The University of Zambia
Department of Mathematics and Statistics
Mat 3110-Engineering Mathematics II

Tutorial Sheet 4- Triple Integrals **July, 2019.**

As part of continuous assessment, submit Question 6 on 2 August, 2019 during class time.

1. Evaluate the following triple integrals.

   (a) $\int_1^2 \int_0^2 \int_{-1}^1 2 + z^2 - xy \ \ \mathrm{d}z\ \mathrm{d}x\ \mathrm{d}y$

   (b) $\int_0^2 \int_{x^2}^2 \int_0^{xz} y^2 - 6z\ \mathrm{d}y\ \mathrm{d}z\ \mathrm{d}x$

   (c) $\int_{-1}^2 \int_0^1 \int_0^{2z} 3x - \sqrt{1+z^2}\ \mathrm{d}x\ \mathrm{d}z\ \mathrm{d}y$

   In each of the above, can you tell over which solid we are integrating?

2. Evaluate the following triple integrals over the given solids.

   (In as much as you are not required to sketch the solids over which you are integrating, you are encouraged to do so as it will enhance you understanding.)

   (a) Evaluate $\int\int\int_E 12y\ \mathrm{d}V$ where E is the region below $6x + 4y + 3z = 12$ in the first octant.

   (b) Evaluate $\int\int\int_E 5x^2\ \mathrm{d}V$ where E is the region below $x + 2y + 4z = 8$ in the first octant.

   (c) Evaluate $\int\int\int_E 10z^2 - x\ \mathrm{d}V$ where E is the solid below $z = 8 - y$ and above the region in the xy-plane bounded by $y = 2x$, $x = 3$ and $y = 0$.

   (d) Evaluate $\int\int\int_E 4y^2\ \mathrm{d}V$ where E is the solid below $z = -3x^2 - 3y^2$ and above $z = -12$.

   (e) Evaluate $\int\int\int_E 2y - 9z\ \mathrm{d}V$ where E is the solid behind the plane $6x + 3y + 3z = 15$ and infront of the triangle in the xz-plane with vertices, in (x, z) form : $(0, 0)$, $(0, 4)$ and $(2, 4)$.

   (f) Evaluate $\int\int\int_E 18x\ \mathrm{d}V$ where E is the solid behind the surface $y = 4 - x^2$ that is in front of the solid in the xz-plane bounded by $z = -3x$, $z = 2x$ and $z = 2$.

   (g) Evaluate $\int\int\int_E 20x^3\ \mathrm{d}V$ where E is the solid bounded by $x = 2 - y^2 - z^2$ and $x = 5y^2 + 5z^2 - 6$.

   (h) Evaluate $\int\int\int_E 6z^2\ \mathrm{d}V$ where E is the solid behind $x + 6y + 2z = 8$ that is in front of the region in the yz-plane bounded by $z = 2y$ and $z = \sqrt{4y}$.

   (i) Evaluate $\int\int\int_E 8y\ \mathrm{d}V$ where E is the solid between $x+y+z = 6$ and $z+y+z = 10$ above the triangle in the xy-plane with vertices, in (x, y) form : $(0, 0)$, $(1, 2)$ and $(1, 4)$.

3. Use triple integrals to find the volumes of the following solids.
   (a) The volume of the solid is below the plane $z = 8 - y$ and above the region in the xy-plane bounded by $y = 2x$, $x = 3$ and $y = 0$.
   (b) The volume of the solid in the 1stoctant that is below $4x + 8y + z = 16$.
   (c) The volume of the sold behind $6x + 3y + 3z = 15$ and in front of the triangle in thexz-plane with vertices, in (x, z) form : $(0, 0)$, $(0, 4)$ and $(2, 4)$.
   (d) The solid enclosed by by $y = x^2 + z^2$ and $y = \sqrt{x^2 + z^2}$.
   (e) The solid behind $x + 6y + 2z = 8$ that is in front of the region in theyz-plane bounded by $z = 2y$ and $z = \sqrt{4y}$.

4. Evaluate the following integrals
   (a) $\int_0^2 \int_{-\sqrt{4-x^2}}^{0} \int_{\sqrt{5x^2+5y^2}}^{\sqrt{24-x^2-y^2}} 7x \ \mathrm{d}z \ \mathrm{d}y \ \mathrm{d}x$
   (b) $\int_{-4}^{4} \int_{0}^{\sqrt{16-y^2}} \int_{0}^{6+x} 6yx^2 \ \mathrm{d}z \ \mathrm{d}x \ \mathrm{d}y$
   (c) $\int_{-\sqrt{5}}^{\sqrt{5}} \int_{0}^{\sqrt{5-y^2}} \int_{-\sqrt{10-x^2-y^2}}^{-\sqrt{x^2+y^2}} 3xz^2 \ \mathrm{d}z \ \mathrm{d}x \ \mathrm{d}y$

   In each of the above, can you tell over which solid we are integrating?

5. Evaluate the following triple integrals over the given solids.
   (a) Evaluate $\iiint_E 8z \ \mathrm{d}V$where E is the solid bounded by $z = 2x^2 + 2y^2 - 4$ and $z = 5 - x^2 - y^2$
   (b) Evaluate $\iiint_E 4y^2 \ \mathrm{d}V$where E is the solid enclosed by the sphere $x^2 + y^2 + z^2 = 9$.
   (c) Evaluate $\iiint_E 9yz^3 \ \mathrm{d}V$where $E$is the solid between $x = -\sqrt{9y^2 + 9z^2}$ and $x = \sqrt{y^2 + z^2}$ inside the cylinder $y^2 + z^2 = 1$.
   (d) Evaluate $\iiint_E 2yz \ \mathrm{d}V$where E is the solid below $x^2 + y^2 + z^2 = 16$ and inside $z = \sqrt{3x^2 + 3y^2}$ that is in the 1^{st} octant.
   (e) Evaluate $\iiint_E x + 2 \ \mathrm{d}V$where E is the solid betweenthe two planes $2x + y + z = 6$ and $6x + 3y + 3z = 12$ inside the cylinder $x^2 + z^2 = 16$.
   (f) Evaluate $\iiint_E 5y^2 \ \mathrm{d}V$where E is the portion of the solid sphere $x^2 + y^2 + z^2 = 4$ with $x \leq 0$.

6. Use triple integrals to find the volumes of the following solids.
   (a) The solid that is below the plane $x = z + 3$, above $x = -z - 6$ and inside the cylinder $y^2 + z^2 = 4$.
   (b) The solid bounded by $y = \sqrt{x^2 + z^2}$, and $x^2 + y^2 + z^2 = 2$ in the 1^{st} octant.

7. Use triple integrals in cylindrical coordinates to derive the volume of a cylinder of height h and radius a.

8. Use a triple integral in spherical coordinates to derive the volume of a sphere with radius a.