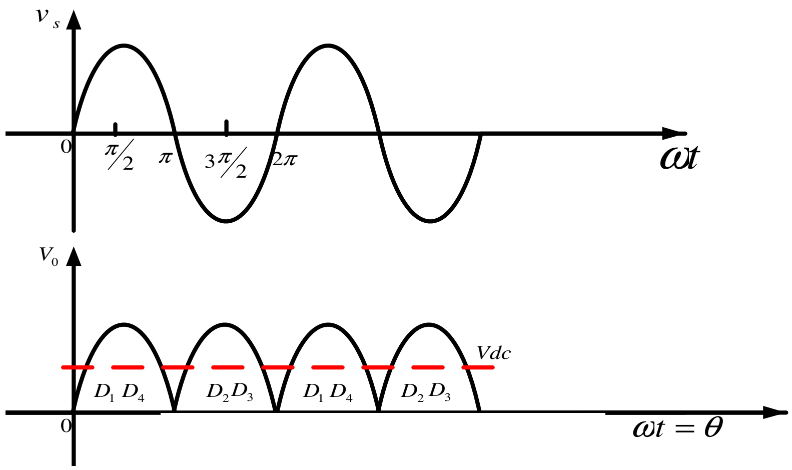


FULL WAVE RECTIFIER





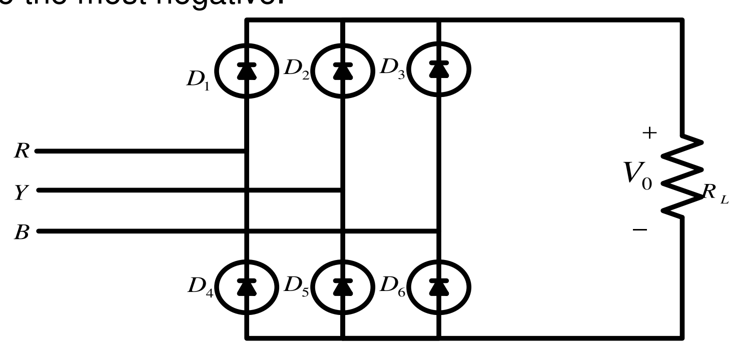


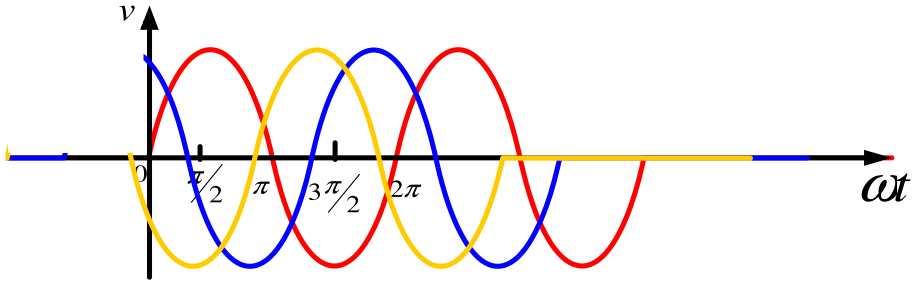


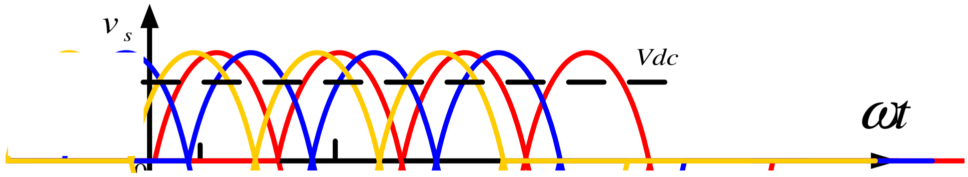
For positive going cycles, the diode is forward biased and reverse biased for negative going cycles. The diode conducts for positive going part of the wave form and blocks the negative part. A thyristor also known as Silicon Controlled Rectifier (SCR) works on the diode principle when triggered or fired. Varying the instant at which the thyristor is triggered varies the dc output. Hence controlled dc output is obtained varying the firing or triggering angle. The rectified output dc contains ripples. Smooth dc output is obtained by filtering the ripple using filter circuits.

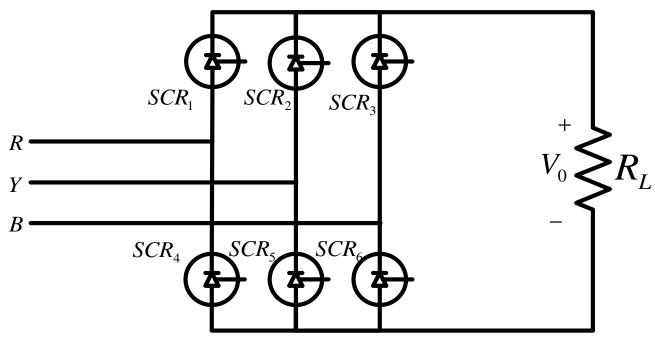
THREE-PHASE RECTIFIER

In a case of three-phase rectifiers, the current flows from the most positive to the most negative.

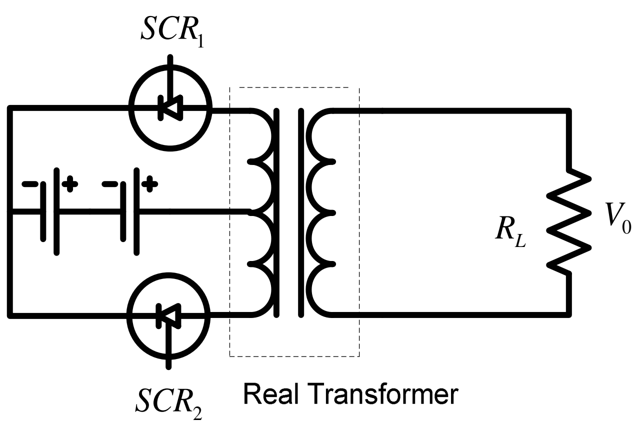


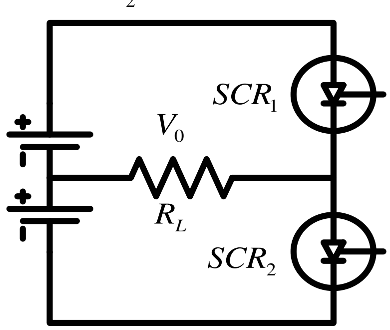


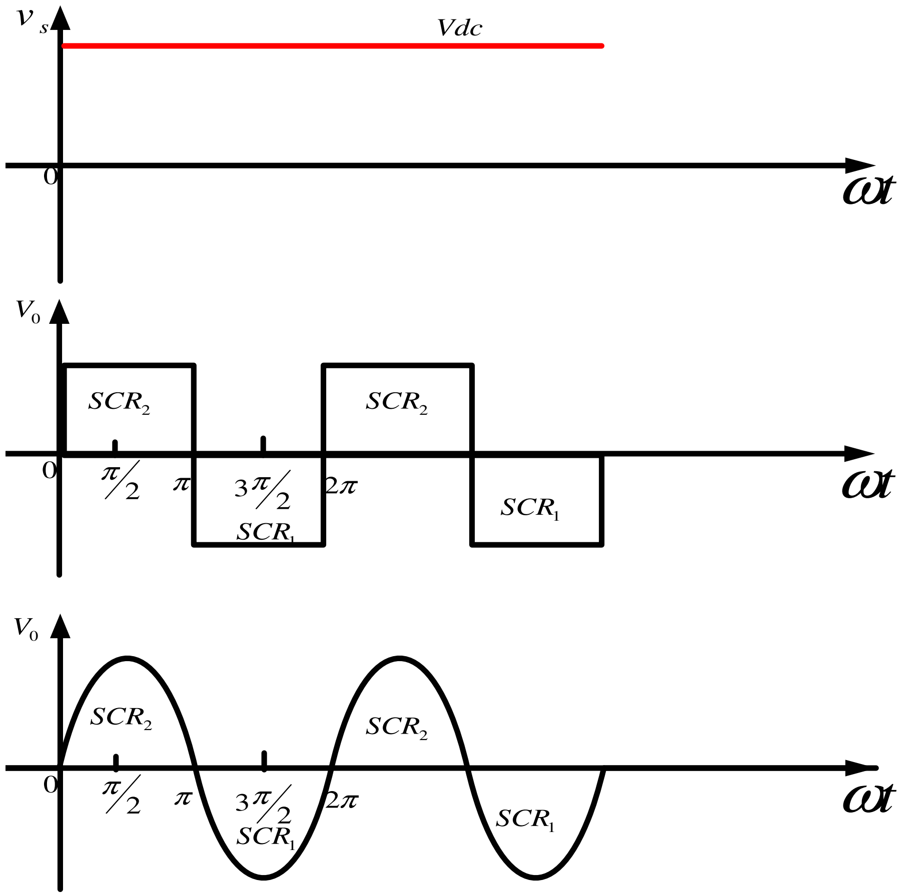




DC-AC Inverters

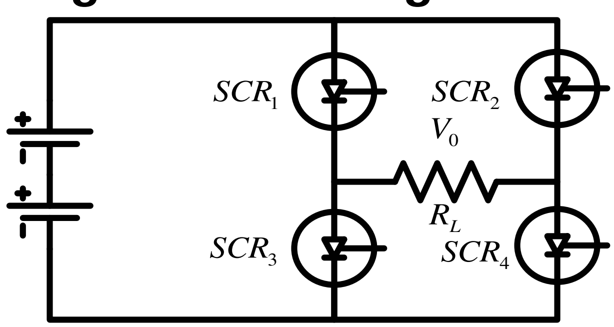




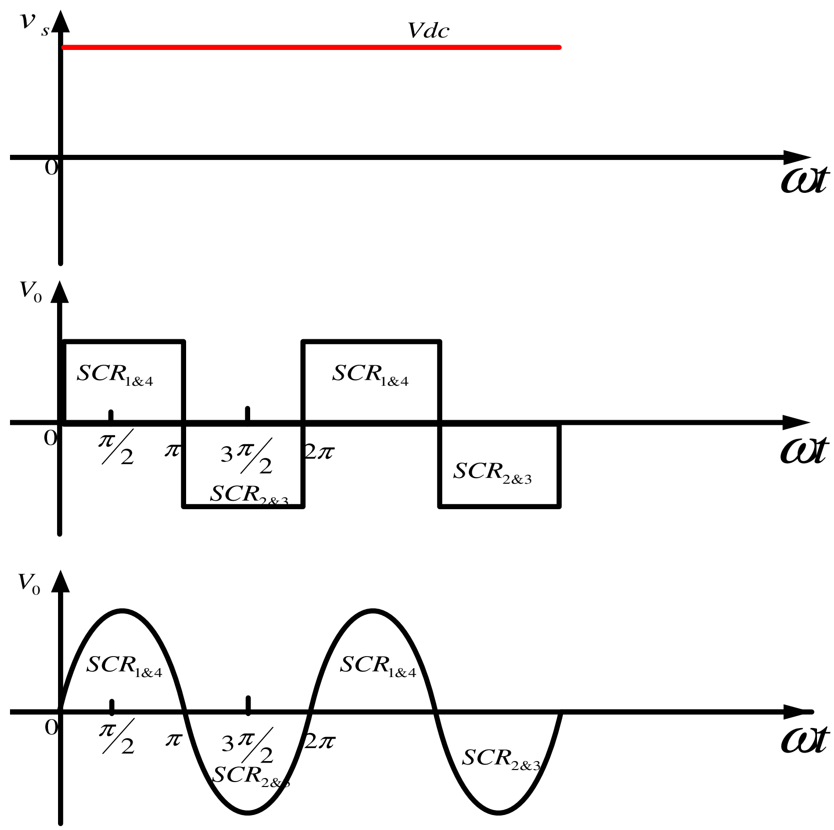


By sequentially switching SCR1 and SCR2 on and off, the voltage across the load is made to change polarity cyclically to produce a square wave alternating current. When the filter circuit is connected across the output, a smooth sinusoidal wave form is obtained.

Single Phase Bridge Inverter



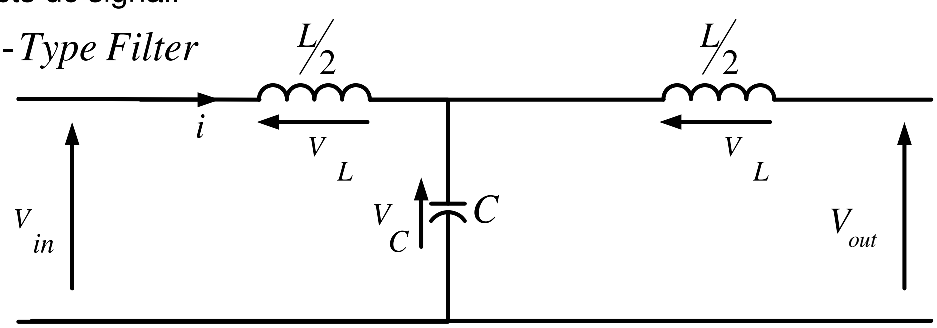
Thyristor 1 and 4 are simultaneously switched on and off sequentially with thyristor 2 and 3 to give ac output through a filter circuit.



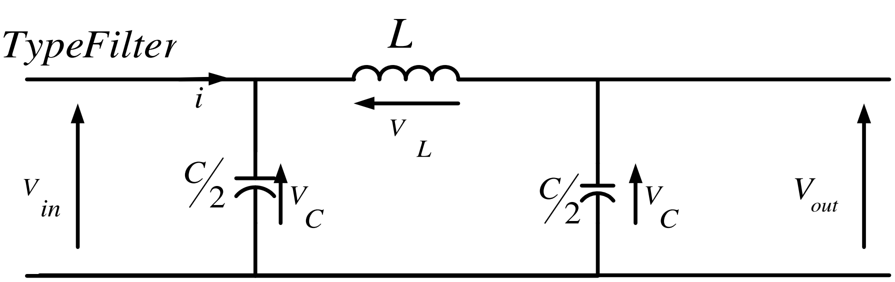
Rectifier Filter Circuits

The filter network shunts ripples with a certain frequency signals and rejects dc signal.

T -Type Filter



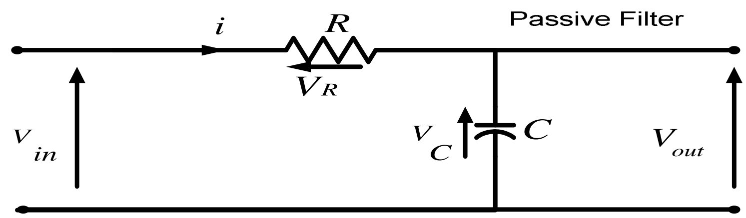
π-TypeFilter

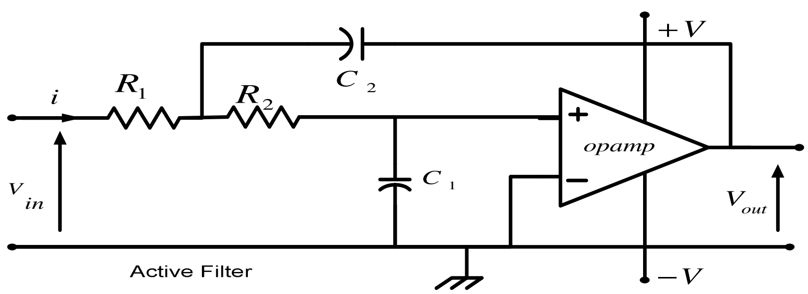


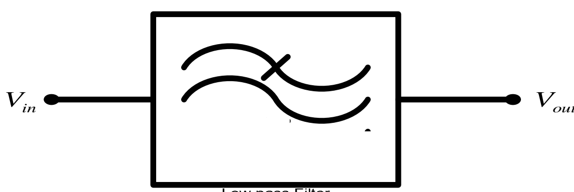
17.0 FILTERS

A filter network allows certain frequency signals to pass and rejects signals at all other frequencies. A filter introduces zero attenuation to selected frequencies called pass-band and blocks all the other bands. Filters are frequency sensitive and employ frequency dependant components i.e. capacitors, inductors or a combination of both. The filters are classified as Low-Pass, High-Pass, Band-Pass and Band-Stop filters. Filters are also classified as passive or active. They are passive if they have no power source within them and active if they contain an operational amplifier (op.amp) which includes its own power supply.

LOW PASS FILTER







Low pass Filter

Using passive filter circuit;

$$V_{out} = V_C = iX_C = -j\frac{i}{\omega C} = -j\frac{i}{2\pi fC}$$

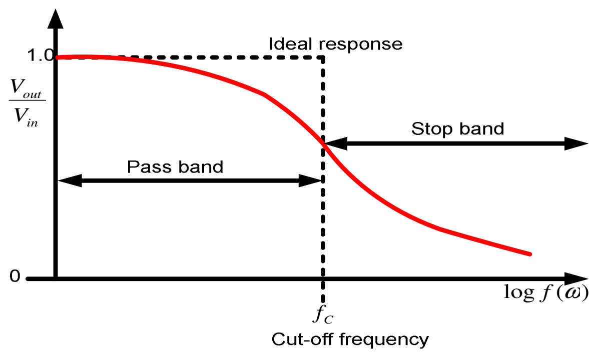
$$V_R = iR$$

$$V_{in} = V_R + V_C = i(R - jX_C) = i\left(R - j\frac{1}{\omega C}\right) = i\left(R - j\frac{1}{2\pi fC}\right)$$

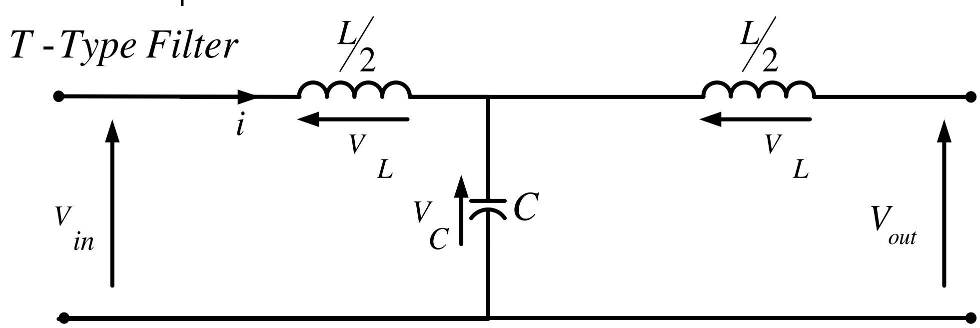
$$Gain = \frac{V_{out}}{V_{in}} = \frac{i\left(-j\frac{1}{2\pi fC}\right)}{i\left(R - j\frac{1}{2\pi fC}\right)} = \left\{\frac{\left(-j\frac{1}{2\pi fC}\right)}{\left(R - j\frac{1}{2\pi fC}\right)}\right\}\left\{\frac{j2\pi fC}{j2\pi fC}\right\} = \frac{1}{1 + j2\pi fCR}$$

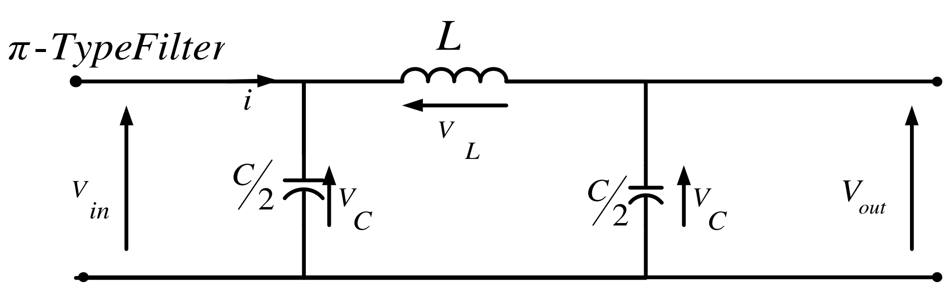
$$\therefore Gain = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j2\pi fCR}$$

$$f = 0, Gain = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j2\pi(0)CR} = 1 \text{ and } f = \infty, Gain = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j2\pi(\infty)CR} = 0$$

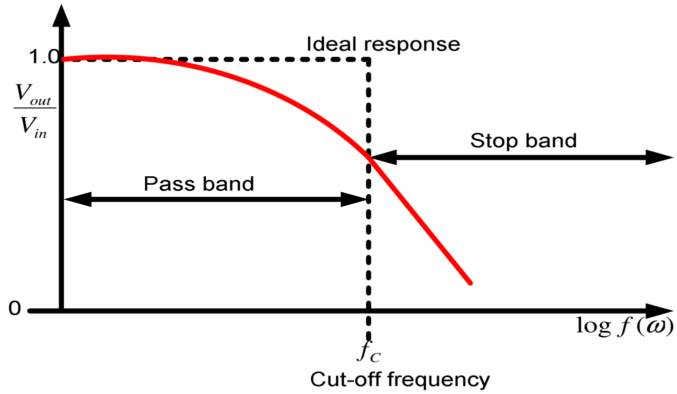


The RC passive filter has unwanted attenuation of the signal in the low frequency pass band due to the resistor R. One method of overcoming this problem is to use an active form of filter or a passive low filter utilizing the capacitors and inductors. The inductors need to have as small a resistance as possible.

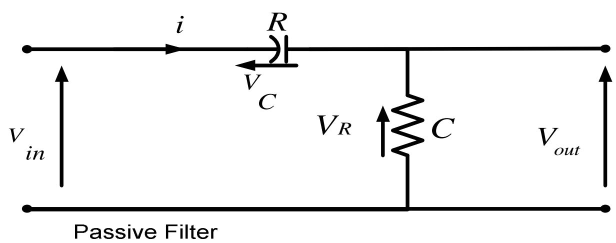


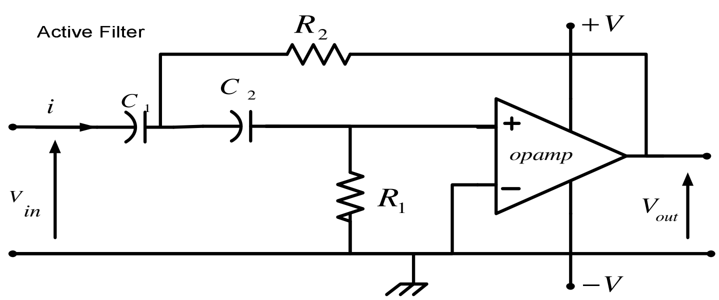


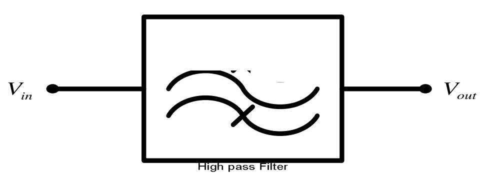
The frequency response for the capacitor-inductor (LC) low passive filter is close to the ideal response.



HIGH PASS FILTER







Using passive filter circuit;

$$V_{out} = V_R = iR$$

$$V_C = iX_C = -j\frac{i}{\omega C} = -j\frac{i}{2\pi fC}$$

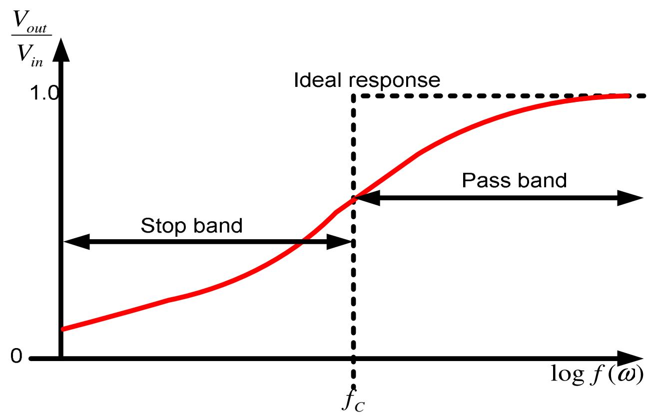
$$V_{in} = V_R + V_C = i(R - jX_C) = i\left(R - j\frac{1}{\omega C}\right) = i\left(R - j\frac{1}{2\pi fC}\right)$$

$$Gain = \frac{V_{out}}{V_{in}} = \frac{iR}{i\left(R - j\frac{1}{2\pi fC}\right)} = \left[\frac{R}{\left(R - j\frac{1}{2\pi fC}\right)}\right]\left\{\frac{\frac{1}{R}}{\frac{1}{R}}\right\} = \frac{1}{1 - j\frac{1}{2\pi fCR}}$$

$$\therefore Gain = \frac{V_{out}}{V_{in}} = \frac{1}{1 - j\frac{1}{2\pi fCR}}$$

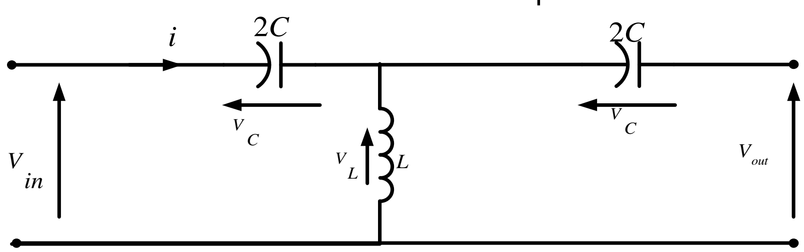
$$f = 0, Gain = \frac{V_{out}}{V_{in}} = \frac{1}{1 - j\frac{1}{2\pi(0)CR}} = 0$$

$$f = \infty, Gain = \frac{V_{out}}{V_{in}} = \frac{1}{1 - j\frac{1}{2\pi(\infty)CR}} = 1$$

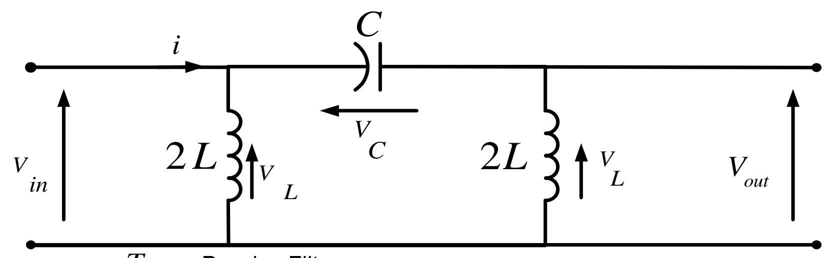


Cut-off frequency

The high pass RC passive filter has also suffers from unwanted attenuation of the signal in the low frequency pass band due to the resistor R. One method of overcoming this problem is to use an active form of filter or a passive low filter utilizing the capacitors and inductors. The inductors need to have as small a resistance as possible.

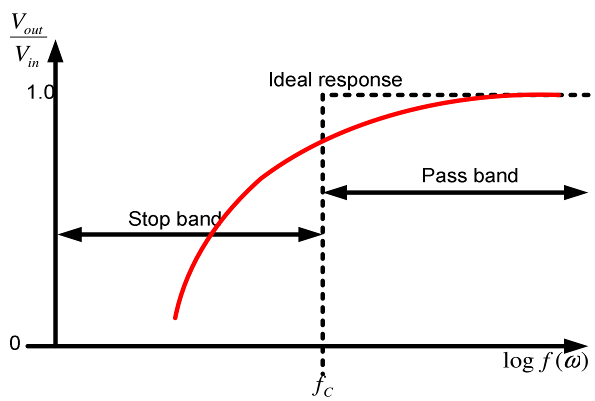


T-Type Passive Filter



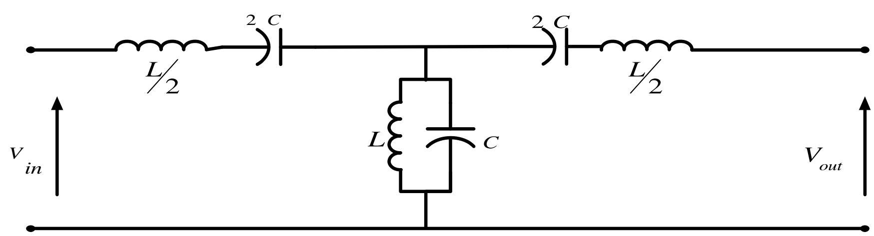
$\pi - Type$ Passive Filter

The frequency response for the capacitor-inductor (LC) high passive filter is close to the ideal response.

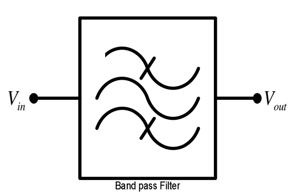


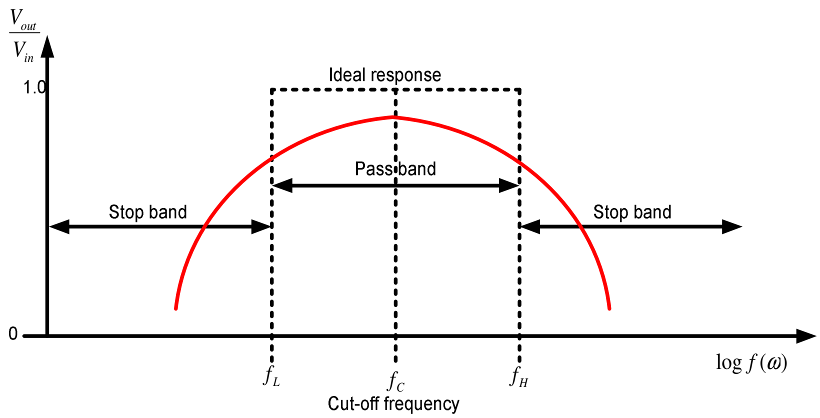
BAND PASS FILTER

The circuit utilizes both the series and parallel resonant circuits. The function is a combination of both the low pass and high pass filters. The frequency response combines both the low and high pass filter characteristics.



Band Passive Filter





BAND-STOP FILTERS (NOTCH FILTERS)

The filter also utilizes both the series and parallel resonant circuits. Compared with the band-pass filter, the roles of the series and parallel tuned elements are reversed.